Mediation, Moderation, and Conditional Process Analysis I

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GLOBAL SCHOOL EMPIRICAL RESEARCH METHODS

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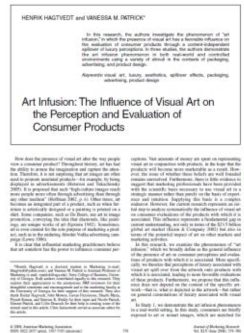
Global School in Empirical Research Methods, Oslo, Norway

Understanding causal effects

Hagtvedt, H., & Patrick, V. M. (2008). Art infusion: The influence of visual art on the perception and evaluation of consumer products. *Journal of Marketing Research, 45*, 379-389.



HENRIK HAGTVEDT and VANESSA M. PATRICK*

Art Infusion: The Influence of Visual Art on the Perception and Evaluation of Consumer Products

Understanding causal effects

Hagtvedt, H., & Patrick, V. M. (2008). Art infusion: The influence of visual art on the perception and evaluation of consumer products. *Journal of Marketing Research, 45*, 379-389.



X → *Y*

PHOTO or PAINTING

PRODUCT EVALUATION

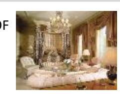
The product with the impressionist painting on the box was evaluated more favorably than the product with a photograph of a similar scene.

Remaining Questions:
- How does this effect occur? What is the *mechanism* that produces it?
- Is the effect consistent across type of product, type of consumer, and so forth.

These are the important questions!

A simple mediation model

PERCEPTIONS OF "LUXURY"




X → *M* → *Y*; *X* → *Y*

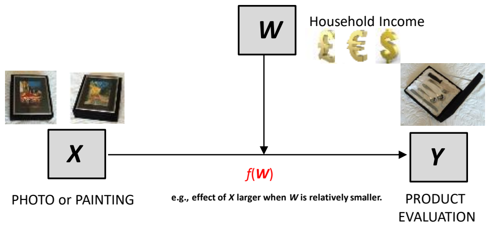
PHOTO or PAINTING

PRODUCT EVALUATION

Art-infused product was perceived as more "luxurious," and this greater perceived luxury translated into a more favorable product evaluation. Thus, the infusion of the product with art influenced product evaluation at least partly through the "mechanism" of perceived luxuriousness.

Mediation analysis is about estimating and making inferences about such *indirect effects*.

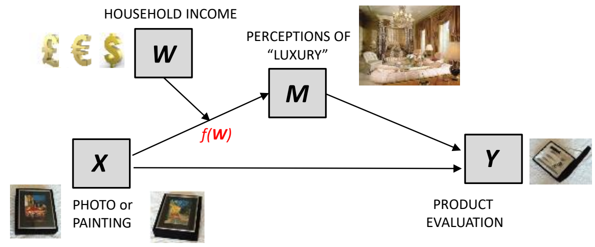
Moderation



Is this effect of art infusion is larger among those with less income? Is the size of the effect dependent on (or a function of) income? In this case, income is a *moderator* of the effect of art infusion on product evaluation.

Moderation analysis is about the estimation of *contingent effects*, i.e., examining the boundary conditions of effects or the factors that make effects large versus small, positive versus negative.

Combining moderation and mediation



Does art infusion result in greater perceptions of luxury more so among those with less income? If so, then the indirect effect of art infusion on product evaluation through perceptions of luxury depends on income. Thus, the strength of this "mechanism" may depend on income. <u>Mediation</u> can be <u>moderated</u>.

In this class

- ❑ After a review of OLS regression, we start with questions of "**HOW**"—**statistical mediation analysis**

  "Direct," "indirect," and "total effects" in path models and how to test hypotheses in such models using OLS regression and various computational tools developed for this purpose.

- ❑ We then move to questions of "**WHEN**"--**moderation analysis**

  Estimation and interpretation of models in which a predictor can have different effects on an outcome depending on the value of another variable in the model.

- ❑ We then explore models that combine moderation and mediation—**"conditional process analysis"**

- ❑ With fundamentals covered, we address more complex models and issues in mediation and moderation analysis, such as multiple mediators and multi-categorical independent variables and repeated-measures mediation.

What you'll need

- ❑ **This course is hands-on. Hopefully you brought a laptop with SPSS 19+ or SAS 9.1+ with PROC IML. If not, that is ok. You'll still benefit.**

  `SPSS Code` `SAS Code`

- ❑ **Various files available on a USB drive I pass around.**
  - ❑ SPSS and SAS data folders. SPSS data files are ready to go. SAS files are programs thus must be executed to make them "work" files.
  - ❑ SPSS and SAS PROCESS folders. This contains the PROCESS macro we'll heavily rely on, and some documents related to it.
  - ❑ Miscellaneous folder. Various files, including some PDFs and other miscellaneous things of relevance to this course.

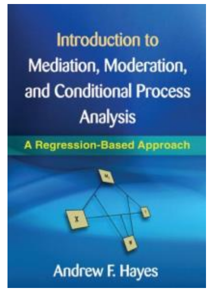
- ❑ **A lot of stamina.**

What we will and won't do

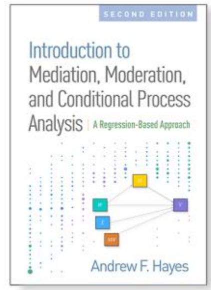
- We will stick with fairly simple models to cover basic principles, with continuous outcomes, and cross-sectional or experimental data.
- Statistical mediation analysis. No discussion of counterfactuals, "potential outcomes," directed graphs, or other approaches to thinking about cause.
- Everything OLS-regression-based.
- No dichotomous outcomes, nothing multilevel.

Although the principles are not software specific, their implementation is facilitated with the use of a "macro" which makes otherwise tedious things very simple and effortless. You will learn about PROCESS.

This course is a companion to…



2013



December 2017

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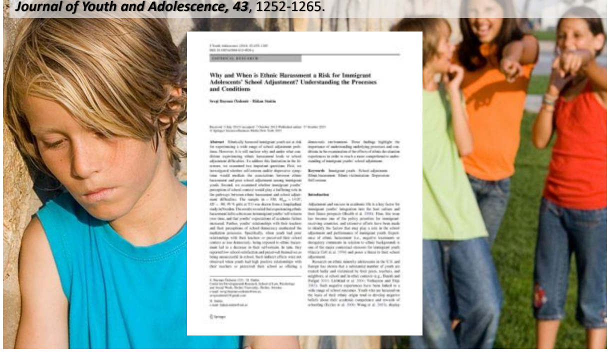
Example for the class "inspired by"…

Bayram-Ozdemir, S. & Stattin, H. (2014). Why and when is ethnic harassment a risk for immigrant adolescents' school adjustment? Understanding the processes and conditions. ***Journal of Youth and Adolescence, 43***, 1252-1265.



Example for the class "inspired by…"

Bayram-Ozdemir, S. & Stattin, H. (2014). Why and when is ethnic harassment a risk for immigrant adolescents' school adjustment? Understanding the processes and conditions. ***Journal of Youth and Adolescence, 43***, 1252-1265.

330 7th to 9th grade students in Finland measured in the spring term (T1) and again one year later (T2).
All were first or second generation immigrants who reported at least some ethnic harassment.

HARASS: 6-item measure of ethnicity-related harassment frequency (scaled 1 to 5). T1 only.

POSREL: 6-item measure of positivity of relationships with teachers (scaled 1 to 4). T1 only.

SE: 10-item Rosenberg self-esteem scale (scaled 1 to 4). T1 and T2.

DEP: 20-item Center for Epidemiological Studies Depression Scale for Children (scaled 1 to 4). T1 and T2.

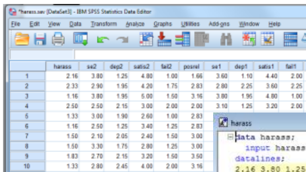
FAIL: 4-item measure of perceived academic failure at school (scaled 1 to 4). T1 and T2.

SATIS: 5-item measure of satisfaction in school (scaled 1 to 5). T1 and T2.

All are continuous variables scaled such that higher = more.

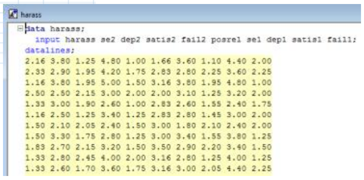
The Data: HARASS

SPSS



"1" and "2" in the variable name refers to time 1 and time 2, respectively. The absence of a number means the data were available only at time 1.

SAS



The SPSS file is ready for analysis. The SAS version is a SAS program that must be executed to produce a temporary work data file.

These are not the actual data from this study. They were generated to produce similar results to the published study.

A quick review of regression analysis

- Linear regression is the foundation of this class.
- Used throughout science as a means of "modeling" the relationship between variables.
- Many of the kinds of analyses and statistics you already know about can be expressed in the form of a linear regression model
  - independent groups *t* test
  - analysis of variance

Pearson's coefficient of correlation (*r*) is the building block of linear regression analysis. Consider the correlation between positivity of relationships with teachers and satisfaction at school (measured contemporaneously).

Correlations

		posrel	satis1
posrel	Pearson Correlation	1	.458
	Sig. (2-tailed)		.000
	N	330	330
satis1	Pearson Correlation	.458	1
	Sig. (2-tailed)	.000	
	N	330	330

SPSS code in black box

```
correlations variables = posrel satis1.
```

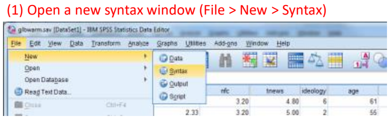
SAS code in white box

```
proc corr data=harass;var posrel satis1;run;
```

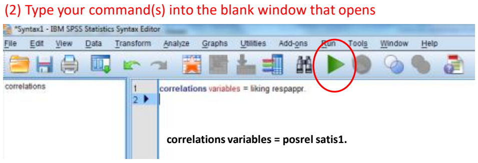
Using SPSS syntax

We will use syntax to instruct SPSS what to do in this class. There are many benefits of learning how to write SPSS syntax.

(1) Open a new syntax window (File > New > Syntax)



(2) Type your command(s) into the blank window that opens



(3) Click and drag to highlight code you want to execute and press the "play" button or select various options under "Run" in the syntax window menu.

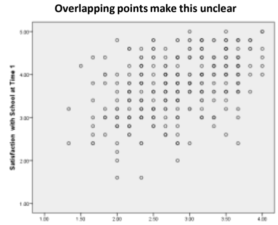
A scatterplot

Consider a scatterplot visually depicting this relationship:

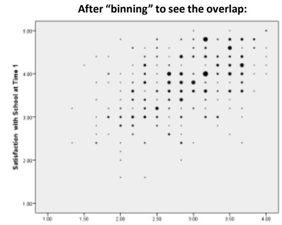
```
graph/scatterplot=posrel with satis1.
```

```
proc sgscatter data=harass;plot satis1*posrel;run;
```

Overlapping points make this unclear



After "binning" to see the overlap:



If you had to draw a single straight line through this plot that "best fits" the relationship, where would you draw it? At its heart, this is the problem regression analysis solves.

OLS (Ordinary Least Squares) linear regression

Goal: Derive the equation ("model") for the line representing the association between independent variable *X* and dependent variable *Y* that "best fits" the data.

The "simple regression model" (i.e., only one variable on the right hand side) takes the form

$$Y_i = b_0 + b_1 X_i + e_i$$

Using the ordinary least squares criterion, there is only one line described by the function

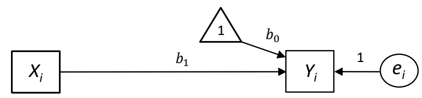
$$\hat{Y}_i = b_0 + b_1 X_i \qquad e_i = Y_i - \hat{Y}_i$$

that "best fits" the data, where $\hat{Y}_i$ is the estimated or fitted value of Y_i, and "best fit" is defined as the line that minimizes the sum of the squared residuals ($SS_{residual}$), summed over all *n* cases in the data. This is called the LEAST SQUARES criterion.

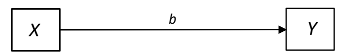
$$SS_{residual} = \sum_{i=1}^{n} \left(Y_i - \hat{Y}_i\right)^2 = \sum_{i=1}^{n} e_i^2$$

A visual representation

$$Y_i = b_0 + b_1 X_i + e_i$$



or, in shorthand,



In a diagram such as this, ⟶ represents "predictor of" or "component of" but not necessarily "cause of," although the association could be causal. So *X* is a predictor of *Y* (and *perhaps* a cause of *Y*) in this diagram.

Easier to do in SPSS and then explain

This is an easy problem for a computer with an OLS regression routine. We estimate *Y* from *X*, or regress *Y* on *X*. *X* = POSREL (positivity of teacher-student relationships), *Y* = SATIS1 (satisfaction with school).

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.458[a]	.210	.207	.61840

a. Predictors: (Constant), posrel

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	33.263	1	33.263	86.980	.000[b]
	Residual	125.433	328	.382		
	Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), posrel

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.	95.0% Confidence Interval for B Lower Bound	95.0% Confidence Interval for B Upper Bound
1	(Constant)	2.245	.166		13.527	.000	1.918	2.571
X	posrel	.531	.057	.458	9.326	.000	.419	.643

a. Dependent Variable: satis1

$SS_{residual}$ = 125.433
No other $\{b_0, b_1\}$pair would produce a smaller value of $SS_{residual}$.

$$\hat{Y}_i = b_0 + b_1 X_i$$

$$\hat{Y}_i = 2.245 + 0.531 X_i$$

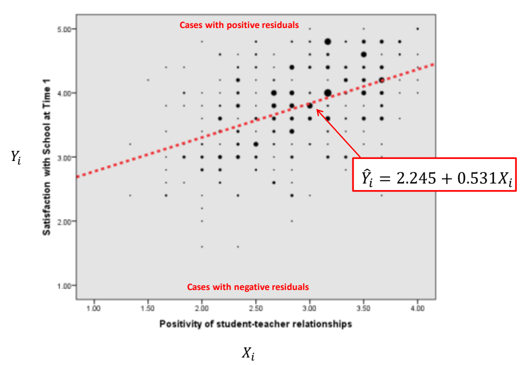
This is the best fitting OLS regression model, assuming a linear association between *X* and *Y*.

```
regression/statistics defaults ci/dep=satis1/method=enter posrel.
```

```
proc reg data=harass;model satis1 = posrel/stb clb;run;
```

Output A

The model in visual form

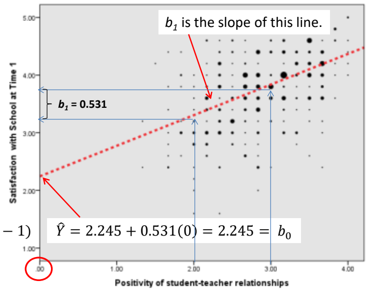


Interpretation of b_0 and b_1

$\hat{Y}_i = b_0 + b_1 X_i$

$\hat{Y}_i = 2.245 + 0.531 X_i$

b_1 = estimated difference in Y between two cases that differ by one unit on X. The sign of b_1 speaks to the sign of the association between X and Y.



$b_1 = \hat{Y}|(X = \theta) - \hat{Y}|(X = \theta - 1)$

b_0 = estimated value of Y when X = 0. This is not meaningful here.

Two kids that differ by one unit in the positivity of their student-teacher relationships are estimated to differ by b_1 = 0.531 units in satisfaction with school. The kid that is **higher** in positivity of those relationships is estimated to be *more* satisfied (because b_1 is positive)

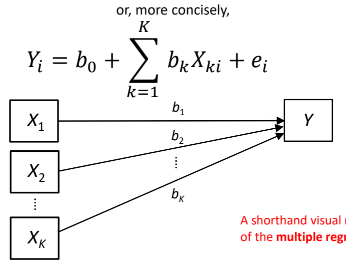
Multiple predictors

Multiple predictors variables are handled with ease, without modification to the estimation process. But this results in some interpretational changes.

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{3i} + \ldots + b_K X_{Ki} + e_i$$

or, more concisely,

$$Y_i = b_0 + \sum_{k=1}^{K} b_k X_{ki} + e_i$$



A shorthand visual representation of the **multiple regression** model.

A multiple regression model (SPSS)

```
regression/statistics defaults ci/dep=satis1/method=enter posrel harass se1.
```

```
proc reg data=harass;model satis1 = posrel harass se1/stb clb;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.545[a]	.297	.290	.58503

a. Predictors: (Constant), se1, posrel, harass

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	47.120	3	15.707	45.891	.000[b]
	Residual	$\boxed{111.576}$	326	.342		
	Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), se1, posrel, harass

$SS_{residual}$ = 111.576

No other set of b values would produce a smaller value of $SS_{residual}$.

X_1 = positivity of teacher-student relationships

X_2 = harassment frequency

X_3 = self-esteem.

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	1.479	.309		4.788	.000	.871	2.087
	posrel	.472	.055	.408	8.519	.000	.363	.581
	harass	-.145	.090	-.078	-1.613	.108	-.322	.032
	se1	.373	.064	.276	5.828	.000	.247	.499

a. Dependent Variable: satis1

$$\hat{Y}_i = 1.479 + 0.472 X_{1i} - 0.145 X_{2i} + 0.373 X_{3i}$$

Output B

The meaning of the values of *b*

$$\hat{Y}_i = 1.479 + 0.472 X_{1i} - 0.145 X_{2i} + 0.373 X_{3i}$$

Satisfaction — Positive relationships — harassment — Self-esteem

Two kids *the same on all predictors except the positivity of their teacher relationships* (X_1) but who differ by one unit in such positivity will differ by b_1 = 0.472 units in estimated satisfaction with school ($\hat{Y}$).

b_1, the partial regression coefficient for X_1, quantifies how differences in positivity of the student-teacher relationship relates to differences in satisfaction with school when *all other predictor variables in the model are held constant*, or "statistically controlling for" those other variables.

Two people the same on all predictors except ethnic harassment (X_2) who *differ by one unit in harassment* will differ by b_2 = 0.145 units in estimated satisfaction with school ($\hat{Y}$).

b_2, the partial regression coefficient for X_2, quantifies how *differences in frequency in the experience of ethnic harassment relate to differences in school satisfaction* when all other predictor variables in the model are held constant, or "statistically controlling for" those other variables. The negative sign for b_2 means those who experience *more* harassment are estimated to be *less* satisfied with school.

Statistical inference

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + b_3 X_{3i} + \ \ldots + b_K X_{Ki} + e_i$$

The constant, values of *b* are sample-specific. They are sample-specific estimates of a corresponding population or process model (the "true" model):

$$Y_i = \widetilde{b_0} + \widetilde{b_1} X_{1i} + \widetilde{b_2} X_{2i} + \widetilde{b_3} X_{3i} + \ \ldots + \widetilde{b_K} X_{Ki} + \widetilde{e_i}$$

Departures between the "true model" and the obtained model resulting from our data are used to test hypotheses about the "true values" of *b*.

Departures between the true and the obtained model are assumed to be driven by "random" processes, such as random sampling, random assignment variation, measurement error, etc., unless the data suggest otherwise. We attempt to estimate the true model using our data, hoping that our estimates of the true values of that model are accurate.

Null hypothesis testing for $\widetilde{b}$

$$Y_i = \widetilde{b_0} + \widetilde{b_1} X_{1i} + \widetilde{b_2} X_{2i} + \widetilde{b_3} X_{3i} + \ \ldots + \widetilde{b_K} X_{Ki} + \widetilde{e_i}$$

In any study, we observe only b_j, the sample estimate of $\widetilde{b_j}$. We often are interested in making an inference about the size of $\widetilde{b_j}$, or testing a hypothesis about its value.

e.g., Null hypothesis test about $\widetilde{b_j}$:

Assume $\widetilde{b_1}$ equals some specific value. Typically, we assume $\widetilde{b_1} = 0$ under the **null hypothesis** (i.e., X_1 is unrelated to *Y* when all other variables in the model are held constant).

$$H_0: \widetilde{b_1} = 0$$
$$H_a: \widetilde{b_1} \neq 0$$

If H_0 is true, then b_1 / s_{b_1} follows the $t(df_{residual})$ distribution, where s_{b_1} is the estimated standard error of b_1. Using the *t* distribution, we generate a *p*-value and reject H_0 in favor of H_a if $p \le \alpha$-level chosen for the test (usually .05). In that case, the result is "statistically significant."

Statistical inference for partial regression coefficients

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	47.120	3	15.707	45.891	.000[b]
	Residual	111.576	326	.342		
	Total	158.696	329			

a. Dependent Variable: satis1

b. Predictors: (Constant), se1, posrel, harass

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	1.479	.309		4.788	.000	.871	2.087
	posrel	.472	.055	.408	8.519	.000	.363	.581
	harass	-.145	.090	-.078	-1.613	.108	-.322	.032
	se1	.373	.064	.276	5.828	.000	.247	.499

a. Dependent Variable: satis1

Output B

$$H_0: \widetilde{b_1} = 0$$
$$H_a: \widetilde{b_1} \neq 0$$

$b_1 = 0.472$, se(b_1) = 0.055,
$t(326) = 8.519$, $p < 0.001$
Reject H_0 in favor of H_a

$$H_0: \widetilde{b_2} = 0$$
$$H_a: \widetilde{b_2} \neq 0$$

$b_2 = -0.145$, se(b_2) = 0.090,
$t(326) = -1.613$, $p = 0.108$
Do not reject H_0

Two kids equal in ethnic harassment frequency and self esteem but who differ in the positivity of their student-teacher relationships differ from each other in satisfaction more than can be explained by chance. The observed and statistically significant positive partial relationship tells us that kids with more positive student-teacher relationships are more satisfied with school.

Two kids equal in the positivity of their student-teacher relationships and their self-esteem but who differ in ethnic harassment frequency do not differ in their satisfaction any more than would be expected by "chance."

Interval estimation

Inferences can also be framed as an interval such that this interval will capture the true value a certain percentage of the time. As a rough rule-of-thumb, we can be 95% confident that the true value resides within <u>about</u> 2 standard errors of the obtained estimate.

$$b_j \ - 2s_{b_j} \le \widetilde{b_j} \le b_j + 2s\_b_i$$

Output B

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	1.479	.309		4.788	.000	.871	2.087
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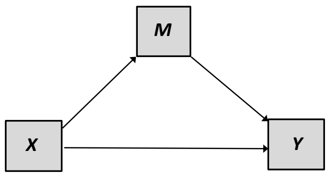
	harass	-.145	.090	-.078	-1.613	.108	-.322	.032
	se1	.373	.064	.276	5.828	.000	.247	.499

a. Dependent Variable: satis1

We can be 95% sure $\widetilde{b_1}$ is somewhere between 0.363 and 0.581
We can be 95% sure $\widetilde{b_2}$ is somewhere between -0.322 and 0.032.

Question: Is it fair to say we have 'no evidence' that kids who are harassed more frequently are less satisfied with school?

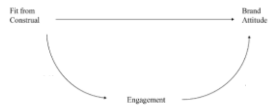
Statistical mediation analysis



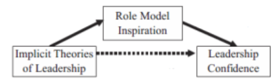
The "simple mediation" model

A mediation model links an assumed cause (*X*) to an assumed effect (*Y*) at least in part via an intermediary variable (*M*). An intermediary variable can be a psychological state, a cognitive process, an affective response, or any other conceivable "mechanism" through which *X* exerts an effect on *Y*. *X* affects *M* which in turn affects *Y*.

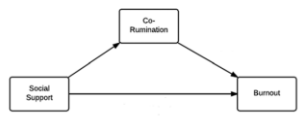
Some examples in the literature



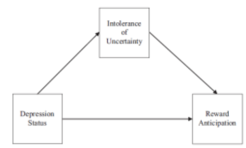
Lee, A. Y., Keller, P. A., & Sternthal, B. (2010). Value from regulatory construal fit: Persuasive impact of fit between consumer goals and message concreteness. ***Journal of Consumer Research***, *36*, 735-747.



Hoyt, C. L., Burnette, J. L., & Innella, A. N. (2012). I can do that: The impact of implicit theories on leadership model effectiveness. ***Personality and Social Psychology Bulletin***, *38*, 257-268.



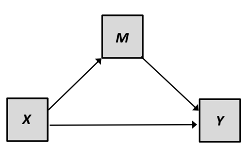
Boren, J. P. (2014). The relationship between co-rumination, social support, stress, and burnout among working adults. ***Management Communication Quarterly***, *28*, 3-25.



Nelson, B. D., Shankman, S. A., & Proudfit, G. H. (2014). Intolerance of uncertainty mediates reduced reward anticipation in major depressive disorder. ***Journal of Affective Disorders***, *158*, 108-113.

The most basic intervening variable model

- For *M* to be an intermediary, it must be located *causally between X* and *Y*.
- *M* is sometimes called a "mediator", but it goes by other names as well.



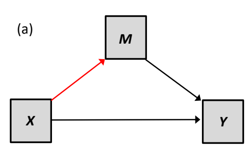
- Mediator models are causal models and carry with them the usual criteria for making causal claims.

  Difficult to establish cause statistically or otherwise. Theory is sometimes the sole foundation upon which our causal claims rest. That's ok so long as we recognize this.

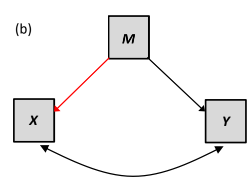
Mediation and spuriousness

Mediation analysis cannot distinguish between (a)mediation and (b)spuriousness. If (b) can be deemed plausible, that weakens the case for (a) regardless of what the data analysis tells you.

(a)



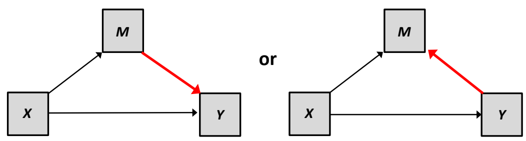
(b)



Inferences are always design-bound.
Mediation is a causal process, but causal claims are only justified if the design allows such claims, regardless of what the statistics say.

Causal order

Manipulation of and random assignment to *X* affords causal inference for the effect of *X* on *M* and *Y*, but not the effect of *M* on *Y*. We cannot establish causal order for the *M*-*Y* path using the methods that are the focus here. Theory is important. Multiple studies can help, one of which involves manipulation of *M*.



When *X* is not experimentally manipulated, all paths are subject to potential alternative causal orders.

MacKinnon, D. P. & Pirlott, A. G. (2015). Statistical Approaches for Enhancing Causal Interpretation of the M to Y Relation in Mediation Analysis. *Personality and Social Psychology Review, 19*(1), 30 – 43.

Path analysis: Total, direct, and indirect effects

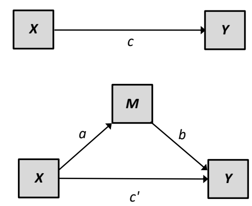
Let *a*, *b*, *c*, and *c'* be quantifications of causal effects, such as regression coefficients in an OLS model (or maximum likelihood path estimates in a structural equation model)

$$\hat{Y} = c_0 + cX$$

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c'_0 + c'X + bM$$

A "simple mediation" model

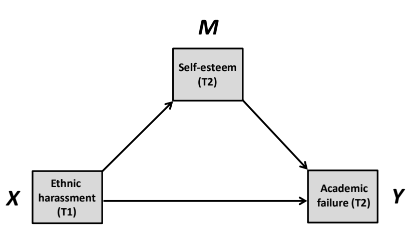


c = "total effect" of *X* on *Y*
a × *b* = "indirect effect" of *X* on *Y*
c' = "direct effect" of *X* on *Y*

total effect = direct effect + indirect effect
c = *c'* + (*a* × *b*)

indirect effect = total effect – direct effect
(*a* × *b*) = *c* – *c'*

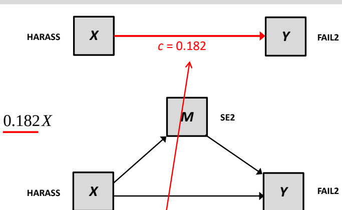
Our question



Does ethnic harassment influence school performance by affecting self-esteem which in turn affects performance.

Asking this question does not require evidence that there is a bivariate relationship between *X* (ethnic harassment) and *Y* (performance)

Using a set of OLS regression analyses



$$\hat{Y} = 1.346 + \underline{0.182}\,X$$

Coefficients[a]

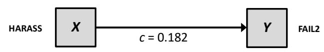
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.346	.113		11.866	.000
	harass	.182	.073	.136	2.478	.014

a. Dependent Variable: fail2

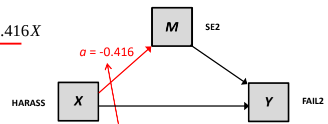
SPSS:
```
regression/dep=fail2/method=enter harass.
```

SAS:
```
proc reg data=harass;model fail2=harass;run;
```

Using a set of OLS regression analyses



$\hat{M} = 3.597 - \underline{0.416}X$



Coefficients[a]

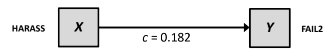
Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.597	.123		29.123	.000
	harass	$\boxed{-.416}$	.080	-.276	-5.209	.000

a. Dependent Variable: se2

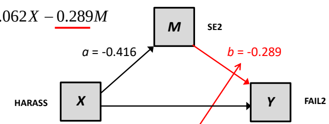
```
regression/dep=se2/method=enter harass.
```

```
proc reg data=harass;model se2=harass;run;
```

Using a set of OLS regression analyses



$\hat{Y} = 2.385 + 0.062X - \underline{0.289}M$



Coefficients[a]

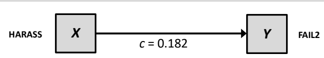
Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.385	.204		11.676	.000
	harass	.062	.072	.046	.850	.396
	se2	$\boxed{-.289}$	.048	-.324	-5.988	.000

a. Dependent Variable: fail2

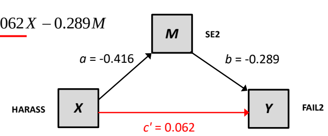
```
regression/dep=fail2/method=enter harass se2.
```

```
proc reg data=harass;model fail2=harass se2;run;
```

Using a set of OLS regression analyses



$\hat{Y} = 2.385 + \underline{0.062}X - 0.289M$



Coefficients[a]

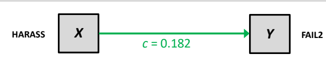
Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.385	.204		11.676	.000
	harass	$\boxed{.062}$	.072	.046	.850	.396
	se2	-.289	.048	-.324	-5.988	.000

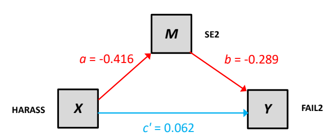
a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass se2.
```

```
proc reg data=harass;model fail2=harass se2;run;
```

Using a set of OLS regression analyses





Direct effect of X on Y = c' = 0.062
Indirect effect of X on Y via M = ab = -0.416(-0.289) = 0.120
Total effect of X on Y = c' + ab = 0.062 + 0.120 = 0.182 = c

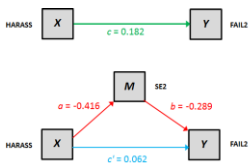
Interpretation of the total, direct, and indirect effects

Generic

Total: Two people who differ by one unit on *X* are estimated to differ by *c* units on *Y* on average.

Indirect: They differ by *ab* units on average as a result of the effect of *X* on *M* which in turn affects *Y*.

Direct: The rest of the difference, the difference of *c'* units, is due to the effect of *X* on *Y* independent of *M*.



Direct effect = *c'* = 0.062
Indirect effect = *ab* = -0.416(-0.289) = 0.120
Total effect = *c* = 0.062 + 0.120 = 0.182

Specific

Total: Two kids who differ by one scale point in ethnic harassment are estimated to differ by 0.182 units in perceived academic failure one year later, with the more frequently-harassed kid perceiving greater failure.

Indirect: They differ by 0.120 units in perceived failure as a result of the negative effect of harassment on self esteem a year later, which in turn increases perceived failure.

Direct: Independent of this mechanism, the more harassed kid is estimated to be 0.062 units higher in perceived failure.

It works for dichotomous *X* too.

Garcia, D. M., Schmitt, M. T., Branscombe, N. R., & Ellemers, N. (2010). Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness. ***European Journal of Social Psychology, 49***, 733-745.

Participants (all female) read a narrative about a female attorney who lost a promotion at her firm to a much less qualified male through unequivocally discriminatory actions of the senior partners.

Participants assigned to the 'protest' condition were then told she protested the decision by presenting an argument to the partners about how unfair the decision was.

Participants assigned to the 'no protest' condition were told that although she was disappointed, she accepted the decision and continued working at the firm.

After reading the narrative, the participants evaluated **how appropriate they perceived her response to be**, and also evaluated the characteristics of the attorney, the responses of which were aggregated to produce a measure of **"liking."** Prior to the study, the participants filled out the Modern Sexism Scale.

The data: PROTEST

*protest.sav [DataSet1] - IBM SPSS Statistics Data Editor

	subnum	cond	sexism	angry	liking	respappr	protest
1	209	2	4.87	2	4.83	4.25	1.00
2	44	0	4.25	1	4.50	5.75	.00
3	124	2	5.00	3	5.50	4.75	1.00
4	232	2	5.50	1	5.66	7.00	1.00
5	30	2	5.62	1	6.16	6.75	1.00
6	140	1	5.75	1	6.00	5.50	1.00
7	27	2	5.12	2	4.66	5.00	1.00
8	64	0	6.62	1	6.50	6.25	.00
9	67	0	5.75	6	1.00	3.00	.00
10	182	0	4.62	1	6.83	5.75	.00
11	85	2	4.75	2	5.00	5.25	1.00
12	109	2	6.12	5	5.66	7.00	1.00
13	122	0	4.87	2	5.83	4.50	.00
14	69	1	5.87	1	6.50	6.25	1.00

SAS users, run this program to make a temporary or "work" data file named PROTEST.

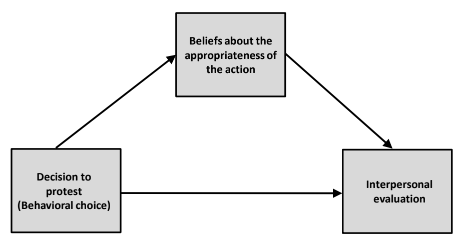
```
data protest;
input subnum cond sexism angry liking respappr protest;
cards;
209 2   4.87   2   4.83   4.25   1.00
44  0   4.25   1   4.50   5.75   .00
124 2   5.00   3   5.50   4.75   1.00
232 2   5.50   1   5.66   7.00   1.00
30  2   5.62   1   6.16   6.75   1.00
140 1   5.75   1   6.00   5.50   1.00
27  2   5.12   2   4.66   5.00   1.00
64  0   6.62   1   6.50   6.25   .00
67  0   5.75   6   1.00   3.00   .00
182 0   4.62   1   6.83   5.75   .00
85  2   4.75   2   5.00   5.25   1.00
109 2   6.12   5   5.66   7.00   1.00
122 0   4.87   2   5.83   4.50   .00
69  1   5.87   1   6.50   6.25   1.00
```

PROTEST: Experimental condition (1 = protest, 0 = no protest)

LIKING : Evaluation (liking) of the lawyer (higher = more positive evaluation, i.e. like more)

RESPAPPR: A measure of how appropriate the lawyer's behavior in response to the action of the partners was perceived to be for the situation (higher = more appropriate)

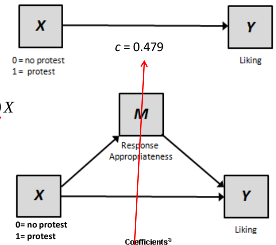
Our question



Do perceptions of the appropriateness of the response act as the mechanism through which that choice influences interpersonal evaluation?

Notice that this question is not asked contingent on evidence of simple association between the choice and the evaluation.

Using a set of OLS regression analyses



$\hat{Y} = 5.313 + \underline{0.479}\, X$

Because the two groups differ by one unit on *X*, this is the difference between the group means. The attorney was liked more when she protested than when she did not.

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	5.313	.161		33.055	.000
	X: experimental condition (0 = no protest, 1 = protest)	$\boxed{.479}$	.195	.213	2.460	.015

a. Dependent Variable: Y: liking of the target

SPSS:
```
regression/dep=liking/method=enter protest.
```
SAS:
```
proc reg data=protest;model liking=protest;run;
```

Interpretation when *X* is dichotomous

$$\boxed{\hat{Y}_i = 5.310 + 0.479X_i}$$

```
means tables = liking by protest.
```

Report

LIKING: liking of the target

PROTEST: experimental condition (0 = no protest, 1 = protest)	Mean	N	Std. Deviation
no protest	5.3102	41	1.30158
protest	5.7889	88	.87669
Total	5.6357	129	1.04970

When *X* = 1 (protest), $\hat{Y}$ = 5.310 + 0.479(1) = 5.789
When *X* = 0 (no protest), $\hat{Y}$ = 5.310 + 0.479(0) = 5.310

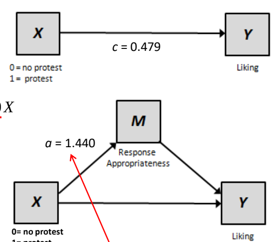
Notice that with X coded 0 and 1, the model yields the group means, *b* is the difference between the group means, and *the regression constant* is the mean for the group coded *X* = 0 (no protest condition).

More generally, if the two groups are coded by a difference of λ units, such that $X = \theta + \lambda$ for group 1 and $X = \theta$ for group 2,

$$b = (\bar{Y}_1 - \bar{Y}_2)/\lambda$$

If you get in the habit of coding a dichotomous variable such that the groups differ by one unit on *X*, *b* will always be a difference between group means.

Using a set of OLS regression analyses



$\hat{M} = 3.884 + \underline{1.440}\, X$

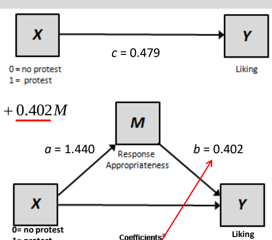
Because the two groups differ by one unit on X, this is the difference between the group means. Protesting was seen as more appropriate for the situation than doing nothing.

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.884	.183		21.208	.000
	X: experimental condition (0 = no protest, 1 = protest)	$\boxed{1.440}$	.222	.499	6.493	.000

a. Dependent Variable: M: appropriateness of response

```
regression/dep=respappr/method=enter protest.
```
```
proc reg data=protest;model respappr=protest;run;
```

Using a set of OLS regression analyses



$\hat{Y} = 3.751 - 0.100\,X + \underline{0.402}\,M$

Holding constant what the attorney did, she was liked more by those who saw her behavior as more appropriate for the situation.

Coefficients[a]

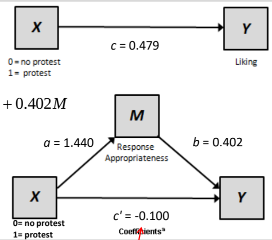
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.751	.306		12.271	.000
	M: appropriateness of response	$\boxed{.402}$	.069	.517	5.789	.000
	X: experimental condition (0 = no protest, 1 = protest)	-.100	.200	-.045	-.501	.617

a. Dependent Variable: Y: liking of the target

```
regression/dep=liking/method=enter respappr protest.
```
```
proc reg data=protest;model liking=respappr protest;run;
```

Using a set of OLS regression analyses

$$\hat{Y} = 3.751 - \underline{0.100}\,X + 0.402\,M$$



Because the two groups differ by one unit on *X*, this is the difference between the group means adjusted for differences between the groups in how appropriate her behavior was perceived as being for the situation (i.e., holding it constant)

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.751	.306		12.271	.000
	M: appropriateness of response	.402	.069	.517	5.789	.000
	X: experimental condition (0 = no protest, 1 = protest)	-.100	.200	-.045	-.501	.617

a. Dependent Variable: Y: liking of the target

```
regression/dep=liking/method=enter respappr protest.
```

```
proc reg data=protest;model liking=respappr protest;run;
```

Interpretation of total, direct, and indirect effects

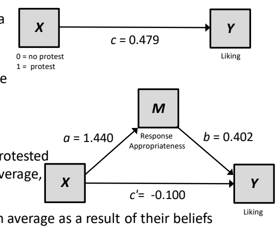
Direct effect = -0.100
Indirect Effect = 1.440(0.402) = 0.579
Total effect = -0.100 + 0.579 = 0.479

Generic

Total: Two people who differ by one unit on *X* are estimated to differ by *c* units on *Y* on average.

Indirect: They differ by *ab* units on average as a result of the effect of *X* on *M* which in turn affects *Y*.

Direct: The rest of the difference, the difference of *c′* units, is due to the effect of *X* on *Y* independent of *M*.



Specific

Total: Participants who were told the lawyer protested (X = 1) liked the lawyer 0.479 units **more**, on average, than those who were told she did not protest.

Indirect: They liked her by 0.579 units **more** on average as a result of their beliefs about the appropriateness of her response, which in turn affected their liking.

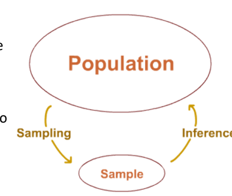
Direct: Among those equal in their beliefs about the appropriateness of her response, those who were told the lawyer protested liker her 0.100 units **less** (because the sign is negative) than those who were told she did not protest the decision.

Statistical Inference

Statistical inference: The indirect effect

Statistical inference is how we take information from our sample and generalize to the population.

Up to this point we have made estimates based on the sample, but we were unable to make any claims about what we think the population looks like.

