Mediation with Repeated-Measures and Multilevel Data

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Please go to https://github.com/akmontoya/SHWithin.git, download the whole folder.

General Outline

Day 1

Introduction

- Within- and between-subject designs
- Overview of between-subject mediation

Within-Subject Mediation

- Estimation & Inference
- MEMORE Macro
- Issues of Causality
- Power Analysis
- Preregistration and writing guides
- Visuals for Publication

If time (Multilevel Mediation)

General Outline

Day 2

Between-subject moderation (review)

Within-Subject Moderation

- Estimation & Inference
- Visualizations
- Probing interactions
- MEMORE Macro
- Power Analysis
- Preregistration and writing guides

(If time) Within-Subject Moderated Mediation

- Estimation & Inference
- Visualizations
- Probing Conditional Effects
- MEMORE Macro
- Preregistration and Writing Guides

Wrap Up

Workshop Procedures

Assuming some familiarity with:

- Linear Regression
- Mediation & Moderation in BS-Design
- SPSS or SAS

Class time:

10:30am – 12:30pm EST

BREAK/Activity

1pm – 3pm EST

Download files at
https://github.com/akmontoya/SHWithin.git

How we will learn:

- Combination of theory and practice
- Follow along with the analysis as we go
  - Use syntax!
  - **Ask questions** about concepts or anything that is confusing

What You'll Need

- **This course is hands-on. Hopefully you brought a laptop with R 3.6.1, SPSS 19+ or SAS 9.1+ with PROC IML. If not, that is ok. You'll still benefit.**

SPSS Code	SAS Code	R Code

- **Various files available on Canvas.**

  - SPSS, SAS, and R data folders. SPSS data files are ready to go. SAS files are programs thus must be executed to make them "work" files. R files are CSV and need to be read into R.
  - SPSS, SAS, and R MEMORE folders. This contains the MEMORE macro we'll heavily rely on, and some documents related to it.
  - Miscellaneous folder. Various files, including some PDFs and other miscellaneous things of relevance to this course.

- **A lot of stamina.**

Setting Up in R

A few packages I'm going to use throughout the workshop:

```
install.packages(c("mosaic", "ggformula", "jtools", "dplyr"))
library(mosaic)
library(ggformula)
library(jtools)
library(dplyr)
```

Loading in data:

```
filelocation <- "C:\\Your\\Path\\Here\\harass.csv"
harass <- read.csv(filelocation, header = TRUE)
```

You are welcome to deviate from the way I do things, if you have a preferred package for certain types of operations, feel free to do so.

Experimental Designs

This course focuses on two types of designs:

- Between-subject
- Within-subject

Whether we are doing mediation or moderation, there will always be a focal predictor (*X*) and an outcome (*Y*).

To determine the type of design, we only need to focus on *X* and *Y*

Between-Subject Design

- X is observed only once per subject
  - Randomly assigned (e.g., condition in a study)
  - Observed (e.g., handedness)

- Y is observed only once per subject
  - Only observed (e.g., typing speed)

Examples of Between-Subject Designs

Human participants are randomly assigned to complete a math test in either a hot or cold (X = temperature) room. Their performance on that math test (Y) is then measured.

Assigned

Researchers want to compare start-ups in finance to those in health (*X* = type of company) on their number of employees in their first two years (*Y*)

Observed

In this course X will typically be categorical, but it can also be continuous.

Within-Subject Design

- X is observed or assigned multiple times per subject
  - Assigned (e.g., participate in both conditions)
    - Every subject is observed under the same values of X
  - Observed (e.g., measured across time)
    - Values of X may differ across participants

- Y is observed under each instance of X
  - Only observed (e.g., typing speed)

Examples of Within-Subject Designs

Human participants are complete a math test both a hot and a cold (X = temperature) room. Their performance on that math test (Y) is then measured.

Assigned

Number of employees (Y) is measured at the end of the first year and second year (X = age of startup) across many start-ups.

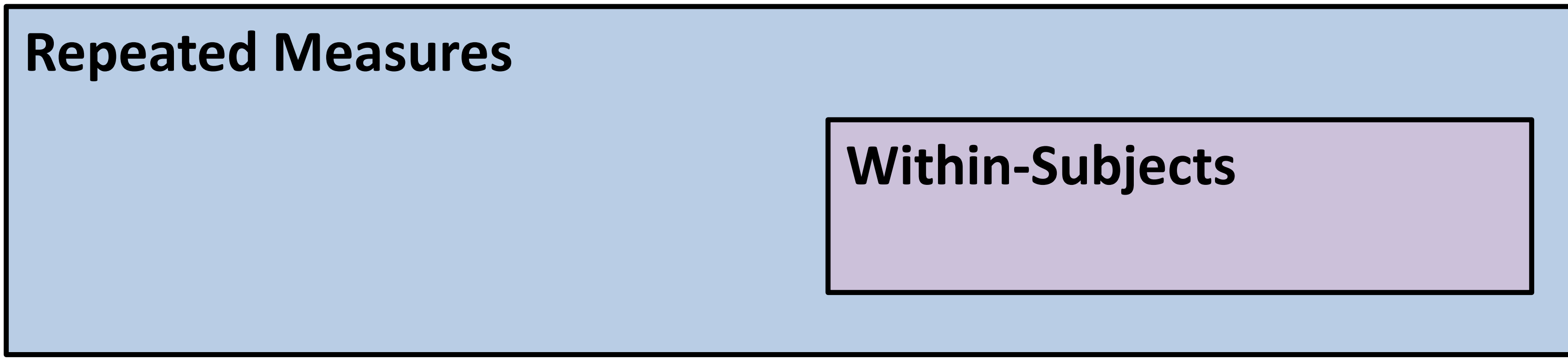
Assigned

Number of employees (*Y*) is measured one year apart for each start-up (X = age of startup)

Observed

Repeated-Measures vs Within-Subjects

Used very similarly



Types of Repeated-Measuresments

- Each person *over time*
- *Nested/Multilevel* data (individuals within schools, cohorts, etc)
- *Dyadic* data (twins, couples, labmates, roommates)
- Each person in a *variety of circumstances*
- and many more…

Repeated Measures Data

It's important to think about how/when/how many times the variables in your mediation model are measured

- Within-Subject: X and Y are assigned/observed repeatedly
- *Multilevel* has a nice system referring to levels (1-1-1 mediation, 2-2-1, mediation etc.
- Is your focal predictor measured repeatedly?
- Is your focal predictor what differentiates your repeated measurements?

Our Focus: Two Instance Repeated-Measures

The focal predictor is the factor which differs by repeated measures.

X: is assigned twice
M: measured in each of the two instances
Y: measured in each of the two instances

Examples:

- Participants read two scenarios. Interested in how scenario influences *Y* through *M*. Measure *M* and *Y* in each scenario.
- Pre-post test: Therapist measures certain symptoms and various outcomes before administering some intervention, and after administering the intervention.
- Researcher interested in if male partners in heterosexual relationships believe fights are less severe because they are less perceptive of small "squabbles". Measure both male and female partners in relationships, self report number of small "squabbles" and severity of last fight.

Counter Examples

- Does calorie consumption (*X*) impact body image (*Y*) through weight gain (*M*) over time?
  - *X* is observed (not assigned), so difference subjects have different *X*s. Can be dealt with in Multilevel or Latent Growth approaches.
- Any instance where repeated-measure factor is a “nuisance” (e.g. studying schools, but not interested in comparing schools directly).
  - *X* is observed and doesn’t vary within subjects of study

Mediation

- Between Subjects
  - Path analytic approach
  - Interpretation
  - Estimation
  - Inference

- Two-Instance Repeated-Measures
  - Judd Kenny and McClelland (2001)
  - Path analytic approach
  - Estimation of Indirect Effects
  - MEMORE
  - Reporting (Writing and Figures)
  - Common Questions

Running Example: Group Work in Computer Science (BS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach.

Between-Subjects Version (CompSci_BS.sav, CompSci_BS.sas) :
Female participants (N = 107) read *one of two* syllabi for a computer science class. One of the syllabi reported the class would have group projects throughout (`cond = 1`), and the other syllabi stated that there would be individual projects (`cond = 0`) throughout the class.

Measured Variables:

- `Interest` in the class ($\alpha = .89$)
  - How interested are you in taking the class you read about?
  - How much would you want to take the class you read about?
  - How likely would you be to choose the class you read about?
  - How interested are you in majoring in computer science?
  - 1 Not at All – 7 Very much

University of Washington
Computer Science & Engineering 142: Introduction to Programming I
Course Syllabus

Instructor

name: John Johnson
email: j.johnson@uw.edu
office: CSE 800
office phone: (206)555-1234
office hours: see course website

Course Overview

This course provides an introduction to computer science using the Java programming language. CSE 142 is primarily a programming course that focuses on common computational problem solving techniques. No prior programming experience is assumed, although students should know the basics of using a computer (e.g., using a web browser and word processing program) and should be competent with math through Algebra 1. The information, concepts, and analytical thinking introduced in lecture provide a unifying framework for the topics covered in CSE 143.

Lecture Time

MWF 12:00 PM - 1:00 PM, Classroom TBA

Discussion Sections

You will be expected to participate in a weekly discussion section, held on Thursdays (see course website for details). The TA who runs your section will grade your homework assignments. In section, we will answer questions, go over common errors in homework, and discuss sample problems in more detail than lecture.

Course Web Site

- http://www.cs.washington.edu/142/

Textbook

- Reges/Stepp, *Building Java Programs: A Back to Basics Approach (2nd Edition)*.

Grading

The primary assessment for your success in this class is exams. There will be 2 midterms and 1 final, and together they make up 85% of your grade. The homework assignments are designed to prepare you for your exams. The exams are designed to assess your ability to utilize the concepts you've learned from your homework and in lecture in new contexts.

5% participation
10% weekly homework assignments
25% midterm 1
25% midterm 2
35% final exam

Exams

Our exams are closed-book and closed-notes, although each student will be allowed to bring a single index card with hand-written notes (no larger than 5" by 8"). No electronic devices may be used, including calculators. Make-up exams will not be given except in case of a serious emergency.

Homework

Homework consists of weekly assignments done in optional groups and submitted electronically on the course web site. Disputes about homework grading must be made within 2 weeks of receiving the grade. If you don't make an honest effort on the homework, your exam score will reflect it.

Academic Integrity and Collaboration

Computer Science is best learned through interacting with your fellow students to ensure that you thoroughly understand each concept. Homework assignments may be completed with other students. You are strongly encouraged to discuss general ideas of how to approach an assignment with other students, and may discuss specific details about what to write with other students. Any help you receive from or provide to classmates should be cited in your assignment. You may seek help from University of Washington CSE 142 TAs, professors, and classmates.

You must abide by the following rules:

- You are highly encouraged to work with another student on homework assignments.
- You may not show another student outside of your class your solution to an assignment, nor look at his/her solution.
- You may not have anyone outside of your class describe in detail how to solve an assignment or sit with you as you write it.
- You may not post online about your homework, other than on the class discussion board, to ask others for help.

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Computer Science & Engineering 142: Introduction to Programming I
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Instructor

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You must abide by the following rules:

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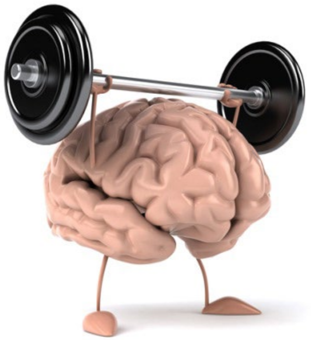
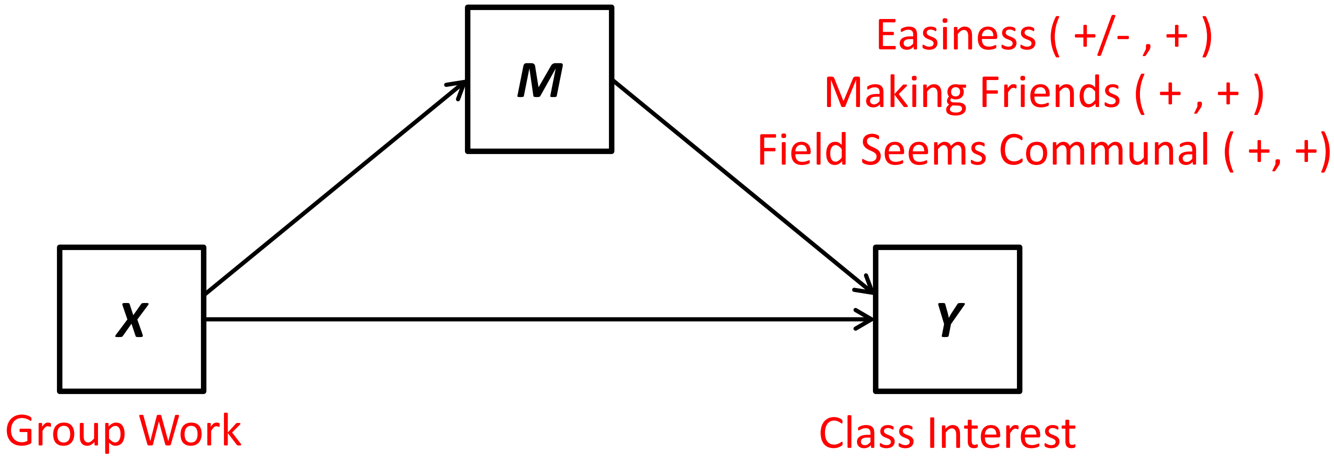
- You may not have anyone describe in detail how to solve an assignment or sit with you as you write it.
- You may not post online about your homework to ask others for help.



Mediation



A simple mediation model connects an **assumed** causal variable (*X*) to an **assumed** outcome variable (*Y*), through some mechanism (*M*).

M is frequently referred to as a *mediator, intermediary variable,* or *surrogate variable*.

Many different kind of variables may act as mediators. Emotional variables, situational, individual level variables, cognitive variables, environmental variables, etc.

Mediation can be found throughout the psychology literature and is particularly common in social psychology

A quick example: Name some possible mediators!

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- `Interest` in the class ($\alpha = .89$)
  - How interested are you in taking the class you read about?
  - How much would you want to take the class you read about?
  - How likely would you be to choose the class you read about?
  - How interested are you in majoring in computer science?
  - 1 Not at All – 7 Very much
- `CSComm`: Perceptions that computer science is communal ($\alpha = .90$)
  - Computer science would assist me in _______________.
  - Helping others, serving the community, working with others, connecting with others, caring for others.
  - 1 Strongly Disagree – 7 Strongly Agree

Mediation: Path Analysis

Consider *a*, *b*, *c*, and *c′* to be measures of the effect of the variables in the mediation model.

These could be measured using regression coefficients from OLS or path estimates in a structural equation model using maximum likelihood estimation.

Indirect effect of *X* on *Y* (through *M*) = $a \times b$

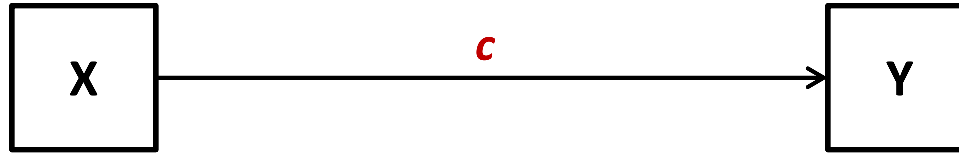
Direct effect of *X* on *Y* (not through *M*) = c'