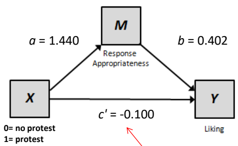


Methods of inference:

- Sobel Test (Normality Test / Delta Method)
- Bootstrap confidence intervals
- Monte Carlo confidence intervals
- Causal Steps Method

Statistical inference: The direct effect

Inference for the direct effect is simple and noncontroversial. The inference can be framed in terms of a hypothesis test or a confidence interval. Any OLS regression program will provide both.



No statistically significant evidence of a direct effect of protest on liking, c' = -0.100, p = 0.617, 95% CI = (-0.497 to 0.296)

```
regression/statistics defaults ci/ dep=liking/method=enter respappr protest.
```

```
proc reg data=protest;model liking=respappr protest/stb clb; run;
```

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	3.747	.306		12.255	.000	3.142	4.352
	RESPAPPR: appropriateness of response	.402	.070	.517	5.788	.000	.265	.540
	PROTEST: experimental condition (0 = no protest, 1 = protest)	-.101	.200	-.045	-.502	.616	-.497	.296

a. Dependent Variable: LIKING: liking of the target

Statistical inference: The indirect effect

The indirect effect estimates the influence of X on Y through the mechanism represented by M (i.e., the $X \rightarrow M \rightarrow Y$ sequence). 21st-century mediation analysis bases claims of mediation on evidence that the indirect effect is different from zero.

A popular "20th-century" approach to inference: The Sobel test

Indirect effect

$$Z = \frac{ab}{\sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2}}$$

"Second order" estimator of the standard error of *ab*

One version eliminates this term ("first order" estimator) ,

A p-value is derived by assuming normality of the sampling distribution of the indirect effect and using the standard normal distribution for derivation of p. A p-value no greater than α leads to the claim that the indirect effect is statistically different from zero at the α level of significance.

Computation with Protest Data

a = 1.440 s_a = 0.222

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	3.884	.183		21.208	.000
	X: Protest	1.440	.222	.499	6.493	.000

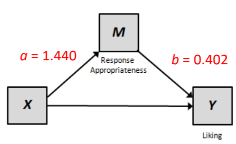
a. Dependent Variable: RESPAPPR: appropriateness of response

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	3.747	.306		12.255	.000
	RESPAPPR: appropriateness of response	.402	.070	.517	5.788	.000
	PROTEST: experimental condition (0 = no protest, 1 = protest)	-.101	.200	-.045	-.502	.616

a. Dependent Variable: LIKING: liking of the target

b = 0.402 s_b = 0.070



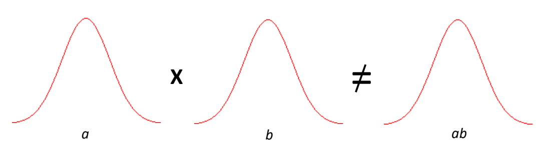
$$Z = \frac{ab}{\sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2}}$$

$$Z = \frac{(1.440)(0.402)}{\sqrt{(0.402)^2(0.222)^2 + (1.440)^2(0.070)^2 + (0.222)^2(0.070)^2}} = \frac{0.579}{0.136} = 4.271, p < .0001$$

The indirect effect is statistically significant. But this test has serious problems.

What's wrong with the Sobel test?

For the Sobel test, the p-value is derived by assuming normality of the sampling distribution of the indirect effect and using the standard normal distribution. **Although this assumption is fairly sensible in large samples, it is not in smaller ones. What is a sufficiently large sample is situationally-specific, and typically you won't know going into the analysis whether or not to trust large sample theory.**



This assumption, which typically will not hold, yields a test that is lower in power than alternatives. **Experts in mediation analysis don't recommend the use of this test, though it remains popular.** Eventually, researchers will get the message.

The bootstrap confidence interval

Bootstrapping allows us to empirically estimate the sampling distribution of the indirect effect and generate a confidence interval (CI) for estimation and hypothesis testing. **It has become the preferred inferential method for estimating and testing indirect effects.**

(1) Take a random sample of size *n* from the sample *with replacement.*

(2) Estimate the indirect effect in this "resample".

(3) Repeat (1) and (2) a total of *k* times, where *k* is at least 1,000. The larger *k*, the better. I recommend at least 5,000.

(4) Use distribution of the indirect effect over multiple resamples as an approximation of the sampling distribution of the indirect effect.

(5) For 95% CI using "percentile" method, lower and upper bounds are 2.5th and 97.5th percentile in *k* bootstrap estimates of the indirect effect. Variations exist (e.g., 'bias corrected' or 'bias-corrected and accelerated' confidence intervals but they do not perform as well as percentile.)

Bootstrapping

Your data

X	*M*	*Y*
4.3	1.4	9.1
1.4	5.4	6.4
4.9	4.3	1.3
5.9	2.3	5.4
6.1	3.3	3.9
3.8	3.1	6.3
2.8	3.2	1.5
9.4	4.1	2.3
4.3	1.3	4.4
4.9	3.7	2.1

a = -0.051 b = -0.844

ab = 0.043

A resampling of your data

X	*M*	*Y*
5.9	2.3	5.4
4.9	4.3	1.3
9.4	4.1	2.3
4.9	4.3	1.3
4.3	1.4	9.1
1.4	5.4	6.4
3.8	3.1	6.3
9.4	4.1	2.3
6.1	3.3	3.9
4.3	1.3	4.4

a = 0.020 b = -0.921

ab = -0.018

Bootstrapping

Your data

X	*M*	*Y*
4.3	1.4	9.1
1.4	5.4	6.4
4.9	4.3	1.3
5.9	2.3	5.4
6.1	3.3	3.9
3.8	3.1	6.3
2.8	3.2	1.5
9.4	4.1	2.3
4.3	1.3	4.4
4.9	3.7	2.1

a = -0.051 b = -0.844

ab = 0.043

Another resampling of your data

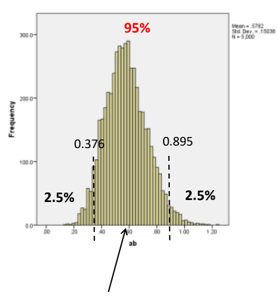
X	*M*	*Y*
6.1	3.3	3.9
4.9	4.3	1.3
2.8	3.2	1.5
4.9	3.7	2.1
3.8	3.1	6.3
9.4	4.1	2.3
2.8	3.2	1.5
4.9	4.3	1.3
4.9	3.7	2.1
1.4	5.4	6.4

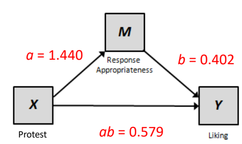
a = -0.034 b = 0.523

ab = -0.017

5,000 bootstrap estimates of the indirect effect

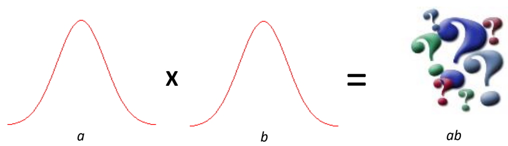
95% of the 5,000 bootstrap estimates of the indirect effect were between 0.376 and 0.895. This is our 95% confidence interval.





Zero is not in the confidence interval, so we **can** claim an indirect effect different from zero with 95% confidence. This is akin to (though not exactly the same as) rejecting the null hypothesis of no indirect effect at the α = 0.05 level of significance.

The Monte Carlo interval



If all of the assumptions of linear regressions are met (or sample sizes are sufficiently large), then we know that *a* and *b* will have normally distributions.

The Monte Carlo confidence intervals takes advantage of this knowledge by simulating normal distributions for *a* and *b* then calculating their product to get an estimated distribution of the indirect effect (*ab*).

The Monte Carlo interval

Monte Carlo empirically estimate the sampling distribution of the indirect effect and generate a confidence interval (CI) for estimation and hypothesis testing. This simulation based method assumes each individual path (*a* and *b*) are normally distributed.

(1) Generate *k* samples from a normal distribution with mean *a* and standard deviation s_a

(2) Generate *k* samples from a normal distribution with mean *b* and standard deviation s_b

(3) Multiply samples together to get a distribution of *k* estimates of *ab.*

(4) Rank order estimates and select estimates which define the lower percentile of sorted *k* estimates and upper percentile of sorted estimates which define CI of interest.

(5) For 95% CI lower and upper bounds are 2.5th and 97.5th percentile in *k* bootstrap estimates of the indirect effect.

The Monte Carlo interval

This method performs well (similarly to bootstrapping) in a variety of simulation studies, but is still less popular.

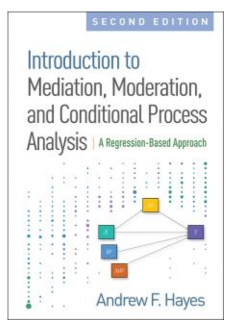
This method makes stronger assumptions than bootstrapping, but does not result in great power.

Add mc = 1 to PROCESS command line to request Monte Carlo CIs

```
Indirect effect(s) of X on Y:
              Effect     MC SE    MC LLCI    MC ULCI
respappr       .5793     .1332     .3369      .8469
```

Using a Monte Carlo confidence interval also suggests that we are confident the indirect effect is not zero. So all three inferential methods come to the same conclusion, which is comforting.

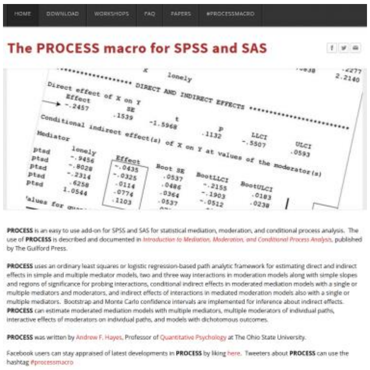
PROCESS



Published in December 2017 and available through The Guilford Press, Amazon.com, and elsewhere.

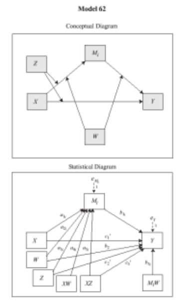
- First released in beta form in March of 2012 and later documented in Hayes (2013, IMMCPA, published by The Guilford Press).
- Available for both SPSS (in macro and "custom dialog" form) and SAS.
- An integration of functions available in my other published macros for mediation and moderation analysis (SOBEL, INDIRECT, MODMED, MODPROBE, MED3C) and a whole lot more, all in one command.
- A handy tool for both "confirmatory" and "exploratory" approaches to data analysis.
- Freely available at www.processmacro.org. The current release is v3.2

processmacro.org



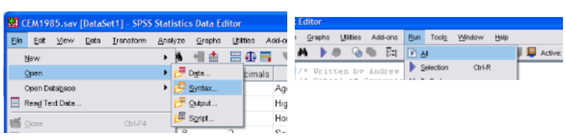
Read the documentation (eventually)

The PROCESS documentation is an eventual must-read. It describes how to use PROCESS, as well as its various options, capabilities, and limitations. It is available as Appendix A in Hayes (2017). *Introduction to Mediation, Moderation, and Conditional Process Analysis (2nd Ed)*. At a minimum, you must have the model templates handy, as PROCESS expects you to tell it which model number you are estimating and which variables play what role. You have a mini version of the templates PDF.



PROCESS as a syntax-driven macro

Open process.sps as a syntax file and run the entire program **exactly as is.** This produces a new SPSS command called PROCESS. See the documentation for details on the syntax structure. PROCESS goes away when you close SPSS.

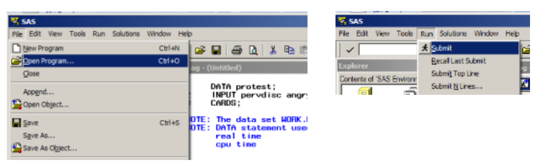


Once PROCESS is defined, you can run a properly-formatted PROCESS command in a new syntax window. Such a command might look something like below. Not all of the arguments below are required.

```
process y=liking/x=protest/m=respappr/total=1/normal=1
      /model=4/boot=10000.
```

PROCESS for SAS

In SAS, open process.sas and submit the entire program **exactly as is.** This produces a new SAS command called %PROCESS. The syntax structure is described in the documentation.



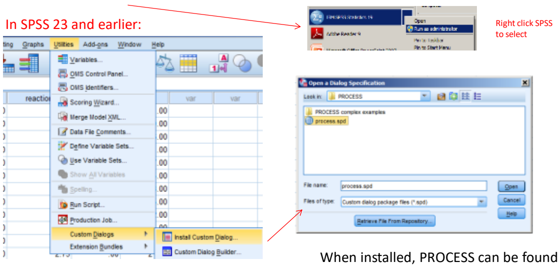
Once PROCESS is defined, you can run a properly-formatted PROCESS command in a new syntax window. Such a command might look something like below. Not all of the arguments below are required.

```
%process (data=protest,y=liking,x=protest,m=respappr,total=1,
          normal=1,boot=10000,model=4);
```

PROCESS “Custom Dialog”

The PROCESS macro must be run at least once in your SPSS session to activate the PROCESS command. Custom Dialog files are permanently installed in SPSS, integrating the procedure into SPSS menus.
Use the procedure below. In Windows, installation requires administrative access to your machine. You **probably** have to open SPSS as an administrator as well. You may not have access to do so.

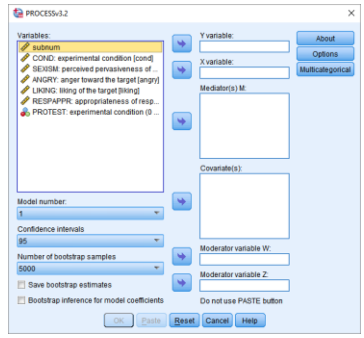
In SPSS 23 and earlier:



In SPSS24, look under “Extensions” for the Utilities option

When installed, PROCESS can be found under “Analyze”→”Regression”

PROCESS “Custom Dialog”

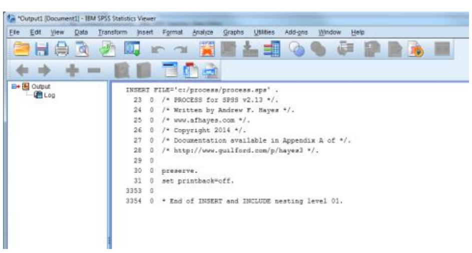


Installing the dialog box does not eliminate the need to run the PROCESS code if you plan on executing with syntax. And don’t use the PASTE button.

Autoexecution

It is possible to get SPSS to execute the PROCESS code on its own when SAS/SPSS executes. A document is provided to you with the course files that provides instructions for SPSS for Windows and Mac.

When successful, SPSS for Windows users typically see something like the following in the output window when SPSS is opened:



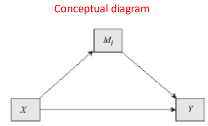
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

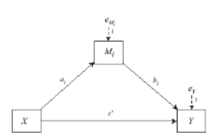
Example #1:

Model 4 is a simple or parallel multiple mediator model, which estimates the direct and indirect effect(s) of *X* on *Y* through one or more mediators (*M*) (up to 10 mediators at once)

Conceptual diagram



Statistical diagram



Model template system

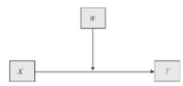
PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

Example #2:

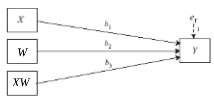
Model 1 is a simple moderation model, with *W* moderating the effect of *X* on *Y*.

The statistical diagram shows the model in the form of a path diagram. This is the form in which the model is estimated.

Conceptual diagram



Statistical diagram



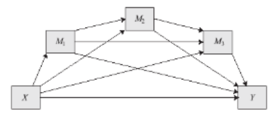
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

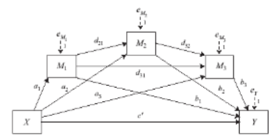
Example #3:

Model 6 is a serial multiple mediator model, which estimates the direct and indirect effect(s) of *X* on *Y* through up to 4 mediators (*M*) chained together in serial. An example with **three** mediators is depicted to the right.

Conceptual diagram



Statistical diagram



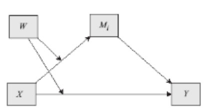
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

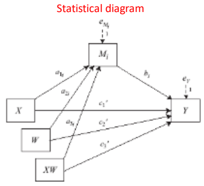
Example #4:

Model 8 is a conditional process model which estimates the conditional direct and indirect effects of *X* on *Y* through *M*, with direct effect and “first stage” moderation by *W*.

Conceptual diagram



Statistical diagram



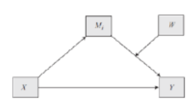
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

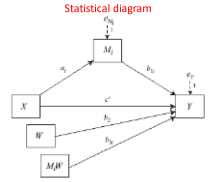
Example #5:

Model 14 is a conditional process model which estimates the direct effect of *X* on *Y* and conditional indirect effects of *X* on *Y* through *M*, with “second stage” moderation by *W*.

Conceptual diagram



Statistical diagram



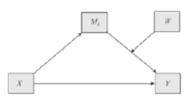
Model template system

PROCESS has 56 model templates, referred to by number in PROCESS, which specify the form of the model (moderation, mediation, moderated mediation, and so forth) and which variables play what roles.

Minimum required specifications

- Which variables play which role in the model (**y= x= m= w =** and so forth)
- Model number (**model=**)
- SAS only: Data file (**data=**)

Conceptual diagram



SPSS

```
PROCESS y=yvar/x=xvar/m=mvlist/w=wvar/model=14.
```

SAS

```
%process (data=filename,y=yvar,x=xvar,m=mvlist,w=wvar,model=14);
```

Limitations and constraints

- Only one *X* and one *Y* allowed in a model.
- PROCESS is an OLS or logistic regression modeling tool. Categorical mediators are not allowed.
- Up to 10 mediators in numbered models, 6 in custom models.
- No more than two moderators can be used in any model.
- A variable can play only one role in the model. For example a variable can't be both a moderator and a covariate, or both a mediator and a moderator.
- PROCESS is a single-level observed variable modeling system. No multilevel problems can be analyzed with PROCESS.
- PROCESS requires complete data. Listwise deletion is used for cases missing on any variable in the model.
- Although PROCESS will accept them, it is safer to restrict variable names to eight characters or fewer.

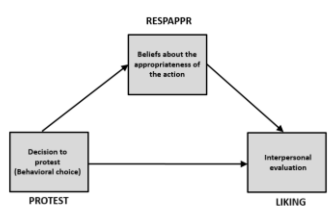
If you are familiar with PROCESS v2, see the "What's new in PROCESS 3" pages in your course book.

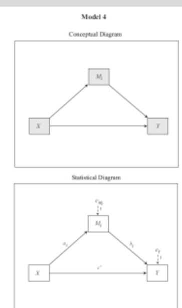
Differences between V2 & V3

- No longer need the `vars` list
- Covariates now listed in `cov` list
- Moderators are always `W` and `Z`, no more `V`, `M` or `Q` moderators
- Dichotomous Y now available for some models
- A variety of models have been cut, but new ability to create and edit models
- New models for serial moderated mediation and serial and parallel mediation
- Probing option now defaults to what used to be `quantiles`, can use `moments` argument for legacy output
- Probing and plotting for models with any moderation
- Default is now percentile bootstrap, no more BC or ABC
- Multicategorical X or Moderators
- `Wmodval` and `zmodval` allow for multiple values
- Covariate assignment
- Bootstrap CIs for regression coefficients
- Model construction
- `Cluster`, `ws`, `varorder`, and `percent` are no longer options

Estimation of the PROTEST model in PROCESS

PROCESS Model 4





What should the PROCESS command look like?

```
process y=liking/x=protest/m=respappr/model=4/boot=10000/normal=1/total=1.
```

not required

```
%process (data=protest, y=liking,x=protest,m=respappr,model=4,boot=10000,normal=1,total=1);
```

PROCESS output

```
**************** PROCESS Procedure for SPSS Version 3.00 *****************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018). www.guilford.com/p/hayes3

**************************************************************************
Model  : 4
    Y  : liking
    X  : protest
    M  : respappr

Sample
Size:  129

**************************************************************************
OUTCOME VARIABLE:
 respappr

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4992      .2492     1.3753    42.1550     1.0000   127.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.8841      .1831    21.2078      .0000     3.5217     4.2466
protest      1.4397      .2217     6.4927      .0000     1.0009     1.8785

**************************************************************************
```

$$\hat{M} = 3.884 + 1.440X$$

path *a*

Output C

PROCESS output

```
**************************************************************************
Outcome: liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4959      .2459      .8441    20.5483     2.0000   126.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.7473      .3058    12.2553      .0000     3.1422     4.3524
respappr      .4024      .0695     5.7884      .0000      .2648      .5400
protest      -.1007      .2005     -.5023      .6163     -.4975      .2960

************************ TOTAL EFFECT MODEL ****************************
Outcome: liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .2131      .0454     1.0601     6.0439     1.0000   127.0000      .0153

Model
              coeff         se          t          p       LLCI       ULCI
constant     5.3102      .1608    33.0244      .0000     4.9921     5.6284
protest       .4786      .1947     2.4584      .0153      .0934      .8639

**************************************************************************
```

$$\hat{Y} = 3.747 - 0.101X + 0.402M$$

$$\hat{Y} = 5.310 + 0.479X$$

path *b*
path *c'*
path *c*

Output C

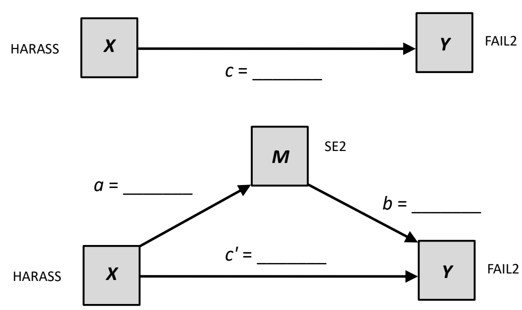
PROCESS output

```
**************** TOTAL, DIRECT, AND INDIRECT EFFECTS ********************

Total effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .4786      .1947     2.4584      .0153      .0934      .8639

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
     -.1007      .2005     -.5023      .6163     -.4975      .2960

Indirect effect of X on Y
             Effect    Boot SE   BootLLCI   BootULCI
respappr      .5793      .1519      .3113      .9067

Normal theory tests for indirect effect
     Effect         se          Z          p
      .5793      .1350     4.2924      .0000
```

Output C

path *c*
path *c'*
ab with 95% bootstrap confidence interval
Sobel test

Her behavior was perceived as more appropriate if she protested relative to when she did not (*a* = 1.440), and the more appropriate her behavior, the more positively she was perceived (*b* = 0.402).   Her choice to protest had a positive effect on how favorably she was perceived indirectly through perceived appropriateness of the response (point estimate: 0.579, 95% CI = 0.311 to 0.907). After accounting for this mechanism, there was no effect of her choice to protest on how she was evaluated (direct effect = -0.101, *p* = 0.62)

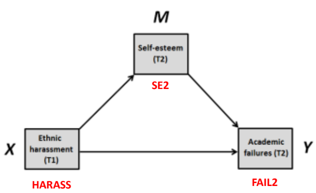


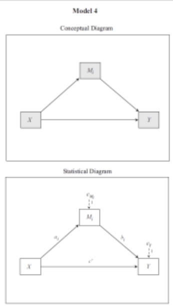
Indirect effect = _______, 95% bootstrap CI = ____________

Your CI will not exactly match. Why?

Estimation of the harassment model in PROCESS

PROCESS Model 4






```
Process y=fail2/x=harass/m=se2/model=4/boot=10000
        /normal=1/total=1.
```

```
%process (data=harass,y=fail2,x=harass,m=se2,model=4,
         boot=10000,normal=1,percent=1,total=1);
```

PROCESS output

```
*************** PROCESS Procedure for SPSS Version 3.00 ****************

          Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018). www.guilford.com/p/hayes3

**************************************************************************
Model  : 4
    Y  : fail2
    X  : harass
    M  : se2

Sample
Size:  330

**************************************************************************
OUTCOME VARIABLE:
 se2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .2764      .0764      .2905    27.1349     1.0000   328.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.5966      .1235    29.1227      .0000     3.3536     3.8395
harass       -.4156      .0798    -5.2091      .0000     -.5725     -.2586
```

$$\hat{M} = 3.597 - 0.416X$$

path *a*

PROCESS output

```
**************************************************************************
Outcome: fail2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3397      .1154      .2215    21.3247     2.0000   327.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.3845      .2042    11.6757      .0000     1.9827     2.7863
se2          -.2887      .0482    -5.9879      .0000     -.3836     -.1939
harass        .0616      .0725      .8499      .3960     -.0810      .2042
```

$$\hat{Y} = 2.385 + 0.062X - 0.289M$$

path *b*
path *c'*

```
************************** TOTAL EFFECT MODEL ****************************
Outcome: fail2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .1356      .0184      .2451     6.1419     1.0000   328.0000      .0137

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.3460      .1134    11.8660      .0000     1.1229     1.5692
harass        .1816      .0733     2.4783      .0137      .0374      .3257

**************************************************************************
```

$$\hat{Y} = 1.346 + 0.182X$$

path *c*

PROCESS output

```
**************** TOTAL, DIRECT, AND INDIRECT EFFECTS ********************

Total effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .1816      .0733     2.4783      .0137      .0374      .3257

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .0616      .0725      .8499      .3960     -.0810      .2042

Indirect effect of X on Y
        Effect    Boot SE   BootLLCI   BootULCI
se2      .1200      .0321      .0629      .1899

Normal theory tests for indirect effect
     Effect         se          Z          p
      .1200      .0308     3.8993      .0001
```

path *c*

path *c'*

ab with 95% bootstrap confidence interval

Sobel test

Kids one unit higher in harassment frequency were 0.416 units lower in self esteem one year later (a = -0.416), and lower self-esteem was related to higher perceived academic failure (*b* = -0.289). So harassment indirectly affected perceived academic failure (point estimate: 0.120, 95% bootstrap CI = 0.063 to 0.190). After accounting for this mechanism, there was no evidence of an effect of harassment on perceived academic failure (direct effect = 0.062, *p* = 0.396, 95% CI = -0.081 to 0.204)

Output C

Some additional options

SPSS

```
process y=liking/x=protest/m=respappr/model=4/boot=10000/normal=1/total=1/
        effsize=1/conf=99/save=1/seed=25545.
```

SAS

```
%process (data=protest, y=liking,x=protest,m=respappr,model=4,boot=10000,
        normal=1,total=1,effsize=1,conf=99,save=boots,seed=25545);
```

EFFSIZE=1: Generates various effect size measures for the indirect effect.
CONF=z: Changes level of confidence to z% for confidence intervals.
SAVE=1 or SAVE=fn: in SPSS, produces a file of all bootstrap estimates of all regression coefficients in the model. In SAS, saves bootstrap estimates to a file named “fn”
SEED=xxxx: Seeds the random number generator for replication of resamples over repeated runs of PROCESS.

See the documentation in Appendix A of IMMCPA for details.

Confounding

Kids who reported greater harassment earlier reported lower in self-esteem later, but they also reported lower self-esteem earlier (r = -0.18), and self-esteem was temporally consistent over time (r = 0.51). Furthermore, students who reported lower self esteem later reported greater academic failure later, but these later low self-esteem students also reported higher greater academic failure earlier (r = -0.26), and academic failure was temporally consistent (r = 0.30)

Correlations

		harass	se1	fail1	se2	fail2
harass	Pearson Correlation	1	-.176	.196	-.276	.136
	Sig. (2-tailed)		.001	.000	.000	.014
	N	330	330	330	330	330
se1	Pearson Correlation	-.176	1	-.306	.505	-.259
	Sig. (2-tailed)	.001		.000	.000	.000
	N	330	330	330	330	330
fail1	Pearson Correlation	.196	-.306	1	-.255	.297
	Sig. (2-tailed)	.000	.000		.000	.000
	N	330	330	330	330	330
se2	Pearson Correlation	-.276	.505	-.255	1	-.337
	Sig. (2-tailed)	.000	.000	.000		.000
	N	330	330	330	330	330
fail2	Pearson Correlation	.136	-.259	.297	-.337	1
	Sig. (2-tailed)	.014	.000	.000	.000	
	N	330	330	330	330	330

Pre-existing self-esteem and failure confound the relationships we believe to be causal. We want to know whether later self-esteem and academic failure are related to ethnic harassment frequency after accounting for initial self-esteem and failure.

Confounding

Some effects in a mediation model are subject to ‘confounding’ <u>even when X is based on random assignment</u>, making causality harder to establish. Partialing out various confounders can help though won’t solve the problem entirely.

$$\hat{Y} = c_0 + cX + c_2 U$$

$$\hat{M} = a_0 + aX + a_2 U$$

$$\hat{Y} = c'_0 + cX + bM + c'_2 U$$

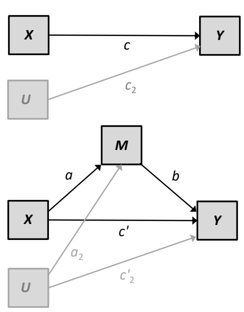
total effect = direct effect + indirect effect

c = c' + ($a \times b$)

indirect effect = total effect – direct effect

$a \times b$ = c – c'

A simple mediation model, adjusting for a potential confounding variable (U)



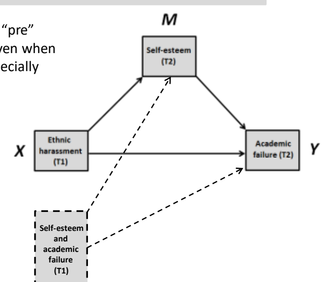
Some rationales for adjusting for prior state

When available, it is desirable to include “pre” measures of M and/or Y as covariates, even when X is experimentally manipulated, but especially when it is not.

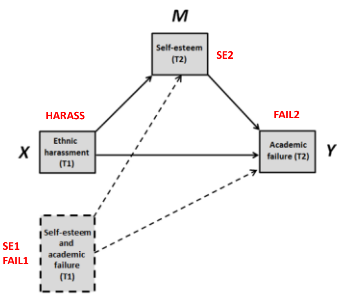
(a) Doing so can increase precision in the estimation of X’s effect on M and/or Y if pre-measures are correlated with later measures (as they typically are).

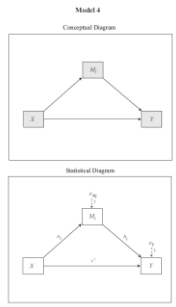
(b) Prior states often are correlated with X, M, or Y, introducing a “self-selection” threat to causal claims. Including prior state helps to reduce that threat.

(c) It gives an interpretation to paths that are closer to a “change” interpretation without regression artifacts that can be introduced with the use of difference scores. In this example, the b path estimates the relationship between later self-esteem and how much higher or lower a student’s failure is given expected later failure from prior self-esteem and failure. Path a estimates the relationship between ethnic harassment frequency and how much lower or higher a student’s self-esteem is later relative to what would be expected given his or her earlier self-esteem and academic failure.



Adding covariates to a model using PROCESS






```
process cov=se1 fail1/y=fail2/x=harass/m=se2/model=4/
  boot=10000/total=1.
```

```
%process (data=harass, cov=se1 fail1,y=fail2,x=harass,m=se2,
  model=4,boot=10000,total=1);
```

PROCESS output

Output D

```
**************** PROCESS Procedure for SPSS Version 3.00 *****************

         Written by Andrew F. Hayes, Ph.D.       www.afhayes.com
    Documentation available in Hayes (2018). www.guilford.com/p/hayes3

**************************************************************************
Model  : 4
    Y  : fail2
    X  : harass
    M  : se2

Covariates:
 se1      fail1

Sample
Size:  330

**************************************************************************
OUTCOME VARIABLE:
 se2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5450      .2971      .2224    45.9259     3.0000   326.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.0280      .2412     8.4095      .0000     1.5536     2.5024
harass       -.2728      .0717    -3.8025      .0002     -.4139     -.1317
se1           .4879      .0536     9.1081      .0000      .3825      .5933
fail1        -.1010      .0606    -1.6661      .0967     -.2202      .0182

**************************************************************************
```

path *a*

PROCESS output

Output D

```
**************************************************************************
Outcome: fail2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4059      .1648      .2105    16.0276     4.0000   325.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.0934      .2588     8.0900      .0000     1.5843     2.6025
se2          -.2175      .0539    -4.0375      .0001     -.3235     -.1115
harass        .0196      .0713      .2754      .7832     -.1207      .1599
se1          -.0672      .0584    -1.1517      .2503     -.1820      .0476
fail1         .2307      .0592     3.8966      .0001      .1142      .3471
```

path *b*

path *c'*

```
************************** TOTAL EFFECT MODEL ****************************
Outcome: fail2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3505      .1229      .2203    15.2219     3.0000   326.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.6523      .2400     6.8842      .0000     1.1801     2.1245
harass        .0790      .0714     1.1060      .2695     -.0615      .2194
se1          -.1733      .0533    -3.2513      .0013     -.2782     -.0685
fail1         .2526      .0603     4.1886      .0000      .1340      .3713
```

path *c*

PROCESS output

Output D

```
**************** TOTAL, DIRECT, AND INDIRECT EFFECTS ********************

Total effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .0790      .0714     1.1060      .2695     -.0615      .2194
```

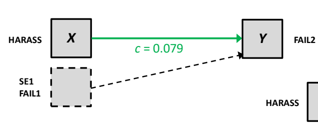
path *c*

```
Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .0196      .0713      .2754      .7832     -.1207      .1599
```

path *c'*

```
Indirect effect of X on Y
       Effect     Boot SE   BootLLCI   BootULCI
se2     .0593      .0229      .0203      .1089
```

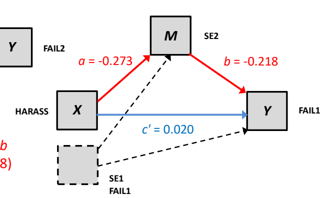
ab with 95% bootstrap confidence interval



$c \; = \; c' \; + \; a \; \times \; b$

$0.079 = 0.020 + (-0.273)(-0.218)$

$0.079 = 0.020 + 0.059$



Discussion: Comparing Results

We ran the mediation analysis with the harass data twice. Once without covariates and once with covariates.

- What are some key differences between the results?
- Which analysis do you think best approximates the "truth"?
- If the results had not changed while including the covariates, what would that mean?

What about Baron & Kenny?

Also called the "causal steps" approach, it was popularized by Baron and Kenny (1986) as a test of mediation.

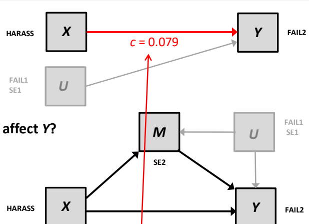
Conditions required to claim *M* functions as a mediator of the relationship between *X* and *Y*:

(1) Does *X* affect *Y?*
(2) Does *X* affect *M* ?
(3) Does *M* affect *Y* holding *X* constant ?

..as gauged by a hypothesis test.

(4) Is the direct effect of *X* closer to zero than the total effect?
  (i) if direct effect is closer to zero than total effect but statistically different from zero, claim "partial mediation"
  (ii) If direct effect is closer to zero than total effect and not statistically different from zero, claim "complete mediation"

Using a set of OLS regression analyses



Condition 1: Does *X* affect *Y*?

NO. THUS, NO MEDIATION BY THE CAUSAL STEPS STRATEGY

Coefficients[a]

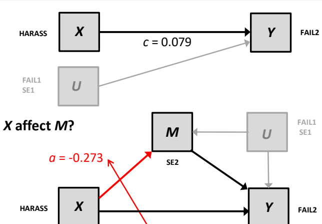
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.652	.240		6.884	.000
	harass	.079	.071	.059	1.106	.270
	fail1	.253	.060	.231	4.189	.000
	se1	-.173	.053	-.179	-3.251	.001

a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass fail1 se1.
```

```
proc reg data=harass;model fail2=harass fail1 se1;run;
```

Using a set of OLS regression analyses



Condition 2: Does *X* affect *M*?

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.028	.241		8.410	.000
	harass	-.273	.072	-.181	-3.802	.000
	se1	.488	.054	.448	9.108	.000
	fail1	-.101	.061	-.082	-1.666	.097

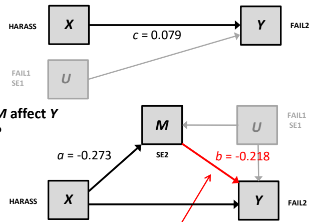
a. Dependent Variable: se2

```
regression/dep=se2/method=enter harass se1 fail1.
```

```
proc reg data=harass;model se2=harass se1 fail1;run;
```

Using a set of OLS regression analyses

Condition 3: Does *M* affect *Y* holding *X* constant?



Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.093	.259		8.090	.000
	harass	.020	.071	.015	.275	.783
	se2	-.218	.054	-.244	-4.038	.000
	fail1	.231	.059	.211	3.897	.000
	se1	-.067	.058	-.069	-1.152	.250

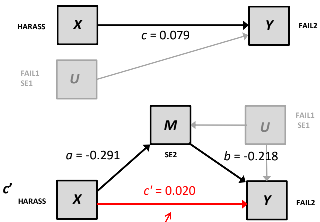
a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass se2 fail1 se1.
```

```
proc reg data=harass;model fail2=harass se2 fail1 se1;run;
```

Using a set of OLS regression analyses

4. **Qualitatively compare *c* to *c'***



Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.093	.259		8.090	.000
	harass	.020	.071	.015	.275	.783
	se2	-.218	.054	-.244	-4.038	.000
	fail1	.231	.059	.211	3.897	.000
	se1	-.067	.058	-.069	-1.152	.250

a. Dependent Variable: fail2

```
regression/dep=fail2/method=enter harass se2 fail1 se1.
```

```
proc reg data=harass;model fail2=harass se2 fail1 se1;run;
```

Direct effect (c') is closer to zero than the total effect (*c*) and is not statistically different from zero. But can we really call this 'complete mediation' given that there was no total effect of *X* in the first place by commonly-used inferential rules?

Problems with the causal steps approach

- Indirect effect is logically inferred rather than directly estimated

  But typically, we make inferences from data using estimates of quantities pertinent to the question. Why should inferences about indirect effects be any different?

  A fallacious rebuttal: if *a* and *b* are both different from zero (as established by rejection of the null hypothesis) so too must their product, so no estimate or test of indirect effect is needed.

  a) Although frequently that will be true, it isn't necessarily true.
  b) An indirect effect may be different from zero even in the absence of evidence that both paths *a* and *b* are.

- If data fail to meet a single criterion, **game over--no indirect effect through *M*.**

  The use of multiple, fallible hypothesis tests gives this approach the lowest power among competing methods for testing intervening variable effects. **Tests or claims of mediation should not be based on the significance of individual paths in the model.**

What about Baron & Kenny?

Consider:

$$X + Y = Z$$

If $Z > 0$ what does this tell us about X?
What does this tell us about Y?

Problems with the causal steps approach

- If total effect (path c) is not detectably different from zero, the **game doesn't even begin.**

  This is logically sensible if you accept one definition of a mediator variable – a variable that is causally between X and Y and that accounts for their association.

  - By this definition, an effect that does not exist can't be mediated. But the significance of c neither constrains nor determines the size of the product of paths a and b, nor does it tell us whether that product is different from zero.
  - Kenny and Judd (2014, *Psychological Science*) illustrate that a hypothesis test about the total effect is generally less powerful than a hypothesis test about the indirect effect.

- Because the indirect effect is not quantified, this method does not lend itself well to comparisons between indirect effects in multiple mediators models, or to modeling of the size of indirect effects ('conditional process analysis')

"Complete"/"full" and "partial" mediation

The causal steps strategy is often used as a means of labeling a process as "complete" or "partial mediation". There is little value to this semantic labeling exercise.

- What if there is no evidence of a total effect (i.e., c non-significant)? This can happen, and actually does more often than people probably realize. Thus, these concepts don't have a place much of the time.
- The reliance on statistical significance criteria means that when power is high for the test on c' partial mediation is the best you can hope for, and when power is relatively low, complete mediation is more likely. So if establishing complete mediation is your goal, <u>you should intentionally limit the size of your sample to as small as necessary</u>
- Establishing complete mediation by your favored mediator does not preclude others from being able to make the same claim with their own favored mediator.
- "Direct effects" don't exist in reality. All effects are mediated by something. Thus, a claim of 'partial mediation' is a claim that one has not specified the model correctly.

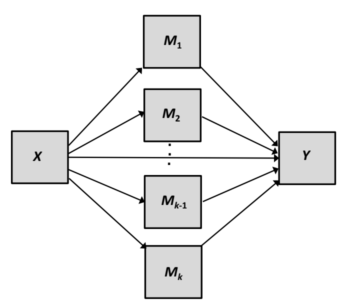
Experts in mediation analysis are abandoning these concepts. They are of historical interest only these days.

Mediation analysis summary thus far

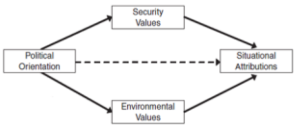
- Mediators are variables which are causally located between two variables X and Y and that explain, in part, the effect of X on Y. X affects M which in turn affects Y.
- The causal steps strategy popularized by Baron and Kenny (1986) remains a popular method for mediation analysis.
  - Yet it is among the lowest in power, in some circumstances, massively so.
  - It is not consistent with modern thinking about mediation analysis.
  - Its use is not recommended. Soon you won't be able to get away with it.
- Tests of mediation should be based on an estimate of the indirect effect.
  - Sobel test for inference in large samples only, but we don't know how large is large enough.
  - Bootstrap confidence intervals in a sample of any size.
- There is no need to condition the hunt for an indirect effect on a statistically significant total effect (path c).
- Focus interpretation on the size and sign of the indirect effect. Tests of significance for the individual paths ($X \rightarrow M$ and $M \rightarrow Y$) are useful as supplemental information but need not be part of the story.

Models with More Than One Mediator

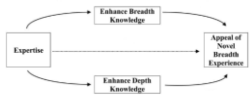
A parallel multiple mediator model



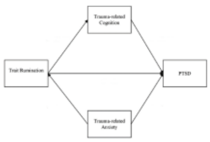
Some examples From the literature with 2 mediators



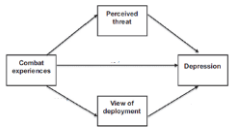
Morgan, G. S., Mullen, E., & Skitka, L. J. (2010). When values and attributions collide: Liberals' and conservatives' values motivate attributions for alleged misdeeds. ***Personality and Social Psychology Bulletin***, 36, 1241-1254.



Clarkson, J. J., Janiszewski, C., & Cinelli, M. D. (2013). The desire for consumption knowledge. ***Journal of Consumer Research, 39***, 1313-1329.

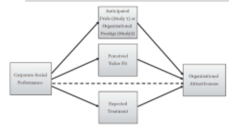


Spinhoven, P., Penninx, B. W., Krempeniou, A., et al. (2015). Trait rumination predicts onset of post-traumatic stress disorder through trauma-related cognitive appraisals: A 4-year longitudinal study. ***Behaviour Research and Therapy, 71***, 101-109.

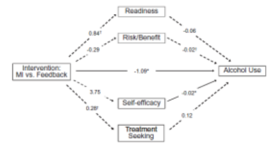


Pitts, B. L., & Safer, M. A. (2016). Retrospective appraisals mediate the effect of combat experiences on PTS and depression symptoms in U.S. Army medics. ***Journal of Traumatic Stress, 29***, 65-71.

Some examples from the literature with several mediators

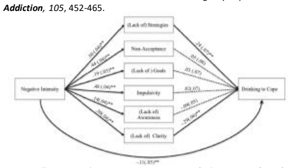


Jones, D. A., Willness, C. R., & Madey, S. (2014). Why are job seekers attracted by corporate social performance? Experimental and field tests of three signal-based mechanisms. ***Academy of Management Journal, 57***, 383-404.



Barnett, N. P., Apodaca, T. R., et al. (2010). Moderators and mediators of two brief interventions for alcohol in the emergency department. ***Addiction***, *105*, 452-465.

Brandt, M. J., & Reyna, C. (2010). The role of prejudice and the need for closure in religious fundamentalism. ***Personality and Social Psychology Bulletin***, *36*, 715-725.



Veilleux, J. C.,Skinner, K. D., Reese, E. D., & Shaver, J. A. (2014). Negative affect intensity influences drinking to cope through facets of emotion dysregulation. ***Personality and Individual Differences***, *49*, 96-101.

Why estimate such a model?

- Many causal effects probably operate through multiple mechanisms simultaneously. Better to estimate a model consistent with such real-world complexities.
- If your proposed mediator is correlated with the real mediator but not caused by the independent variable, a model with only your proposed mediator in it will be a misspecification and will potentially misattribute the process to your proposed mediator rather than the real mediator—"epiphenomenality."
- Different theories may postulate different mediators as mechanisms. Including them all in a model simultaneously allows for a formal statistical comparison of indirect effects representing different theoretical mechanisms.

Path Analysis: Total, Direct, and Indirect Effects

$$\hat{Y} = c_0 + cX$$

$$\widehat{M_j} = a_{0j} + a_j X$$

$$\hat{Y} = c'_0 + c'X + \sum_{j=1}^{K} b_j M_j$$

c = "total effect" of X on Y

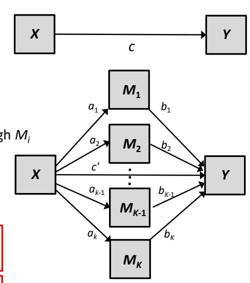
$a_j \times b_j$ = "specific indirect effect" of X on Y through M_i

$\Sigma\,(a_j \times b_j)$= "total indirect effect" of X on Y

c' = "direct effect" of X on Y

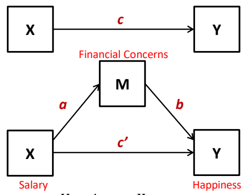
total effect = direct effect + total indirect effect $c \quad = \quad c' \quad + \quad \Sigma\,(a_j \times b_j)$

total indirect effect = total effect – direct effect $\Sigma\,(a_j \times b_j) \quad = \quad c \quad – \quad c'$



Path Analysis: Exercise Example

Suppose the true state of the world is such, and salary is measured in thousands of dollars per year (i.e. a one unit increase in salary corresponds to a $1000 increase in salary/year): An increase in salary of **$2,000/year** is associated with an overall increase in happiness of **3**. Suppose also that an increase in salary of **$1,000/year** is associated with a decrease in financial concerns by **2**. It is known that increasing financial concerns by **1** decreases happiness by **.5** when controlling for salary.



$$Y_i = i_{Y*} + cX_i + e_{Y_i^*}$$

$$M_i = i_M + aX_i + e_{M_i}$$

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

What are the values for:

a = ___________ total = ____________

b = ___________ direct = ____________

c = ___________ indirect = _______________

c' = ___________

Direct effect of X on Y (not through M) = c'

Indirect effect of X on Y (through M) = $a \times b$

Total effect = direct effect + indirect effect

$c = c' + a \times b$

Indirect effect = total effect - direct effect

$a \times b = c - c'$

113

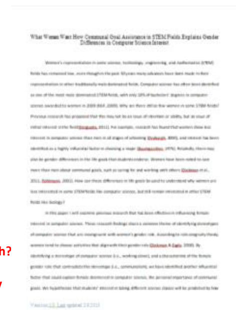
Example: Science

Participants read a syllabus for a computer science class. The syllabus either had a policy that was **procollaboration** or one which required **no collaboration**. Participants were randomly assigned to condition. This is the main independent variable.

Participants were asked a series of questions about the class they read about including four questions which assessed **interest in the class** (this is the primary DV). Higher = greater interest. They were also asked a set of questions to assess (1) how much they felt the class would help them in achieving **communal goals** (helping others, working with others) and (2) how **difficult** they expected the class to be. Higher = greater communal fulfillment /difficulty.

Question: Does group work in computer science classes increase interest in the class indirectly through perceived communal goal fulfillment, through class difficulty, or both?

Would people who read about the procollaboration policy think the class is more communal and would that communality then predict greater interest? Would the procollaboration policy make students think the course is easier, and this would increase interest?



The data: Science

ProNo	comm	diff	interest
1.00	5.20	5	6.0
1.00	1.00	4	2.2
1.00	4.00	4	2.5
1.00	4.00	2	3.5
1.00	7.00	7	7.0
1.00	6.00	1	6.0
1.00	4.00	7	2.7
1.00	3.40	6	4.2
1.00	4.20	3	3.5
1.00	4.60	6	1.5
1.00	4.20	5	2.0
1.00	4.40	5	1.0
1.00	5.40	3	7.0
1.00	5.00	4	2.2
1.00	4.60	6	2.7
1.00	3.40	4	1.2

```
data science;
   input Subject Cond sex ProNo comm diff interest;
   datalines;
   106   1   1   1   5.2   5   6
   109   1   1   1   1     4   2.25
   112   1   1   1   4     4   2.5
   114   1   1   1   4     2   3.5
   115   1   1   1   7     7   7
   121   1   1   1   6     1   6
   131   1   1   1   4     7   2.75
   132   1   1   1   3.4   6   4.25
   148   1   1   1   4.2   3   3.5
   161   1   1   1   4.6   6   1.5
   162   1   1   1   4.2   5   2
   164   1   1   1   4.4   5   1
   174   1   1   1   5.4   3   7
   176   1   1   1   5     4   2.25
   177   1   1   1   4.6   6   2.75
   178   1   1   1   3.4   4   1.25
   190   1   1   1   3.8   5   2.75
   206   1   1   1   4.2   6   1.75
   216   1   1   1   7     7   6
   217   1   1   1   5     4   4.75
```

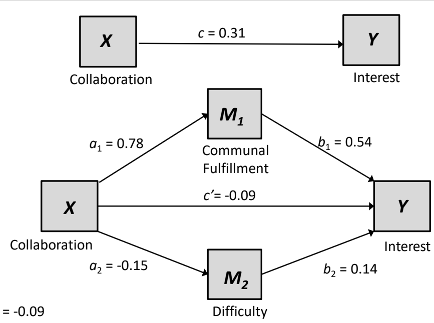
ProNo: Experimental condition (1 = procollaboration, 0 = no collaboration)

interest : interest in class (higher = greater interest)

comm: Perceived fulfillment of communal goals (higher = more fulfillment)

diff: Perceived difficulty of the class (higher = more difficult)

Example: Science



Direct effect = -0.09

Specific indirect effect via Comm: 0.78(0.54) = 0.42

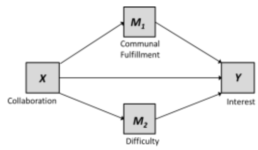
Specific indirect effect via Import: -0.15(0.14) = -0.02

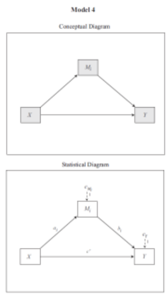
Total indirect effect = 0.42 – 0.02 = 0.40

Total effect = -0.09+ .42 – 0.02= 0.31

Estimation and inference using PROCESS

PROCESS model 4 is used for the parallel multiple mediator model.





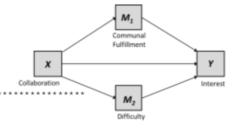
Up to 10 mediators can be listed in the "m =" list. Order does not matter.

```
process y=interest/x=ProNo/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

```
%process (data=science,y=interest,x=ProNo,m=comm diff,total=1,boot=10000,model=4,
        normal=1,contrast=1);
```

PROCESS output

Output E




```
Model  : 4
    Y  : interest
    X  : ProNo
   M1  : comm
   M2  : diff
Sample  Size:  232
**************************************************************************
OUTCOME VARIABLE:
 comm

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3031      .0919     1.5279    23.2670     1.0000   230.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.1160      .1133    27.4994      .0000     2.8927     3.3392
ProNo         .7831      .1624     4.8236      .0000      .4632     1.1030

**************************************************************************
OUTCOME VARIABLE:
 diff

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .0594      .0035     1.6760      .8155     1.0000   230.0000      .3674

Model
              coeff         se          t          p       LLCI       ULCI
constant     4.9412      .1187    41.6352      .0000     4.7073     5.1750
ProNo        -.1536      .1700     -.9031      .3674     -.4886      .1815
```

$\widehat{M_1} = 3.12 + 0.78X$

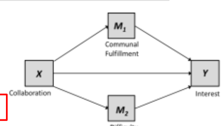
$\widehat{M_2} = 4.94 - 0.15X$

a_1 path

a_2 path

PROCESS output

Output E




```
OUTCOME VARIABLE:
 interest

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4418      .1952     1.9659    18.4348     3.0000   228.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant      .4898      .4618     1.0607      .2899     -.4201     1.3996
ProNo        -.0895      .1933     -.4630      .6438     -.4705      .2914
comm          .5367      .0752     7.1418      .0000      .3886      .6848
diff          .1364      .0718     1.9008      .0586     -.0050      .2778

************************** TOTAL EFFECT MODEL ****************************
OUTCOME VARIABLE:
 interest

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .1000      .0100     2.3974     2.3217     1.0000   230.0000      .1290

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.8361      .1419    19.9817      .0000     2.5565     3.1158
ProNo         .3099      .2034     1.5237      .1290     -.0908      .7106
```

$\hat{Y} = 0.49 - 0.09X + 0.54M_1 + 0.14M_2$

$\hat{Y} = 2.84 + 0.31X$

c' path

b_1 path

b_2 path

c path

PROCESS output

Output E

```
************** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Total effect of X on Y
     Effect         se          t          p       LLCI       ULCI
      .3099      .2034     1.5237      .1290     -.0908      .7106

Direct effect of X on Y
     Effect         se          t          p       LLCI       ULCI
     -.0895      .1933     -.4630      .6438     -.4705      .2914

Indirect effect(s) of X on Y:
         Effect     BootSE   BootLLCI   BootULCI
TOTAL     .3994      .1135      .1950      .6399
comm      .4203      .1128      .2171      .6594
diff     -.0209      .0282     -.0858      .0280
(C1)      .4413      .1189      .2251      .6927

   Normal theory test for indirect effect(s):
        Effect         se          Z          p
comm     .4203      .1059     3.9706      .0001
diff    -.0209      .0284     -.7367      .4613

Specific indirect effect contrast definition(s):
(C1)          comm      minus   diff
```

c path

c' path

$a_1b_1 + a_2b_2$ with bootstrap CI

a_1b_1 with bootstrap CI

a_2b_2 with bootstrap CI

Sobel tests (less trustworthy than bootstrap CIs)

The data are consistent with the claim that group work influences interest indirectly through communal goal fulfillment controlling for difficulty (0.420; 95% CI = 0.217 to 0.659) but not through difficulty controlling for goal fulfillment (-0.0209; 95% CI = -0.086 to 0.028).

Things to consider

(1) In a multiple mediator model, the specific indirect effect through M_k quantifies the component of the total indirect effect that is unique to M_k. Each specific indirect effect is estimated **controlling for all other mediators.**

M_k may function as a mediator variable when considered in isolation but not when considered with other mediator variables in the same model. If the intervening variables are highly intercorrelated, they can "cancel out" each others' effects.

(2) It is possible for a total indirect effect to be not detectably different from zero even when one or more specific indirect effects is.

total indirect effect = sum of specific indirect effects

$\Sigma(a_i b_i) \quad = \quad a_1b_1 + a_2b_2 + \ldots a_kb_k.$

Scenario (a): A single large specific indirect combined with several tiny ones.

Scenario (b): Specific indirect effects that have different signs and add to near zero.

In multiple mediator models, the total indirect effect is rarely of much interest.

Comparing specific indirect effects

Indirect effects quantify how *Y* changes as *X* changes by one unit through a mediator. **They are free of the scale of measurement of the mediators**. So in multiple mediator models, indirect effects linking the same *X* to the same *Y* are directly comparable even If the mediators are measured on different scales. We can statistically compare them if so desired. No standardization or other arithmetic gymnastics is required.

Approach #1: Calculate the ratio of the difference between the indirect effect through M_i and the indirect effect through M_j to its standard error. Assuming a normally distributed sampling distribution of the difference, a *p*-value for the null hypothesis that the difference equals zero can be derived from the standard normal distribution.

$$Z = \frac{a_ib_i - a_jb_j}{se_{a_ib_i - a_jb_j}}$$

Approach #2: Bootstrap a confidence interval for the $a_ib_i - a_jb_j$ and ascertain whether 0 is in the confidence interval as a pseudo null hypothesis test that the difference is zero.

PROCESS can generate a bootstrap confidence interval for all possible pairwise comparisons between specific indirect effects

PROCESS output

```
process y=interest/x=prono/m=comm diff/total=1/boot=10000/model=4/normal=1/contrast=1.
```

```
%process (data=science,y=interest,x=prono,m=comm diff,total=1,boot=10000,model=4,
         normal=1,contrast=1);
```

```
Indirect effect(s) of X on Y:
            Effect     BootSE   BootLLCI   BootULCI
TOTAL        .3994      .1135      .1950      .6399
comm         .4203      .1128      .2171      .6594
diff        -.0209      .0282     -.0858      .0280
(C1)         .4413      .1189      .2251      .6927

   Normal theory test for indirect effect(s):
           Effect          se          Z          p
comm        .4203       .1059     3.9706      .0001
diff       -.0209       .0284     -.7367      .4613

Specific indirect effect contrast definition(s):
(C1)            comm       minus   diff
```

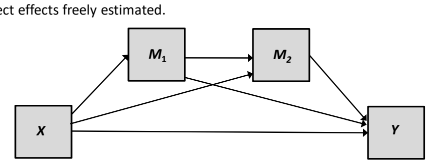
$a_1b_1 - a_2b_2$

Output E

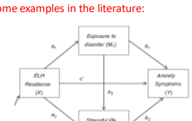
The specific indirect effect of collaboration on interest through perceived Communal goal fulfillment is different from the specific indirect effect through perceived class difficulty (difference = 0.441; 95% CI = 0.225 to .6927).

The serial multiple mediator model

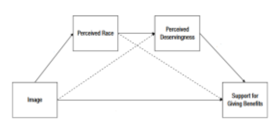
A serial multiple mediator model with two mediators and all possible direct and indirect effects freely estimated.



Some examples in the literature:

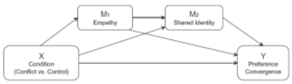


Casciano, R., & Massey, D. S. (2012). Neighborhood disorder and anxiety symptoms: New evidence from a quasiexperimental study. ***Health and Place,*** *18*, 180-190.

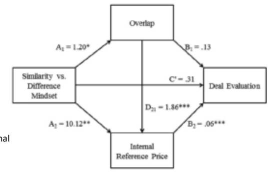


Brown-Iannuzzi, J. L., Dotsch, R., Cooley, E., & Payne, B. K. (2017). The relationship between mental representations of welfare recipients and attitudes toward welfare. ***Psychological Science, 28***, 92-103.

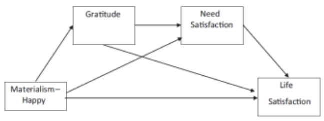
More examples from the literature



Schrift, R. Y., & Moty, A. (2015). Pain and preferences: Observed decisional conflict and the convergence of preferences. ***Journal of Consumer Research,*** *42*, 515-534..

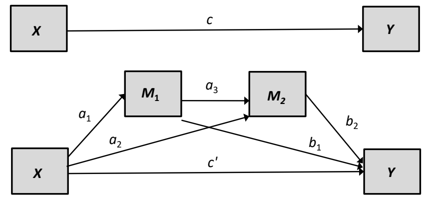


Kan, C., Lichtenstein, D. R., Grant, S. J., & Janiszwski, C. (2014). Strengthening the influence of advertised reference prices through information priming. ***Journal of Consumer Research,*** *40*, 1078-1096.



Tsang,J-A., Carpenter, T. P., Roberts, J. A., Frisch, M. B., & Carlisle, R. D. (2014). Why are materialists less happy? The role of gratitude and need satisfaction in the relationship between materialism and life satisfaction. ***Personality and Individual Differences****, 64*, 62-66.

Serial mediation: Path analysis rules



The total effect of *X* on *Y* is equal to the direct effect of *X* plus the sum of all specific indirect effects (there are three of them here).

$$\hat{Y} = c_0 + cX$$
$$\widehat{M_1} = a_{01} + a_1X$$
$$\widehat{M_2} = a_{02} + a_2X + a_3M_1$$
$$\hat{Y} = c'_0 + c'X + b_1M_1 + b_2M_2$$

Direct effect of *X*: c'
Specific indirect effect of *X* through M_1: a_1b_1
Specific indirect effect of *X* through M_2: a_2b_2
Specific indirect effect of *X* through M_1 and M_2: $a_1a_3b_2$
Total indirect effect of *X*: $a_1b_1 + a_2b_2 + a_1a_3b_2$
Total effect of *X*: $c = c' + a_1b_1 + a_2b_2 + a_1a_3b_2$

Example

May 1st, 2011, 11:30PM

Professor Erik Nisbet (OSU School of Communication) had an national telephone survey in the field examining perceptions of Muslims in the U.S. when Obama announces the death of bin Laden; 390 respondents prior to announcement (BINLADEN = 0) and 271 after announcement (BINLADEN = 1). See report in materials provided for details.

Measures

STEREO: Stereotype endorsement, 4 items (5-pt semantic differential)

"Please tell us how much you associate each of the following sets of characteristics with Muslims"

e.g., Peaceful – Violent
Tolerant – Fanatical

RTHREAT: Realistic threat, 5 items (5-pt Likert)

"Below are a few statements expressing different views about Muslims living in the U.S. Please read and tell us how much you agree with each statement"

e.g., "Muslims in the U.S. sympathize with terrorists"
"Muslims make America a more dangerous place to live"

MCIVIL: Restriction of Muslim civil liberties, 5 items (5-pt Likert)

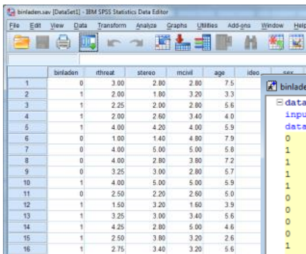
"Below are some statements people have expressed about Muslim civil liberties and terrorism in the U.S. Please read each and tell us how much you agree or disagree…"

e.g., "All Muslims in the U.S. should be required to carry a special ID card"
"Muslims in the U.S. should register their whereabouts with the U.S. government"

Higher reflect greater negative stereotype endorsement/threat/willingness to restrict…"

The Data: BINLADEN

BINLADEN.SAV



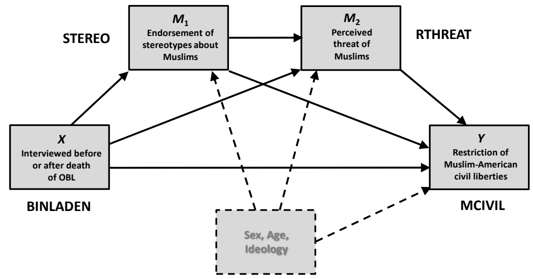
	binladen	rthreat	stereo	mcivil	age	ideo
1	0	3.00	2.80	2.80	7.5	
2	1	2.00	1.80	3.20	3.3	
3	1	2.25	2.00	2.80	5.6	
4	1	2.00	2.60	3.40	4.0	
5	1	4.00	4.20	4.00	5.9	
6	0	1.00	1.40	4.80	7.9	
7	0	4.00	5.00	5.00	5.8	
8	0	4.00	2.80	3.80	7.2	
9	0	3.25	3.00	2.80	5.7	
10	1	4.00	5.00	5.00	5.9	
11	0	2.50	2.20	2.60	5.0	
12	1	1.50	3.20	1.60	3.9	
13	1	3.25	3.00	3.40	5.6	
14	1	4.25	2.80	5.00	4.6	
15	1	2.50	3.80	3.20	2.6	
16	1	2.75	3.40	3.20	5.6	

BINLADEN.SAS

```
binladen
data binladen;
input binladen rthreat stereo mcivil age ideo sex;
datalines;
0   3.00    2.80    2.80    7.5 8   0
1   2.00    1.80    3.20    3.3 4   1
1   2.25    2.00    2.80    5.6 6   0
1   2.00    2.60    3.40    4.0 5   1
1   4.00    4.20    4.00    5.9 8   1
0   1.00    1.40    4.80    7.9 6   0
0   4.00    5.00    5.00    5.8 5   0
0   4.00    2.80    3.80    7.2 6   1
0   3.25    3.00    2.80    5.7 4   1
1   4.00    5.00    5.00    5.9 5   1
0   2.50    2.20    2.60    5.0 7   1
1   1.50    3.20    1.60    3.9 4   1
1   3.25    3.00    3.40    5.6 9   0
1   4.25    2.80    5.00    4.6 4   1
```

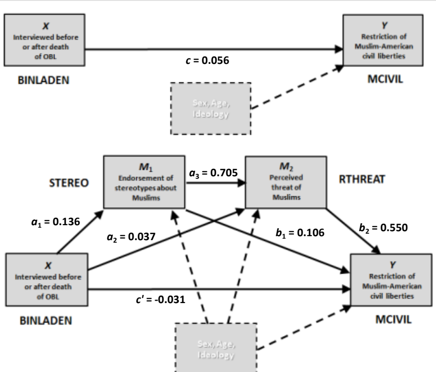
Also included in the data file are respondent **age** in <u>decades</u>, political **ideo**logy (7 point scale, higher = more conservative) and **gender** (0 = female, 1 =male).

Example

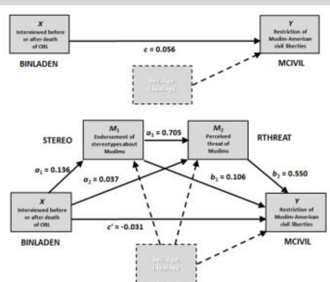


This model includes four pathways of influence of news coverage of OBL death, two through a single mediator, one through both mediators in serial, and one direct.

Example

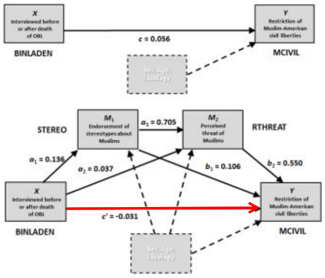


Example



Direct effect = -0.031
Specific indirect effect via stereotype endorsement: 0.136(0.106) = 0.014
Specific indirect effect via perceived threat: 0.037(0.550) = 0.020
Specific indirect effect via stereotype endorsement and threat: 0.136(0.705)(0.550) = 0.053
Total indirect effect = 0.014 + 0.020 + 0.053 = 0.087
Total effect = -0.031 + 0.087 = 0.056

Example



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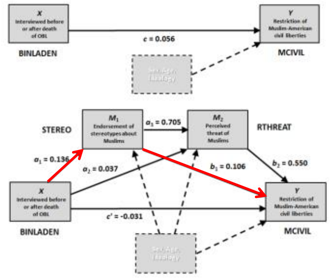
Example



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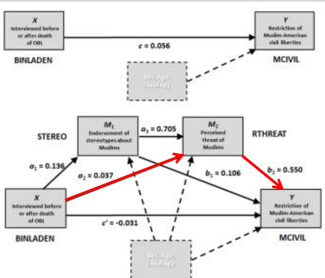
Example



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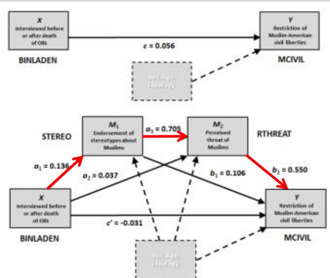
Example



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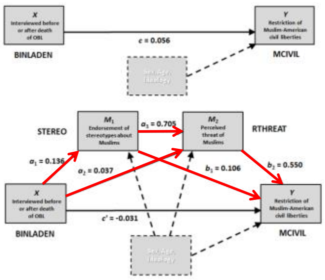
Example



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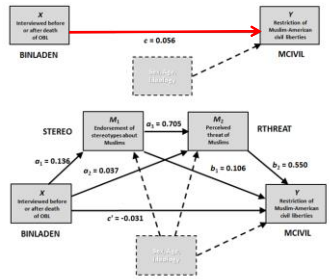
Example



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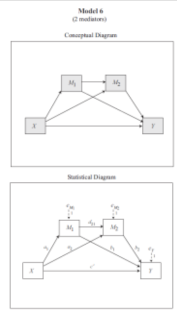
Estimation and inference using PROCESS



PROCESS model 6 is the serial multiple mediator model.

In model 6, order of the variables in the "m=" list matters. Variables listed earlier are causally prior to those listed later. PROCESS allows up to four mediators to be linked in a causal chain. All possible indirect and direct effects are estimated.

```
process cov=sex age ideo/y=mcivil/x=binladen/m=stereo rthreat /boot=10000/model=6/total=1.
```

```
%process (data=binladen,cov=sex age ideo,y=mcivil,x=binladen,m=stereo rthreat,boot=10000,
         model=6,total=1);
```

PROCESS output

Output F

```
**************************************************************************
Model = 6
    Y = mcivil
    X = binladen
   M1 = stereo
   M2 = rthreat

Statistical Controls:
CONTROL= sex      age      ideo

Sample size
       661
**************************************************************************
Outcome: stereo

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3557      .1265      .6495    23.7609     4.0000   656.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.9045      .1322    14.4084      .0000     1.6449     2.1640
binladen      .1358      .0639     2.1258      .0339      .0104      .2613
sex           .0398      .0635      .6262      .5314     -.0849      .1644
age           .0504      .0192     2.6220      .0089      .0127      .0882
ideo          .1293      .0143     9.0483      .0000      .1012      .1574

**************************************************************************
```

$$\hat{M}_1 = 1.905 + 0.136X + \ldots$$

a_1 path

PROCESS output

Output F

```
**************************************************************************
Outcome: rthreat
```

$$\hat{M}_2 = -0.255 + 0.037X + 0.705M_1 + \ldots$$

R	R-sq	MSE	F	df1	df2	p
.6764	.4575	.6076	110.4916	5.0000	655.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI	
constant	-.2548	.1467	-1.7369	.0829	-.5428	.0332	
stereo	.7047	.0378	18.6630	.0000	.6306	.7789	a_3 path
binladen	.0374	.0620	.6038	.5462	-.0843	.1592	a_2 path
sex	.1286	.0614	2.0938	.0367	.0080	.2492	
age	.0451	.0187	2.4135	.0161	.0084	.0818	
ideo	.0898	.0147	6.1257	.0000	.0610	.1186	

```
**************************************************************************
```

PROCESS output

Output F

```
**************************************************************************
Outcome: mcivil
```

$$\hat{Y} = 0.717 - 0.031X + 0.106M_1 + 0.549M_2 + \ldots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.6727	.4526	.5890	90.1100	6.0000	654.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI	
constant	.7165	.1448	4.9499	.0000	.4323	1.0008	
stereo	.1057	.0460	2.2965	.0220	.0153	.1960	b_1 path
rthreat	.5491	.0385	14.2732	.0000	.4736	.6247	b_2 path
binladen	-.0311	.0611	-.5095	.6106	-.1510	.0888	c' path
sex	-.1001	.0607	-1.6504	.0993	-.2193	.0190	
age	-.0103	.0185	-.5599	.5758	-.0466	.0259	
ideo	.0545	.0148	3.6696	.0003	.0253	.0836	

```
**************************************************************************
```

PROCESS output

Output F

```
************************** TOTAL EFFECT MODEL ****************************
Outcome: mcivil
```

$$\hat{Y} = 1.515 + 0.056X + \ldots$$

Model Summary

R	R-sq	MSE	F	df1	df2	p
.3675	.1351	.9278	25.6100	4.0000	656.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI	
constant	1.5149	.1580	9.5894	.0000	1.2047	1.8251	
binladen	.0564	.0764	.7380	.4608	-.0936	.2063	c path
sex	-.0099	.0759	-.1310	.8958	-.1589	.1391	
age	.0393	.0230	1.7085	.0880	-.0059	.0844	
ideo	.1675	.0171	9.8053	.0000	.1339	.2010	

```
**************************************************************************
```

PROCESS output

Output F

```
**************** TOTAL, DIRECT, AND INDIRECT EFFECTS ********************
```

Total effect of X on Y

Effect	SE	t	p	LLCI	ULCI	
.0564	.0764	.7380	.4608	-.0936	.2063	c path

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI	
-.0311	.0611	-.5095	.6106	-.1510	.0888	c' path

Indirect effect(s) of X on Y:

	Effect	BootSE	BootLLCI	BootULCI	
TOTAL	.0875	.0460	.0005	.1798	$a_1b_1 + a_2b_2 + a_1a_3b_2$
Ind1	.0144	.0098	-.0002	.0373	a_1b_1
Ind2	.0206	.0338	-.0446	.0895	a_2b_2
Ind3	.0526	.0248	.0050	.1021	$a_1a_3b_2$

and bootstrap CIs

```
Indirect effect key
 Ind1 :   binladen ->      stereo   ->      mcivil
 Ind3 :   binladen ->      stereo   ->      rthreat  ->      mcivil
 Ind2 :   binladen ->      rthreat  ->      mcivil
```

The data are consistent with the claim that coverage of OBL's death increased endorsement of restriction of Muslim civil liberties serially through stereotype endorsement and perceived threat of Muslims (0.053, 95% CI=0.005 to 0.102) but not through stereotype endorsement independent of perceived threat (.014, 95% CI = -0.0002 to 0.037) or perceived threat independent of stereotype endorsement (0.021, 95% CI = -0.045 to 0.089). There is no evidence of a direct effect of his death independent of these pathways of influence.

Moderation

Moderation. The effect of X on Y can be said to be *moderated* if its size or direction is dependent on some third variable W. It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or what the effect is large vs. small, present vs. absent, positive vs. negative vs. zero.

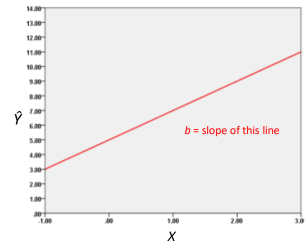
The simple regression coefficient

Consider a simple regression model with predictor variable X.

$$\hat{Y} = b_0 + bX \quad \text{such as} \quad \hat{Y} = 5.00 + 2.00X$$

Two cases that differ by one unit on X are estimated to differ by b = 2.00 units on Y. b is a "global property" of the model, in that makes no difference which value of X you start at---b is the estimated difference in Y between two cases who differ by a unit on X. Most generally, $b = \hat{Y}|(X = \omega + 1) - \hat{Y}|(X = \omega)$ for all ω.

X	$\hat{Y}$
-1	3.00
0	5.00
1	7.00
2	9.00
3	11.00



Partial regression coefficients as unconditional effects

Consider a multiple regression model with two predictors, X and W.

$$\hat{Y} = b_0 + b_1X + b_2W \quad \text{such as} \quad \hat{Y} = 4.50 + 2.00X + 0.50W$$

Regardless of W, a one unit difference in X is associated with the same expected difference on Y. And regardless of the value of X, a one unit difference in W is associated with the same expected difference on Y. This is true regardless of which value of X or W you choose. b_1 and b_2 are global properties of the model.

Most generally,

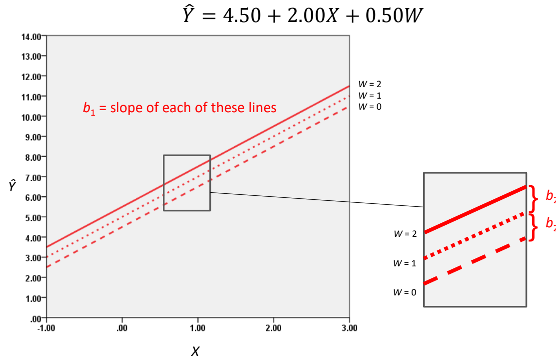
$b_1 = \hat{Y}|(X = \omega + 1, W = \lambda) - \hat{Y}|(X = \omega, W = \lambda)$ for all ω, λ

$b_2 = \hat{Y}|(W = \lambda + 1, X = \omega) - \hat{Y}|(W = \lambda, X = \omega)$ for all λ, ω

X	W	$\hat{Y}$
-1	0	2.50
-1	1	3.00
-1	2	3.50
0	0	4.50
0	1	5.00
0	2	5.50
1	0	6.50
1	1	7.00
1	2	7.50
2	0	8.50
2	1	9.00
2	2	9.50

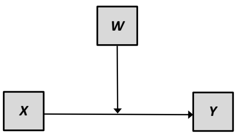
Partial regression coefficients as unconditional effects

$$\hat{Y} = 4.50 + 2.00X + 0.50W$$



Moderation

Moderation. The effect of *X* on *Y* can be said to be *moderated* if its size or direction is dependent on some third variable *W*. It tells us about the conditions that facilitate, enhance, or inhibit the effect, or for whom or what the effect is large vs. small, present vs. absent, positive vs. negative vs. zero.



In this diagram, *W* is depicted to *moderate* the size of the effect of *X* on *Y*, meaning that the size of the effect of *X* on *Y* depends on *W*. In such a case, we say *W* is the *moderator* of the *X* → *Y* relationship, or that *X* and *W* *interact* in their influence on *Y*. *X* is sometimes called the focal predictor, and *W* the moderator.

Releasing this constraint on the model

Suppose we let *X*'s effect be a function of *W*, *f*(*W*), as in

$$\hat{Y} = b_0 + f(W)X + b_2W$$

For instance, let *f*(*W*) be a linear function of *W*, $b_1 + b_3W$. Thus,

$$\hat{Y} = b_0 + (b_1 + b_3W)X + b_2W$$

This can be rewritten in an equivalent form as

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

This model, the "simple moderation model," allows *X*'s effect on *Y* to depend linearly on *W*. Other forms of moderation are possible, but this form is the one most frequently estimated.

Moderation

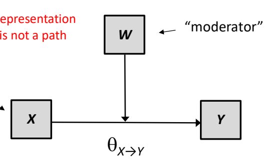
$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

can be written as

$$\hat{Y} = b_0 + (b_1 + b_3W)X + b_2W$$

This is a conceptual representation of moderation. This is not a path diagram.

"focal predictor"



$\theta_{X\to Y} = b_1 + b_3W$

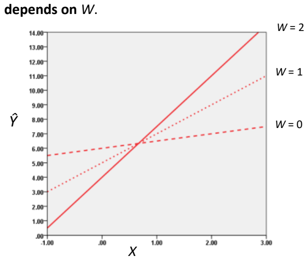
$\theta_{X\to Y}$ is the "conditional effect of *X*" defined by the function $b_1 + b_3W$

X's effect as a function of *W*

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on *X* are estimated to differ on *Y* **depends on** *W*.



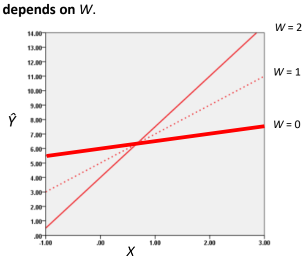
X	*W*	$\hat{Y}$
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

X's effect as a function of W

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y **depends on** W.



X	W	$\hat{Y}$
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

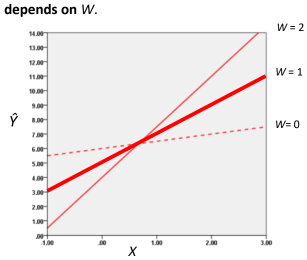
$\theta_{X \rightarrow Y} = b_1 + b_3W$
$= 0.50 + 1.50W$
$= 0.50 + 1.50(0) = 0.50$

X's effect as a function of W

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y **depends on** W.



X	W	$\hat{Y}$
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

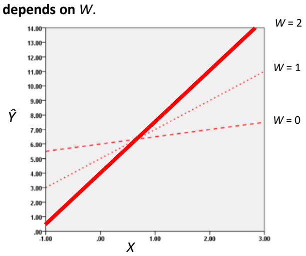
$\theta_{X \rightarrow Y} = b_1 + b_3W$
$= 0.50 + 1.50W$
$= 0.50 + 1.50(1) = 2.00$

X's effect as a function of W

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$

Observe that the amount by which two cases that differ by one unit on X are estimated to differ on Y **depends on** W.



X	W	$\hat{Y}$
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

$\theta_{X \rightarrow Y} = b_1 + b_3W$
$= 0.50 + 1.50W$
$= 0.50 + 1.50(2) = 3.50$

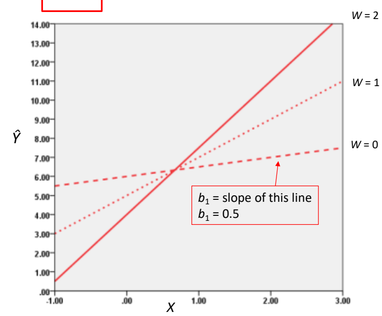
Interpretation of b_1 as a conditional effect

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + \boxed{0.50X} - 1.00W + 1.50XW$$

b_1 is the effect of X on Y when W = 0. It quantifies how much two cases that differ by one unit on X but with W = 0 are estimated to differ on Y.

b_1 is a local property of the model. It characterizes the association between X and Y only when W = 0.



$$\boxed{b_1 = \hat{Y} | (X = \omega + 1, W = 0) - \hat{Y} | (X = \omega, W = 0) \text{ for all } \omega.}$$

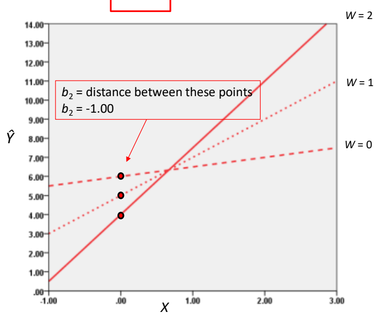
Interpretation of b_2 as a conditional effect

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y} = 6.00 + 0.50X \boxed{- 1.00}W + 1.50XW$$

b_2 is the effect of W when X = 0. It quantifies how much two cases that differ by one unit on W but with X = 0 are estimated to differ on Y.

b_2 is a local property of the model. It characterizes the association between W and Y only when X = 0.



$\boxed{b_2 = \hat{Y}|(W = \lambda + 1, X = 0) - \hat{Y}|(W = \lambda, X = 0) \text{ for all } \lambda}$

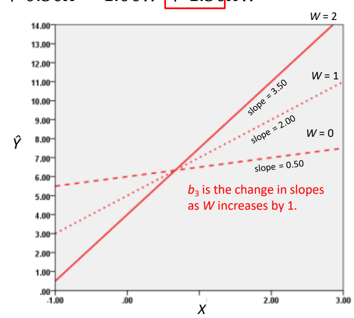
Interpretation of b_3

$$\hat{Y} = 6.00 + 0.50X - 1.00W \boxed{+ 1.50}XW$$

b_3 is the amount by which the conditional effect of X changes as W changes by one unit.

$\theta_{X \to Y} = b_1 + b_3 W$
$= 0.50 + 1.50W$

$\theta_{X \to Y}$	W
0.50	0
2.00	1
3.50	2



$$b_3 = (\theta_{X \to Y} | W = \lambda + 1) - (\theta_{X \to Y} | W = \lambda) \text{ for all } \lambda$$

Differences in interpretation

	$\hat{Y} = b_0 + b_1 X + b_2 W$	$\hat{Y} = b_0 + b_1 X + b_2 W + b_3 XW$
b_0	The estimated value of Y when X and W = 0.	The estimated value of Y when X and W = 0.
b_1	The effect of X on Y holding W constant. This is a *partial* effect.	The effect of X on Y when W = 0. This is a *conditional* effect. It is Not a “main effect” or “average effect” of X.
b_2	The effect of W on Y holding X constant. This is a *partial* effect.	The effect of W on Y when X = 0. This is a *conditional* effect. It is not a “main effect” or “average effect” of W.
b_3		How much the effect of X on Y changes as W changes by 1 unit.

Symmetry in moderation

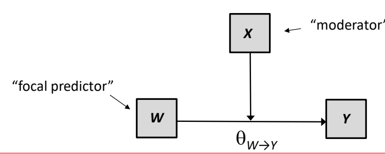
$$\boxed{\hat{Y} = b_0 + b_1 X + b_2 W + b_3 XW}$$

We saw that this is alternative representation of

$$\hat{Y} = b_0 + (b_1 + b_3 W)X + b_2 W$$

But it is also an alternative representation of

$$\hat{Y} = b_0 + (b_2 + b_3 X)W + b_1 X$$

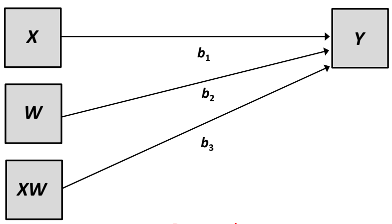


$\theta_{W \to Y} = b_2 + b_3 X$

Here, X moderates the size of the effect of W on Y. Now X is the moderator. Ultimately, which variable X or W we think of as the moderator depends on substantive concerns. Statistically, it makes no difference as they are mathematically equivalent models.

In path diagram form

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

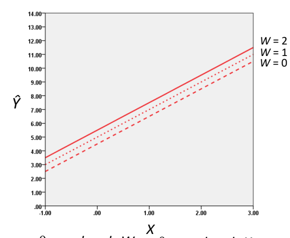


Remember

b_1 is NOT the effect of *X* on *Y*. The effect of *X* is $b_1 + b_3W$
b_2 is NOT the effect of *W* on *Y*. The effect of *W* is $b_2 + b_3X$

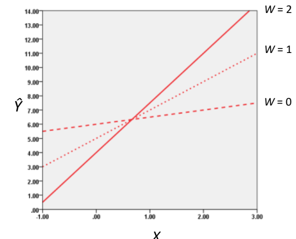
The importance of b_3 when testing a moderation hypothesis

$$\hat{Y} = 4.50 + 2.00X + 0.50W + 0XW$$



$\theta_{X\to Y} = b_1 + b_3W = 2.00 + 0W$
$\theta_{W\to Y} = b_2 + b_3X = 0.50 + 0X$

$$\hat{Y} = 6.00 + 0.50X - 1.00W + 1.50XW$$



$\theta_{X\to Y} = b_1 + b_3W = 0.50 + 1.50W$
$\theta_{W\to Y} = b_2 + b_3X = -1.00 + 1.50X$

When $b_3 = 0$, a one unit change in *X* has the same effect on *Y* regardless of *W*, and a one unit change in *W* has the same effect on *Y* regardless of *X*. When $b_3 \neq 0$, the effect of a change in *X* on *Y* depends on *W*, and the effect of a change in *W* on *Y* depends on *X*. So we test a moderation hypothesis by testing whether b_3 is different from zero.

Example inspired by …

Witkiewitz, K., & Bowen, S. (2010). Depression, craving, and substance use following a randomized trial of mindfulness-based relapse prevention. ***Journal of Consulting and Clinical Psychology, 78***, 362-374.

168 clients of a public service agency providing treatment for alcohol and substance use disorders.

MBRP : Randomly assigned to treatment as usual (0) or mindfulness-based relapse prevention therapy (1)

BDI0: Beck Depression Inventory at start of therapy (0 to 3; multiply by 21 to see BDI in its original 0 to 63 metric). This is also available at the termination of therapy (**BDIP**)

CRAVE2: Score on the Penn Alcohol Craving Scale at 2 month follow-up (0 to 6). Also available at baseline, prior to start of therapy (**CRAVE0**)

USE4: Alcohol and other substance use at 4-month follow-up. (0 to 5)

TREATHRS: Hours of therapy administered.

The data file is MBRP

The Data: MBRP

SPSS

mbrp.sav [DataSet1] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Graphs Utilities Add-ons Window

	mbrp	bdi0	bdip	crave0	crave2	use4
1	0	1.28	1.09	4.0	.8	.9
2	0	1.47	1.52	2.4	3.8	1.3
3	0	.66	1.14	2.2	1.4	1.0
4	1	1.66	1.23	2.2	2.4	1.1
5	0	1.28	.85	4.2	2.4	.9
6	1	.95	1.04	1.0	1.0	1.3
7	0	1.38	.85	2.0	.8	.3
8	0	1.76	.95	3.2	2.0	.8
9	0	.80	.71	3.0	1.2	.5
10	1	1.47	1.14	1.2	1.6	1.2
11	1	1.38	2.00	2.4	2.6	1.5
12	0	1.00	.61	1.0	.6	.6
13	1	1.38	1.66	3.8	1.2	.5
14	1	1.09	.95	1.8	1.2	1.8

SAS

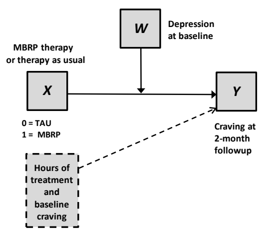
mbrp *