Indirect effect = total effect - direct effect

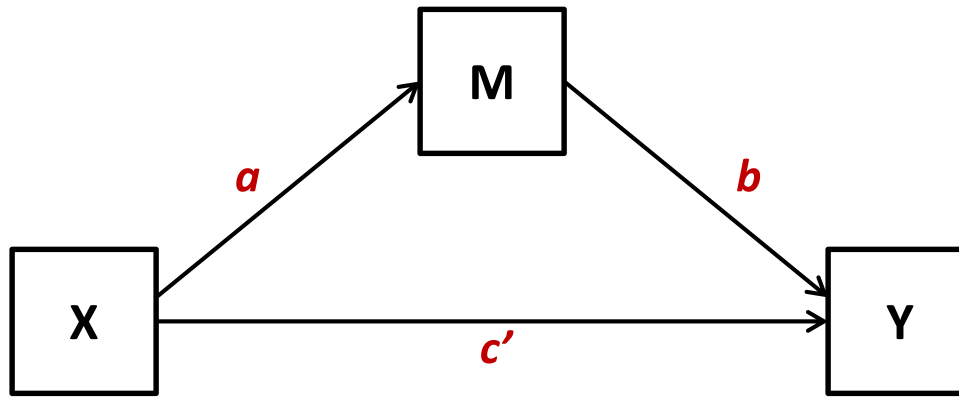
$$a \times b = c - c'$$

Total effect = direct effect + indirect effect

$$c = c' + a \times b$$



$$Y_i = i_{Y^*} + cX_i + e_{Y_i^*}$$

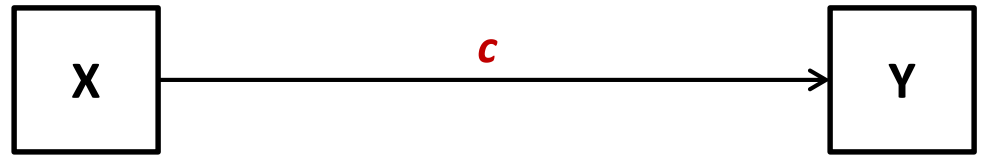


$$M_i = i_M + aX_i + e_{M_i}$$

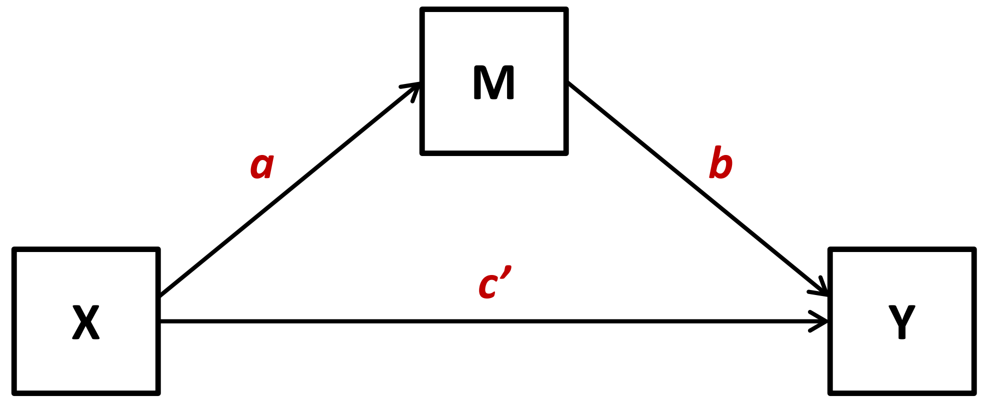
$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

Interpreting the Coefficients

Total Effect (*c*): The effect of our presumed cause (*X*) on our outcome (*Y*), without controlling for any other variables.



***a*-path:** The effect of our presumed cause (*X*) on our mediator *(M).*



***b*-path:** The effect of our mediator (*M*) on the outcome (*Y*) while controlling for *X*. (i.e. predicted difference in *Y* for two people with the same score on *X* but who differ on *M* by one unit).

Direct effect (*c’*): The effect of our presumed cause (*X*) on *Y* while controlling for *M*. (i.e. predicted difference in *Y* for two people who differ by one unit on *X* but with the same score on M)

Indirect Effect (*ab*): Product of effect of *X* on *M*, and effect of *M* on *Y* controlling for *X*. The effect of *X* on *Y* through *M*.

The Data: HARASS

SPSS

*harass.sav [DataSet3] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Gra

	harass	se2	dep2	satis2
1	2.16	3.80	1.25	4.80
2	2.33	2.90	1.95	4.20
3	1.16	3.80	1.95	5.00
4	2.50	2.50	2.15	3.00
5	1.33	3.00	1.90	2.60
6	1.16	2.50	1.25	3.40
7	1.50	2.10	2.05	2.40
8	1.50	3.30	1.75	2.80
9	1.83	2.70	2.15	3.20
10	1.33	2.80	2.45	4.00

The SPSS file is ready for analysis.

SAS

harass

```
data harass;
  input harass se2 dep2 satis2 fa
datalines;
2.16 3.80 1.25 4.80 1.00 1.66 3.6
2.33 2.90 1.95 4.20 1.75 2.83 2.8
1.16 3.80 1.95 5.00 1.50 3.16 3.8
2.50 2.50 2.15 3.00 2.00 2.00 3.1
1.33 3.00 1.90 2.60 1.00 2.83 2.6
1.16 2.50 1.25 3.40 1.25 2.83 2.8
1.50 2.10 2.05 2.40 1.50 3.00 1.8
1.50 3.30 1.75 2.80 1.25 3.00 3.4
1.83 2.70 2.15 3.20 1.50 3.50 2.9
1.33 2.80 2.45 4.00 2.00 3.16 2.8
1.33 2.60 1.70 3.60 1.75 3.16 3.0
```

The SAS version is
a SAS program that must be executed to produce a temporary work data file.

R

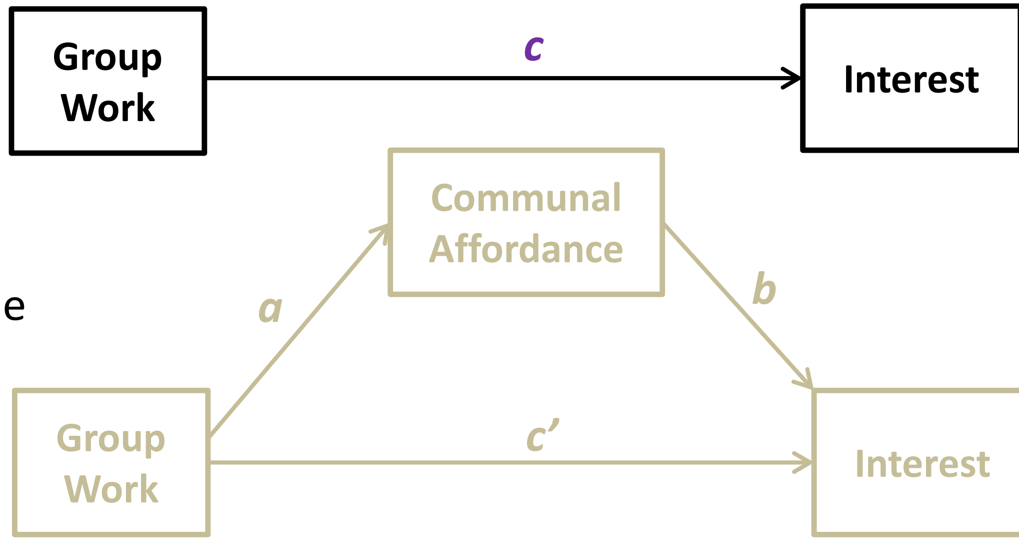
	harass	se2	dep2	satis2	fail2	posrel	se1	dep1	sa
1	2.16	3.8	1.25	4.8	1.00	1.66	3.6	1.10	
2	2.33	2.9	1.95	4.2	1.75	2.83	2.8	2.25	
3	1.16	3.8	1.95	5.0	1.50	3.16	3.8	1.95	
4	2.50	2.5	2.15	3.0	2.00	2.00	3.1	1.25	
5	1.33	3.0	1.90	2.6	1.00	2.83	2.6	1.55	
6	1.16	2.5	1.25	3.4	1.25	2.83	2.8	1.45	
7	1.50	2.1	2.05	2.4	1.50	3.00	1.8	2.10	
8	1.50	3.3	1.75	2.8	1.25	3.00	3.4	1.55	
9	1.83	2.7	2.15	3.2	1.50	3.50	2.9	2.20	
10	1.33	2.8	2.45	4.0	2.00	3.16	2.8	1.25	
11	1.33	2.6	1.70	3.6	1.75	3.16	3.0	2.05	
12	1.00	2.1	2.30	2.8	1.25	1.33	3.0	2.10	
13	1.66	3.1	2.00	3.8	2.00	2.66	3.2	1.55	
14	1.16	3.5	1.55	3.8	1.25	2.50	3.2	1.75	
15	1.33	2.7	1.35	3.4	1.50	3.00	3.8	1.15	
16	1.66	3.1	1.20	4.4	1.75	4.00	3.4	1.80	

```
filelocation <-
"C:\\Data\\harass.csv"
harass <- read.csv(filelocation,
header = TRUE)
```

These are not the actual data from this study. They were generated to produce similar results to the published study.