```
data mbrp;
input mbrp bdi0 bdip crave0 crave2 use4 treathrs;
datalines;
0 1.28 1.09 4.0 .8 .95 32
0 1.47 1.52 2.4 3.8 1.30 36
0 .66 1.14 2.2 1.4 1.09 34
1 1.66 1.23 2.2 2.4 1.17 37
0 1.28 .85 4.2 2.4 .91 27
1 .95 1.04 1.0 1.0 1.31 32
0 1.38 .85 2.0 .8 .36 38
0 1.76 .95 3.2 2.0 .83 26
0 .80 .71 3.0 1.2 .57 42
1 1.47 1.14 1.2 1.6 1.24 32
1 1.38 2.00 2.4 2.6 1.54 25
0 1.00 .61 1.0 .6 .63 24
1 1.38 1.66 3.8 1.2 .54 35
1 1.09 .95 1.8 1.2 1.81 34
```

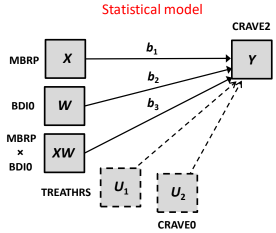
These aren't their actual data. But the analyses we do yield similar results to what they report.

Example

Conceptual diagram



Statistical model



Does the effect of MBRP therapy relative to therapy as usual on craving depend on initial depression? That is, is the therapy more or less effective as a function of depression prior to start of therapy?

Estimation using OLS regression

```
compute mbrpdep = mbrp*bdi0.
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp;set mbrp;mbrpdep=mbrp*bdi0;run;
proc reg data=mbrp;model crave2=mbrp bdi0 mbrpdep treathrs crave0;run;
```

X **= MBRP**
W **= BDI0**
Y **= CRAVE2**

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$

Output G

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.038	.470		2.209	.029
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
	mbrpdep	-.948	.423	-.598	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

"conditional effects", not "main effects"

b_1 = 0.587
b_2 = 1.122
b_3 = -0.948

The coefficient for the product is statistically different from zero. This means that the effect of MBRP therapy on craving depends on the person's level of depression at the start of therapy. But to really understand what is happening, we need a picture.

Visualizing the model

Rejecting the null hypothesis that "true b_3" is equal to zero tells you that the focal predictor's effect is indeed moderated by the proposed moderator. But moderation can take many different forms. We need to visualize the effect in order to interpret the result.

Step 1: Select various combinations of values of the focal predictor and moderator. The selection is sometimes arbitrary, but it may not be. Just make sure the values chosen are within the range of the data.

Step 2: Using the model, generate the estimates of Y using your selected values of the focal predictor and moderator. If your model includes covariates, use the sample mean for each of those.

Step 3: Graph, using whatever graphics program you prefer.

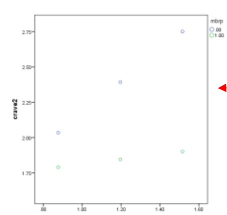
$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW - 0.018U_1 + 0.192U_2$$

MBRP (X)	BDI0 (W)	TREATHRS (U_1)	CRAVE0(U_2)	$\hat{Y}$
0	0.877	30.685	2.943	2.035
0	1.196	30.685	2.943	2.393
0	1.515	30.685	2.943	2.751
1	0.877	30.685	2.943	1.790
1	1.196	30.685	2.943	1.846
1	1.515	30.685	2.943	1.901

I used one standard deviation below the mean, the mean, and one standard deviation above the mean. It really makes no difference what you choose, except you want to make sure that your resulting graph is not extrapolating beyond the available data.

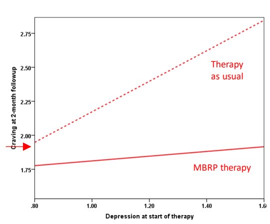
Example code in SPSS

```
data list free/mbrp bdi0.
begin data.
 0  0.877
 0  1.196
 0  1.515
 1  0.877
 1  1.196
 1  1.515
end data.
compute crave2=1.038+0.587*mbrp+1.122*bdi0-0.948*mbrp*bdi0-0.018*30.685+0.192*2.943.
graph/scatterplot = bdi0 with crave2 by mbrp.
```



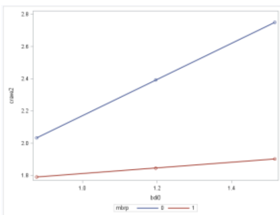
Before editing

After editing



Example code in SAS

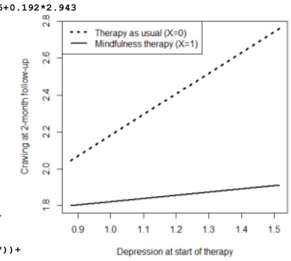
```
data;
input mbrp bdi0;
crave2=1.038+0.587*mbrp+1.122*bdi0-0.948*mbrp*bdi0-0.018*30.685+0.192*2.943;
datalines;
0  0.877
0  1.196
0  1.515
1  0.877
1  1.196
1  1.515
run;
proc sgplot;reg x=bdi0 y=crave2/group=mbrp;run;
```



Example code in R

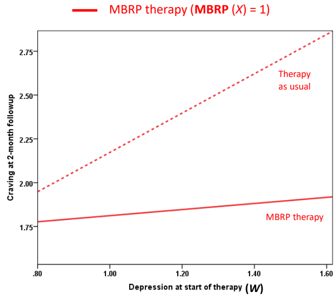
Although hard to learn at first, once you learn how to use R, you will find it very helpful
In the construction of visual depictions of models.

```
x<-c(0,1,0,1,0,1)
w<-c(0.877,0.877,1.196,1.196,1.515,1.515)
y<-1.038+0.587*x+1.122*w-0.948*x*w-0.018*30.685+0.192*2.943
plot(y=y,x=w,pch=15,col="white",
xlab="Depression at start of therapy",
ylab="Craving at 2-month follow-up")
legend.txt<-c("Therapy as usual (X=0)",
"Mindfulness therapy (X=1)")
legend("topleft",legend=legend.txt,
lty=c(3,1),lwd=c(3,2))
lines(m[x==0],y[x==0],lwd=3,lty=3)
lines(m[x==1],y[x==1],lwd=2,lty=1)

                              OR

library(ggplot2)
qplot(x = w, y = y, linetype = as.factor(x),
      geom = "line")+
  xlab("Depression at start of therapy")+
  ylab("Craving at 2-month follow-up")+
  scale_linetype_discrete(name=element_blank(),
        breaks=c("0", "1"),
        labels=c("Therapy as Usual (X = 0)",
                 "Mindfulness Therapy (X = 1)"))+
  theme(legend.justification=c(-0.1,1.1),
         legend.position=c(0,1),
        panel.background = element_rect("white", "black"),
        panel.grid.major = element_blank())
```



Substantive interpretation of the pattern

Therapy as usual (**MBRP** (X) =0)
MBRP therapy (**MBRP** (X) = 1)



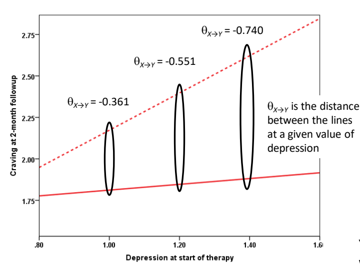
Those who receive MBRP therapy crave substances less than those who receive MBRP therapy, but this difference is larger among those more depressed at the start of therapy.

A graphical depiction of the model

$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + ...$ or, equivalently,

Therapy as usual (**MBRP** (X) =0)
MBRP therapy (**MBRP** (X) = 1)

$\hat{Y} = 1.038 + (0.587 - 0.948W)X + 1.122W + ...$



The conditional effect of MBRP therapy ($\theta_{X \to Y}$) is defined by the function

$\theta_{X \to Y} = \mathbf{0.587 - 0.948}W$

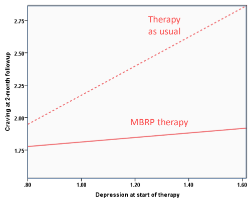
BDI0 (W)	$\theta_{X \to Y}$
1.00	-0.361
1.20	-0.551
1.40	-0.740

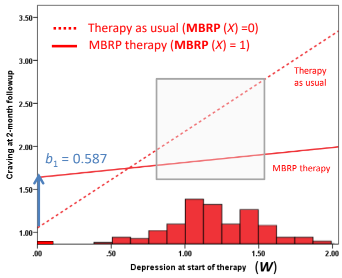
You can plug any value of BDI0 you want into the function to get the conditional effect of MBRP therapy

Interpretation of b_1

b_1 = 0.587

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$





b_1 is the effect of X on Y when W = 0. It is a conditional effect, and a local term of the model.

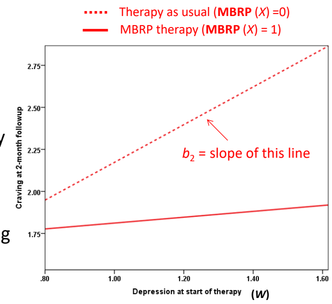
Interpretation of b_2

b_2 = 1.122

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$

b_2 is the conditional effect of W when X = 0. It is a conditional effect and a local term of the model.

Among those given therapy as usual, those who were relatively more depressed at the start of therapy had relatively higher craving at two months follow-up



Interpretation of b_3

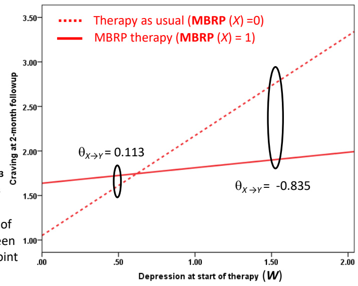
b_3 = -0.948

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$

$\theta_{X\to Y}$ = 0.587 – 0.948W

W	$\theta_{X\to Y}$
0.50	**0.113**
1.00	-0.361
1.50	**-0.835**
2.00	-1.309

-0.835 – (0.113) = -0.948 = b_3
-1.309 – (-0.361) = -0.948 = b_3



b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.

$\theta_{X\to Y}|(W = \lambda+1) - \theta_{X\to Y}|(W = \lambda)$ = -0.948, for all λ

Interpretation of b_3

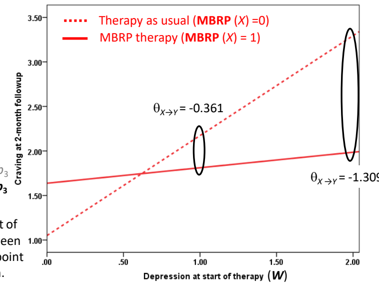
b_3 = -0.948

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$

$\theta_{X\to Y}$ = 0.587 – 0.948W

W	$\theta_{X\to Y}$
0.50	0.113
1.00	**-0.361**
1.50	-0.835
2.00	**-1.309**

-0.835 – (0.113) = -0.948 = b_3
-1.309 – (-0.361) = -0.948 = b_3



b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.

$\theta_{X\to Y}|(W=\lambda+1) - \theta_{X\to Y}|(W = \lambda)$ = -0.948, for all λ

Interpretation of b_3

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$

b_3 = -0.948

$\theta_{X \to Y} = 0.587 - 0.948W$

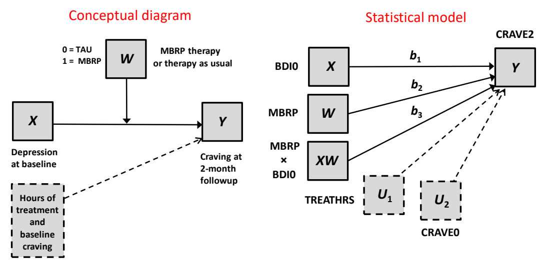
b_3 is the difference in the effect of MBRP therapy on craving between those who differ by one scale point in their pre-therapy depression.

$$\boxed{\theta_{X \to Y} | (W = \lambda + 1) - \theta_{X \to Y} | (W = \lambda) = -0.948, \text{ for all } \lambda}$$

Select 3 values of λ then calculate the conditional effect of X on Y at λ+1, λ, and the difference between the two.

λ	$\theta_{X \to Y} \| (W=\lambda+1)$	$\theta_{X \to Y} \| (W = \lambda)$	$\theta_{X \to Y} \| (W=\lambda+1) - \theta_{X \to Y} \| (W = \lambda)$

A Dichotomous Moderator



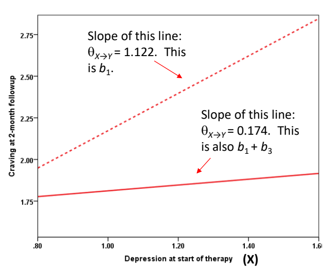
Does the association between pre-treatment depression and later craving differ between those who receive MBRP therapy versus therapy as usual?

A graphical depiction of the model

$$\hat{Y} = 1.038 + 1.122X + 0.587W - 0.948XW + \ldots$$ or, equivalently,

$$\hat{Y} = 1.038 + (1.122 - 0.948W)X + 0.587W + \ldots$$

Therapy as usual (**MBRP** (W) =0)
MBRP therapy (**MBRP** (W) = 1)



The conditional effect of pre-therapy depression ($\theta_{X \to Y}$) is defined by the function

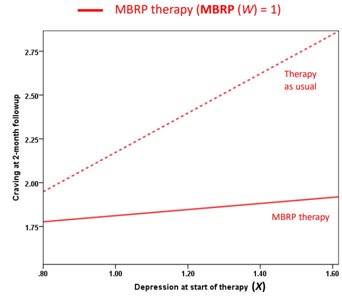
$$\theta_{X \to Y} = b_1 + b_3 W = 1.122 - 0.948W$$

MBRP (W)	$\theta_{X \to Y}$
0	1.122
1	0.174

Two cases that differ by one unit on W are estimated to differ by $\theta_{X \to Y}$ units on Y. $\theta_{X \to Y}$ depends on W.

Substantive interpretation of the pattern

Therapy as usual (**MBRP** (W) =0)
MBRP therapy (**MBRP** (W) = 1)

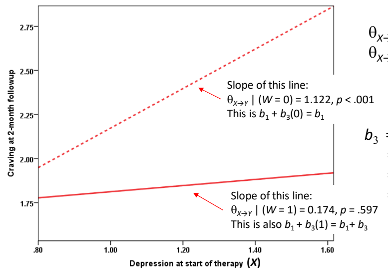


A larger effect of pre-therapy depression on later craving among those who experienced therapy as usual compared to those who received mindfulness behavioral relapse prevention therapy. MBRP therapy seems to have disrupted the link between depression and craving.

Interpreting b_3

Therapy as usual (**MBRP** (W) =0)
MBRP therapy (**MBRP** (W) = 1)

$$\hat{Y} = 1.038 + 1.122X + 0.587W - 0.948XW$$
$$= 1.038 + (1.122 - 0.948W)X + 0.587W + ...$$



$\theta_{X\to Y} | W = b_1 + b_3 W$
$\theta_{X\to Y} | W = 1.122 - 0.948W$

$b_3 = \theta_{X\to Y} | (W = 1) - \theta_{X\to Y} | (W = 0)$
$= (b_1 + b_3) - (b_1)$
$= (0.174) - (1.122)$
$= -0.948$

So b_3 is the difference in the slopes of these two lines. As W increases by one unit, $\theta_{X\to Y}$ decreases by 0.948 units. This difference is statistically different from zero.

Probing an interaction

The coefficient for the product term carries information about how changes in one variable are related to changes in the effect of the other. A picture helps to understand how the focal variable's effect changes as a function of the moderator variable.

It is typically desirable to conduct statistical tests of the focal predictor variable's effect at values of the moderator. This allows you to make more definitive claims about where the focal predictor variables effect is zero versus where it is not.

"Pick-a-Point" Approach

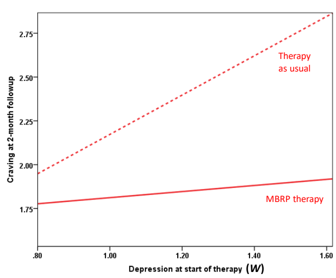
Select values of the moderator and estimate the conditional effect of the focal predictor at those values of the moderator, along with a hypothesis test or confidence interval.

Johnson-Neyman Technique

Derive mathematically where on the moderator variable continuum the focal variable's effect transitions between statistically significant and nonsignificant.

Substantive interpretation of the pattern

Therapy as usual (**MBRP** (X) =0)
MBRP therapy (**MBRP** (X) = 1)



Those who receive MBRP therapy crave substances less than those who receive MBRP therapy, but this difference is larger among those more depressed at the start of therapy.

Pick-a-point approach

$$\hat{Y} = b_0 + b_1 X + b_2 W + b_3 XW$$

Select a value of the moderator (W) at which you'd like to have an estimate of $\theta_{X\to Y}$, the focal predictor variable's (X) effect. Then derive its standard error. The ratio of the effect to its standard error is distributed as $t(df_{residual})$ under the null hypothesis that the effect of the focal predictor is zero at that moderator value, where df_{residual} is the residual degrees of freedom from the regression model.

We already know that $\theta_{X\to Y} = b_1 + b_3 W$

The estimated standard error of $\theta_{X\to Y}$ is

$$s_{\theta_{X\to Y}} = \sqrt{s^2_{b_1} + 2W s_{b_1 b_3} + W^2 s^2_{b_3}}$$

Squared standard error of b_1 — Covariance of b_1 and b_3 — Squared standard error of b_3

You could do this by hand, and instructions are available in various books on regression analysis (e.g., Aiken and West, 1991; Cohen et al., 2003). But there is no reason to, and the potential for mistakes is high. It is made easier using "**regression centering**."

Pick-a-point: Regression centering approach

$$\hat{Y} = b_0 + b_1X + b_2W + b_3XW$$

In the above model, b_1 estimates the conditional effect of X when W = 0. If we desire the conditional effect of X when W equals some value λ, we can produce a new variable W' that is W centered around λ, such that W' = 0 when $W = \lambda$. Then substitute W' for W in the model above. That is, we will estimate

$$\hat{Y} = b_0 + b_1X + b_2(W - \lambda) + b_3X(W - \lambda)$$

as

$$\hat{Y} = b_0 + b_1X + b_2W' + b_3XW' \quad \text{where} \quad W' = W - \lambda$$

In this model, b_1 is the conditional effect of X when W' = 0. But W' = 0 when $W = \lambda$. So b_1 estimates the conditional effect of X when $W = \lambda$. A common (but arbitrary) convention is to use $\lambda = \overline{W}$, $\lambda = \overline{W} - SD_W$, and $\lambda = \overline{W} + SD_W$

Pick-a-point: Regression centering approach

```
compute bdi0_p = bdi0-1.196.
compute interact = bdi0_p*mbrp.
regression/dep = crave2/method = enter mbrp bdi0_p interact treathrs crave0.
```

λ= 1.196
(the sample mean)

```
data mbrp;set mbrp;
bdi0_p=bdi0-1.196;
interact=bdi0_p*mbrp;
proc reg data=mbrp;model crave2=mbrp bdi0_p interact treathrs crave0;run;
```

Coefficients[a]

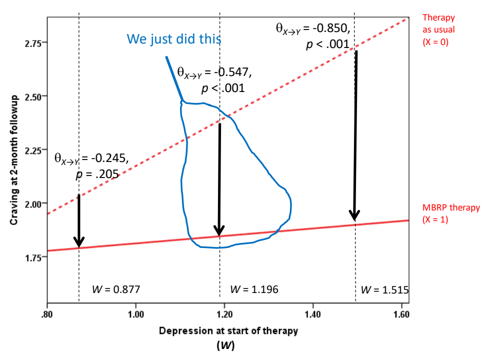
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.380	.364		6.534	.000
	MBRP: Therapy as usual (0) or MBRP therapy (1)	-.547	.137	-.279	-3.980	.000
	bdi0_p	1.122	.276	.366	4.063	.000
	interact	-.948	.423	-.197	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

$\theta_{X\rightarrow Y} \,|(W = 1.196)$ $\quad s_{\theta_{X\rightarrow Y}}$

MBRP therapy reduces craving relative to therapy as usual among people "average" in pre-therapy depression, $\theta_{X\rightarrow Y}$ = -0.547, p < .001.

Repeat for other values of the moderator



Pick-a-point: Regression centering approach

```
compute bdi0_p = bdi0-0.877.
compute interact = bdi0_p*mbrp.
regression/dep = crave2/method = enter mbrp bdi0_p interact treathrs crave0.
```

λ= 0.877
(One SD below the sample mean)

```
data mbrp;set mbrp;
bdi0_p=bdi0-0.877;
interact=bdi0_p*mbrp;
proc reg data=mbrp;model crave2=mbrp bdi0_p interact treathrs crave0;run;
```

Coefficients[a]

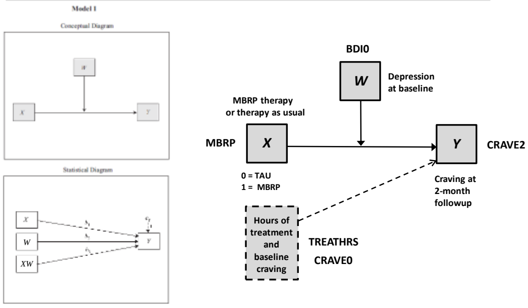
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.023	.367		5.506	.000
	MBRP: Therapy as usual (0) or MBRP therapy (1)	-.245	.192	-.124	-1.272	.205
	bdi0_p	1.122	.276	.366	4.063	.000
	interact	-.948	.423	-.242	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

$\theta_{X\rightarrow Y} \,|(W = 0.877)$ $\quad s_{\theta_{X\rightarrow Y}}$

MBRP therapy does not reduce craving relative to therapy as usual among people "relatively low" in pre-therapy depression, $\theta_{X\rightarrow Y}$ = -0.245, p = .21.

Using PROCESS




```
process cov = treathrs crave0/y=crave2/x=mbrp/w=bdi0
  /model=1/jn=1/plot=1/moments=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=mbrp,
  w=bdi0,model=1,jn=1,plot=1,moments=1);
```

PROCESS output

Output H

```
Model  : 1
    Y  : crave2
    X  : mbrp
    W  : bdi0

Covariates:
 treathrs crave0

Sample
Size:  168

**************************************************************************
OUTCOME VARIABLE:
 crave2
```

$$\hat{Y} = 1.038 + 0.587X + 1.122M - 0.948XM + \ldots$$

PROCESS generates the product term for you.

```
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5140      .2642      .7277    11.6319     5.0000   162.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.0385      .4701     2.2090      .0286      .1102     1.9668
mbrp          .5872      .5241     1.1204      .2642     -.4478     1.6222
bdi0         1.1221      .2762     4.0625      .0001      .5767     1.6675
Int_1        -.9485      .4235    -2.2398      .0265    -1.7847     -.1122
treathrs     -.0177      .0103    -1.7190      .0875     -.0380      .0026
crave0        .1920      .0735     2.6138      .0098      .0470      .3371

Product terms key:
 Int_1    :        mbrp     x        bdi0

Test(s) of highest order unconditional interaction(s):
       R2-chng          F        df1        df2          p
X*W      .0228     5.0166     1.0000   162.0000      .0265
```

PROCESS output

PROCESS sees that the moderator is quantitative (because it has more than 2 values) so it automatically implements the pick-a-point procedure. When `moments = 1` moderator values equal to the mean of the moderator as well as ± one SD from the mean.

```
**************************************************************************

Conditional effect of X on Y at values of the moderator(s):
       bdi0     Effect         se          t          p       LLCI       ULCI
      .8772     -.2447      .1922    -1.2733      .2047     -.6243      .1348
     1.1963     -.5473      .1375    -3.9818      .0001     -.8188     -.2759
     1.5153     -.8500      .1933    -4.3973      .0000    -1.2317     -.4683

Values for quantitative moderators are the mean and plus/minus one SD from mean.
Values for dichotomous moderators are the two values of the moderator.

**************************************************************************
```

$$\theta_{X \rightarrow Y} = 0.587 - 0.948W$$

Output H

MBRP therapy resulted in lower craving than did therapy as usual among those relatively "moderate" ($\theta_{X \rightarrow Y|W=1.196}$ = -0.547, $p < .001$) or "relatively high" ($\theta_{X \rightarrow Y|W=1.515}$ = -0.850, $p < .001$) in pre-therapy depression. Among those "relatively low" in pre-therapy depression, MBRP therapy had no statistically significant effect on craving relative to therapy as usual. ($\theta_{X \rightarrow Y|W=0.877}$ = -0.245, p = .205)

PROCESS output: PLOT option

```
process cov = treathrs crave0/y=crave2/x=mbrp/w=bdi0
  /model=1/jn=1/plot=1/moments=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=mbrp,
  w=bdi0,model=1,jn=1,plot=1,moments=1);
```

Both the SPSS and SAS versions produce a table of estimated values of *Y* for different combinations of *X* and *W*. Plug these into your preferred graphing program to generate a plot, or use SPSS or SAS's graphics features. SPSS writes the code for you. Just cut and paste this into an SPSS syntax file and execute:

```
DATA LIST FREE/mbrp bdi0 crave2.
BEGIN DATA.
      .0000      .8772     2.0456
     1.0000      .8772     1.8009
      .0000     1.1963     2.4037
     1.0000     1.1963     1.8563
      .0000     1.5153     2.7617
     1.0000     1.5153     1.9117
END DATA.
GRAPH/SCATTERPLOT=bdi0 WITH crave2 BY mbrp.
```

Output H

Generating a graph from PROCESS "PLOT" option: SAS

```
data;
input mbrp bdi0 crave2;
datalines;
 .0000      .8772     2.0456
1.0000      .8772     1.8009
 .0000     1.1963     2.4037
1.0000     1.1963     1.8563
 .0000     1.5153     2.7617
1.0000     1.5153     1.9117
run;
proc sgplot;reg x=bdi0 y=crave2/group=mbrp;run;
```

Output generated by the PLOT option.

Generating a graph from PROCESS "PLOT" option: R

```
x<-c(0,1,0,1,0,1)
w<-c(0.877,0.877,1.196,1.196,1.515,1.515)
y<-c(2.046,1.801,2.404,1.856,2.762,1.912)
plot(y=y,x=w,pch=15,col="white",
xlab="Depression at start of therapy",
ylab="Craving at 2-month follow-up")
legend.txt<-c("Therapy as usual (X=0)","Mindfulness therapy (X=1)")
legend("topleft",legend=legend.txt,
lty=c(3,1),lwd=c(3,2))
lines(w[x==0],y[x==0],lwd=3,lty=3)
lines(w[x==1],y[x==1],lwd=2,lty=1)
```

From the PLOT option in PROCESS.

Additional probing options

Setting `moments = 0` or leaving it out, produces estimates of the conditional effect of *X* at the 16th, 50th, and 84th percentiles of the moderator rather than the mean and plus/minus one standard deviation. Or use the **wmodval** option to request a specific value of the moderator at which you'd like the conditional effect of *X.*

```
process y = ... /moments = 0.
```

```
%process (data = ... , moments = 0);
```

```
Conditional effects of the focal predictor at values of the moderator(s):

     bdi0     Effect         se          t          p       LLCI       ULCI
    .9020     -.2683      .1850    -1.4500      .1490     -.6336      .0971
   1.1900     -.5414      .1375    -3.9384      .0001     -.8129     -.2699
   1.5180     -.8525      .1941    -4.3923      .0000    -1.2358     -.4692

W values in conditional tables are the 16th, 50th, and 84th percentiles.
```

```
process y = ... /wmodval = 1.5.
```

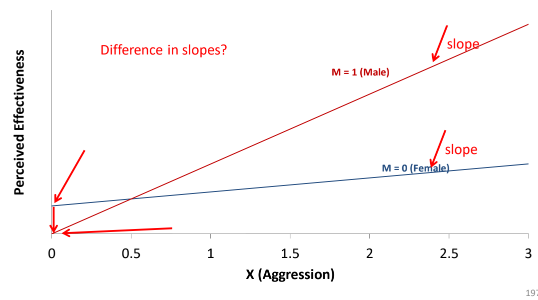
```
%process (data = ... , wmodval = 1.5);
```

```
            Conditional effect of X on Y at values of the moderators(s)
     bdi0   Effect         se          t          p       LLCI       ULCI
   1.5000   -.8354      .1888    -4.4253      .0000    -1.2082     -.4626
```

Practice Interpreting Coefficients

Among upper level managers, does the relationship between aggression and perceived effectiveness depends on gender?

$$Y_i = 1 + (.5 + 2M_i)X_i - 1M_i$$



The Johnson-Neyman technique

The Johnson-Neyman technique seeks to find the value or values of the moderator *(W)* within the data, if they exist, such that the *p*-value for the ratio of the conditional effect of the focal predictor at that value or values of *W* is exactly equal to some chosen level of significance α

To do so, we ask what value of W produces a ratio exactly equal to the critical t value (t_{crit}) required to reject the null hypothesis that the conditional effect of X is equal to zero?

$$t_{crit} = \frac{b_1 + b_3 W}{\sqrt{\{s_{b_1}^2 + 2W s_{b_1 b_3}^2 + W^2 s_{b_3}^2\}}}$$

Isolate W and solve the polynomial that results. The quadratic formula finds the solutions:

$$W = \frac{-2(t_{crit}^2 s_{b_1 b_3} - b_1 b_3) \pm \sqrt{(2t_{crit}^2 s_{b_1 b_3} - 2b_1 b_3)^2 - 4(t_{crit}^2 s_{b_3}^2 - b_3^2)(t_{crit}^2 s_{b_1}^2 - b_1^2)}}{2(t_{crit}^2 s_{b_3}^2 - b_3^2)}$$

The Johnson-Neyman technique

This will produce no values, one value, or two values of *W* that are within the range of the moderator variable data.

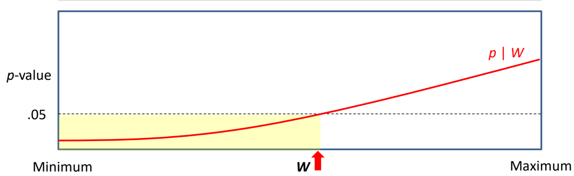
- If one value, this defines a single point of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that $p \leq .05$ for either values of the moderator (1) equal to above W or (2) equal to and below W.

- If two values, this defines the two points of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that the conditional effect is statistically significant for either (1) values of the moderator between the two values of W, or (2) values of the moderator at least as large as the larger W and at least as small as the smaller W.

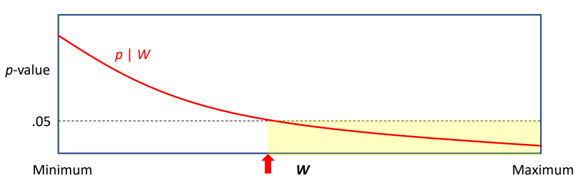
- If no values, that means the conditional effect is statistically significant for ALL values of the moderator within the range of the data, or it NEVER is.

We would not attempt to do this by hand

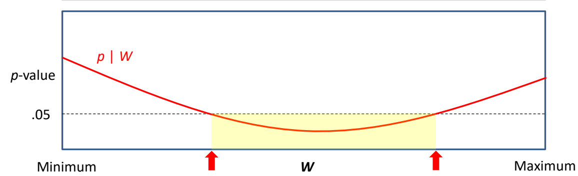
Examples of one solution



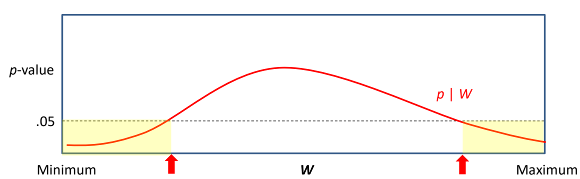
OR



Examples of two solutions

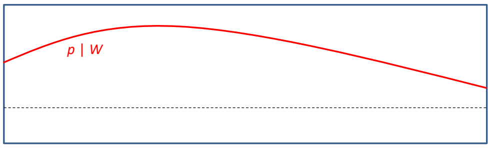


OR



Examples of no solutions

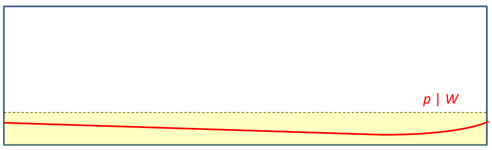
p-value



Minimum *W* Maximum

OR

p-value



Minimum *W* Maximum

Johnson-Neyman output from PROCESS

```
process cov = treathrs crave0/y=crave2/x=mbrp/w=bdi0
  /model=1/jn=1/plot=1/moments=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=mbrp,
   w=bdi0,model=1,jn=1,plot=1,moments=1);
```

Output H

Moderator value(s) defining Johnson-Neyman significance region(s)

Value	% below	% above
.9681	21.4286	78.5714

$\theta_{X\to Y}|W$

W

Conditional effect of X on Y at values of the moderator:

bdi0	Effect	se	t	p	LLCI	ULCI
.0000	.5872	.5241	1.1204	.2642	-.4478	1.6222
.1070	.4858	.4806	1.0108	.3136	-.4632	1.4347
.2140	.3843	.4373	.8787	.3809	-.4793	1.2479
.3210	.2828	.3946	.7167	.4746	-.4964	1.0620
.4280	.1813	.3525	.5144	.6077	-.5147	.8773
.5350	.0798	.3112	.2565	.7979	-.5348	.6944
.6420	-.0217	.2713	-.0798	.9365	-.5574	.5141
.7490	-.1231	.2334	-.5276	.5985	-.5840	.3377
.8560	-.2246	.1986	-1.1312	.2596	-.6167	.1675
.9630	-.3261	.1688	-1.9318	.0551	-.6595	.0072
.9681	-.3309	.1676	-1.9747	.0500	-.6618	.0000
1.0700	-.4276	.1472	-2.9047	.0042	-.7183	-.1369
1.1770	-.5291	.1377	-3.8435	.0002	-.8009	-.2573
1.2840	-.6306	.1426	-4.4220	.0000	-.9122	-.3490
1.3910	-.7321	.1607	-4.5553	.0000	-1.0494	-.4147
1.4980	-.8335	.1882	-4.4288	.0000	-1.2052	-.4619
1.6050	-.9350	.2216	-4.2186	.0000	-1.3727	-.4973
1.7120	-1.0365	.2587	-4.0063	.0001	-1.5474	-.5256
1.8190	-1.1380	.2981	-3.8178	.0002	-1.7266	-.5494
1.9260	-1.2395	.3389	-3.6571	.0003	-1.9088	-.5702
2.0330	-1.3410	.3808	-3.5216	.0006	-2.0929	-.5890
2.1400	-1.4424	.4234	-3.4072	.0008	-2.2784	-.6064

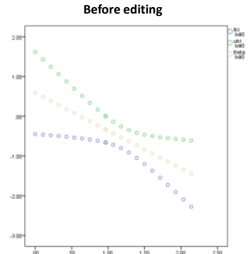
78.6% of the data are up here

A visual representation (SPSS)

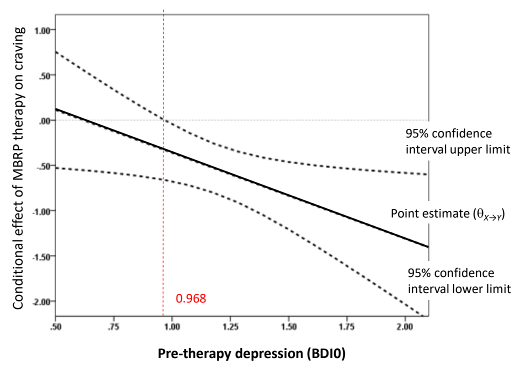
bdi0 effect LLCI ULCI (from PROCESS JN output)