Estimation with CompSci_BS Data

Research Question: **Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**



The *c*-path can be estimated in a sample using the regression equation below.

$$Y_i = i_{Y^*} + cX_i + e_{Y_i^*}$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.701	.193		14.002	.000
	Cond	.462	.285	.156	1.621	.108

a. Dependent Variable: Interest

Overall women were .462 units more interested in the class with group work.

$c = .462$

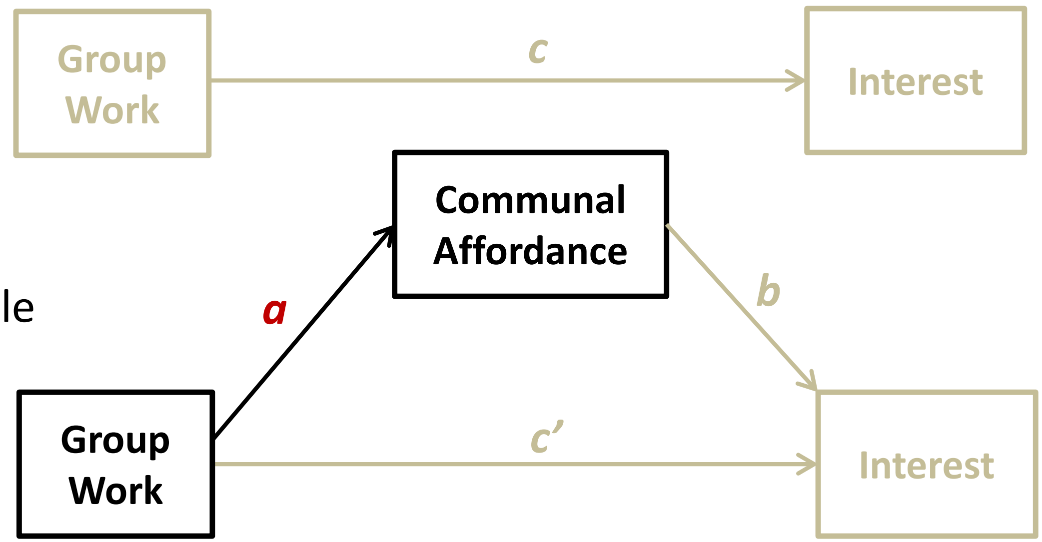
```
regression /dep = interest /method = enter cond.
```

```
proc reg data=CompSci_BS;model interest = cond/stb clb;run;
```

```
summary(lm(interest~cond, data = CompSci_BS)
```

Estimation with CompSci_BS Data

Research Question: **Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**



The *a*-path can be estimated in a sample using the regression equation below.

$$M_i = i_M + aX_i + e_{M_i}$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	3.421	.159		21.472	.000
	Cond	.488	.237	.198	2.060	.042

a. Dependent Variable: CSComm

Women saw computer science as .488 units more communal after reading a syllabus with group work.

a = .488

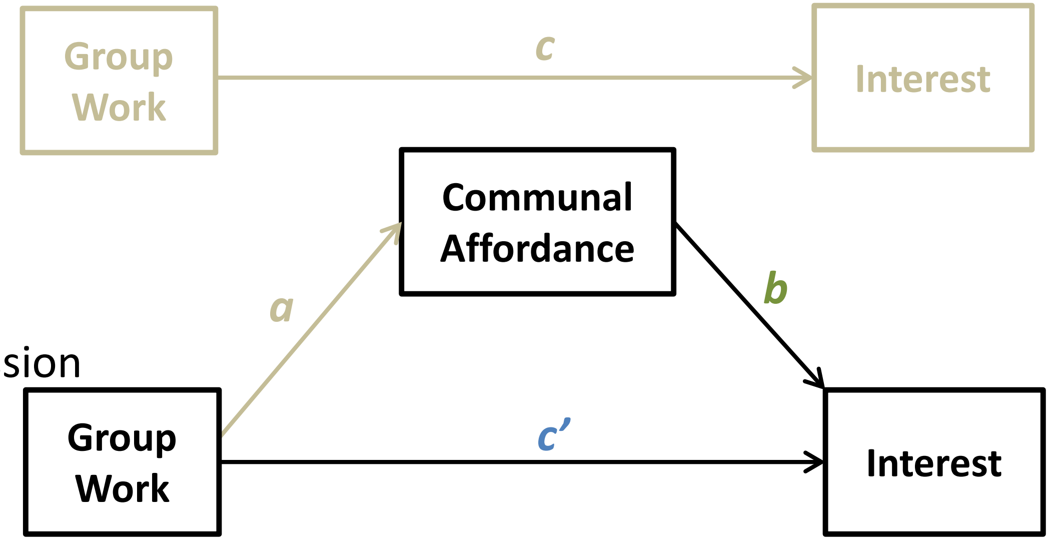
```
regression /dep = CScomm /method = enter cond.
```

```
proc reg data=CompSci_BS;model CScomm = cond/stb clb;run;
```

```
summary(lm(CScomm~cond, data = CompSci_BS)
```

Estimation with CompSci_BS Data

Research Question: **Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**



The *b*-path and direct effect can be estimated in a sample using the regression equation below.

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	.964	.413		2.336	.021
	Cond	.218	.268	.073	.812	.419
	CSComm	.508	.109	.421	4.663	.000

a. Dependent Variable: Interest

Controlling for communal affordance, women in the group work condition were .218 units more interested in the class with group work.

$c' = 0.218$

$b = .508$

```
regression /dep = interest /method = enter cond  CScomm.
```

```
proc reg data=CompSci_BS;model interest = cond CScomm/stb clb;run;
```

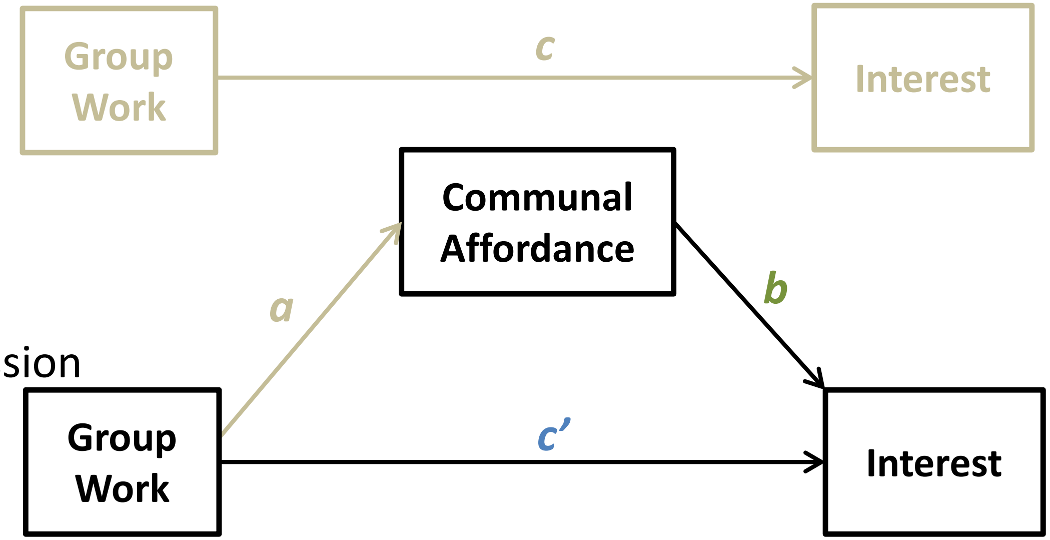
```
summary(lm(interest~cond+CScomm, data = CompSci_BS)
```

Estimation with CompSci_BS Data

Research Question: **Can group work in computer science classes increase women's interest by increasing their perception that computer science is communal?**

The *b*-path and direct effect can be estimated in a sample using the regression equation below.

$$Y_i = i_Y + c'X_i + bM_i + e_{Y_i}$$



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		B	Std. Error	Beta		
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	Cond	.218	.268	.073	.812	.419
	CSComm	.508	.109	.421	4.663	.000

a. Dependent Variable: Interest

c' = 0.218

For two people in the same condition, a one unit difference in communal goals results in a 0.51 unit difference in interest, on average.

```
regression /dep = interest /method = enter cond  CScomm.
```

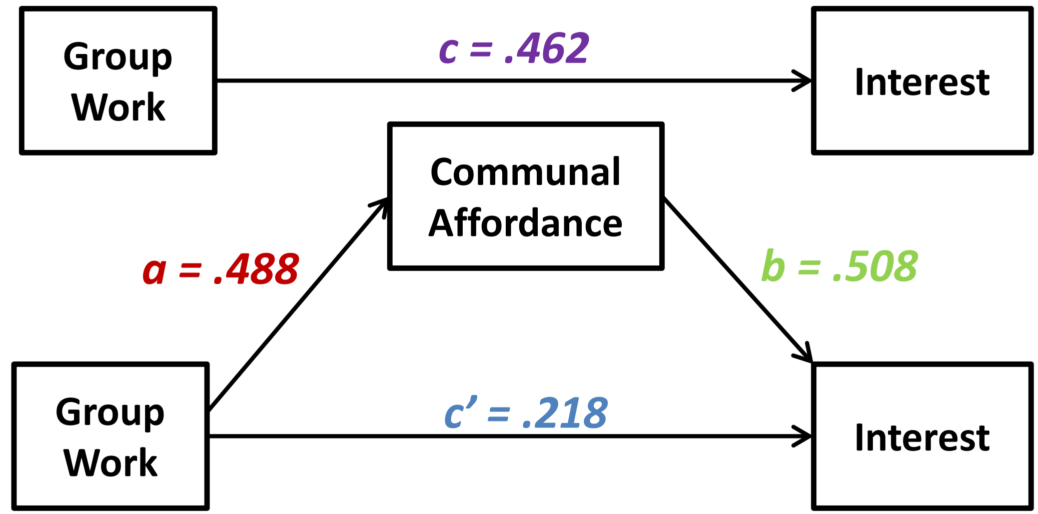
```
proc reg data=CompSci_BS;model interest = cond CScomm/stb clb;run;
```

```
summary(lm(interest~cond+CScomm, data = CompSci_BS)
```

Interpreting the Coefficients

Research Question: Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?

On average, women were .46 units more interested in the class with group work (p = .108). Similarly, computer science was perceived as .49 units more communal after reading a syllabus with group work (p = .042). Controlling for condition, a one unit increase in communal affordance resulted in a .508 unit increase in interest (p < .001). Controlling for communal affordance, group work did not predict additional interest (c' = .22, p = .42).