```
data list free/bdi0 theta llci ulci.
begin data.
 .0000      .5872     -.4478    1.6222
 .1070      .4858     -.4632    1.4347
 .2140      .3843     -.4793    1.2479
 .3210      .2828     -.4964    1.0620
 .4280      .1813     -.5147     .8773
 .5350      .0798     -.5348     .6944
 .6420     -.0217     -.5574     .5141
 .7490     -.1231     -.5840     .3377
 .8560     -.2246     -.6167     .1675
 .9630     -.3261     -.6595     .0072
 .9681     -.3309     -.6618     .0000
1.0700     -.4276     -.7183    -.1369
1.1770     -.5291     -.8009    -.2573
1.2840     -.6306     -.9122    -.3490
1.3910     -.7321    -1.0494    -.4147
1.4980     -.8335    -1.2052    -.4619
1.6050     -.9350    -1.3727    -.4973
1.7120    -1.0365    -1.5474    -.5256
1.8190    -1.1380    -1.7266    -.5494
1.9260    -1.2395    -1.9088    -.5702
2.0330    -1.3410    -2.0929    -.5890
2.1400    -1.4424    -2.2784    -.6064
end data.
graph
  /scatterplot(overlay)=bdi0 bdi0 bdi0 WITH llci ulci theta (pair).
```

Before editing

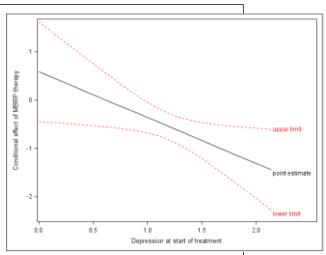


After some editing in SPSS



A visual representation (SAS)

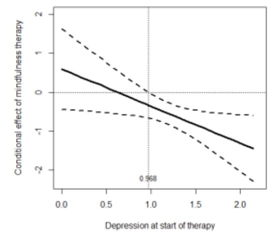
```
data;
input bdi0 effect llci ulci;
datalines;
 .0000      .5872      -.4478     1.6222
 .1070      .4858      -.4632     1.4347
 .2140      .3843      -.4793     1.2479
 .3210      .2828      -.4964     1.0620
 .4280      .1813      -.5147      .8773
 .5350      .0798      -.5348      .6944
 .6420     -.0217      -.5574      .5141
 .7490     -.1231      -.5840      .3377
 .8560     -.2246      -.6167      .1675
 .9630     -.3261      -.6595      .0072
 .9681     -.3309      -.6618      .0000
1.0700     -.4276      -.7183     -.1369
1.1770     -.5291      -.8009     -.2573
1.2840     -.6306      -.9122     -.3490
1.3910     -.7321     -1.0494     -.4147
1.4980     -.8335     -1.2052     -.4619
1.6050     -.9350     -1.3727     -.4973
1.7120    -1.0365     -1.5474     -.5256
1.8190    -1.1380     -1.7266     -.5494
1.9260    -1.2395     -1.9088     -.5702
2.0330    -1.3410     -2.0929     -.5890
2.1400    -1.4424     -2.2784     -.6064
run;
proc sgplot;
  series x=bdi0 y=ulci/curvelabel = "upper limit" lineattrs=(color=red pattern=ShortDash);
  series x=bdi0 y=effect/curvelabel = "point estimate" lineattrs=(color=black pattern=Solid);
  series x=bdi0 y=llci/curvelabel = "lower limit" lineattrs=(color=red pattern=ShortDash);
  yaxis label = "Conditional effect of MBRP therapy";
  xaxis label = "Depression at start of treatment";
run;
```



A visual representation (R)

```
bdi0<-c(0,.107,.214,.321,.438,.535,.642,.749,.856,.963,.968,1.070,1.177,
  1.284,1.391,1.498,1.605,1.712,1.819,1.926,2.033,2.140)
effect<-c(.587,.486,.384,.283,.181,.080,-.022,-.123,-.225,-.326,-.331,-.428,
  -.529,-.631,-.732,-.834,-.935,-1.037,-1.138,-1.240,-1.341,-1.442)
llci<-c(-.448,-.463,-.479,-.496,-.515,-.535,-.557,-.584,-.617,-.660,-.662,
  -.718,-.801,-.912,-1.049,-1.205,-1.373,-1.547,-1.727,-1.909,-2.092,-2.278)
ulci<-c(1.622,1.435,1.248,1.062,.877,.684,.515,.338,.168,.007,0,-.137,
  -.257,-.349,-.415,-.462,-.497,-.526,-.549,-.571,-.589,-.606)
plot(x=bdi0,y=effect,type="l",pch=19,ylim=c(-2.3,2),xlim=c(0,2.2),lwd=3,
ylab="Conditional effect of mindfulness therapy",
xlab="Depression at start of therapy")
points(bdi0,llci,lwd=2,lty=2,type="l")
points(bdi0,ulci,lwd=2,lty=2,type="l")
abline(h=0,untf = FALSE,lty=3,lwd=1)
abline(v=0.968,untf=FALSE,lty=3,lwd=1)
text(0.968,-2.2,"0.968",cex=0.8)
```

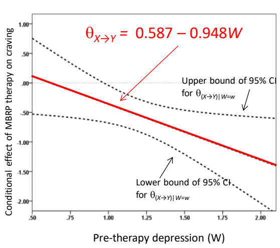
From the JN option in PROCESS.



Testing moderation versus testing a conditional effect

A test of moderation is a test as to whether the size of the effect of *X* on *Y* is related to the proposed moderator. This is different than asking whether a conditional effect is different from zero.

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ldots$$



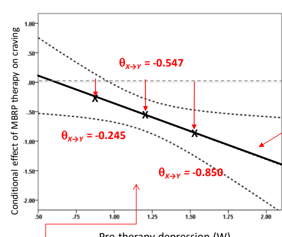
$\theta_{X\to Y} = b_1 + b_3M = 0.587 - 0.948W$

b_3 is the slope of this line. It is statistically different from zero, meaning that the effect of *X* depends on *W*---moderation.

Moderation does **not** imply that the conditional effect of *X* is different from zero at some, any, or all specific values of the moderator that you choose. Often it will be, perhaps for some values of the moderator but not others. But this is not a requirement of moderation.

Testing moderation versus testing a conditional effect

A test of moderation is a test as to whether the size of the effect of *X* on *Y* is related to the proposed moderator. This is different than asking whether a conditional effect is different from zero.



When testing a conditional effect, we are asking whether the effect of *X* on Y at a specific value of *W* is statistically different from zero. This is the difference between the point estimate of $\theta_{X\to Y}$ and zero.

$\theta_{X\to Y} = b_1 + b_3W = 0.587 - 0.948W$

Difference in significance does not imply significantly different. The pattern of significance or lack thereof across values of *M* does not say anything about moderation.

```
Conditional effect of X on Y at values of the moderator(s):
      bdi0     Effect         se          t          p       LLCI       ULCI
     .8772     -.2447      .1922    -1.2733      .2047     -.6243      .1348
    1.1963     -.5473      .1375    -3.9818      .0001     -.8188     -.2759
    1.5153     -.8500      .1933    -4.3973      .0000    -1.2317     -.4683
```

Comparing conditional effects

We want to know whether the conditional effect of X on Y when $W = w_1$ is different from the conditional effect of X on Y when $W = w_2$.

$$\theta_{(X\to Y)|W=w_2} - \theta_{(X\to Y)|W=w_1} = (b_1 + b_3 w_2) - (b_1 + b_3 w_1) = (w_2 - w_1) b_3$$

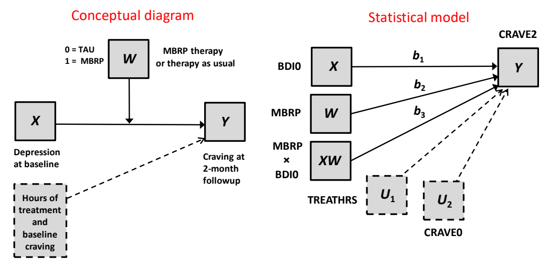
and the standard error of the difference is $(w_2 - w_1) \times$ standard error of b_3. Under the null hypothesis that the difference in conditional effects is zero, the ratio

$$\frac{(w_2 - w_1) b_3}{(w_2 - w_1) se_{b_3}}$$

is distributed as $t(df_{residual})$. But notice that *regardless of the values of* w_1 *and* w_2, this ratio simplifies to b_3 / se_{b_3}. **We already have the *p*-value for this**. It is the p-value for b_3 from the regression model.

A test of linear moderation of *X*'s effect on *Y* by *W* is equivalent to a test of the difference between *any* two conditional effects of *X*. Moderation = any two conditional effects of *X* are different from each other. No moderation = no two conditional effects of *X* are different from each other. It doesn't matter what values of w_1 and w_2 you choose.

A Dichotomous Moderator



Does the association between pre-treatment depression and later craving differ between those who receive MBRP therapy versus therapy as usual?

Probing the Interaction

When the moderator is dichotomous, the pick-a-point procedure is the only option available, as the Johnson-Neyman technique is meaningful only with a quantitative moderator. Typically, you'd want to estimate the effect of the focal predictor at the two values of the moderator and conduct an inferential test for each conditional effect.

$$\hat{Y} = 1.038 + 1.122X + 0.587W - 0.948XW$$
$$= 1.038 + (1.122 - 0.948W)X + 0.587W + \ldots$$
$$\theta_{X\to Y} = b_1 + b_3 W = 1.122 - 0.948W$$

When one of the moderator categories is coded 0, we already have an estimate of $\theta_{X\to Y}$ when $W = 0$. That estimate is b_1. And the regression output provides a test of significance.

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.038	.470		2.209	.029
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
	mbrpdep	-.948	.423	-.598	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

Among those given therapy as usual, those who were relatively more depressed at the start of therapy had relatively higher craving at two months follow-up, $\theta_{X\to Y} = 1.122$, $t(162) = 4.063$, $p < .001$

Probing the Interaction

We already know effect of pre-therapy depression on later craving among those given MBRP therapy. That is $1.122 - 0.948(1) = 0.174$. We can use the regression centering approach, constructing $W' = W - 1$ and reestimating the model in order to get a test of significance.

```
compute mbrp_p = mbrp-1.
compute mbrpdep = mbrp_p*bdi0.
regression/dep = crave2/method = enter mbrp_p bdi0 mbrpdep treathrs crave0.
```

```
data mbrp;set mbrp;mbrp_p=mbrp-1;mbrpdep=mbrp_p*bdi0;
proc reg data=mbrp;model crave2=mbrp_p bdi0 mbrpdep treathrs crave0;run;
```

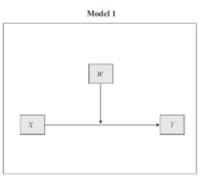
Coefficients[a]

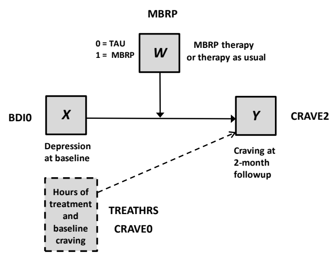
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.626	.535		3.041	.003
	mbrp_p	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	.174	.328	.057	.529	.597
	mbrpdep	-.948	.423	-.639	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

Among those given MBRP therapy, there was no statistically significant relationship between pre-therapy depression and later craving, $\theta_{X\to Y} = 0.174$, $t(162) = 0.529$, $p = .597$

Estimation Using PROCESS






```
process cov = treathrs crave0/y=crave2/x=bdi0/w=mbrp/model=1.
```

```
%process (data=mbrp,cov=treathrs crave0,y=crave2,x=bdi0,w=mbrp,model=1);
```

PROCESS Output

Output I

```
Model = 1
    Y = crave2
    X = bdi0
    M = mbrp

Statistical Controls:
CONTROL= treathrs crave0

Sample size
        168

**************************************************************************
Outcome: crave2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5140      .2642      .7277    11.6319     5.0000   162.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.0385      .4701     2.2090      .0286      .1102     1.9668
mbrp          .5872      .5241     1.1204      .2642     -.4478     1.6222
bdi0         1.1221      .2762     4.0625      .0001      .5767     1.6675
int_1        -.9485      .4235    -2.2398      .0265    -1.7847     -.1122
treathrs     -.0177      .0103    -1.7190      .0875     -.0380      .0026
crave0        .1920      .0735     2.6138      .0098      .0470      .3371

Interactions:

 int_1    bdi0        X     mbrp

R-square increase due to interaction(s):
        R2-chng          F        df1        df2          p
int_1      .0228     5.0166     1.0000   162.0000      .0265

*************************************************************************

Conditional effect of X on Y at values of the moderator(s):
       mbrp     Effect         se          t          p       LLCI       ULCI
      .0000     1.1221      .2762     4.0625      .0001      .5767     1.6675
     1.0000      .1736      .3281      .5291      .5974     -.4744      .8216
```

PROCESS detects that the moderator is dichotomous and generates the conditional effect of the focal predictor at the two values of the moderator.

Myths and truths about mean centering

In a model such as

$$\hat{Y}_i = b_0 + b_1 X_i + b_2 W_i + b_3 X_i W_i$$

there is much ado in the literature about the need to mean center *X* and *W* first, thereby estimating the following model instead:

$$\hat{Y}_i = b'_0 + b'_1(X_i - \bar{X}) + b'_2(W_i - \bar{W}) + b_3(X_i - \bar{X})(W_i - \bar{W})$$

There are three reasons commonly offered for why this should be done.

(1) Estimation accuracy and statistical power is increased by reducing collinearity. **A MYTH!**
(2) The interpretations of b_1 and b_2 are more meaningful. **TYPICALLY TRUE!**
(3) Rounding error is less likely to affect computations. **NOT A CONCERN THESE DAYS**

There is no need to mean center in this fashion, although you may do so if you choose. As standardization is a form of mean centering (combined with a rescaling), all these arguments apply to standardization as well.

Illustration

Without mean centering

```
compute mbrpdep = mbrp*bdi0.
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp;set mbrp;mbrpdep=mbrp*bdi0;run;
proc reg data=mbrp;model crave2=mbrp bdi0 mbrpdep treathrs crave0;run;
```

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + \ \ldots$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.038	.470		2.209	.029
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
	mbrpdep	-.948	.423	-.598	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

b_3 = -0.948
se_{b_3} = 0.423
t = -2.240,
p = 0.026

b_1 = 0.587. This is the effect of *X* (MBRP) when *M* (pre-therapy depression) = 0.

b_2 = 1.122. This is the effect of *W* (Pre-therapy depression) when *X* = 0 (therapy-as-usual condition)

Illustration

With mean centering

```
compute mbrp_c = mbrp-0.554.
compute bdi0_c = bdi0-1.196.
compute mbrpdep_c = mbrp_c*bdi0_c.
regression/dep = crave2/method = enter mbrp_c bdi0_c mbrpdep_c treathrs crave0.
```

```
data mbrp;set mbrp;bdi0_c=bdi0-1.196;mbrp_c=mbrp-0.554;mbrpdep_c=mbrp_c*bdi0_c;run;
proc reg data=mbrp;model crave2=mbrp_c bdi0_c mbrpdep_c treathrs crave0;run;
```

$$\hat{Y} = 2.077 - 0.547X + 0.597W - 0.948XW$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.077	.369		5.635	.000
	mbrp_c	-.547	.137	-.279	-3.980	.000
	bdi0_c	.597	.222	.194	2.685	.008
	mbrpdep_c	-.948	.423	-.158	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

a. Dependent Variable: CRAVE2: Craving at two month follow-up

b_3 = -0.948
se_{b_3} = 0.423
t = -2.240,
p = 0.026

Mean centering has had no effect on b_3.

b_1 = -0.547. This is the effect of *X* (MBRP) among people average in *W* (pre-therapy depression)

b_2 = 0.597. This is the group-weighted average effect of *W* (Pre-therapy depression)

The interaction is unaffected by centering. b_1 and b_2 have changed because the meaning of "0" changes when *X* and *W* are mean centered. This change has nothing to do with multicollinearity being reduced by mean centering.

Collinearity and regression standard errors

The estimated standard error (s_{b_j}) for predictor variable *j* is

$$s_{b_j} = \sqrt{\frac{1}{1-R_j^2}}\sqrt{\frac{MS_{residual}}{n(s_j^2)}} = \sqrt{\frac{MS_{residual}(VIF)}{n(s_j^2)}}$$

where R^2_j is the squared multiple correlation in a model estimating predictor variable *j* from the other predictor variables and s^2_j is the variance of predictor *j*.

- $1 - R^2_j$ is called predictor variable *j*'s **tolerance**. It quantifies the proportion of the variance in variable *j* unexplained by the other predictor variables. Larger is better.
- The inverse of a variable's tolerance is its **variance inflation factor** (VIF). It quantifies how much the sampling variance of predictor *j*'s regression coefficient is affected by the correlation between it and the other predictor variables (larger is worse)

In general, the weaker the correlation between predictor variables, the larger a variable's tolerance, the smaller its variation inflation factor, the smaller the standard error, and the more power the hypothesis test for that predictor. Thus, anything you can do to reduce the correlation between predictors would seem to be a good thing.

Collinearity and regression standard errors

In moderated multiple regression, some therefore argue that you should mean center *X* and *M* prior to computing their product, because this will lower the intercorrelation between *X*, *M*, and their product, reduce the standard error for the regression coefficient for the product, and therefore increase the power of the hypothesis test for interaction.

Correlations between variables in their original metric:

Correlations

		MBRP: Therapy as usual (0) or MBRP therapy (1)	BDI0: Beck Depression Inventory baseline	mbrpdep
MBRP: Therapy as usual (0) or MBRP therapy (1)	Pearson Correlation	1	-.091	.945
	Sig. (2-tailed)		.242	.000
	N	168	168	168
BDI0: Beck Depression Inventory baseline	Pearson Correlation	-.091	1	.123
	Sig. (2-tailed)	.242		.113
	N	168	168	168
mbrpdep	Pearson Correlation	.945	.123	1
	Sig. (2-tailed)	.000	.113	
	N	168	168	168

	Tolerance	VIF
MBRP	0.064	15.674
BDI0	0.561	1.782
mbrpdep	0.064	15.704

Some say you should NEVER include two predictors in a model that are so highly correlated.

Collinearity and regression standard errors

In moderated multiple regression, some therefore argue that you should mean center *X* and *M* prior to computing their product, because this will lower the intercorrelation between *X*, *M*, and their product, reduce the standard error for the regression coefficient for the product, and therefore increase the power of the hypothesis test for interaction.

Correlations between variables after mean centering *X* and *M*:

Correlations

		mbrp_c	bdi0_c	mbrpdep_c
mbrp_c	Pearson Correlation	1	-.091	.020
	Sig. (2-tailed)		.242	.798
	N	168	168	168
bdi0_c	Pearson Correlation	-.091	1	-.287
	Sig. (2-tailed)	.242		.000
	N	168	168	168
mbrpdep_c	Pearson Correlation	.020	-.287	1
	Sig. (2-tailed)	.798	.000	
	N	168	168	168

	Tol.	VIF
mbrp_c	0.928	1.078
bdi0_c	0.867	1.154
mbrpdep_c	0.915	1.093

The offensively large correlation has been reduced to near zero, and all the VIFs are near the minimum possible value of 1. Certainly this is a good thing, right?

But other things have changed too.

$$s_{b_j} = \sqrt{\frac{MS_{residual}(VIF)}{n(s_j^2)}}$$

Descriptive Statistics

	N	Variance
mbrpdep	168	.3816
mbrpdep_c	168	.0266
Valid N (listwise)	168	

Without mean centering

$$s_{b_3} = \sqrt{\frac{MS_{residual}(15.704)}{n(0.3816)}} = \sqrt{\frac{15.704}{0.3816}}\sqrt{\frac{MS_{residual}}{n}} = \sqrt{41.1}\sqrt{\frac{MS_{residual}}{n}}$$

With mean centering

$$s_{b_3} = \sqrt{\frac{MS_{residual}(1.093)}{n(0.0266)}} = \sqrt{\frac{1.093}{0.0266}}\sqrt{\frac{MS_{residual}}{n}} = \sqrt{41.1}\sqrt{\frac{MS_{residual}}{n}}$$

The variance of the product changes by the same factor as the variance inflation factor is changed after mean centering. The result is no change in the standard error of the interaction. (The mean squared residual and sample size are unaffected by mean centering)

To mean center or not to mean center

- The choice is yours to make. It is not required for the purpose of estimation.
- Mean centering does nothing to the test of the interaction. Although mean centering does reduce multicollinearity, this has no consequence on the estimate of the interaction or its statistical test.
- If you mean center, you run no risk of interpreting the coefficients for the predictor and the moderator when they estimate quantities beyond the range of the data. This is a good reason for doing it.
- Mean centering does change the coefficient for the focal predictor and moderator. But this has nothing to do with reducing multi-collinearity. Mean centering changes the conditioning from "0" to the sample mean.

All these arguments apply to standardization as well.

Mean centering in PROCESS

If you wish to mean center, you may do so before using PROCESS. But PROCESS has an option built in which does it for you. Use the **center = 1** option to automatically mean center variables which define product terms.

```
process cov = treathrs crave0
 /y=crave2/x=mbrp /w=bdi0/model=1
 /center=1.
```

```
%process (data=mbrp,cov=treathrs
 crave0,y=crave2, x=mbrp,m=bdi0,
 model=1,center=1);
```

```
**************************************************************************
Outcome: crave2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5140      .2642      .7277    11.6319     5.0000   162.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     2.0778      .3686     5.6363      .0000     1.3498     2.8057
bdi0          .5970      .2221     2.6876      .0079      .1584     1.0357
mbrp         -.5473      .1375    -3.9818      .0001     -.8188     -.2759
int_1        -.9485      .4235    -2.2398      .0265    -1.7847     -.1122
treathrs     -.0177      .0103    -1.7190      .0875     -.0380      .0026
crave0        .1920      .0735     2.6138      .0098      .0470      .3371

Interactions:

 int_1    mbrp        X     bdi0

R-square increase due to interaction(s):
         R2-chng          F        df1        df2          p
int_1      .0228     5.0166     1.0000   162.0000      .0265

*************************************************************************

Conditional effect of X on Y at values of the moderator(s):
       bdi0     Effect         se          t          p       LLCI       ULCI
     -.3191     -.2447      .1922    -1.2733      .2047     -.6243      .1348
      .0000     -.5473      .1375    -3.9818      .0001     -.8188     -.2759
      .3191     -.8500      .1933    -4.3973      .0000    -1.2317     -.4683

Values for quantitative moderators are the mean and plus/minus one SD from mean.
Values for dichotomous moderators are the two values of the moderator.

******************** ANALYSIS NOTES AND WARNINGS ************************

Level of confidence for all confidence intervals in output:
    95.00

NOTE: The following variables were mean centered prior to analysis:
 mbrp     bdi0
```

You get the same results. But interpret in terms of centered metrics rather than uncentered.

Moderation analysis summary

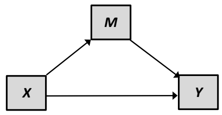
- A moderator of the effect of *X* on *Y* is a variable which influences or otherwise is related to the size of *X*'s effect on *Y*.
- Including a variable defined as the product *X* of *W* to a regression model that includes *X* and *W* allows *X*'s effect on *Y* to be a linear function of *W*.
- The regression coefficient for *XW* in such a model is hard to interpret without a picture. Draw a picture of your model before attempting to interpret.
- We can dissect or "probe" interactions in a few different ways:
  - The pick-a-point approach requires us to select values of *W* at which to estimate the conditional effect of *X* on *Y*. Usually the selection is arbitrary.
  - The Johnson-Neyman technique avoids the need to choose values of the moderator arbitrarily.
- Care must be taken when interpreting the regression coefficients for *X* and *W* in a model that includes *XW*. They are not "main effects" and they may not have any substantive interpretation. Their interpretation will be influenced by their scaling and whether a value of zero is meaningful on the measurement scale. We can make it meaningful by centering.

Combining moderation and mediation
"Conditional Process Analysis"

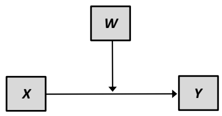
"Conditional process analysis" is a general modeling strategy undertaken with the goal of describing the *conditional* nature of the *mechanism*(*s*) by which a variable transmits its effect on another, and testing hypotheses about such contingent effects.

A merging of two ideas conceptually and analytically:

"**Process analysis**", used to quantify and examine the direct and indirect pathways through which an antecedent variable *X* transmits its effect on a consequent variable *Y* through an intermediary *M*. Better known as "mediation analysis" these days.



"**Moderation analysis**" used to examine how the effect of an antecedent *X* on an consequent *Y* depends on a third moderator variable *W* (a.k.a. "interaction")



History

Idea is not new (e.g., Judd & Kenny, 1981; James & Brett, 1984; Baron and Kenny, 1986). It goes by various names that often confuse, including "moderated mediation" and "mediated moderation."

More recently:

Muller, Judd, and Yzerbyt (2005): Describe analytical models and steps for assessing when "mediation is moderated" and "moderation is mediated."

Edwards and Lambert (2007): Take a path analysis perspective and show how various effects in a simple mediation model can be conditioned on a third variable.

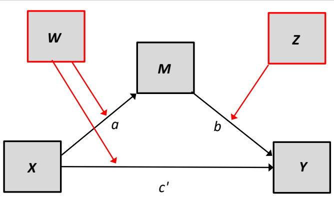
Preacher, Rucker, and Hayes (2007): Provide a formal definition of the *conditional indirect effect* and give formulas, standard errors, and a bootstrap approach for estimating and testing hypotheses about moderated mediation in five different models.

MacKinnon and colleagues (e.g., Fairchild & MacKinnon, 2009): Explicate various analytical approaches to testing hypotheses about mediated moderation and moderated mediation.

Hayes (2013). Introduces the term "conditional process modeling" (also see Hayes and Preacher, 2013) and provides tools for SPSS and SAS to make it easy to do.

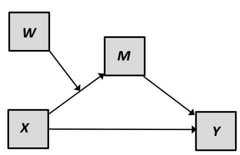
Hayes (2015): Introduces the *index of moderated mediation* which provides a formal test for moderated mediation in a variety of models.

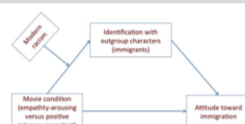
"Moderated mediation"



- The indirect effect of *X* on *Y* through *M* is estimated as the product of the *a* and *b* paths
- But what if the size of *a* or *b* (or both) depends on another variable (i.e., is moderated)?
- If so, then the magnitude of the indirect effect therefore depends on a third variable, meaning that "mediation is moderated".
- When *a* or *b* is moderated, it is sensible then to estimate "conditional indirect effects"— values of indirect effect conditioned on values of the moderator variable that moderates *a* and/or *b*.
- Direct effects can also be conditional. For instance, above, *W* moderates *X*'s direct effect on *Y*.

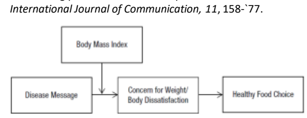
Examples: *X* to *M* path moderated by *W*





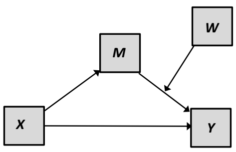
Igartua, J-J., & Frutos, F. J. (2017). Enhancing attitudes toward stigmatized groups with movies. Mediating and moderating processes of narrative persuasion. *International Journal of Communication, 11*, 158-`77.

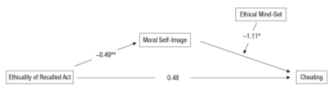
Wout, D. A., Murphy, M. C., & Steele, C. M. (2010). When your friends matter: The effect of White students' racial friendship networks on meta-perceptions and Perceived identity contingencies. ***Journal of Experimental Social Psychology**, 46*, 1035-1041.



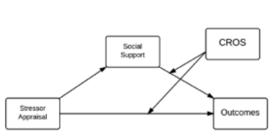
Hoyt, C. L., Burnette, J. L., & Auster-Gussman, L. (2014). "Obesity is a disease": Examining the self-regulatory impact of this public-health message. ***Psychological Science**, 25*, 997-1002.

Examples: *M* to *Y* path moderated by *W*

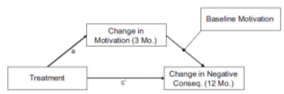




Cornelissen, G., Bashshur, M. R., Rode, J., & Le Menestrel, M. (2013). Rules or consequences? The role of ethical mind-sets in moral dynamics. ***Psychological Science*, *24***, 492-488.

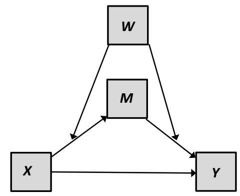


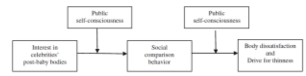
Boren, J. P., & Veksler, A. E. (2015). Communicatively restricted organizational stress (CROS) I: Conceptualization and overview. ***Management Communication Quarterly***, *29*, 28-55.



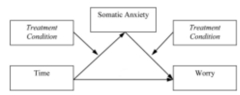
Stein, L. A. R., Minugh, P. A. et al. (2009). Readiness to change as a mediator of the effect of a brief motivational intervention on posttreatment alcohol-related consequences of injured emergency department hazardous drinkers. ***Psychology of Addictive Behaviors*, *23***, 185-195.

Examples: *X* to *M* and *M* to *Y* path moderated by *W*

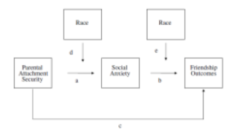




Chae, J. (2014). Interest in celebrities' post-baby bodies and Korean women's body image disturbance after childhood. ***Sex Roles***, *71*, 419-435.



Donegan, E., & Dugas, M. (2012). Generalized anxiety disorder: A comparison of symptom change in adults receiving cognitive-behavioral therapy or applied relaxation. ***Journal of Consulting and Clinical Psychology***, *80*, 490-496.



Parade, S. H., Leerkes, E. M., & Blankson, A. (2010). Attachment to parents, social anxiety, and close relationships of female students over the transition to college**.** ***Journal of Youth and Adolescence***, *39*, 127-137.

"Conditional direct effect"

In a mediation model, the direct effect of *X* on *Y* quantifies *X*'s effect independent of the intervening variable or variables. If that direct effect is moderated, then the direct effect is conditional on the variable that moderates *X*'s effect. For example,

$$\hat{M} = a_0 + aX$$

$$\hat{Y} = c_0' + c_1'X + c_2'W + c_3'XW + bM$$

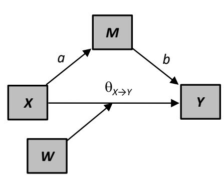
or, equivalently,

$$\hat{Y} = c_0' + (c_1' + c_3'W)X + c_2'W + bM$$

or, equivalently,

$$\hat{Y} = c_0' + \theta_{X\rightarrow Y}X + c_2'W + bM$$

where $\theta_{X\rightarrow Y} = (c_1' + c_3'W)$



In this model, $\theta_{X\rightarrow Y}$ is the conditional direct effect of *X*, which is defined by the function $c'_1 + c'_3W$. Holding *M* constant, two cases that differ by one unit on *X* are estimated to differ by $c'_1 + c'_3W$ units on *Y*.

This is a very basic conditional process model. It models two pathways through which *X* affects *Y*. One is unconditional and indirect via *M*, and the other is direct but conditional—the size of the direct effect depends on *W*.

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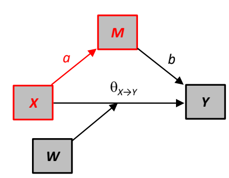

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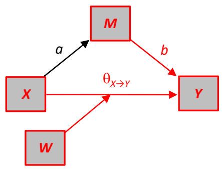

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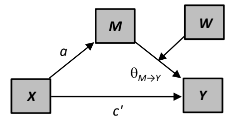
where $\theta_{M\rightarrow Y} = (b_1 + b_3W)$



The indirect effect of *X* on *Y* via *M* is $a\theta_{M\rightarrow Y}$, but as $\theta_{M\rightarrow Y}$ is a conditional effect (the conditional effect of *M*), then $a\theta_{M\rightarrow Y}$ is the *conditional indirect effect* of *X* on *Y* via *M*: $a\theta_{M\rightarrow Y} = a(b_1 + b_3W) = ab_1 + ab_3W$. It depends on *W*.

This is also a basic conditional process model, and potentially more interesting one. It allows for the process or ‘mechanism’ linking *X* to *Y* via *M* to differ systematically as a function of *W*. This model allows “mediation to be moderated.”

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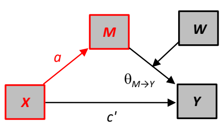

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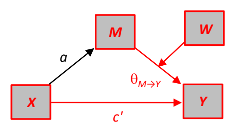

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Condition process analysis: Example 1

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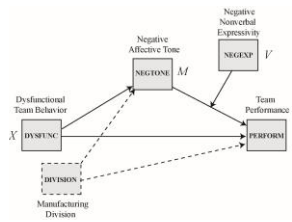
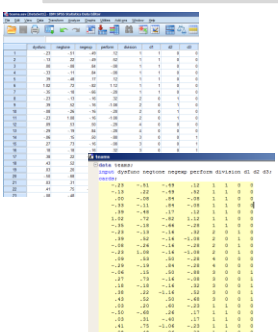



Affective Mechanisms Linking Dysfunctional Behavior to Performance in Work Teams: A Moderated Mediation Study

Michael S. Cole
Frank Walter
Heike Bruch

This is a model of **negative affective tone (*M*)** as the mechanism by which **dysfunctional team behavior (*X*)** influences **performance *(Y)***, with that mechanism being contingent on the extent to which **team members hide their negative feelings (*V*)** from the team. This "nonverbal expressivity" is postulated as moderating the effect of negative tone on performance. This is a "second stage" moderated mediation model.

The Data: TEAMS



60 teams working in an automobile parts manufacturing facility.

DYSFUNC: **Dysfunctional team behavior**, i.e., How often members of the team do things to weaken the work of others or hinder change and innovation.

NEGTONE: **Negative affective group tone**. How often team members report feeling negative emotions at work such as "angry", "disgust", etc.

NEGEXP: **Negative nonverbal expressivity.** Supervisor's perception as to how easy it is to tell how team members are feeling.

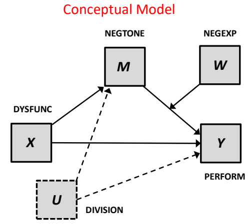
PERFORM: **Team performance.** Supervisor's judgment as to the team's efficiency, ability to get task done in a timely fashion, etc.

All variables are scaled arbitrarily, but higher = "more"

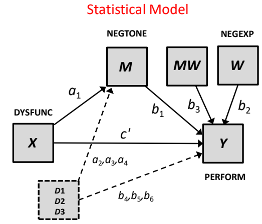
Also available is which of four parts divisions the team worked in, as a single categorical variable (division) as well as three dummy variables (d1, d2, d3).

Conceptual and statistical models

Conceptual Model



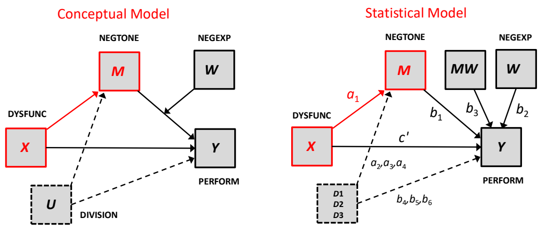
Statistical Model



$$\hat{M} = a_0 + a_1 X + a_2 D_1 + a_3 D_2 + a_4 D_3$$
$$\hat{Y} = c'_0 + c' X + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3$$

The number of equations needed is equal to the number of variables with arrows pointing **at** them in the conceptual or statistical diagram.

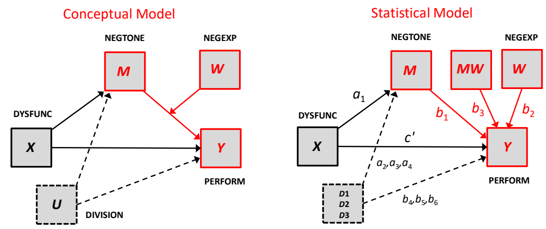
Conceptual and statistical models



$$\hat{M} = a_0 + \boxed{a_1 X} + a_2 D_1 + a_3 D_2 + a_4 D_3$$
$$\hat{Y} = c'_0 + c'X + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3$$

The effect of dysfunctional team behavior (*X*) on negative affective tone of the work environment (*M*).

Conceptual and statistical models



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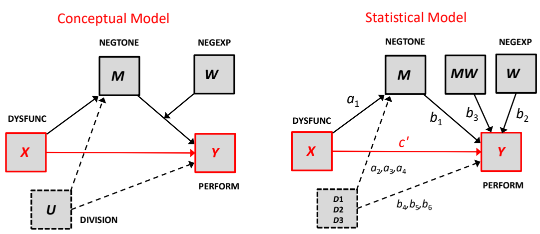
$$\hat{Y} = c'_0 + c'X + \boxed{b_1 M + b_2 W + b_3 MW} + b_4 D_1 + b_5 D_2 + b_6 D_3$$

The moderation of the effect of the negative affective tone of the work environment (*M*) on team performance (*Y*) by negative nonverbal expressivity (*W*).

Conceptual and statistical models



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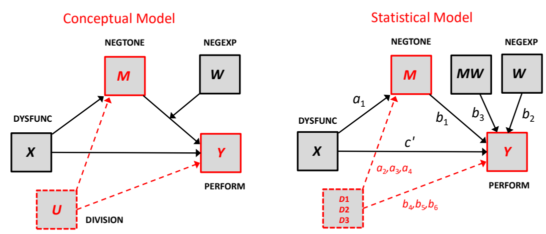
$$\hat{Y} = c'_0 + \boxed{c'X} + b_1 M + b_2 W + b_3 MW + b_4 D_1 + b_5 D_2 + b_6 D_3$$

The direct effect of dysfunctional team behavior (*X*) on team performance (*Y*).

Conceptual and statistical models



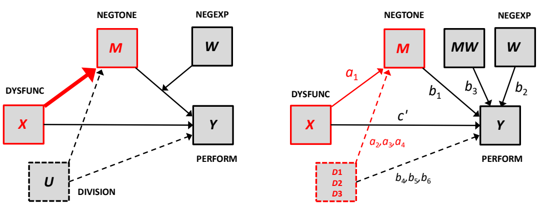
$$\hat{M} = a_0 + a_1 X + \boxed{a_2 D_1 + a_3 D_2 + a_4 D_3}$$
$$\hat{Y} = c'_0 + c'X + b_1 M + b_2 W + b_3 MW + \boxed{b_4 D_1 + b_5 D_2 + b_6 D_3}$$

Covariates to account for potential confounding by divisional differences (*U*) in negative tone of the work environment (*M*) and performance (*Y*)

Estimating the a_1 path

Let's first estimate the effect of dysfunctional team behavior on the negative affective tone of the team environment: Path a_1 in the statistical model.

Conceptual Model　　　　　Statistical Model



$$\hat{M} = a_0 + a_1X + a_2D_1 + a_3D_2 + a_4D_3$$

Emphasis in not on statistical significance, as neither the direct or indirect effects of X are defined entirely in terms of a_1.

Estimating the a_1 path

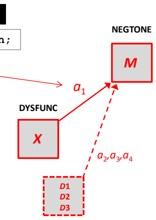
```
regression/dep=negtone/method=enter dysfunc d1 d2 d3.
```

```
proc reg data=teams;model negtone=dysfunc d1 d2 d3;run;
```

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-.206	.130		-1.576	.121
	Dysfunctional team behavior	.609	.167	.431	3.655	.001
	d1	.349	.171	.307	2.033	.047
	d2	.295	.212	.193	1.391	.170
	d3	.251	.166	.230	1.508	.137

a. Dependent Variable: Negative affective tone



$a_1 = 0.609$

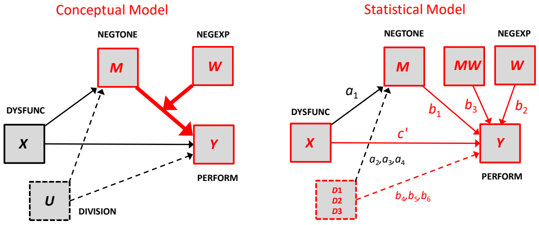
$$\hat{M} = -0.206 + 0.609X + 0.349D_1 + 0.295D_2 + 0.251D_3$$

Teams whose members exhibit relatively more dysfunctional behavior tend to operate in a work environment characterized by relatively more negative affective tone (i.e., members report more negative affect)

Estimating the moderation component of the model

The conceptual model proposes that the effect of negative work tone on performance depends on negative nonverbal expressivity. Let's see whether there is evidence of this.

Conceptual Model　　　　　Statistical Model



$$\hat{Y} = c'_0 + c'X + \boxed{b_1M + b_2W + b_3MW} + b_4D_1 + b_5D_2 + b_6D_3$$

We most care about the moderation components of the model of Y: b_1, b_2, and b_3. But these must be estimated in the context of the complete model of Y, which includes X as well.

Estimating the moderation component of the model

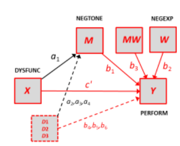
```
compute toneexp=negtone*negexp.
regression/dep=perform/method=enter dysfunc negtone negexp toneexp d1 d2 d3.
```