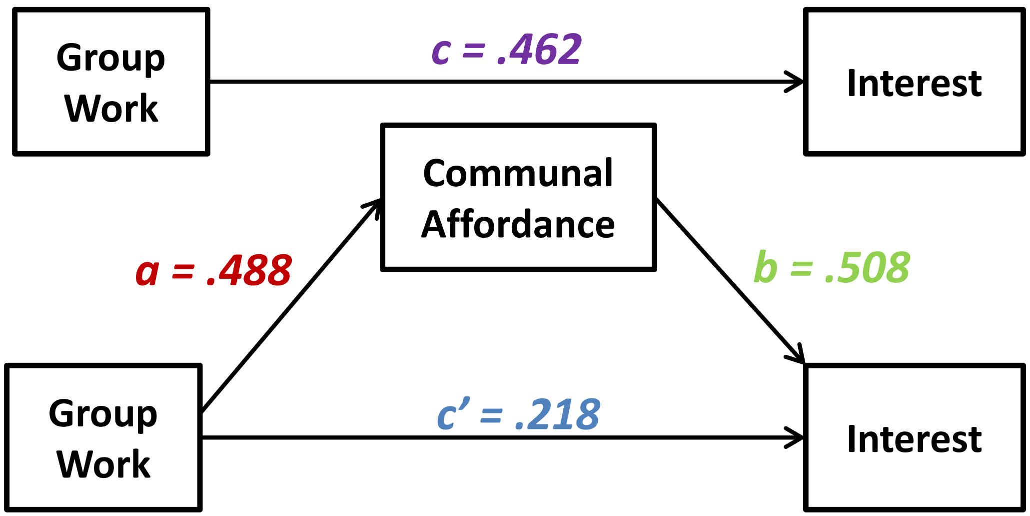
But what about the indirect effect?

Interpreting Indirect, Direct, and Total Effects

Indirect Effect

$a \times b = .488 \times .508 = .249$

Group work increased interest by .249 units indirectly through communal affordance. Where group work increased perceptions of communal affordance by .488 units, and a one unit increase in communal affordance resulted in a .508 unit increase in interest.



Direct Effect

$c' = .218$

Group work increased interest by .218 units directly (not through communal affordance).

$p = .419$

Total Effect

$c = .462$

Group work increased interest by .462 units in total.

$p = .108$

Inference for the direct and total effects can be drawn from the regression results because these are based on a single regression parameter.

Inference about the Indirect Effect

- How to make proper inference about the indirect effect may be the most active area of research in mediation analysis
- Some methods you may have heard of
    - Causal Steps / Baron and Kenny Method / Baron and Kenny Steps
    - Test of Joint Significance
    - Sobel Test / Multivariate Delta Method
    - Monte Carlo Confidence Intervals
    - Distribution of the Product Method
    - Bootstrap Confidence Intervals
        - Percentile Bootstrap
        - Bias-Corrected Bootstrap
        - Bias Corrected and Accelerated Bootstrap

- Why is this so hard?
    - The product of two normal distributions is not necessarily normal. The shape of the distribution of the indirect effect depends on the true indirect effect.
    - There are many instances where the indirect effect could be zero (either *a* or *b* could be zero, or both could be zero).

Causal Steps Method

Method

1. Test if there is a significant total effect ($c \neq 0$).
2. Test if there is a significant effect of X on M ($a \neq 0$).
3. Test if there is a significant effect of M on Y controlling for X ($b \neq 0$).
4. If all three steps are confirmed, test for partial vs. complete mediation.
    1. If X still has an effect on Y controlling for M ($c' \neq 0$), this is partial mediation
    2. If X does not have a significant effect on Y controlling for M, complete mediation

Appeal

- Easy to do, just need regression
- Intuitive

What's wrong with it?

- No estimate of the indirect effect
- No quantification of uncertainty about conclusion
    - *p*-value
    - Confidence Interval
- Requirement that the total effect is significant before looking for indirect effect
- Issues with *complete* and *partial* mediation

Joint Significance

Method

1. Test if there is a significant effect of X on M ($a \neq 0$).
2. Test if there is a significant effect of M on Y controlling for X ($b \neq 0$).

Appeal

- Easy to do, just need regression
- Intuitive
- Solves issues of requirement of significant total effect to claim an indirect effect.
- Good method balance Type I Error and Power

What's wrong with it?

- No estimate of the indirect effect
- No quantification of uncertainty about conclusion
    - *p*-value
    - Confidence Interval

Sobel Test / Normal Theory / Delta Method

Method

1. Calculate a Z-statistic

$$Z = \frac{ab}{\sqrt{b^2 s_a^2 + a^2 s_b^2 + s_a^2 s_b^2}}$$

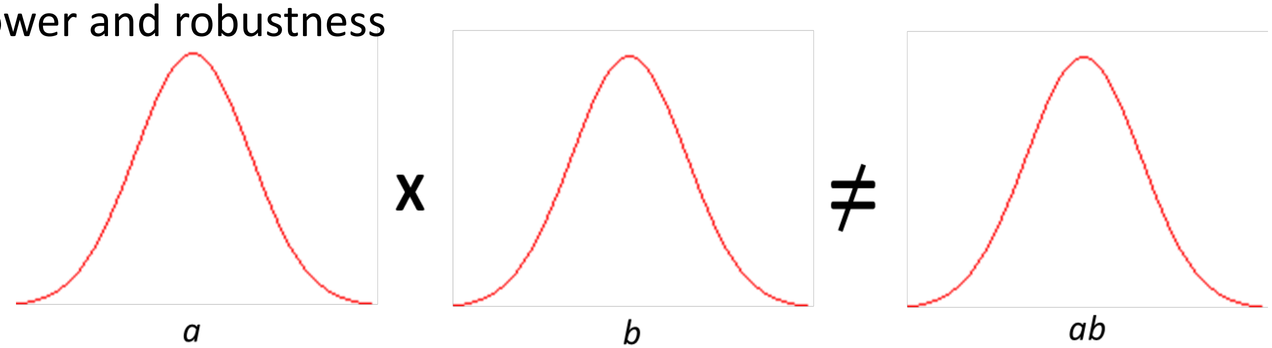
2. Calculate a p-value from Z-statistic

Appeal

- Very similar to process from regression
- Single test of the indirect effect
- Follows from asymptotic theory

What's wrong with it?

- Assumes indirect effect is normally distributed, which is not the case at finite sample sizes
- Poor power and robustness



Bootstrap Confidence Intervals (Percentile)

Empirically estimate sampling distribution of the indirect effect. From this distribution compute confidence intervals which can be used for estimation and hypothesis testing.

Method

1. Randomly sample *n* cases from your dataset with replacement.
2. Estimate the indirect effect using resampled dataset, call this $ab^{(1)}$
3. Repeat steps 1 and 2 a total of *K* times where *K* is many (10,000 recommended), each time calculated $ab^{(k)}$.
4. The sampling distribution of the $ab^{(i)}$'s can be used as an estimate of the sampling distribution of the indirect effect.
5. For a 95% confidence interval the lower and upper bounds will be the 2.5th and 97.5th percentiles of the *K* estimates of the indirect effect.