```
data teams;set teams;toneexp=negtone*negexp;run;
proc reg data=teams;model perform=dysfunc negtone negexp toneexp d1 d2 d3;run;
```

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-.175	.130		-1.344	.185
	Dysfunctional team behavior	.373	.181	.265	2.062	.044
	Negative affective tone	-.489	.138	-.491	-3.549	.001
	Negative expressivity	-.022	.118	-.023	-.188	.852
	toneexp	-.450	.245	-.240	-1.835	.072
	d1	.182	.172	.161	1.056	.296
	d2	.084	.210	.055	.400	.690
	d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance



b_1 = -0.489, b_2=-0.022, b_3 = -0.450

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$

"Marginally significant" evidence that the effect of negative tone of the work environment on team performance depends on the negative nonverbal expressivity of team members. To better understand this, dissect this model.

Estimating the moderation component of the model

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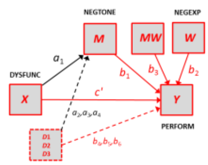
	d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance



b_1 = 0.489, b_3 = -0.450

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$

which can be written as

$$\hat{Y} = -0.175 + 0.373X + (-0.489 - 0.450W)M - 0.022W + 0.182D_1 + 0.084D_2 + 0.282D_3$$

or

$$\hat{Y} = -0.175 + 0.373X + \theta_{M \to Y}M - 0.022W + 0.182D_1 + 0.084D_2 + 0.282D_3$$

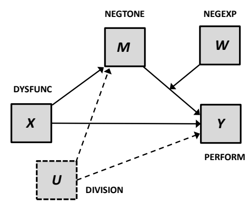
where

$$\theta_{M \to Y} = -0.489 - 0.450W = b_1 + b_3W$$

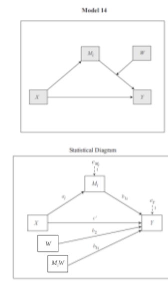
Let's visualize and probe this. PROCESS will take the work out of it.

Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.



This is PROCESS model 14




```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/w=negexp/boot=10000/model=14/plot
= 1.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform,w=negexp,
         boot=10000,model=14, plot = 1);
```

PROCESS output

Output J

```
OUTCOME VARIABLE:
 perform

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5937      .3524      .2006     4.0428     7.0000    52.0000      .0013

Model
              coeff         se          t          p       LLCI       ULCI
constant     -.1754      .1305    -1.3444      .1847     -.4373      .0864
dysfunc       .3729      .1808     2.0622      .0442      .0100      .7357
negtone      -.4886      .1377    -3.5485      .0008     -.7649     -.2123
negexp       -.0221      .1176     -.1875      .8520     -.2581      .2140
Int_1        -.4498      .2451    -1.8353      .0722     -.9417      .0420
d1            .1815      .1720     1.0556      .2960     -.1635      .5266
d2            .0841      .2099      .4004      .6905     -.3372      .5053
d3            .2816      .1648     1.7087      .0935     -.0491      .6123

Product terms key:
 Int_1    :        negtone  x        negexp

Test(s) of highest order unconditional interaction(s):
       R2-chng          F        df1        df2          p
M*W      .0419     3.3684     1.0000    52.0000      .0722
```

b_2 = -0.022
b_1 = -0.489
b_3 = -0.450

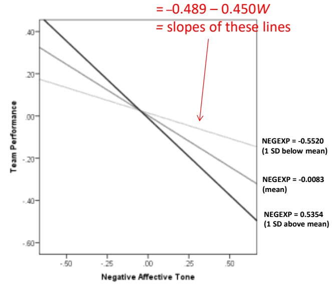
Visualizing the interaction

Use of the PLOT option in SPSS (**`plot=1`**) produces a table of estimated values of the outcome for various combinations of the focal predictor and moderator, and an SPSS program to generate a skeleton of the plot that can be edited.

M *W* $\hat{Y}$

```
DATA LIST FREE/negtone negexp perform.
BEGIN DATA.
     -.4782     -.5520      .1288
      .0472     -.5520      .0026
      .5726     -.5520     -.1237
     -.4782     -.0083      .2338
      .0472     -.0083     -.0210
      .5726     -.0083     -.2757
     -.4782      .5354      .3387
      .0472      .5354     -.0445
      .5726      .5354     -.4278
END DATA.
GRAPH/SCATTERPLOT=negtone WITH perform
   BY negexp.
```

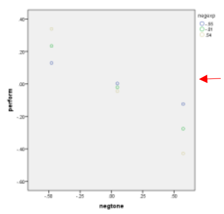
$\theta_{M \to Y} = b_1 + b_3W$ =
= -0.489 – 0.450*W*
= slopes of these lines



Visualizing the interaction

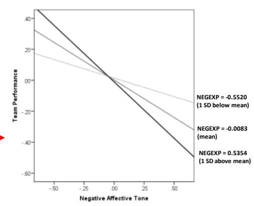
```
DATA LIST FREE/negtone negexp perform.
BEGIN DATA.
     -.4782     -.5520      .1288
      .0472     -.5520      .0026
      .5726     -.5520     -.1237
     -.4782     -.0083      .2338
      .0472     -.0083     -.0210
      .5726     -.0083     -.2757
     -.4782      .5354      .3387
      .0472      .5354     -.0445
      .5726      .5354     -.4278
END DATA.
GRAPH/SCATTERPLOT=negtone WITH perform BY negexp.
```

Use of the PLOT option (**`plot=1`**) produces a table of estimated values of the outcome for various combinations of the focal predictor and moderator.



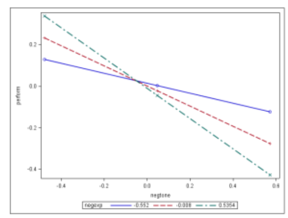
Before editing

After editing



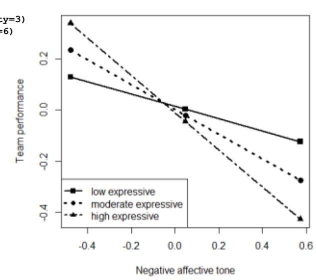
Example code in SAS

```
data;
input negtone negexp perform;
cards;
-.4782     -.5520      .1288
 .0472     -.5520      .0026
 .5726     -.5520     -.1237
-.4782     -.0083      .2338
 .0472     -.0083     -.0210
 .5726     -.0083     -.2757
-.4782      .5354      .3387
 .0472      .5354     -.0445
 .5726      .5354     -.4278
run;
proc sgplot;reg x=negtone y=perform/group=negexp;run;
```



Example code in R

```
m<-c(-.478,.047,.573,-.478,.047,.573,-.478,.047,.573)
w<-c(-.552,-.552,-.552,-.008,-.008,-.008,.535,.535,.535)
y<-c(.129,.003,-.124,.234,-.021,-.276,.339,-.045,-.428)
wmarker<-c(15,15,15,16,16,16,17,17,17)
plot(y=y,x=m,cex=1.2,pch=wmarker,xlab="Negative affective tone",
ylab="Team performance")
legend.txt<-c("low expressive","moderate expressive","high expressive")
legend("bottomleft", legend = legend.txt,cex=1,lty=c(1,3,6),lwd=c(2,3,2),pch=c(15,16,17))
lines(m[w==-.552],y[w==-.552],lwd=2)
lines(m[w==-.008],y[w==-.008],lwd=3,lty=3)
lines(m[w==.535],y[w==.535],lwd=2,lty=6)
```



Probing the interaction: Pick-a-point

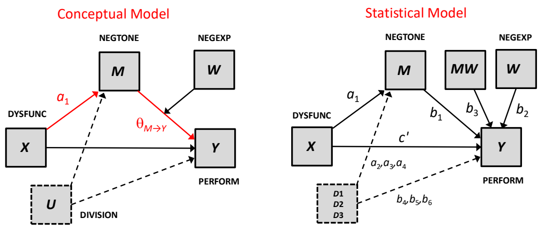
PROCESS sees that the moderator is quantitative (i.e., it has more than 2 values) so it implements the pick-a-point procedure with moderator values equal to 14th, 50th, and 84th percentile.

$$\theta_{M \to Y} = b_1 + b_3 W = -0.489 - 0.450W$$

```
*************************************************************************
 Focal predict: negtone  (M)
       Mod var: negexp   (W)

Conditional effects of the focal predictor at values of the moderator(s):

     negexp     Effect         se          t          p       LLCI       ULCI
     -.5308     -.2498      .2196    -1.1379      .2604     -.6904      .1907
     -.0600     -.4616      .1434    -3.2188      .0022     -.7494     -.1738
      .6000     -.7585      .1633    -4.6451      .0000    -1.0862     -.4308
```

Negative affective tone is significantly negatively related to performance among teams relatively “moderate” and “relatively high” in negative nonverbal expressivity but not among teams “relatively low” in negative nonverbal expressivity.

The conditional indirect effect of *X*

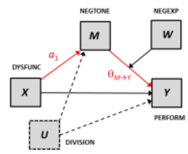


The conditional indirect effect of *X* on *Y* through *M* is the product of the effect of X on *M* (a_1) and the conditional effect of *M* on *Y* given *W* ($\theta_{M\to Y} = b_1 + b_3W$):

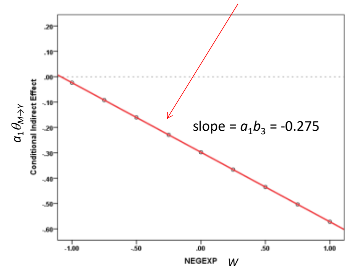
$$a_1\theta_{M\to Y} = a_1(b_1 + b_3W) = 0.610(-0.489 - 0.450W)$$

The indirect effect of dysfunctional team behavior on team performance through negative tone is allowed to be a function of negative nonverbal expressivity.

A visual representation of the indirect effect



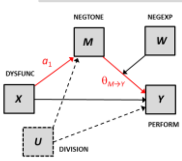
$$a_1\theta_{M\to Y} = a_1(b_1 + b_3W) = 0.610(-0.489 - 0.450W)$$
$$= a_1b_1 + a_1b_3W = -0.298 - 0.275W$$



The indirect effect is more negative as negative nonverbal expressivity increases. The "**index of moderated mediation**" is a_1b_3 = -0.275. It quantifies the relationship between the moderator and the indirect effect in this model.

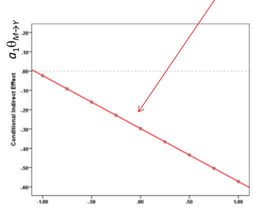
As will be seen, a hypothesis test that the slope of this function is equal to 0 is a formal test of moderated mediation….moderation of the indirect effect.

The conditional indirect effect of *X*



$$a_1\theta_{M\to Y} = a_1(b_1 + b_3W) = 0.610(-0.489 - 0.450W)$$
$$= a_1b_1 + a_1b_3W = -0.298 - 0.275W$$

The indirect effect is more negative as negative nonverbal expressivity increases.



Using this function, we can estimate the indirect effect for any value of the moderator we choose:

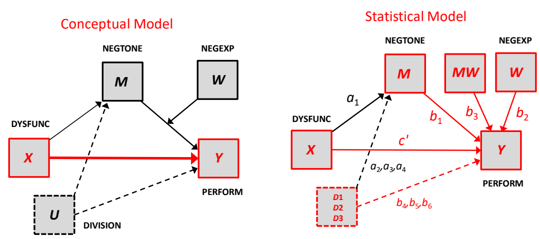
If we had used `moments = 1`

	NEGEXP (*W*)	a_1	$\theta_{M\to Y}$	$a_1\theta_{M\to Y}$
$\bar{W} - SD_W$	-0.552	0.610	-0.240	-0.146
$\bar{W}$	-0.008	0.610	-0.485	-0.296
$\bar{W} + SD_W$	0.535	0.610	-0.730	-0.445

Conditional indirect effects

The direct effect of *X*

The direct effect of *X* is the effect of *X* of *Y* that does not operate through *M*.



$$\hat{Y} = i_1 \boxed{+ c'X} + b_1M + b_2W + b_3MW + b_4D_1 + b_5D_2 + b_6D_3$$

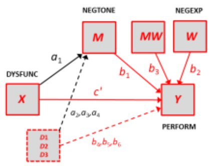
In this model, the direct effect is fixed to be unmoderated. It is a constant rather than a function of another variable in the model.

The direct effect of *X* (we estimated earlier)

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-.175	.130		-1.344	.185
	Dysfunctional team behavior	.373	.181	.265	2.062	.044
	Negative affective tone	-.489	.138	-.491	-3.549	.001
	Negative expressivity	-.022	.118	-.023	-.188	.852
	toneexp	-.450	.245	-.240	-1.835	.072
	d1	.182	.172	.161	1.056	.296
	d2	.084	.210	.055	.400	.690
	d3	.282	.165	.259	1.709	.093

a. Dependent Variable: Team performance



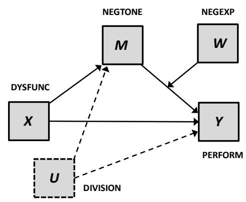
$c' = 0.373$, $t(55) = 2.062$, $p < .05$.

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + 0.182D_1 + 0.084D_2 + 0.282D_3$$

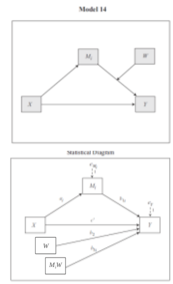
Holding constant negative affective tone and negative nonverbal expressivity, teams that exhibit more dysfunctional behavior perform *better*.

Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.



This is PROCESS model 14

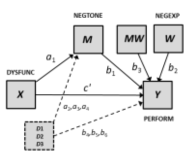



```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/w=negexp/boot=10000
        /model=14/plot = 1.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform,w=negexp,
        boot=10000,model=14, plot = 1);
```

PROCESS output

```
Model = 14
    Y = perform
    X = dysfunc
    M = negtone
    W = negexp

Statistical Controls:
CONTROL= d1       d2       d3

Sample size
        60

**************************************************************************
Outcome: negtone

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5026      .2526      .2213     4.6462     4.0000    55.0000      .0027

Model
              coeff         se          t          p       LLCI       ULCI
constant     -.2057      .1305    -1.5760      .1208     -.4672      .0559
dysfunc       .6095      .1668     3.6546      .0006      .2753      .9437
d1            .3487      .1715     2.0332      .0469      .0050      .6923
d2            .2951      .2122     1.3906      .1700     -.1302      .7204
d3            .2507      .1663     1.5078      .1373     -.0825      .5840

**************************************************************************
```



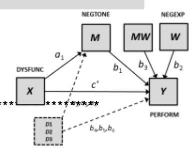
$$\hat{M} = -0.206 + 0.610X + \ldots$$

$a_1 = 0.610$

Output J

PROCESS output

$$\hat{Y} = -0.175 + 0.373X - 0.489M - 0.022W - 0.450MW + \ldots$$




```
**************************************************************************

Outcome: perform

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5937      .3524      .2006     4.0428     7.0000    52.0000      .0013

Model
              coeff         se          t          p       LLCI       ULCI
constant     -.1754      .1305    -1.3444      .1847     -.4373      .0864
negtone      -.4886      .1377    -3.5485      .0008     -.7649     -.2123
dysfunc       .3729      .1808     2.0622      .0442      .0100      .7357
negexp       -.0221      .1176     -.1875      .8520     -.2581      .2140
int_1        -.4498      .2451    -1.8353      .0722     -.9417      .0420
d1            .1815      .1720     1.0556      .2960     -.1635      .5266
d2            .0841      .2099      .4004      .6905     -.3372      .5053
d3            .2816      .1648     1.7087      .0935     -.0491      .6123

Interactions:

 int_1    negtone      X      negexp

**************************************************************************
```

$b_1 = -0.489$
$c' = 0.373$
$b_2 = -0.022$
$b_3 = -0.450$

Output J

PROCESS output

```
******************** DIRECT AND INDIRECT EFFECTS *************************

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .3729      .1808     2.0622      .0442      .0100      .7357

INDIRECT EFFECT:
 dysfunc     ->    negtone     ->    perform

     negexp     Effect    BootSE   BootLLCI   BootULCI
     -.5308     -.1523     .1540     -.4335      .1943
     -.0600     -.2813     .1249     -.5432     -.0549
      .6000     -.4623     .1678     -.8095     -.1503

      Index of moderated mediation:
            Index    BootSE   BootLLCI   BootULCI
negexp     -.2742     .1791     -.7172     -.0234


W values in conditional tables are the 16th, 50th, and 84th percentiles.
```

Direct effect
$c' = .373, p < .05$

Output J

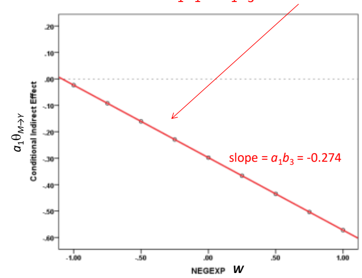
$a_1\theta_{M\to Y} = a_1(b_1 + b_3W)$
$= 0.610(-0.489 - 0.450W)$
$= a_1b_1 + a_1b_3W$
$= -0.298 - 0.274W$

PROCESS sees that the moderator is continuous so, without instruction otherwise, prints the conditional indirect effect at the 16th, 50th, and 84th percentiles. For the mean +/- 1 SD add `moments = 1` to the command line.

A statistical test of moderated mediation

in the second stage moderated mediation model

$a_1\theta_{M\to Y} = a_1(b_1 + b_3W) = 0.610(-0.489 - 0.450W)$
$= a_1b_1 + a_1b_3W = -0.298 - 0.274W$



The indirect effect is a function of W (negative nonverbal expressivity) in our model. This function is a line.

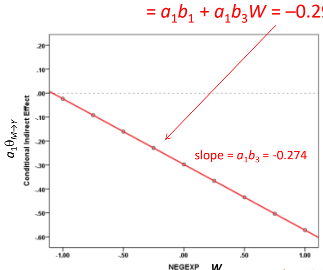
$a_1\theta_{M\to Y} = a_1(b_1 + b_3W)$
$= a_1b_1 + a_1b_3W$
$= -0.298 - 0.274W$

An inference about the slope of this line—the "index of moderated mediation"—is an inference about whether the indirect effect is moderated: Is this slope significantly different from zero? If so, that supports a claim of "moderated mediation."

As a_1b_3 is a product of regression coefficients, an inferential test should respect the nonnormality of the sampling distribution of the product. A bootstrap confidence interval is a good choice.

This test is provided automatically by PROCESS

$a_1\theta_{M\to Y} = a_1(b_1 + b_3W) = 0.610(-0.489 - 0.450W)$
$= a_1b_1 + a_1b_3W = -0.298 - 0.274W$



The indirect effect is a function of V (negative nonverbal expressivity) in our model. This function is a line.

$a\theta_{M\to Y} = a_1(b_1 + b_3W)$
$= a_1b_1 + a_1b_3W$
$= -0.298 - 0.274W$

Output J

a_1b_3 = -0.274, 95% bootstrap CI = -0.683 to -0.024

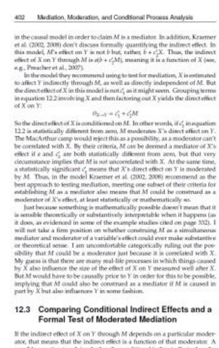
```
******************** INDEX OF MODERATED MEDIATION ************************

             Index    SE(Boot)   BootLLCI   BootULCI
Negexp      -.2742      .1727     -.6833     -.0243
```

This slope is statistically different from zero. The indirect effect depends on negative nonverbal expressivity…. The mediation is moderated.

Where is this test discussed?


An Index and Test of Linear Moderated Mediation
Andrew F. Hayes

Provided with your materials


402 Mediation, Moderation, and Conditional Process Analysis
12.3 Comparing Conditional Indirect Effects and a Formal Test of Moderated Mediation

Chapter 12 of IMMCPA

PROCESS output

We need an inferential test for these conditional indirect effects. Bootstrap confidence intervals are perfect for the job. PROCESS does this for us automatically.

```
******************* DIRECT AND INDIRECT EFFECTS ************************

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .3729      .1808     2.0622      .0442      .0100      .7357

INDIRECT EFFECT:
 dysfunc     ->    negtone     ->    perform

     negexp     Effect     BootSE   BootLLCI   BootULCI
     -.5308     -.1523      .1540     -.4335      .1943
     -.0600     -.2813      .1249     -.5432     -.0549
      .6000     -.4623      .1678     -.8095     -.1503

      Index of moderated mediation:
            Index     BootSE   BootLLCI   BootULCI
negexp     -.2742      .1791     -.7172     -.0234
```

Conditional indirect effects with 95% bias-corrected bootstrap CIs based on 10,000 bootstrap samples.

The indirect effect of dysfunctional behavior on performance through negative tone is negative among teams relatively moderate (point estimate: -0.28, 95% CI from -0.54 to -0.05) and relatively high (point estimate: -0.46, 95% CI from -0.81 to -0.14) in negative nonverbal expressivity but not different from zero among those low in negative nonverbal expressivity (point estimate: -0.15, 95% CI from -0.43 to 0.19).

Comparing conditional indirect effects (2nd stage model)

A seemingly sensible question to ask is whether the conditional indirect effect of *X* when the moderator equals some value $W = w_1$ is different than the conditional indirect effect of *X* when the moderator is some different value $W = w_2$. For example, is the indirect effect among teams low in negative nonverbal expressivity different from the indirect effect among teams high in negative nonverbal expressivity?

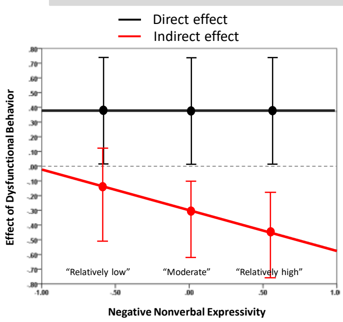
Rejection of the null hypothesis of no moderated mediation based on the index of moderated mediation implies that **any** two conditional indirect effects are different! No additional test is needed.

For example, for the second stage moderated mediation model just estimated:

$$\begin{aligned} a_1(b_1+b_3w_1) - a_1(b_1+b_3w_2) &= a_1(b_1+b_3w_1) - a_1(b_1+b_3w_2) \\ &= a_1b_1+a_1b_3w_1 - a_1b_1 - a_1b_3w_2 \\ &= a_1b_3w_1 - a_1b_3w_2 \\ &= a_1b_3(w_1 - w_2) \end{aligned}$$

If a bootstrap confidence interval for a_1b_3 does not contain zero, then neither will a confidence interval for $a_1b_3(w_1 - w_2)$, ***regardless*** of values of w_1 and w_2 chosen, so long as $w_1 \neq w_2$. And if a bootstrap confidence interval for a_1b_3 contains zero, then so too will a confidence interval for $a_1b_3(w_1 - w_2)$, ***for any two values*** of w_1 and w_2, ($w_1 \neq w_2$).

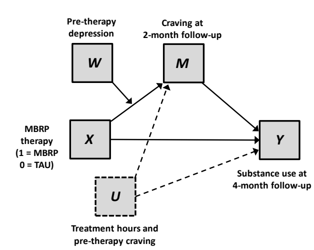
Putting it all together



Point estimates with 95% confidence intervals (OLS confidence intervals for direct effects, bootstrap confidence intervals for indirect effects).

More dysfunctional behavior tends to lead to more negative affective tone, yet this negative affective tone seems to lower performance only among teams that are more demonstrative of their negative feelings. Such a process does not operate among teams that hide their feelings. Independent of differences between teams in the negative affective tone of the work environment, teams that exhibit more dysfunctional behavior otherwise perform *better*.

Conditional Process Modeling Example #2

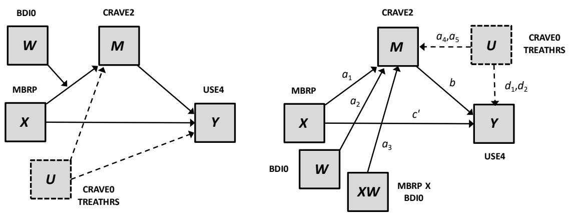


This is a model of **craving (*M*)** as the mechanism by which **mindfulness relapse prevention therapy (*X*)** affects **substance use (*Y*)** relative to therapy as usual. In this model, moderation of the mechanism is proposed as operating in the "first stage" of the mediation process via the moderation of the effect of mindfulness relapse prevention therapy on craving by **pre-therapy depression level (*W*)**.

Conceptual and Statistical Models

Conceptual model

Statistical model



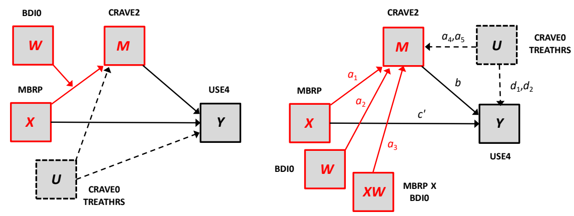
$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$
$$\hat{Y} = c_0' + c'X + bM + d_1U_1 + d_2U_2$$

The number of equations needed is equal to the number of variables with arrows pointing at them in the conceptual or statistical diagram.

Conceptual and Statistical Models

Conceptual model

Statistical model



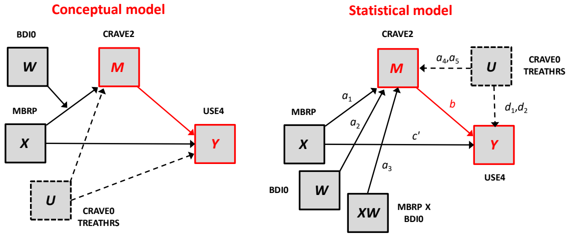
$$\hat{M} = a_0 + \boxed{a_1X + a_2W + a_3XW} + a_4U_1 + a_5U_2$$
$$\hat{Y} = c_0' + c'X + bM + d_1U_1 + d_2U_2$$

The moderation of the effect of mindfulness behavioral relapse prevention therapy relative to therapy as usual on craving by pre-therapy depression level.

Conceptual and Statistical Models

Conceptual model

Statistical model



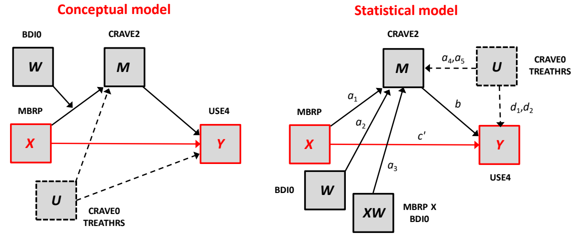
$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$
$$\hat{Y} = c_0' + c'X + \boxed{bM} + d_1U_1 + d_2U_2$$

The effect of craving at two month follow-up on substance use after 4 months.

Conceptual and Statistical Models

Conceptual model

Statistical model

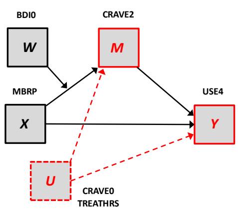


$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$
$$\hat{Y} = c_0' + \boxed{c'X} + bM + d_1U_1 + d_2U_2$$

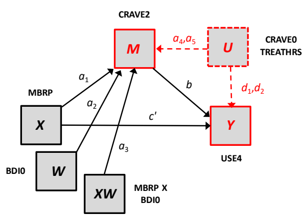
The direct effect of mindfulness behavioral relapse prevention therapy on substance use at four month follow up accounting for the mechanism through craving.

Conceptual and Statistical Models

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + \boxed{a_4U_1 + a_5U_2}$$

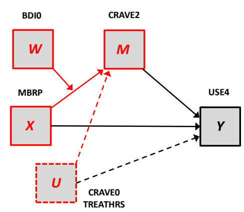
$$\hat{Y} = c_0' + c'X + bM + \boxed{d_1U_1 + d_2U_2}$$

Covariates to account for potential confounding by treatment hours and pre-therapy craving.

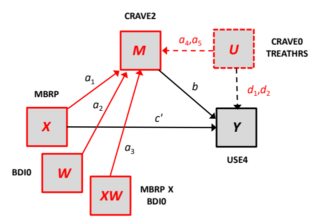
Estimating the moderation component of the model

The conceptual model proposes that the effect of mindfulness behavioral relapse prevention therapy depends on pre-therapy depression.

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$

We most care about the moderation components of the model of M: a_1 and a_3

We did this already

```
compute mbrpdep = mbrp*bdi0.
regression/dep = crave2/method = enter mbrp bdi0 mbrpdep treathrs crave0.
```

```
data mbrp;set mbrp;mbrpdep=mbrp*bdi0;run;
proc reg data=mbrp;model crave2=mbrp bdi0 mbrpdep treathrs crave0;run;
```

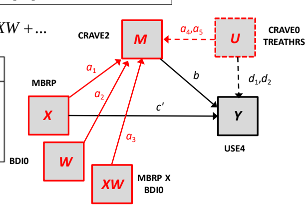
***X* = MBRP**
***W* = BDI0**
***Y* = CRAVE2**

$$\hat{Y} = 1.038 + 0.587X + 1.122W - 0.948XW + ...$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.038	.470		2.209	.029
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.587	.524	.299	1.120	.264
	BDI0: Beck Depression Inventory baseline	1.122	.276	.366	4.063	.000
	mbrpdep	-.948	.423	-.598	-2.240	.026
	TREATHRS: Hours of therapy	-.018	.010	-.120	-1.719	.088
	CRAVE0: Baseline craving	.192	.073	.183	2.614	.010

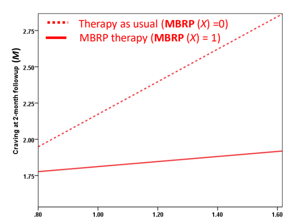
a. Dependent Variable: CRAVE2: Craving at two month follow-up



a_1 = 0.587, a_3 = -0.948

Pre-therapy depression moderates the effect of mindfulness behavioral relapse prevention therapy on craving. We can say that this moderation/interaction is statistically significant, but this doesn't matter for our purposes because neither the direct nor indirect effects in this model are determined entirely by a_3, and it is the direct and indirect effects we care about. We need a_1 and a_3 to estimate the indirect effects.

Recall the pattern from the earlier analysis



The conditional effect of MBRP therapy ($\theta_{X \to M}$) is defined by the function

$$\boldsymbol{\theta_{X \to M} = 0.587 - 0.948W}$$

BDI0 (W)	$\theta_{X \to M}$
0.877	-0.245
1.196	-0.547
1.515	-0.850

Recall these from our Implementation of the pick-a-point approach.

$$\hat{M} = 1.038 + 0.587X + 1.122W - 0.948XW + ...$$

which can be written as

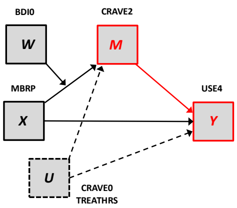
$$\hat{M} = 1.038 + (0.587 - 0.948W)X + 1.122W + ...$$

or

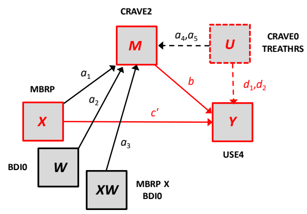
$$\hat{M} = 1.038 + \theta_{X \to M}X + 1.122W + ...$$ where $$\theta_{X \to M} = 0.587 - 0.948W = a_1 + a_3W$$

Estimating the *b* and *c'* paths

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$

$$\hat{Y} = c'_0 + \boxed{c'X + bM} + d_1U_1 + d_2U_2$$

Estimating the *b* and *c'* paths

```
regression/dep=use4/method=enter crave2 mbrp crave0 treathrs.
```

```
proc reg data=mbrp;model use4 = crave2 mbrp crave0 treathrs;run;
```

Coefficients[a]

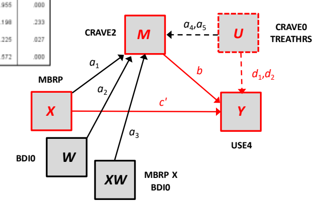
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.130	.215		5.254	.000
	CRAVE2: Craving at two month follow-up	.481	.040	.710	11.955	.000
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.093	.077	.070	1.198	.233
	CRAVE0: Baseline craving	-.088	.040	-.124	-2.225	.027
	TREATHRS: Hours of therapy	-.020	.006	-.200	-3.572	.000

a. Dependent Variable: USE4: Substance use at four month follow-up

$$\hat{Y} = 1.130 + 0.093X + 0.481M + ...$$

$b = 0.481$, $c' = 0.093$

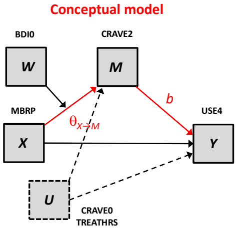
Statistical model



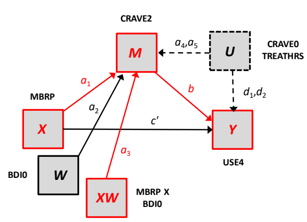
Emphasis in not on statistical significance of the *b* path, as the indirect effect of *X* is not defined entirely in terms of *b*. *c'* is the direct effect (discussed in a bit).

The conditional indirect effect of *X*

Conceptual model



Statistical model

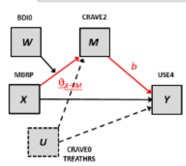


The conditional indirect effect of *X* on *Y* through *M* is the product of the conditional effect of *X* on *M* ($\theta_{X \to M} = a_1 + a_3W$) and effect of *M* on *Y* (*b*):

$$\theta_{X \to M}b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$$

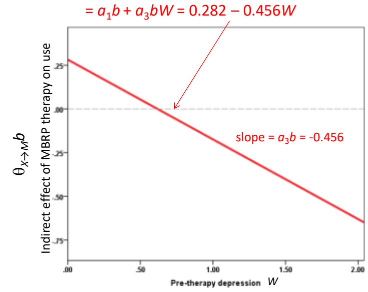
The indirect effect of MBRP therapy relative to therapy as usual on later substance use through craving is a function of pre-therapy depression.

A visual representation of the indirect effect



$\theta_{X \to M}b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$
$= a_1b + a_3bW = 0.282 - 0.456W$

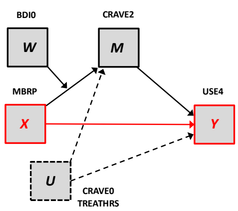
The indirect effect declines with increasing pre-therapy depression. The "**index of moderated mediation**" is $a_3b = -0.456$. It quantifies the relationship between the moderator and the indirect effect in this model.



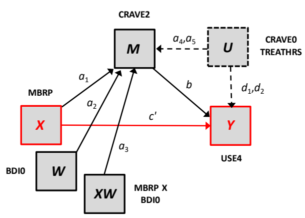
As will be seen, a hypothesis test that the slope of this function is equal to 0 is a formal test of moderated mediation....moderation of the indirect effect.

The direct effect of *X*

Conceptual model



Statistical model



$$\hat{M} = a_0 + a_1X + a_2W + a_3XW + a_4U_1 + a_5U_2$$

$$\hat{Y} = c_0' + \boxed{c'X} + bM + d_1U_1 + d_2U_2$$

In this model, the direct effect is fixed to be unmoderated. It is a constant rather than a function of another variable in the model. This is a modeling or theoretical decision, not a requirement.

The direct effect of *X* (estimated earlier)

```
regression/dep=use4/method=enter crave2 mbrp crave0 treathrs.
```

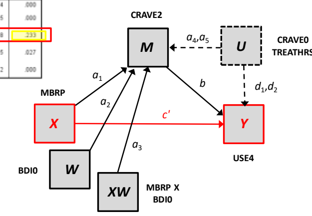
```
proc reg data=mbrp;model use4 = crave2 mbrp crave0 treathrs;run;
```

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	1.130	.215		5.254	.000
	CRAVE2: Craving at two month follow-up	.481	.040	.710	11.955	.000
	MBRP: Therapy as usual (0) or MBRP therapy (1)	.093	.077	.070	1.198	.233
	CRAVE0: Baseline craving	-.088	.040	-.124	-2.225	.027
	TREATHRS: Hours of therapy	-.020	.006	-.200	-3.572	.000

a. Dependent Variable: USE4: Substance use at four month follow-up

Statistical model



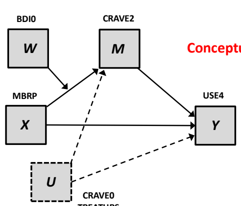
$$\hat{Y} = 1.130 + 0.093X + 0.481M + ...$$

$c' = 0.093$, $t(163) = 1.198$, $p = 0.233$

Mindfulness behavioral relapse prevention therapy has no apparent effect on later substance use relative to therapy as usual after accounting for the mechanism through craving.

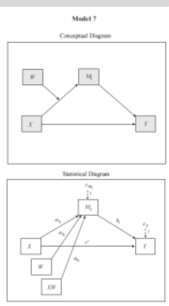
Estimation of the model in PROCESS

PROCESS takes most of the computational burden off our shoulders.



Conceptual model

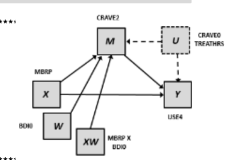
This is PROCESS model 7




```
process cov = crave0 treathrs/x=mbrp/m=crave2/y=use4/
        w=bdi0/boot=10000/model=7.
```

```
%process (data=mbrp,cov=crave0 treathrs,x=mbrp,m=crave2,y=use4,w=bdi0,
        boot=10000,model=7);
```

PROCESS output




```
**************************************************************
Model = 7
    Y = use4
    X = mbrp
    M = crave2
    W = bdi0

Statistical Controls:
CONTROL= crave0   treathrs

Sample size
        168

**************************************************************
Outcome: crave2

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5140      .2642      .7277    11.6319     5.0000   162.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     1.0385      .4701     2.2090      .0286      .1102     1.9668
mbrp          .5872      .5241     1.1204      .2642     -.4478     1.6222
bdi0         1.1221      .2762     4.0625      .0001      .5767     1.6675
int_1        -.9485      .4235    -2.2398      .0265    -1.7847     -.1122
crave0        .1920      .0735     2.6138      .0098      .0470      .3371
treathrs     -.0177      .0103    -1.7190      .0875     -.0380      .0026

Interactions:

 int_1    mbrp        X      bdi0

**************************************************************************
```

$$\hat{M} = 1.039 + 0.587X + 1.122W - 0.948XW + ...$$

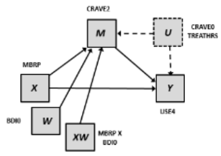
$a_1 = 0.587$

$a_2 = 1.122$

$a_3 = -0.948$

Output K

PROCESS output



$$\hat{Y} = 1.130 + 0.093X + 0.481M + \ldots$$

```
**************************************************************************
Outcome: use4
```

Model Summary

R	R-sq	MSE	F	df1	df2	p
.7304	.5335	.2105	46.6070	4.0000	163.0000	.0000

Model

	coeff	se	t	p	LLCI	ULCI
constant	1.1298	.2150	5.2545	.0000	.7052	1.5544
crave2	.4810	.0402	11.9547	.0000	.4015	.5604
mbrp	.0926	.0773	1.1979	.2327	-.0601	.2453
crave0	-.0884	.0397	-2.2246	.0275	-.1668	-.0099
treathrs	-.0199	.0056	-3.5720	.0005	-.0309	-.0089

$b = 0.481$

$c' = 0.093$

Output K

PROCESS output

```
******************** DIRECT AND INDIRECT EFFECTS *************************
```

Direct effect

$c' = .093$, $p = .232$

Direct effect of X on Y

Effect	SE	t	p	LLCI	ULCI
.0926	.0773	1.1979	.2327	-.0601	.2453

INDIRECT EFFECT:

mbrp -> crave2 -> use4

W

bdi0	Effect	BootSE	BootLLCI	BootULCI
.9020	-.1290	.0770	-.2869	.0183
1.1900	-.2604	.0862	-.4458	-.1090
1.5180	-.4100	.1367	-.7080	-.1767

Output K

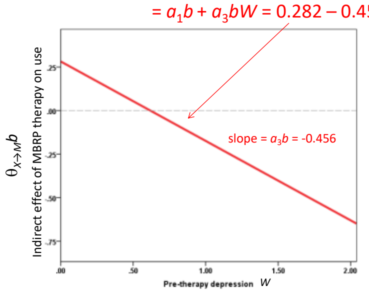
$$\theta_{X \to M} b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$$
$$= a_1b + a_3bW = 0.282 - 0.456W$$

PROCESS sees that the moderator is continuous so, without instruction otherwise, prints the conditional indirect effect at 16th, 50th, and 84th percentile of the moderator.

A statistical test of moderated mediation

in the first stage moderated mediation model

$$\theta_{X \to M} b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$$
$$= a_1b + a_3bW = 0.282 - 0.456W$$



The indirect effect is a function of W (pre-therapy depression) in our model. This function is a line.

$\theta_{X \to M} b = (a_1 + a_3W)b$

$= a_1b + a_3bW$

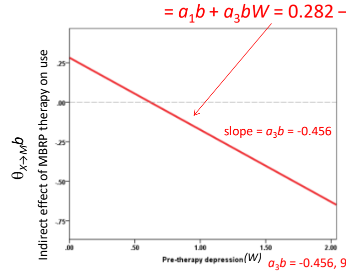
$= 0.282 - 0.456W$

An inference about the slope of this line—the "index of moderated mediation"—is an inference about whether the indirect effect is moderated: Is this slope significantly different from zero? If so, that supports a claim of "moderated mediation."

As a_3b is a product of regression coefficients, an inferential test should respect the nonnormality of the sampling distribution of the product. A bootstrap confidence interval is a good choice.

This test is provided automatically by PROCESS

$$\theta_{X \to M} b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$$
$$= a_1b + a_3bW = 0.282 - 0.456W$$



The indirect effect is a function of W (pre-therapy depression) in our model. This function is a line.

$\theta_{X \to M} b = (a_1 + a_3W)b$

$= a_1b + a_3bW$

$= 0.282 - 0.456W$

Output K

$a_3b = -0.456$, 95% bootstrap CI = -0.960 to -0.104

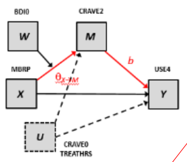
```
******************** INDEX OF MODERATED MEDIATION ************************
```

Mediator

	Index	SE(Boot)	BootLLCI	BootULCI
crave2	-.4562	.2190	-.9596	-.1044

This slope is statistically different from zero. The indirect effect depends on pre-therapy depression....the mediation is moderated.

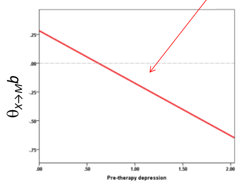
Probing the moderation of mediation



$\theta_{X\to M}\, b = (a_1 + a_3W)b = (0.587 - 0.948W)(0.481)$

The indirect effect decreases with increased pre-therapy depression.

With evidence of moderation of the indirect effect, we can now probe this moderation of mediation through an analogue of the pick-a-point approach used in moderation analysis.



	Pre-therapy depression (W)	$\theta_{X\to M}$	b	$\theta_{X\to M}\, b$
16th %	.9020	-.2681	0.481	-0.118
50th %	1.1900	-.5411	0.481	-0.263
84th %	1.5180	-.8521	0.481	-0.409

Conditional indirect effects

We need an inferential test for these conditional indirect effects. Bootstrap confidence intervals are perfect for the job. PROCESS does this for us automatically.

PROCESS output

```
******************* DIRECT AND INDIRECT EFFECTS ***********************

Direct effect of X on Y
     Effect         SE          t          p       LLCI       ULCI
      .0926      .0773     1.1979      .2327     -.0601      .2453

Conditional indirect effect(s) of X on Y at values of the moderator(s):

Mediator
  bdi0      Effect     BootSE    BootLLCI   BootULCI
  .9020     -.1290      .0770     -.2869      .0183
 1.1900     -.2604      .0862     -.4458     -.1090
 1.5180     -.4100      .1367     -.7080     -.1767
```

Conditional indirect effects with 95% bootstrap CIs based on 10,000 bootstrap samples.

Output K

The indirect effect of MBRP therapy relative to therapy as usual on substance use through craving is negative among the relatively moderate (point estimate: -0.260, 95% CI from -0.446 to -0.109) and relatively highly depressed (point estimate: -0.410, 95% CI from -0.708 to -0.177) but not different from zero among the relatively less depressed (point estimate: -0.129, 95% CI from -0.287 to 0.018).

Comparing conditional indirect effects (1st stage model)

A seemingly sensible question to ask is whether the conditional indirect effect of X when the moderator equals some value $W = w_1$ is different than the conditional indirect effect of X when the moderator is some different value $W = w_2$. For example, is the indirect effect of MBRP therapy through craving different for those relatively low in pre-therapy depression relative those relatively high?

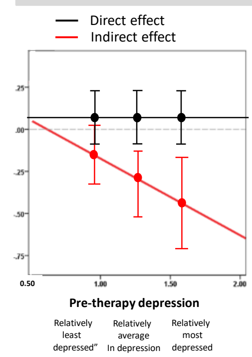
For this model, the difference between any two conditional indirect effects defined by different values of W equal to w_1 and w_2 is

$$
\begin{aligned}
(a_1+a_3w_1)b - (a_1+a_3w_2)b &= a_1b+a_3bw_1 - a_1b - a_3bw_2 \\
&= a_3bw_1 - a_3bw_2 \\
&= a_3b\,(w_1 - w_2)
\end{aligned}
$$

Rejection of the null hypothesis of no moderated mediation based on the index of moderated mediation implies that **any** two conditional indirect effects are different! No additional test is needed.

If a bootstrap confidence interval for a_3b does not contain zero, then neither will a confidence interval for $a_3b\,(w_1 - w_2)$, ***regardless*** of values of w_1 and w_2 chosen, so long as $w_1 \neq w_2$. And if a bootstrap confidence interval for a_3b contains zero, then so too will a confidence interval for $a_3b\,(w_1 - w_2)$, ***for any two values*** of w_1 and w_2, $(w_1 \neq w_2)$.

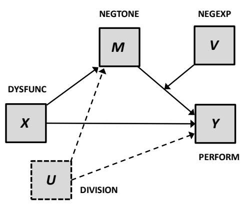
Putting it all together

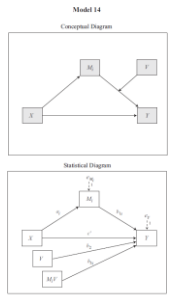


MBRP seems to reduce substance use through a reduction in craving which in turn lowers use, but more so among those who are more depressed at the start of therapy. Among those relatively lower in depression, we cannot say definitively that this mechanism is in operation. Independent of this mechanism, there is no evidence of an effect of MBRP therapy on later substance use.

Point estimates with 95% confidence intervals (OLS confidence intervals for direct effects, bootstrap confidence intervals for indirect effects).

PROCESS versus SEM





This is PROCESS model 14

```
process cov = d1 d2 d3/x=dysfunc/m=negtone/y=perform/v=negexp/boot=10000/model=14.
```

```
%process (data=teams,cov=d1 d2 d3,x=dysfunc,m=negtone,y=perform, v=negexp, boot=10000,
         model=14);
```

In Mplus

COMPARE TO OUTPUT THAT PROCESS GENERATES

```
DATA:
  file is 'c:\mplus\teams.csv';
VARIABLE:
  names are dysfunc negtone negexp perform division d1 d2 d3;
  usevariables are dysfunc negexp negtone perform d1 d2 d3 toneexp;
DEFINE:
  toneexp = negtone*negexp;
ANALYSIS:
  !bootstrap=10000;
MODEL:
  perform ON d1 d2 d3
             dysfunc (cp)
             negtone (b1)
             negexp (b2)
             toneexp (b3);
  negtone ON d1 d2 d3
             dysfunc (a1);
  d1;
  d2;
  d3;
  negexp;
  dysfunc;
  toneexp;
  negtone with negexp;
  negtone with toneexp;
MODEL CONSTRAINT:
  new (a1thetaL a1thetaM a1thetaH a1b3);
  a1thetaL=a1*(b1+b3*(-.5308));
  a1thetaM=a1*(b1+b3*(-0.0600));
  a1thetaH=a1*(b1+b3*(0.6000));
  a1b3=a1*b3;
OUTPUT:
  !cinterval(bootstrap);
```

	Estimate	S.E.	Est./S.E.	P-Value
PERFORM ON				
D1	0.182	0.147	1.231	0.218
D2	0.084	0.252	0.333	0.739
D3	0.282	0.170	1.659	0.097
DYSFUNC	0.373	0.195	1.908	0.056
NEGTONE	-0.489	0.131	-3.728	0.000
NEGEXP	-0.022	0.102	-0.217	0.829
TONEEXP	-0.450	0.243	-1.851	0.064
NEGTONE ON				
D1	0.349	0.167	2.093	0.036
D2	0.295	0.206	1.436	0.151
D3	0.251	0.116	2.167	0.030
DYSFUNC	0.609	0.216	2.822	0.005
Intercepts				
NEGTONE	-0.206	0.087	-2.368	0.018
PERFORM	-0.175	0.127	-1.380	0.167
New/Additional Parameters				
A1THETAL	-0.152	0.152	-1.003	0.316
A1THETAM	-0.281	0.126	-2.235	0.025
A1THETAH	-0.462	0.170	-2.722	0.006
A1B3	-0.274	0.176	-1.557	0.119

In Mplus

Removing exclamation points from the code generates bootstrap confidence intervals for all parameter estimates, including conditional indirect effects.

COMPARE TO OUTPUT THAT PROCESS GENERATES

CONFIDENCE INTERVALS OF MODEL RESULTS

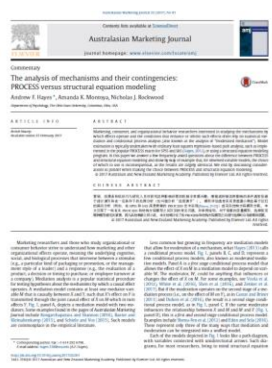
	Lower .5%	Lower 2.5%	Lower 5%	Estimate	Upper 5%	Upper 2.5%	Upper .5%
New/Additional Parameters							
A1THETAL	-0.568	-0.429	-0.377	-0.152	0.113	0.187	0.352
A1THETAM	-0.664	-0.544	-0.488	-0.281	-0.082	-0.052	0.010
A1THETAH	-0.954	-0.816	-0.753	-0.462	-0.194	-0.146	-0.064
A1B3	-0.886	-0.702	-0.617	-0.274	-0.051	-0.022	0.030

Point estimates

End points of a 95% bootstrap confidence intervals
for the conditional indirect effect based on 10,000 bootstrap samples.

Observe that the results using an SEM program are largely the same as the results that PROCESS produces.

PROCESS versus Structural Equation Modeling



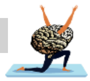
Hayes, A. F., Montoya, A. K., & Rockwood, N. J. (2017). The analysis of mechanisms and their contingencies: PROCESS versus structural equation modeling. *Australasian Marketing Journal, 25,* 76-81.

- For observed variable models, it makes no difference whether you use SEM or PROCESS. You get the same results and with a lot less effort.

Some reasons you might choose SEM:

- More options for dealing with missing data
- More sophisticated means of managing the effects of measurement error.
- Latent variables and blends of latent and observed variables.
- Greater flexibility for model specification.

CPA Activity



The following slides have examples of conditional process analyses from the literature. Try to draw the path diagram for these models and write out the PROCESS code to analyze the data.

Mawritz, M. B., Mayer, D. M., Hoobler J. M., Wayne, S. J., & Marinova, S. V. (2012) A trickle-down model of abusive supervision, *Personnel Psychology*

> "Specifically, we find that abusive manager behavior is positively related to abusive supervisor behavior, which in turn is positively related to work group interpersonal deviance. In addition, hostile climate moderates the relationship between abusive supervisor behavior and work group interpersonal deviance such that the relationship is stronger when hostile climate is high."

HINT: You need to choose a model from the templates that matches the hypothesis

Activity:

- Draw a path diagram and label *X*, *M*, *Y*, and *W* with variable names (e.g., pg 128)
- Variable Names: Abusive Manager Behavior (`Manage`), Abusive Supervisor Behavior (`Super`), Work Group Deviance (`Deviance`), and Hostile Climate (`Hostile`). What would the PROCESS command be to run this analysis? Use percentile bootstrapping with 7,000 bootstraps.
- Write the equations for the model of *M* and the model of *Y*. What would the conditional indirect effect be? What would the index of moderated mediation be? (e.g., pg 128)
- Draw a graphical representation of the indirect effect across the moderator which fits the above hypotheses. (e.g., pg 133)

302

Mediation analysis in the 2-condition within-subject design

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods, 6*, 115 –134.

JUDD.sav JUDD.sas

20 participants with chronic pain symptoms participated in a pain drug trial. Each was measured twice, once after administration of a placebo **and** once after a administration of a pain inhibiting drug. Order is randomized.

Measurement 1 = Following placebo
Measurement 2 = Following drug

Y = pain sensations (0 to 100, higher = more)
M = pain enhancing hormone (0 to 100, higher = *more*)

Analytical goal: Determine if the effect of the drug on pain experienced operates through the mechanism of reducing pain enhancing hormone levels.

pain1	pain2	hormone1	hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00	75.00	37.00	35.00
56.00	60.00	33.00	34.00
72.00	73.00	38.00	35.00
81.00	65.00	43.00	29.00
61.00	59.00	33.00	31.00
67.00	48.00	20.00	17.00
74.00	64.00	43.00	41.00
70.00	55.00	34.00	27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

Advantages of such a design…

Designs such as this are common:

- Participants might read two different scenarios that vary on some manipulated feature *X* and offer emotional reactions, make predictions about the own behavior, and so forth, in each.
- A therapist might measure certain symptoms and various outcomes when clients arrive in the office for the first time and after a few months of treatment. So *X* is "pre" or "post", i.e., the passage of time.

Some advantages relative to "between-subjects" design and analysis:

- Fewer participants needed. Rather than having *n* people (n_1 in one condition, n_2 in the other), we need about 0.5*n* in total. That saves effort, time, money, labor, and so forth.
- Greater statistical power. Each person serves as his or her own control. "Noise" due to individual differences that increases standard errors in estimates of effects is reduced, sometimes substantially.
- Reduces (but doesn't eliminate) the fundamental problem of causal inference. We don't have to think counterfactually. We know how a person responds in *each* condition rather than having to make an assumption about he or she might have if assigned to the other condition instead.

Judd, Kenny, and McClelland (2001)

Estimating and Testing Mediation and Moderation in Within-Subject Designs

Charles M. Judd
University of Colorado at Boulder

David A. Kenny
University of Connecticut

Gary H. McClelland
University of Colorado at Boulder

One of the few treatments of mediation analysis in this common research design.

A "causal steps", Baron and Kenny type logic to determining whether *M* is functioning as a mediator of *X*'s effect on *Y* when both *M* and *Y* are measured twice in difference circumstances but on the same people.

Judd, Kenny, and McClelland (2001)

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods, 6*, 115 –134.

Analytical goal. Determine if the effect of a pain-inhibiting drug on pain experienced operates through the mechanism of reducing pain-enhancing hormone levels.

(1) On average, is pain following the drug lower than pain following the placebo?

(2) On average, are there fewer pain enhancing hormones in the blood following the drug relative to the placebo?

(3) Does difference in hormone level predict differences in pain experienced?

(4) Do differences in hormone levels account for differences in pain?

Y_1 pain1	Y_2 pain2	M_1 hormone1	M_2 hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00	75.00	37.00	35.00
56.00	60.00	33.00	34.00
72.00	73.00	38.00	35.00
81.00	65.00	43.00	29.00
61.00	59.00	33.00	31.00
67.00	48.00	20.00	17.00
74.00	64.00	43.00	41.00
70.00	55.00	34.00	27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

Application of Judd et al. (2001)

(1) On average, is pain following the drug significantly lower than pain following the placebo?

```
ttest pairs=pain2 pain1.
```

```
proc ttest data=judd;paired pain2*pain1;run;
```

$\overline{Y}_{\text{drug}}$ = 61.250, *SD* = 8.583
$\overline{Y}_{\text{placebo}}$ = 67.750, *SD* = 11.889

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	pain2	61.2500	20	8.58318	1.91926
	pain1	67.7500	20	11.88929	2.65853

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper			
Pair 1	pain2 - pain1	-6.50000	9.23665	2.06538	-10.82289	-2.17711	-3.147	19	.005

Pain experienced following the administration of the drug was 6.500 units lower compared to pain experienced following the placebo, $t(19) = -3.147$, $p < .01$.

Application of Judd et al. (2001)

(2) On average, are there fewer pain-enhancing hormones in the blood following the drug relative to the placebo?

```
ttest pairs=hormone2 hormone1.
```

```
proc ttest data=judd;paired hormed2*hormone1;run;
```

$\overline{M}_{\text{drug}}$ = 32.000, *SD* = 5.964
$\overline{M}_{\text{placebo}}$ = 34.250, *SD* = 6.414

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	hormone2	32.0000	20	5.96481	1.33377
	hormone1	34.2500	20	6.41442	1.43431

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper			
Pair 1	hormone2 - hormone1	-2.25000	4.10231	.91730	-4.16994	-.33006	-2.453	19	.024

Pain-enhancing hormone levels were 2.25 units lower on average following the administration of the drug compared to following the placebo, $t(19) = -2.453$, $p < .05$.

Application of Judd et al. (2001)

(3) Does difference in hormone level following the drug relative to placebo predict the difference in pain experienced following the drug relative to placebo?

Regress $Y_2 - Y_1$ on both $M_2 - M_1$ and <u>mean centered</u> $(M_2 + M_1)$, as such:

$$Y_{2i} - Y_{1i} = b_0 + b_1(M_{2i} - M_{1i}) + b_2\left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^{n}(M_{2i} + M_{1i})}{n}\right) + e_i$$

```
compute ydiff=pain2-pain1.
compute mdiff=hormone2-hormone1.
compute msumc=(hormone2+hormone1)-66.25.
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;ydiff=pain2-pain1;mdiff=hormone2-hormone1;
msumc=(hormone2+hormone1)-66.25;run;
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Coefficients[a]

Model		Unstandardized Coefficients: B	Std. Error	Standardized Coefficients: Beta	t	Sig.
1	(Constant)	-3.765	2.087		-1.804	.089
	mdiff	1.215	.457	.540	2.658	.017
	msumc	-.065	.160	-.082	-.406	.690

a. Dependent Variable: ydiff

Relatively fewer pain-enhancing hormones following the drug relative to placebo is associated with less pain following the drug relative to placebo, $b_1 = 1.215$, $p < 0.05$.

Why include $(M_2 + M_1)$ in the model?

The impulse is to model $Y_2 - Y_1$ from $M_2 - M_1$ to assess the effect of difference in M on difference in Y. ***This is appropriate only if the regression weight estimating Y_2 from M_2 is the same as the regression weight estimating Y_1 from M_1.***

If Y_1 and Y_2 are linked to M_1 to M_2 as

$$Y_1 = d_{01} + dM_1 + e_1$$
$$Y_2 = d_{02} + dM_2 + e_2 \text{, then}$$

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} + dM_2 + e_2) - (d_{01} + dM_1 + e_1) \\ &= (d_{02} - d_{01}) + d(M_2 - M_1) + (e_2 - e_1) \\ &= d_0 + d(M_2 - M_1) + e_3 \end{aligned}$$

where $d_0 = d_{02} - d_{01}$ and $e_3 = e_2 - e_1$

But if Y_1 and Y_2 are linked to M_1 and M_2 as

$$Y_1 = d_{01} + d_1 M_1 + e_1$$
$$Y_2 = d_{02} + d_2 M_2 + e_2 \text{, then}$$

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} + d_2 M_2 + e_2) - (d_{01} + d_1 M_1 + e_1) \\ &= (d_{02} - d_{01}) + (d_2 M_2 - d_1 M_1) + (e_2 - e_1) \end{aligned}$$

It can be shown that this is equivalent to

$$\begin{aligned} Y_2 - Y_1 &= (d_{02} - d_{01}) + 0.5(d_2 - d_1)(M_2 + M_1) + 0.5(d_2 + d_1)(M_2 + M_1) + (e_2 - e_1) \\ &= d_0 + 0.5(d_2 - d_1)(M_2 + M_1) + 0.5(d_2 + d_1)(M_2 - M_1) + e_3 \end{aligned}$$

where $d_0 = d_{02} - d_{01}$ and $e_3 = e_2 - e_1$

$$Y_{2i} - Y_{1i} = b_0 + \boxed{b_1(M_{2i} - M_{1i})} + b_2\left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^{n}(M_{2i} + M_{1i})}{n}\right) + e_i$$

These are nearly the same, with the exception of the centering, which has no effect on b_1. $b_1 = 0.5(d_2 + d_1)$. So b_1 estimates the effect of $M_2 - M_1$ on $Y_2 - Y_1$ without assuming $d_1 = d_2$. It is also the average within period regression weight estimating Y from X. **$b_1 = d$ if $d_1 = d_2$. Otherwise, b_1 and d will likely be different.**

Why mean center $M_2 + M_1$?

Mean centering $(M_2 + M_1)$ yields an intercept that estimates the average difference in Y not attributable to differences in M.

$$Y_{2i} - Y_{1i} = b_0 + b_1(M_{2i} - M_{1i}) + b_2\left(M_{2i} + M_{1i} - \frac{\sum_{i=1}^{n}(M_{2i} + M_{1i})}{n}\right) + e_i$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-3.765	2.087		-1.804	.089
	mdiff	1.215	.457	.540	2.658	.017
	msumc	-.065	.160	-.082	-.406	.690

a. Dependent Variable: ydiff

(4) After accounting for differences in hormone levels, there is no statistically significant difference in pain experienced after the drug relative to the placebo, i = -3.765, p = 0.089.

Observations

(1) This method is squarely rooted in the causal steps tradition to mediation analysis that has been severely criticized. Compare it to the "Baron and Kenny" criteria:

- Is Y_2 statistically different than Y_1? This is like asking whether there is a total effect of X (drug) on Y (pain).
- Is M_2 statistically different than M_1? This is like asking whether X affects the mediator.
- Does difference in M significantly predict difference in Y? This is like asking whether the mediator affects the outcome.
- Is there still evidence of a difference in Y after accounting for the mediator? This is like asking whether the mediator completely or partially accounts for the effect of X on Y.

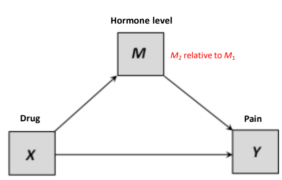
(2) There is no explicit quantification of the indirect effect, but it is the indirect effect that is the primary focus in 21st century mediation analysis.

All of the criticisms of the causal steps approach apply to the Judd, Kenny, and McClelland (2001) method of within-subject mediation analysis.

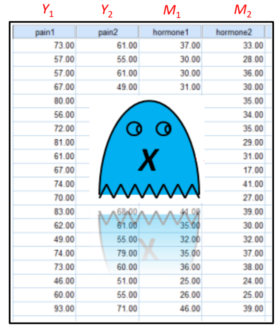
In a path analytic mediation framework

Data are from Judd et al. (2001). Estimating and testing mediation and moderation in within-subjects designs. *Psychological Methods, 6*, 115 –134.

Goal: Model the effect of the pain-facilitating drug on pain sensations, directly as well as indirectly through the effect of the drug on pain-enhancing hormone level.

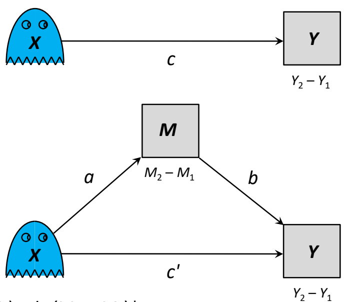


Where is X in the data?



Y_1 pain1	Y_2 pain2	M_1 hormone1	M_2 hormone2
73.00	61.00	37.00	33.00
57.00	55.00	30.00	28.00
57.00	61.00	30.00	36.00
67.00	49.00	31.00	30.00
80.00			35.00
56.00			34.00
72.00			35.00
81.00			29.00
61.00			31.00
67.00			17.00
74.00			41.00
70.00			27.00
83.00	68.00	41.00	39.00
62.00	61.00	35.00	30.00
49.00	55.00	32.00	32.00
74.00	79.00	35.00	37.00
73.00	60.00	36.00	38.00
46.00	51.00	25.00	24.00
60.00	55.00	26.00	25.00
93.00	71.00	46.00	39.00

In a path analytic mediation framework



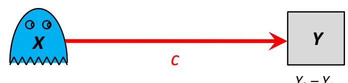
Y_1 = pain1
Y_2 = pain2
M_1 = hormone1
M_2 = hormone2

$$Y_2 - Y_1 = c + e_1$$
$$M_2 - M_1 = a + e_2$$
$$Y_2 - Y_1 = c' + b(M_2 - M_1) + b_2(M_2 + M_1)^* + e_3$$

$c = c' + ab$ $\qquad$ $ab = c - c'$

* mean centered

Estimating the total effect (path *c*)



$$Y_2 - Y_1 = c + e_1$$

The total effect is the intercept in a "constant only" model of the difference between pain following the drug (Y_2) and pain following the placebo (Y_1). This is equivalent to the mean difference in pain experienced. No regression analysis needed. Just calculate the mean difference in Y (i.e., $Y_2 - Y_1$).

```
compute ydiff=pain2-pain1.
descriptives variables=ydiff.
```

```
data judd;set judd;ydiff=pain2-pain1;run;
proc means data=judd;var ydiff;run;
```

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
ydiff	20	-22.00	6.00	-6.5000	9.23665
Valid N (listwise)	20				

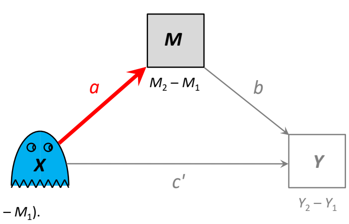
c = -6.500

6.50 units less pain following the drug compared to placebo

Estimating the *a* path

$$M_2 - M_1 = a + e_2$$

The *a* path is the intercept in a "constant only" model of the difference in hormone following the drug (M_2) and following the placebo (M_1). This is equivalent to the mean difference in hormone level. No regression analysis needed. Just calculate the mean difference in M (i.e., $M_2 - M_1$).




```
compute mdiff=hormone2-hormone1.
descriptives variables= mdiff.
```

```
data judd;set judd;mdiff=hormone2-hormone1;run;
proc means data=judd;var mdiff;run;
```

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
mdiff	20	-14.00	6.00	-2.2500	4.10231
Valid N (listwise)	20				

a = -2.250

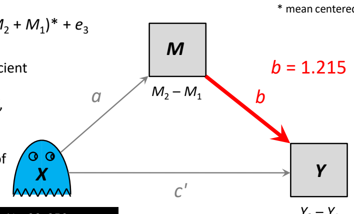
2.50 units less of the pain enhancing hormone following the drug compared to placebo

Estimating the *b* path

* mean centered

$$Y_2 - Y_1 = c' + b(M_2 - M_1) + b_2(M_2 + M_1)^* + e_3$$

The *b* path is the regression coefficient for the mean difference in M in a model of the mean difference in Y, including the mean centered sum of M_1 and M_2 as a covariate. In these data, the mean of the sum of M_1 and M_2 is 66.250.



b = 1.215

```
compute msumc=(hormone2+hormone1)-66.250.
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;mdiff=hormone2-hormone1;msumc=(hormone2+hormone1)-66.250;run;
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-3.765	2.087		-1.804	.089
	mdiff	1.215	.457	.540	2.658	.017
	msumc	-.065	.160	-.082	-.406	.690

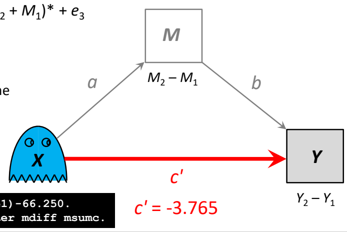
a. Dependent Variable: ydiff

On average, each fewer unit of pain enhancing hormone following drug relative to placebo is associated with 1.215 fewer units of pain following drug relative to placebo.

Estimating the direct effect (path c')

$Y_2 - Y_1 = c' + b(M_2 - M_1) + d_2(M_2 + M_1)^* + e_3$

The direct effect (path c') is the regression constant in the model of the mean difference in Y from the mean difference in M and the mean centered sum of M_1 and M_2. This is the same model used to estimate path b.




```
compute msumc=(hormone2+hormone1)-66.250.
regression/dep=ydiff/method=enter mdiff msumc.
```

```
data judd;set judd;mdiff=hormone2-hormone1;msumc=(hormone2+hormone1)-66.250;run;
proc reg data=judd;model ydiff=mdiff msumc;run;
```

Coefficients[a]

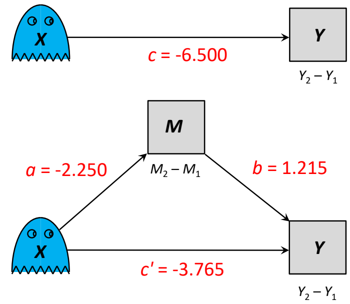
Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-3.765	2.087		-1.804	.089
	mdiff	1.215	.457	.540	2.658	.017
	msumc	-.065	.160	-.082	-.406	.690

a. Dependent Variable: ydiff

Independent of the effect of drug vs. placebo hormone difference on pain difference, the drug results in 3.765 units less pain relative to placebo.

Putting it all together

In this form, it is clear that the effect of *X* partitions into two components direct and indirect in the usual way. We can conduct inferential tests on these estimates as in any mediation analysis.



$c = c' + ab$

$c = \underbrace{-3.765}_{\text{Direct effect}} + \underbrace{(-2.250)(1.215)}_{\text{Indirect effect}} = \underbrace{-3.765}_{\text{Direct effect}} + \underbrace{-2.735}_{\text{Indirect effect}} = \underbrace{-6.500}_{\text{Total effect}}$

Statistical inference

- **The total effect is just the average difference between Y_2 and Y_1. Inference can proceed in the usual way (paired *t*-test, a confidence interval for the difference, etc.)**

  In these data, c = -6.500, $t(19)$ = -3.147, p < 0.01. On average, the drug appears to have an effect on the experience of pain. But we should not insist on evidence of a total effect to proceed with a mediation analysis, as we no longer do elsewhere in modern mediation analysis.

- **The direct effect is just the average difference between Y_2 and Y_1 not accounted for by differences in *M* and their sum. A hypothesis test or confidence interval will do.**

  Here, c' = -3.765, $t(17)$ = -1.804, p = 0.089. Accounting for the effect of change in *M* and individual differences in *M*, the remaining effect of the drug on pain is not statistically different from zero.

- **Whereas the Judd et al. (2001) approach requires the joint significance of *a* and *b* to claim mediation, modern mediation analysis bases claims of mediation on a quantification of the indirect effect and inference about it. The indirect effect is *ab*, which is equivalent to $c - c'$. The sampling distribution of this difference is not normally distributed.**

  a*b* = -2.250(1.215) = -2.785. This is the reduction in pain due to the drug that results from the effect of the drug on reducing pain-enhancing hormones. A bootstrap CI for inference is a good choice.

MEMORE

This method is described in more detail in Montoya and Hayes (2017*).* The paper includes a description of a new macro (MEMORE, pronounced like "memory").

- Single and multiple mediator models.
- Single and multiple moderator models.
- Various inferential methods for indirect effects
- Contrasts between indirect effects
- moderated mediation analysis functions coming soon.

```
memore y=pain2 pain1/m=hormone2 hormone1/samples=10000.
```

```
%memore (data=judd,y=pain2 pain1,m=hormone2 hormone1,samples=10000);
```

MEMORE Output

```
****************** MEMORE Procedure for SPSS Version 1.1 ********************

                       Written by Amanda Montoya

                  Documentation available at afhayes.com

*****************************************************************************

Variables:
Y = pain2    pain1
M = hormone2 hormone1

Computed Variables:
Ydiff =         pain2    -       pain1
Mdiff =         hormone2 -       hormone1
Mavg =  (       hormone2 +       hormone1 )       /2       Centered

Sample Size:
  20

*****************************************************************************
Outcome: Ydiff =  pain2     -       pain1

Model
        Effect        SE         t        df         p      LLCI      ULCI
'X'    -6.5000    2.0654   -3.1471   19.0000     .0053  -10.8233   -2.1767

*****************************************************************************
Outcome: Mdiff =  hormone2  -       hormone1

Model
        Effect        SE         t        df         p      LLCI      ULCI
'X'    -2.2500     .9173   -2.4528   19.0000     .0240   -4.1701   -.3299

*****************************************************************************
```

MEMORE constructs differences and averages for you.

c = -6.500

a = -2.250

MEMORE Output

```
*****************************************************************************
Outcome: Ydiff =  pain2     -       pain1

Model Summary
          R      R-sq       MSE         F       df1       df2         p
      .5554     .3085   65.9384    3.7918    2.0000   17.0000     .0435

Model
           coeff        SE         t        df         p      LLCI      ULCI
'X'      -3.7654    2.0869   -1.8043   17.0000     .0889   -8.1689     .6381
Mdiff     1.2154     .4572    2.6583   17.0000     .0166     .2506    2.1801
Mavg      -.1302     .3209    -.4057   17.0000     .6900    -.8074     .5470

******************** TOTAL, DIRECT, AND INDIRECT EFFECTS ********************

Total effect of X on Y
     Effect        SE         t        df         p      LLCI      ULCI
    -6.5000    2.0654   -3.1471   19.0000     .0053  -10.8233   -2.1767

Direct effect of X on Y
     Effect        SE         t        df         p      LLCI      ULCI
    -3.7654    2.0869   -1.8043   17.0000     .0889   -8.1689     .6381

Indirect Effect of X on Y through M
          Effect    BootSE  BootLLCI  BootULCI
Ind1     -2.7346    1.3120   -5.6521   -.5230

Indirect Key
Ind1  X        ->       M1diff   ->       Ydiff

************************ ANALYSIS NOTES AND WARNINGS ************************

Bootstrap confidence interval method used: Percentile bootstrap.

Number of bootstrap samples for bootstrap confidence intervals: 10000
```

c' = -3.765

b = 1.215

c = -6.500

c' = -3.765

ab with 95% bootstrap confidence interval

A study of sex discrimination in the workplace

Garcia, D. M., Schmitt, M. T., Branscombe, N. R., & Ellemers, N. (2010). Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness. *European Journal of Social Psychology*, *49*, 733-745.


Research article

Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness

All participants (129 females) read a narrative about a female attorney who lost a promotion at her firm to a less qualified male through unequivocally discriminatory actions of the senior partners. The participants where randomly assigned to one of three conditions:

"Individual protest": Participants were told that she protested by describing her qualifications for the job, how much she deserved it, and the unfairness and harm to her own career.

"Collective protest": Participants were told that she protested by describing how the firm has not been fair to women, women are just as qualified as men, and they should be treated equally.

"No protest": These participants were told that although she was disappointed, she accepted the decision and continued working at the firm.

A study of sex discrimination in the workplace

Garcia, D. M., Schmitt, M. T., Branscombe, N. R., & Ellemers, N. (2010). Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness. *European Journal of Social Psychology*, *49*, 733-745.


Research article

Women's reactions to ingroup members who protest discriminatory treatment: The importance of beliefs about inequality and response appropriateness

After reading the narrative, the participants evaluated how **appropriate** they perceived her response to be for the situation. i.e., was it a positive response for dealing with the firm's discrimination.

They also responded to various questions about the attorney that were used to produce a measure of her evaluation which we'll simply call **liking**, such as "I like Catherine," and "Catherine has many positive traits."

Prior to the study, the participants filled out the **modern sexism scale**, used to score each participant with respect to how pervasive she believes sexism and sex discrimination are in society.

The Data: PROTEST

*protest.sav [DataSet1] - IBM SPSS Statistics Data Editor

	subnum	cond	sexism	angry	liking	respappr
1	209	2	4.87	2	4.83	4.25
2	44	0	4.25	1	4.50	5.75
3	124	2	5.00	3	5.50	4.75
4	232	2	5.50	1	5.66	7.00
5	30	2	5.62	1	6.16	6.75
6	140	1	5.75	1	6.00	5.50
7	27	2	5.12	2	4.66	5.00
8	64	0	6.62	1	6.50	6.25
9	67	0	5.75	6	1.00	3.00
10	182	0	4.62	1	6.83	5.75
11	85	2	4.75	2	5.00	5.25
12	109	2	6.12	5	5.66	7.00
13	122	0	4.87	2	5.83	4.50
14	69	1	5.87	1	6.50	6.25

protest

```
data protest;
input subnum cond sexism angry liking respappr protest;
cards;
209 2   4.87   2   4.83   4.25   1.00
44  0   4.25   1   4.50   5.75   .00
124 2   5.00   3   5.50   4.75   1.00
232 2   5.50   1   5.66   7.00   1.00
30  2   5.62   1   6.16   6.75   1.00
140 1   5.75   1   6.00   5.50   1.00
27  2   5.12   2   4.66   5.00   1.00
64  0   6.62   1   6.50   6.25   .00
67  0   5.75   6   1.00   3.00   .00
182 0   4.62   1   6.83   5.75   .00
85  2   4.75   2   5.00   5.25   1.00
109 2   6.12   5   5.66   7.00   1.00
122 0   4.87   2   5.83   4.50   .00
69  1   5.87   1   6.50   6.25   1.00
```

COND: Experimental condition (0 = no protest, 1 = individual protest, 2 = collective protest)

LIKING : Evaluation (liking) of the lawyer (higher = more positive evaluation, i.e. like more)

SEXISM: Score on the modern sexism scale: Beliefs about the pervasiveness of sex discrimination in society (higher = sex discrimination perceived as more pervasive in society)

RESPAPPR: A measure of how appropriate the lawyer's behavior in response to the action of the partners was perceived to be for the situation (higher = more appropriate)

Our objectives with these data

	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	$\bar{Y}^*$
No protest (n = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (n = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (n = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

$\bar{Y}^*$ = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

We'll examine whether the effect of the attorney's response on how she is evaluated (3 conditions)…

… operates through the mechanism of how appropriate her response to the situation is perceived as being (mediation)

Course II….

… depends on perceived pervasiveness of sex discrimination in society (moderation).

… as mediated by the mechanism of perceived appropriateness of her response is moderated by perceived pervasiveness of sex discrimination (moderated mediation)

Single-factor ("one-way") analysis of variance

Did the lawyer's choice as to how to respond (not at all, individual protest, collective protest) influence how she was perceived? That is, is there a difference between conditions, on average, in how much she was liked? Most would answer this using a single-factor (a.k.a. "one-way") analysis of variance (ANOVA).

```
means tables = liking by cond/statistics anova.
```

```
proc anova data=protest;
class cond;model liking = cond;means cond;run;
```

Report

LIKING: liking of the target

COND: experimental condition	Mean	N	Std. Deviation
no protest	5.3102	41	1.30158
individual	5.8260	43	.81943
collective	5.7533	45	.93601
Total	5.6367	129	1.04970

H_0: all means equal

H_a: all means not equal

ANOVA Table

			Sum of Squares	df	Mean Square	F	Sig.
LIKING: liking of the target * COND: experimental condition	Between Groups	(Combined)	6.523	2	3.262	3.055	.051
	Within Groups		134.515	126	1.068		
	Total		141.039	128			

The lawyer's response to the discrimination affected how much she was liked on average, $F(2,126)$ = 3.055, p = .051. She was most liked when she protested individually (Mean = 5.83, SD = 0.82), next most when protesting collectively (Mean = 5.75, SD = 0.94), and least when she didn't protest at all (Mean = 5.31, SD = 1.30).

Mediation

Does perceived response appropriateness mediate this effect?

	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	$\bar{Y}^*$
No protest (n = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (n = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (n = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

$\bar{Y}^*$ = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

Here, the presumed cause is a multicategorical variable with three levels. How does one conduct a mediation analysis in such a design?

Mediation of the effect of a multicategorical independent variable

Does perceived response appropriateness mediate this effect?

The causal steps approach

	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	$\bar{Y}^*$
No protest (n = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (n = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (n = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

$\bar{Y}^*$ = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

From a single-factor ANOVA, the effect of experimental condition **on**…liking: $F(2,126) = 3.055$, $p = .051$ (**total effect**)
…response appropriateness: $F(2,126) = 22.219$, $p < .001$ (‘**a**’ **effect**)
From a single-factor ANCOVA…the relationship between perceived response appropriateness and liking is positive, $b = 0.412$, $p < 0.001$ (‘***b***’ **effect**), and…
…the effect of condition on liking disappears after controlling for response appropriateness $F(2,125) = 0.729$, $p = .485$ (**direct effect**)

This approach has all the problems of the causal steps (“Baron and Kenny”) approach. There is a better way.

Representing a multicategorical predictor in a linear model

Predictor variables in a linear model must be quantitative or dichotomous (i.e., categorical with only two values). What if we want to include multicategorical variables? **How do we proceed?**

Any multicategorical variable with k categories can be represented with $k – 1$ Variables in a regression model. For example, we can code membership in a category with a set of *dummy variables* and all of these $k – 1$ dummy variables in the model.

Indicator Coding (AKA “Dummy Coding”)

Set D_1 to 1 for cases in category 1, 0 otherwise
D_2 to 1 for cases in category 2, 0 otherwise
.
.
$D_{(k-1)}$ to 1 for cases in category $k – 1$, 0 otherwise

Category k is called the “reference category,” for reasons that will be clear soon. It is represented here in the coding system, but it doesn’t seem so.

Indicator coding condition

```
frequencies variables = cond.
```

```
proc freq data=protest;tables cond;run;
```

COND: experimental condition

			Frequency	Percent	Valid Percent	Cumulative Percent
cond = 0	Valid	no protest	41	31.8	31.8	31.8
cond = 1		individual	43	33.3	33.3	65.1
cond = 2		collective	45	34.9	34.9	100.0
		Total	129	100.0	100.0	

One possible dummy variable coding system for condition (k = 3 categories):

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

The “reference category” is the one with zeros on all $k – 1$ dummy variables. In this example, those told the lawyer did not protest are the reference category.

Constructing Indicator variables

There is a variety of ways of constructing indicator codes in a computing platform, each with Its dangers, assumptions, and conveniences.

		D_1	D_2
cond = 0	**No protest**	0	0
cond = 1	**Individual**	1	0
cond = 2	**Collective**	0	1

Here is one way

```
compute d1 = 0.
compute d2 = 0.
if (cond = 1) d1 = 1.
if (cond = 2) d2 = 1.
execute.
```

```
data protest;set protest;
d1=0;d2=0;
if (cond=1) then d1 = 1;
if (cond=2) then d2 = 1;
run;
```

This approach can be dangerous. Any cases missing on condition will be coded as if they were assigned to the no protest condition. Use this approach with caution. As a general rule, know your data before you start manipulating it.

Constructing Indicator variables

There is a variety of ways of constructing indicator codes in a computing platform, each with its dangers, assumptions, and conveniences.

		D_1	D_2
cond = 0	**No protest**	0	0
cond = 1	**Individual**	1	0
cond = 2	**Collective**	0	1

Here is a safer approach:

```
compute d1 = (cond=1).
compute d2 = (cond=2).
execute.
```

```
data protest;set protest;
d1 = (cond=1);
d2 = (cond=2);
if (cond=.) then d1=.;
if (cond=.) then d2=.;
run;
```

In SPSS, this is very efficient. SAS requires the explicit coding of missing data as such. The SPSS version will leave cases missing on cond missing on d1 and d2.

Estimating liking from experimental condition using regression

```
compute d1 = (cond=1).
compute d2 = (cond=2).
regression/dep = liking/method = enter d1 d2.
```

```
data protest;set protest;
d1 = (cond=1);d2 = (cond=2);run;
proc reg data=protest;model liking = d1 d2;run;
```

We know there are no missing data on **cond**, so this is ok.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215[a]	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6.523	2	3.262	3.055	.051[b]
	Residual	134.515	126	1.068		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, d1

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	5.310	.161		32.908	.000
	d1	.516	.226	.233	2.287	.024
	d2	.443	.223	.202	1.986	.049

a. Dependent Variable: LIKING: liking of the target

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

The model reproduces the group means

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

No protest $\hat{Y} = 5.310 + 0.516(0) + 0.443(0) = 5.310 = \bar{Y}_{NP}$

Individual protest $\hat{Y} = 5.310 + 0.516(1) + 0.443(0) = 5.826 = \bar{Y}_{IP}$

Collective protest $\hat{Y} = 5.310 + 0.516(0) + 0.443(1) = 5.753 = \bar{Y}_{CP}$

Report

LIKING: liking of the target

COND: experimental condition	Mean	N	Std. Deviation
no protest	5.3102	41	1.30158
individual	5.8260	43	.81943
collective	5.7533	45	.93601
Total	5.6367	129	1.04970

Interpretation of the coefficients

$$\hat{Y} = b_0 + b_1D_1 + b_2D_2$$

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

b_0 = 5.310 This is the mean liking among those assigned to the no protest condition (D_1 = 0, D_2 = 0).

b_1 = 0.516 This is the mean difference in liking between those in the individual protest condition (D_1 = 1, D_2 = 0) and those in the no protest condition (D_1 = 0, D_2 = 0)

$$b_1 = \bar{Y}_{IP} - \bar{Y}_{NP} = 5.826 - 5.310 = 0.516$$

b_2 = 0.443 This is the mean difference in liking between those in the collective protest condition (D_1 = 0, D_2 = 1) and those in the no protest condition (D_1 = 0, D_2 = 0)

$$b_2 = \bar{Y}_{CP} - \bar{Y}_{NP} = 5.753 - 5.310 = 0.443$$

When D_1 and D_2 are indicator codes constructed in this fashion, b_1 estimates the mean difference in Y between the group coded by D_1 and the reference group, and b_2 estimates the mean difference in Y between the group coded by D_2 and the reference group.

Statistical inference

We are estimating the coefficients of a model of the form

$$\hat{Y} = \widetilde{b_0} + \widetilde{b_1}D_1 + \widetilde{b_2}D_2$$

If there is no actual difference, on average, between these groups on *Y*, this implies that both "true" regression coefficients $\widetilde{b_1}$ and $\widetilde{b_2}$ are both equal to zero. The null hypothesis can be tested by converting the obtained R^2 to an *F*-ratio and then deriving a *p*-value.

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215[a]	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

H_0: $\widetilde{b_1} = \widetilde{b_2} = 0$

H_a: at least one is different from zero

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6.523	2	3.262	3.055	.051[b]
	Residual	134.515	126	1.068		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, d1

$$F(k-1, df_{residual}) = \frac{df_{residual}R^2}{(k-1)(1-R^2)}$$

$$F(2,126) = \frac{126(0.046)}{2(1-0.046)}$$

$$F(2,126) = 3.055$$

F(2,126) = 3.055, *p* = .051. Reject H_0. The three group means differ from each other by more than be explained by just 'chance'. Compare this to the one-way ANOVA from earlier.

Statistical inference

We are estimating the coefficients of a model of the form

$$\hat{Y} = \widetilde{b_0} + \widetilde{b_1}D_1 + \widetilde{b_2}D_2$$

b_1 and b_2 can also be used to test hypotheses about differences between groups—specifically, between the group a dummy variable codes and the reference group.

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	5.310	.161		32.908	.000
	d1	.516	.226	.233	2.287	.024
	d2	.443	.223	.202	1.986	.049

a. Dependent Variable: LIKING: liking of the target

H_0: $\widetilde{b_1} = 0$

H_a: $\widetilde{b_1} \neq 0$

b_1 = 0.516, *t*(126) = 2.287, *p* = .024

Those told she individually protested liked her more on average than those told she did not protest.

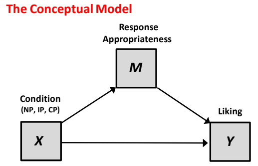
H_0: $\widetilde{b_2} = 0$

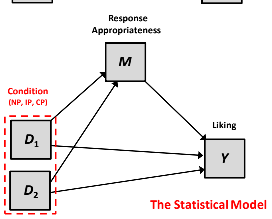
H_a: $\widetilde{b_2} \neq 0$

b_2 = 0.443, *t*(126) = 1.986, *p* = .049

Those told she collectively protested liked her more on average than those told she did not protest.

Mediation analysis with a multicategorical independent variable






The British Psychological Society

Expert Tutorial

Statistical mediation analysis with a multicategorical independent variable

Andrew F. Hayes[1*] and Kristopher J. Preacher[2]

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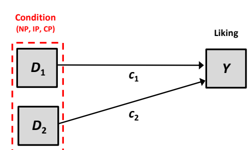
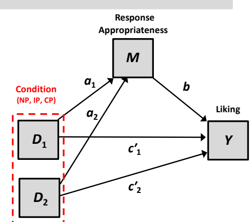
[2]Department of Psychology and Human Development, Vanderbilt University, Nashville, Tennessee, USA

1. Introduction

Hayes and Preacher (2014, BJMSP) available in your materials.

(Relative) total, direct, and indirect effects





c_1 and c_2: *Relative* total effects of experimental condition on liking

c'_1 and c'_2: *Relative* direct effects of experimental condition on liking

a_1b and a_2b: *Relative* indirect effects of condition on liking through perceived response appropriateness.

$$c_1 = c'_1 + a_1b; \text{ therefore, } a_1b = c_1 - c'_1$$

$$c_2 = c'_2 + a_2b; \text{ therefore, } a_2b = c_2 - c'_2$$

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Coding the groups

We'll use indicator codes setting the no protest condition to the reference group. Condition (variable name COND) is coded 0 (no protest condition), 1 (individual protest condition), and 2 (collective protest condition).

Condition	D_1	D_2
No protest	0	0
Individual	1	0
Collective	0	1

```
compute d1 = (cond=1).
compute d2 = (cond=2).
execute.
```

```
data protest;set protest;
d1 = (cond=1);
d2 = (cond=2);
if (cond=.) then d1=.;
if (cond=.) then d2=.;
run;
```

So effects for D_1 will compare individual protest to no protest, and effects for D_2 will compare collective protest to no protest.

The total effect of experimental condition on liking (*c* paths)

```
regression/dep = liking/method = enter d1 d2.
```

We did this already!

```
proc reg data=protest;model liking=d1 d2;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.215[a]	.046	.031	1.03324

a. Predictors: (Constant), d2, d1

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	6.523	2	3.262	3.055	.051[b]
	Residual	134.515	126	1.068		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

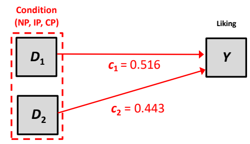
b. Predictors: (Constant), d2, d1

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	5.310	.161		32.908	.000
	d1	.516	.226	.233	2.287	.024
	d2	.443	.223	.202	1.986	.049

a. Dependent Variable: LIKING: liking of the target

Relative total effects



Relative to those told she did not protest, those told she individually protested liked her more on average (c_1 = 0.516, p = .024). Relative to those told she did not protest, those told she collectively protested also liked her more on average (c_2 = 0.443, p = .049).

The effect of experimental condition on perceived response appropriateness (*a* paths)

```
regression/dep = respappr/method = enter d1 d2.
```

```
proc reg data=protest;model respappr=d1 d2;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.511[a]	.261	.249	1.16829

a. Predictors: (Constant), d2, d1

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	60.653	2	30.327	22.219	.000[b]
	Residual	171.977	126	1.365		
	Total	232.631	128			

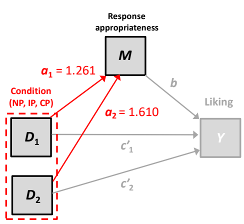
a. Dependent Variable: RESPAPPR: appropriateness of response

b. Predictors: (Constant), d2, d1

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.884	.182		21.288	.000
	d1	1.261	.255	.443	4.946	.000
	d2	1.610	.252	.572	6.384	.000

a. Dependent Variable: RESPAPPR: appropriateness of response



Relative to those told she did not protest, those told she individually protested felt her response was more appropriate on average (a_1 = 1.261 p < .001). Relative to those told she did not protest, those told she collectively protested felt her response was more appropriate on average (a_2 = 1.610, p <.001).

The direct effect of condition on liking (*c′* paths)

along with the effect of response appropriateness on liking (*b* path)

```
regression/dep = liking/method = enter respappr d1 d2.
```

```
proc reg data=protest;model liking=respappr d1 d2;run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.503[a]	.253	.235	.91738

a. Predictors: (Constant), d2, RESPAPPR: appropriateness of response, d1

ANOVA[a]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	35.703	3	11.901	14.123	.000[b]
	Residual	105.336	125	.843		
	Total	141.039	128			

a. Dependent Variable: LIKING: liking of the target

b. Predictors: (Constant), d2, RESPAPPR: appropriateness of response, d1

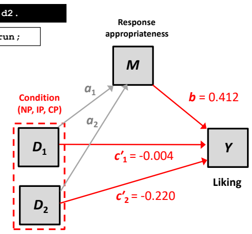
Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.710	.307		12.071	.000
	RESPAPPR: appropriateness of response	.412	.070	.529	5.884	.000
	d1	-.004	.219	-.002	-.017	.987
	d2	-.220	.228	-.100	-.966	.336

a. Dependent Variable: LIKING: liking of the target

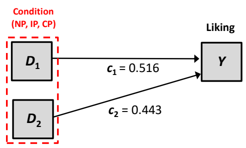
b path

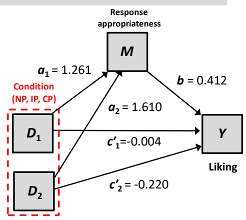
Relative direct effects



Controlling for perceived responses appropriateness, those told she individually protested did not like her any more, on average, than those told she did not protest (c'_1 = -0.004, p = .987). And those told she collectively protested did not like her any more, on average, than those told she did not protest (c'_2 = -0.220, p = .336). Holding condition constant, those who perceived her behavior as relatively more appropriate likely her relatively more (b = 0.412).

(Relative) total, direct, and indirect effects





c_1 and c_2: *Relative* total effects of condition on liking (c_1 = 0.516, c_2 = 0.443).
c'_1 and c'_2: *Relative* direct effects of condition on liking (c'_1 = -0.004, c'_2 = -0.220).
a_1b and a_2b: *Relative* indirect effects of condition on liking through perceived response appropriateness
a_1b = 1.261(0.412) = 0.520, a_2b = 1.610(0.412) = 0.663

$c_1 = c'_1 + a_1b$: 0.516 = -0.004 + 1.261(0.412) = -0.004 + 0.520
$c_2 = c'_2 + a_2b$: 0.443 = -0.220 + 1.610(0.412) = -0.220 + 0.663

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Relative total, direct, and indirect effects

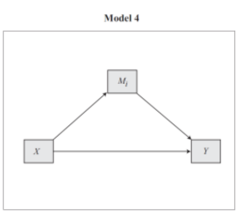
	Perceived Response Appropriateness (M)		Liking (Y)		
	M	SD	Y	SD	$\bar{Y}^*$
No protest (n = 41)	3.884	1.457	5.310	1.302	5.715
Individual protest (n = 43)	5.145	1.075	5.826	0.819	5.711
Collective protest (n = 45)	5.494	0.936	5.753	0.936	5.495
All groups combined	4.866	1.348	5.637	1.050	5.637

a_1 = 1.261
a_2 = 1.610
c_1 = 0.516
c_2 = 0.443
c'_1=-0.004
c'_2=-0.220

$\bar{Y}^*$ = adjusted mean, adjusted to the sample mean of perceived response appropriateness.

$c_1 = c'_1 + a_1b$: 0.516 = -0.004 + 1.261(0.412) = -0.004 + 0.520
$c_2 = c'_2 + a_2b$: 0.443 = -0.220 + 1.610(0.412) = -0.220 + 0.663

Estimation using PROCESS



PROCESS V3 has an option for specifying X as a multicategorical variable with up to 9 categories. Four options are available for coding the groups.

MCX=1 tells PROCESS that X is a multicategorical variable and to use dummy coding to represent the groups. Other coding options are available. See the PROCESS documentation.

MCX	Coding system
1	Simple dummy coding
2	Sequential ("adjacent categories") coding
3	Helmert coding
4	Effect coding

```
process y=liking/m=respappr/x=cond/mcx=1/model=4/total=1/boot=10000.
```

```
%process (data=protest,y=liking,m=respappr,x=cond,mcx=1,model=4,
    total=1,boot=10000);
```

PROCESS output

```
Model  : 4
    Y  : liking
    X  : cond
    M  : respappr

Sample
Size:  129

Coding of categorical X variable for analysis:
   cond      X1      X2
   .000    .000    .000
  1.000   1.000    .000
  2.000    .000   1.000
```

X1 codes individual protest, X2 codes collective protest.
No protest Is the reference group. (The group with the numerically smallest value on the categorical variable is always the reference)

```
**************************************************************************
OUTCOME VARIABLE:
 respappr

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5106      .2607     1.3649    22.2190     2.0000   126.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.8841      .1825    21.2881      .0000     3.5231     4.2452
X1           1.2612      .2550     4.9456      .0000      .7565     1.7659
X2           1.6103      .2522     6.3842      .0000     1.1111     2.1095
```

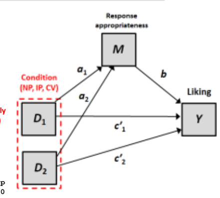
$\hat{M} = 3.884 + 1.261D_1 + 1.610D_2$

a_1 path
a_2 path

```
**************************************************************************
OUTCOME VARIABLE:
 liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5031      .2531      .8427    14.1225     3.0000   125.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.7103      .3074    12.0711      .0000     3.1020     4.3187
X1           -.0037      .2190     -.0169      .9865     -.4371      .4297
X2           -.2202      .2280     -.9658      .3360     -.6715      .2310
respappr      .4119      .0700     5.8844      .0000      .2734      .5504
```

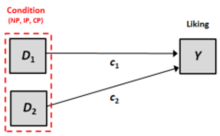
$\hat{Y} = 3.710 - 0.004D_1 - 0.220D_2 + 0.412M$

c'_1 path
c'_2 path
b path



Output L

PROCESS output




```
************************** TOTAL EFFECT MODEL ****************************
OUTCOME VARIABLE:
 liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .2151      .0463     1.0676     3.0552     2.0000   126.0000      .0506

Model
              coeff         se          t          p       LLCI       ULCI
constant     5.3102      .1614    32.9083      .0000     4.9909     5.6296
X1            .5158      .2255     2.2870      .0239      .0695      .9621
X2            .4431      .2231     1.9863      .0492      .0016      .8845
```

c_1 path

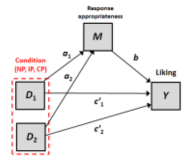
c_2 path

$$\hat{Y} = 5.310 + 0.516D_1 + 0.443D_2$$

Output M

PROCESS output

```
************* TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Relative total effects of X on Y:
       Effect         se          t          p       LLCI       ULCI
X1      .5158      .2255     2.2870      .0239      .0695      .9621
X2      .4431      .2231     1.9863      .0492      .0016      .8845

Omnibus test of total effect of X on Y:
    R2-chng          F        df1        df2          p
      .0463     3.0552     2.0000   126.0000      .0506
----------

Relative direct effects of X on Y
       Effect         se          t          p       LLCI       ULCI
X1     -.0037      .2190     -.0169      .9865     -.4371      .4297
X2     -.2202      .2280     -.9658      .3360     -.6715      .2310

Omnibus test of direct effect of X on Y:
    R2-chng          F        df1        df2          p
      .0087      .7286     2.0000   125.0000      .4846
----------

Relative indirect effects of X on Y

 cond        ->    respappr    ->    liking

       Effect     BootSE   BootLLCI   BootULCI
X1      .5195      .1524      .2590      .8536
X2      .6633      .1671      .3684     1.0187
```



Indirect effect a_1b with bootstrap confidence interval

Indirect effect a_2b with bootstrap confidence interval

Output M

Those told she individually protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (point estimate = 0.520, 95% CI: 0.259 to 0.854). There is no direct effect of individually protesting on liking. Those told she collectively protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (point estimate= 0.663, 95% CI: 0.368 to 1.019). There is no direct effect of collectively protesting on liking.

Omnibus inference

PROCESS gives us tests of the k-1 relative total effects. It also provides a test of equality of the k group means on Y---the "omnibus" total effect. This is equivalent to a single-factor ANOVA.

```
************* TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Relative total effects of X on Y:
       Effect         se          t          p       LLCI       ULCI
X1      .5158      .2255     2.2870      .0239      .0695      .9621
X2      .4431      .2231     1.9863      .0492      .0016      .8845

Omnibus test of total effect of X on Y:
    R2-chng          F        df1        df2          p
      .0463     3.0552     2.0000   126.0000      .0506
----------

Relative direct effects of X on Y
       Effect         se          t          p       LLCI       ULCI
X1     -.0037      .2190     -.0169      .9865     -.4371      .4297
X2     -.2202      .2280     -.9658      .3360     -.6715      .2310

Omnibus test of direct effect of X on Y:
    R2-chng          F        df1        df2          p
      .0087      .7286     2.0000   125.0000      .4846
----------

Relative indirect effects of X on Y

 cond        ->    respappr    ->    liking

       Effect     BootSE   BootLLCI   BootULCI
X1      .5195      .1524      .2590      .8536
X2      .6633      .1671      .3684     1.0187
```

Output M

Test of the "omnibus" total effect.
The three conditions differ on average in liking of the attorney , $F(2,126) = 3.055$, $p = .051$.

Omnibus inference

PROCESS gives us tests of the $k - 1$ relative direct effects. It also provides a test of equality of the k group adjusted means on Y when the mediator is held constant---the "omnibus" direct effect. This is equivalent to a single-factor ANCOVA.

Output M

```
************* TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Relative total effects of X on Y:
       Effect         se          t          p       LLCI       ULCI
X1      .5158      .2255     2.2870      .0239      .0695      .9621
X2      .4431      .2231     1.9863      .0492      .0016      .8845

Omnibus test of total effect of X on Y:
    R2-chng          F        df1        df2          p
      .0463     3.0552     2.0000   126.0000      .0506
----------

Relative direct effects of X on Y
       Effect         se          t          p       LLCI       ULCI
X1     -.0037      .2190     -.0169      .9865     -.4371      .4297
X2     -.2202      .2280     -.9658      .3360     -.6715      .2310

Omnibus test of direct effect of X on Y:
    R2-chng          F        df1        df2          p
      .0087      .7286     2.0000   125.0000      .4846
----------

Relative indirect effects of X on Y

 cond        ->    respappr    ->    liking

       Effect     BootSE   BootLLCI   BootULCI
X1      .5195      .1524      .2590      .8536
X2      .6633      .1671      .3684     1.0187
```

Test of the "omnibus" direct effect.
The three conditions do not differ on average in how much they liked her after accounting for group differences in perceived response appropriateness $\Delta R^2 = 0.009$, $F(2,125) = 0.729$, $p = .485$. ΔR^2 is the change in R^2 when the $k - 1$ variables coding group are added to the model of Y that already contains the mediator.

Omnibus inference about the indirect effect

- The omnibus tests for the total and direct effect of *X* are not dependent on the system used for coding the groups, even though the relative direct and total effects are.
- The rule that *X* indirectly affects *Y* if at least one relative indirect effect is different from zero means our conclusion will depend on the system used for coding groups, since the relative indirect effects are dependent on that choice.
- If all of the bootstrap confidence intervals for the relative indirect effects straddle zero, that does NOT mean *X* does not indirectly affect *Y*. It could be that a different coding choice produces a different outcome.
- The rule can confirm that *X* indirect affects *Y* if at least one relative indirect effect is different from zero. But a failure to meet this criterion does not disconfirm the existence of an indirect effect of *X* on *Y* through *M*.
- Moral: Choose your coding system wisely, so that it produces relative indirect effects you care about and that are sensitive to the question you are trying to answer.
- There are omnibus tests of the indirect effect that are not sensitive to the coding choice. This must be done in SEM and can require problematic assumptions.

A different coding system

Other systems for coding groups can be used. For instance, we might instead choose to estimate the direct and indirect effects of protesting (regardless of form) relative to not, and the effects of collectively protesting relative to individually protesting.

	Condition	D_1	D_2
cond = 0	No protest	-2/3	0
cond = 1	Individual	1/3	-1/2
cond = 2	Collective	1/3	1/2

You may recognize these as two *orthogonal contrasts*. It is also called "Helmert coding" when the multicategorical variable is ordinal.

```
if (cond = 0) d1 = -2/3.
if (cond > 0) d1 = 1/3.
if (cond = 0) d2 = 0.
if (cond = 1) d2 = -1/2.
if (cond = 2) d2 = 1/2.
```

```
data protest;set protest;
  if (cond = 0) then d1 = -2/3;
  if (cond > 0) then d1 = 1/3;
  if (cond = 0) then d2 = 0;
  if (cond = 1) then d2 = -1/2;
  if (cond = 2) then d2 = 1/2;
run;
```

Effects for D_1 will compare no protest to the average of the two protest conditions, and effects for D_2 will compare collective protest to individual protest.

The total effect of experimental condition on liking (*c* paths)

```
regression/dep = liking/method = enter d1 d2.
```

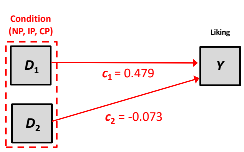
```
proc reg data=protest;model liking=d1 d2;run;
```

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	5.630	.091		61.841	.000
	d1	.479	.195	.214	2.454	.016
	d2	-.073	.220	-.029	-.330	.742

a. Dependent Variable: LIKING: liking of the target

Relative total effects



$$c_1 = \frac{(\bar{Y}_{IP} + \bar{Y}_{CP})}{2} - \bar{Y}_{NP} = \frac{(5.826 + 5.753)}{2} - 5.310 = 5.789 - 5.310 = 0.479$$

$$c_2 = \bar{Y}_{CP} - \bar{Y}_{IP} = 5.753 - 5.826 = -0.073$$

Relative to those told she did not protest, those told she protested liked her more on average (c_1 = 0.479, p = .016). Those told she collectively protested did not differ, on average, in how much they liked her relative to those told she individually protested (c_2 = -0.073, p = .742).

The effect of experimental condition on perceived response appropriateness (*a* paths)

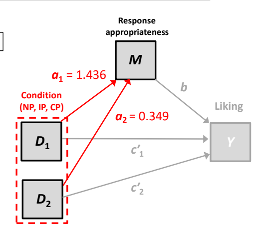
```
regression/dep = respappr/method = enter d1 d2.
```

```
proc reg data=protest;model respappr=d1 d2;run;
```

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	4.841	.103		47.032	.000
	d1	1.436	.221	.498	6.499	.000
	d2	.349	.249	.107	1.401	.164

a. Dependent Variable: RESPAPPR: appropriateness of response



$$a_1 = \frac{(\bar{M}_{IP} + \bar{M}_{CP})}{2} - \bar{M}_{NP} = \frac{(5.145 + 5.494)}{2} - 3.884 = 5.320 - 3.884 = 1.436$$

$$a_2 = \bar{M}_{CP} - \bar{M}_{IP} = 5.494 - 5.145 = 0.349$$

Relative to those told she did not protest, those told she protested felt this was a more appropriate response, on average (a_1 = 1.436, p = .016). Those told she collectively protested did not perceive this as any more or less appropriate, on average, relative to those told she individually protested (a_2 = 0.349, p = .164).

The direct effect of condition on liking (c' paths)

along with the effect of response appropriateness on liking (b path)

```
regression/dep = liking/method = enter respappr d1 d2.
```

```
proc reg data=protest;model liking=respappr d1 d2;run;
```

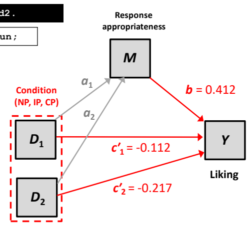
Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.636	.348		10.435	.000
	RESPAPPR: appropriateness of response	.412	.070	.529	[illegible]	.000
	d1	-.112	.201	-.050	-.558	[illegible]
	d2	-.217	.197	-.085	-1.097	.275

a. Dependent Variable: LIKING: liking of the target

b path

Relative direct effects

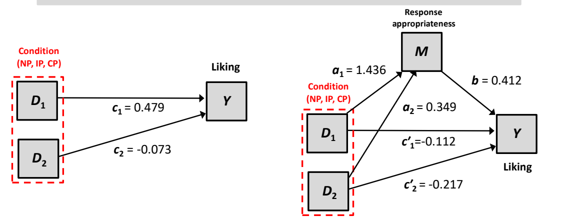


$$c'_1 = \frac{(\bar{Y}^*_{IP} + \bar{Y}^*_{CP})}{2} - \bar{Y}^*_{NP} = \frac{(5.711 + 5.495)}{2} - 5.715 = 5.603 - 5.715 = -0.112$$

$$c'_2 = \bar{Y}^*_{CP} - \bar{Y}^*_{IP} = 5.495 - 5.711 = -0.217$$

Controlling for perceived responses appropriateness, on average, those told she protested did not like her any more than those told she did not protest (c'_1 = -0.112, p = .578). And those told she collectively protested did not like her any more, on average, than those told she individually protested (c'_2 = -0.217, p = .275). Holding condition constant, those who perceived her behavior as relatively more appropriate likely her relatively more (b = 0.412).

(Relative) total, direct, and indirect effects



c_1 and c_2: *Relative* total effects of condition on liking (c_1 = 0.479, c_2 = -0.073).

c'_1 and c'_2: *Relative* direct effects of condition on liking (c'_1 = -0.112, c'_2 = -0.217).

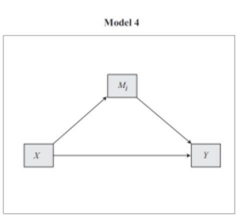
a_1b and a_2b: *Relative* indirect effects of condition on liking through perceived response appropriateness a_1b = 1.436(0.412) = 0.591, a_2b = 0.349(0.412) = 0.144

$c_1 = c'_1 + a_1b$: 0.479 = -0.112 + 1.436(0.412) = -0.112 + 0.591

$c_2 = c'_2 + a_2b$: -0.073 = -0.217 + 0.349(0.412) = -0.217 + 0.144

The relative total effects partition perfectly into relative direct and relative indirect effects. The relative indirect effects are the relative total effects minus the relative direct effects.

Estimation using PROCESS



PROCESS has an option for specifying X as a multicategorical variable with up to 9 categories. Four options are available for coding the groups.

MCX=3 tells PROCESS that X is a multicategorical variable and to use "Helmert coding" to represent the groups. This is equivalent to the orthogonal contrasts we set up manually in this example.

MCX	Coding system
1	Simple dummy coding
2	Sequential ("adjacent categories") coding
3	Helmert coding
4	Effect coding

```
process y=liking/m=respappr/x=cond/mcx=3/model=4/total=1/boot=10000.
```

```
%process (data=protest,y=liking,m=respappr,x=cond,mcx=3,model=4,
    total=1,boot=10000);
```

PROCESS output

```
Model  : 4
    Y  : liking
    X  : cond
    M  : respappr

Sample
Size:  129

Coding of categorical X variable for analysis:
   cond     X1      X2
   .000   -.667    .000
  1.000    .333   -.500
  2.000    .333    .500

**************************************************************************
OUTCOME VARIABLE:
 respappr

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5106      .2607     1.3649    22.2190     2.0000   126.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     4.8413      .1029    47.0321      .0000     4.6376     5.0450
X1           1.4358      .2209     6.4988      .0000      .9985     1.8730
X2            .3491      .2491     1.4012      .1636     -.1440      .8421

**************************************************************************
OUTCOME VARIABLE:
 liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5031      .2531      .8427    14.1225     3.0000   125.0000      .0000

Model
              coeff         se          t          p       LLCI       ULCI
constant     3.6357      .3484    10.4351      .0000     2.9461     4.3252
X1           -.1120      .2006     -.5581      .5778     -.5089      .2850
X2           -.2165      .1973    -1.0974      .2746     -.6070      .1739
respappr      .4119      .0700     5.8844      .0000      .2734      .5504
```

X1 codes protest versus no protest. X2 codes collective versus individual protest.

$$\hat{M} = 4.841 + 1.436D_1 + 0.349D_2$$

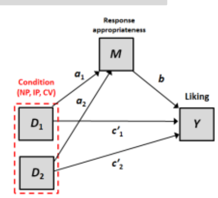
a_1 path

a_2 path

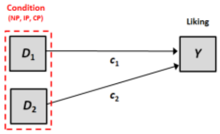
$$\hat{Y} = 3.636 - 0.112D_1 - 0.217D_2 + 0.412M$$

c'_1 path

c'_2 path



PROCESS output




```
************************** TOTAL EFFECT MODEL ****************************
OUTCOME VARIABLE:
 liking

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .2151      .0463     1.0676     3.0552     2.0000   126.0000      .0506

Model
              coeff         se          t          p       LLCI       ULCI
constant     5.6299      .0910    61.8414      .0000     5.4497     5.8100
X1            .4794      .1954     2.4538      .0155      .0928      .8661
X2           -.0727      .2203     -.3300      .7419     -.5088      .3633
```

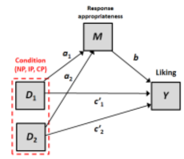
c_1 path

c_2 path

$$\boxed{\hat{Y} = 5.630 + 0.479D_1 - 0.073D_2}$$

PROCESS output

```
************** TOTAL, DIRECT, AND INDIRECT EFFECTS OF X ON Y **************

Relative total effects of X on Y:
       Effect         se          t          p       LLCI       ULCI
X1      .4794      .1954     2.4538      .0155      .0928      .8661
X2     -.0727      .2203     -.3300      .7419     -.5088      .3633

Omnibus test of total effect of X on Y:
    R2-chng          F        df1        df2          p
      .0463     3.0552     2.0000   126.0000      .0506
----------

Relative direct effects of X on Y
       Effect         se          t          p       LLCI       ULCI
X1     -.1120      .2006     -.5581      .5778     -.5089      .2850
X2     -.2165      .1973    -1.0974      .2746     -.6070      .1739

Omnibus test of direct effect of X on Y:
    R2-chng          F        df1        df2          p
      .0087      .7286     2.0000   125.0000      .4846
----------

Relative indirect effects of X on Y

 cond       ->    respappr    ->    liking

       Effect     BootSE   BootLLCI   BootULCI
X1      .5914      .1529      .3273      .9221
X2      .1438      .0937     -.0261      .3446
```



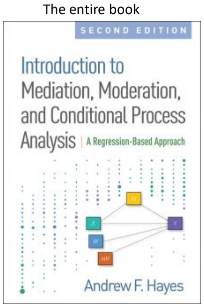
Indirect effect a_1b with bootstrap confidence interval

Indirect effect a_2b with bootstrap confidence interval

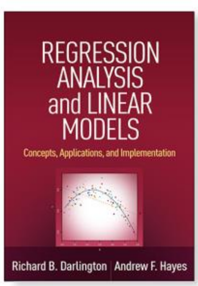
Those told she protested liked her more than those told she did not protest because protesting was perceived as more appropriate than not, which in turn enhanced liking (relative indirect effect = 0.591, 95% CI: 0.327 to 0.922). The was no direct effect of protesting relative to not on liking (relative direct effect = -0.112, *p* = .578). The relative direct and indirect effects of collectively protesting relative to individually protesting were not significantly different from zero (relative indirect effect = 0.144, 95% CI = -0.026 to 0.345; relative direct effect = -0.217, *p* = .274)

Where to learn more

The entire book



Chapters 13, 14, and 15



http://www.guilford.com/

MMCPA II

Typically Taught in GSERM St. Gallen

- Review of the fundamentals of mediation, moderation, and conditional process analysis.
- Testing whether an indirect effect is moderated and probing moderation of indirect effects.
- Partial and conditional moderated mediation.
- Mediation analysis with a multicategorical independent variable.
- Moderation analysis with a multicategorical (3 or more groups) independent variable or moderator.
- Conditional process analysis with a multicategorical independent variable
- Moderation of indirect effects in the serial mediation model.
- New features available in PROCESS v3.0, such as how to modify a numbered model or customize your own model.

Sample Size Planning

How many people do I need to have reasonable power to detect an indirect effect?

Need approximate estimates of all paths involved in the analysis (e.g., *a*, *b*, and *c* for mediation).

Previous simulation work:

- Fritz & MacKinnon (2007)
- Hayes & Scharkow (2013)
- Biesanz, Falk, & Savalei (2016) normal and nonnormal data
- Williams & MacKinnon (2008) serial models and

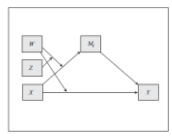
Monte Carlo Simulations:

- Thoemmes , MacKinnon & Reiser (2010) – specific to mediation
- Sigal & Chalmers (2016)
  - Really great R package called 'simdesign' which can be used for Monte Carlo Simulations to understand power.

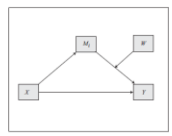
Constructing and editing models in PROCESS

Historically, PROCESS has operated by a model number system. The model numbers and the models those numbers represent can be found in the documentation. Choose the model number that corresponds to the model you would like to estimate.

Model 13



Model 14



Many of the preprogrammed numbered models you will find useful.

But what if the model you want to estimate does not correspond to any preprogrammed model represented by a model number?

Version 2: Too bad. Nothing you can do about it (unless you know some tricks).

Version 3: Within certain constraints, you can create your own model from scratch, or edit an existing model number to make it correspond to the model you want to estimate.

Putting it all together

This system allows for the construction of some very complex models.

B matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	1	0	■	■
M_3	1	0	0	■
Y	1	1	1	1

W matrix

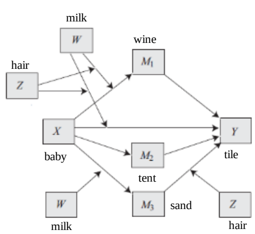
	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	1	0	0	■
Y	0	0	0	0

Z matrix

	X	M_1	M_2	M_3
M_1	0	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	0	0	0	1

WZ matrix

	X	M_1	M_2	M_3
M_1	1	■	■	■
M_2	0	0	■	■
M_3	0	0	0	■
Y	1	0	0	0

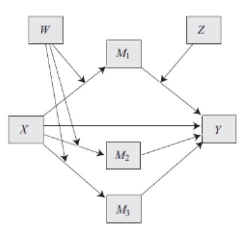



```
process y=tile/x=baby/m=wine tent sand/w=milk/z=hair
  /bmatrix=1,1,0,1,0,0,1,1,1,1/wmatrix=0,0,0,1,0,0,0,0,0,0
  /zmatrix=0,0,0,0,0,0,0,0,0,1/wzmatrix=1,0,0,0,0,0,1,0,0,0.
```

Editing a numbered model

Many preprogrammed numbered models are likely to be close to the model you want to estimate. You can edit a modeled number, adding a desired interaction, or removing one you don't want. This is done by reprogramming the *W*, *Z*, and/or *WZ* matrices.

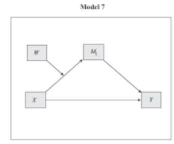
Example



NOTE: When using a model number, the *B* matrix cannot be edited.

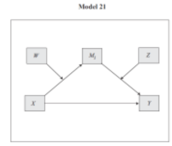
This is like model 7, except model 7 doesn't include moderation of any *M* to *Y* paths.

Model 7



This is like model 21, except model 21 would includes moderation by *Z* of the M_2 and M_3 to *Y* paths as well as the M_1 to *Y* path.

Model 21



The MATRICES statement

If you want to check to make sure you have programmed the matrices correctly, or you want to see what the matrices of a preprogrammed model look like, add **matrices=1** to a PROCESS command.

```
process y=tile/x=baby/m=wine tent
  sand/w=milk/z=hair/model=7/
  zmatrix=0,0,0,0,0,0,0,1,0,0
  /matrices=1.
```

```
********** MODEL DEFINITION MATRICES **********

BMATRIX: Paths freely estimated (1) and fixed
to zero (0):
     baby wine tent sand
wine  1
tent  1    0
sand  1    0    0
tile  1    1    1    1

WMATRIX: Paths moderated (1) and not moderated
(0) by W:
     baby wine tent sand
wine  1
tent  1    0
sand  1    0    0
tile  0    0    0    0

ZMATRIX: Paths moderated (1) and not moderated
(0) by Z:
     baby wine tent sand
wine  0
tent  0    0
sand  0    0    0
tile  0    1    0    0
```

Matrices that don't appear in the output have zeros in all cells. If your model includes covariates, the *C* matrix will appear here too.

Pertinent Publications

Rockwood, N. J., & Hayes, A. F. (2018). Mediation, moderation, and conditional process analysis: Regression-based approaches for clinical research. Draft submitted and to appear in A. G. C. Wright and M. N. Hallquist (Eds.) *Handbook of research methods in clinical psychology.* Cambridge University Press.

Rockwood, N. J., & Hayes, A. F. (2018). Multilevel mediation analysis. Draft submitted and to appear in A. A. O'Connell, D. B. McCoach, and B. Bell (Eds). *Multilevel modeling methods with introductory and advanced applications.* Information Age Publishing.

Hayes, A. F. (2018). Partial, conditional, and moderated moderated mediation: Quantification, inference, and interpretation. *Communication Monographs*, *85*, 4-40. [PDF]

Hayes, A. F., & Rockwood, N. J. (2017). Regression-based statistical mediation and moderation analysis in clinical research: Observations, recommendations, and implementation. *Behaviour Research and Therapy, 98*, 39-57. [paper and data]

Hayes, A. F., Montoya, A. K., & Rockwood, N. J. (2017). The analysis of mechanisms and their contingencies: PROCESS versus structural equation modeling. *Australasian Marketing Journal, 25,* 76-81. [PDF and Mplus code]

Hayes, A. F., & Montoya, A. K. (2017). A tutorial on testing, visualizing, and probing interaction involving a multicategorical variable in linear regression analysis. *Communication Methods and Measures, 11*, 1-30 [paper and data]

Montoya, A. K., & Hayes, A. F. (2017). Two condition within-participant statistical mediation analysis: A path-analytic framework. *Psychological Methods, 22*, 6-27. [paper]

Hayes, A. F. (2015). *An index and test of linear moderated mediation*. *Multivariate Behavioral Research, 50,* 1-22.
Hayes, A. F., & Preacher, K. J. (2014). Statistical mediation analysis with a multicategorical independent variable. *British Journal of Mathematical and Statistical Psychology*, *67*, 451-470.

Hayes, A. F., & Scharkow, M. (2013). The relative trustworthiness of inferential tests of the indirect effect in statistical mediation analysis: Does method really matter? *Psychological Science, 24*, 1918-1927.

Pertinent Publications

Hayes, A. F., & Preacher, K. J. (2013). Conditional process modeling: Using structural equation modeling to examine contingent causal processes. In G. R. Hancock & R. O. Mueller (Eds.) *Structural equation modeling: A second course* (2nd Ed). Greenwich, CT: Information Age Publishing.

Hayes, A. F., Glynn, C. J., & Huge, M. E. (2012). Cautions regarding the interpretation of regression coefficients and hypothesis tests in linear models with interactions. *Communication Methods and Measures, 6,* 1-11.

Hayes, A. F., Preacher, K. J., & Myers, T. A. (2011). Mediation and the estimation of indirect effects in political communication research. In E. P. Bucy & R. L. Holbert (Eds), *Sourcebook for political communication research: Methods, measures, and analytical techniques.* (p. 434-465). *New York:* Routledge.

Hayes, A. F., & Preacher, K. J. (2010). Estimating and testing indirect effects in simple mediation models when the constituent paths are nonlinear. *Multivariate Behavioral Research, 45,* 627-660.

Hayes, A. F. (2009). Beyond Baron and Kenny: Statistical mediation analysis in the new millenium. *Communication Monographs, 76,* 408-420.

Hayes, A. F., & Matthes, J. (2009). Computational procedures for probing interactions in OLS and logistic regression: SPSS and SAS implementations. *Behavior Research Methods, 41,* 924-936.

Preacher, K. J., & Hayes, A. F. (2008). Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. *Behavior Research Methods, 40*, 879-891.

Preacher, K. J., Rucker, D. D., & Hayes, A. F. (2007). Assessing moderated mediation hypotheses: Theory, methods, and prescriptions. *Multivariate Behavioral Research, 42,* 185-227.

Preacher, K. J., & Hayes, A. F. (2004). SPSS and SAS procedures for estimating indirect effects in simple mediation models. *Behavior Research Methods, Instruments, and Computers, 36,* 717-731.