Appeal

- No assumptions about the sampling distribution of the indirect effect
- Provides point estimate of indirect effect
- Can calculate confidence intervals
- Good method balance Type I Error and Power

What's wrong with it?

- Not all software has this built in
- Requires original data

Bootstrap Confidence Intervals

Original Data

X	M	Y
-0.35	-0.58	0.25
0.31	-0.50	1.89
-0.19	2.61	2.08
-1.30	-1.49	-0.54
0.59	1.14	1.74
-0.29	-0.29	1.04
1.80	0.08	1.23
-0.01	1.20	1.30
0.30	1.35	1.31
-0.98	0.90	-0.76

a = .2931 b = .3099

ab = .0908

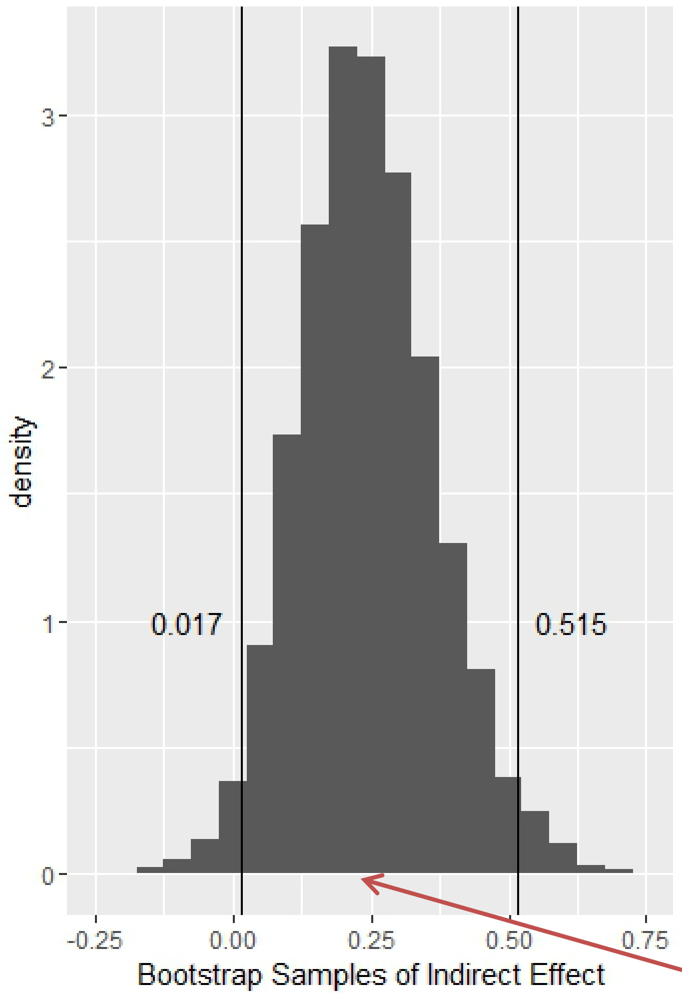
Bootstrap Sample

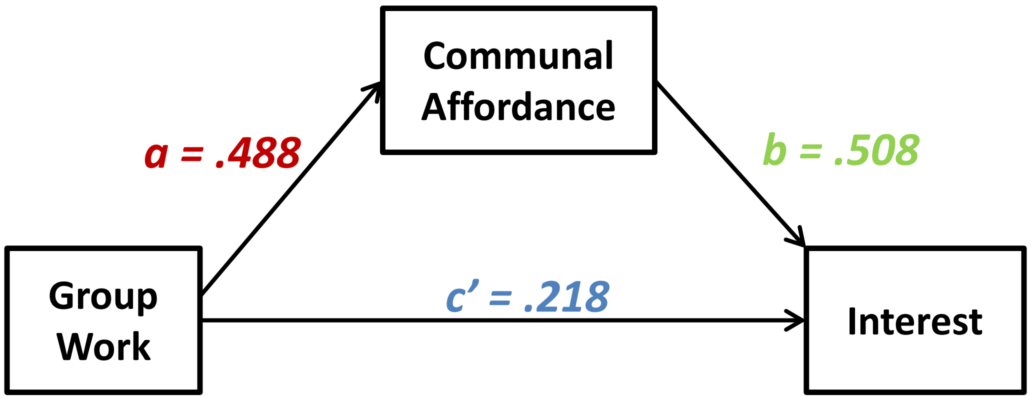
X	M	Y

a = -.1035 b = .1495

ab = -.0155

Bootstrap Confidence Intervals (CompSci Data)





ab = .249

Zero is not contained in the confidence interval [0.017, 0.515] so we conclude the indirect effect is different from zero with 95% confidence. This is similar to rejecting the null hypothesis at $\alpha = .05$.

ab = .249

Other Kinds of Bootstrap Confidence Intervals

All bootstrap confidence intervals use the same basic sampling technique, just use different methods for choosing the end points of the confidence intervals

Bias-Corrected Confidence Interval

- Percentile bootstrapping assumes that your sample estimate (*ab*) is unbiased in estimating the population indirect effect
- Bias-corrected reduces this assumption to assuming that the bias of *ab* is a constant (i.e. as N goes to infinity *ab* will go to the population indirect effect plus some constant)
- Bias-corrected confidence intervals estimate the bias of *ab* then adjust edges of confidence interval to be "bias-corrected" (i.e. centered not around your original estimate of *ab*), but around the point based on the bias estimation.

Bias-Corrected and Accelerated

- Same principles as BC regarding bias correction
- Acceleration allows for the assumption that the standard error of the indirect effect depends on the population value of the indirect effect
- Acceleration parameter, which is used to adjust the ends of the confidence interval is estimated using leave-one-out estimates of skew of the estimates of the indirect effect.

BC and BCA Bootstrap Confidence Intervals have been shown to have inflated Type I Error rates compared to other methods and are **not recommended**

The Monte Carlo Interval

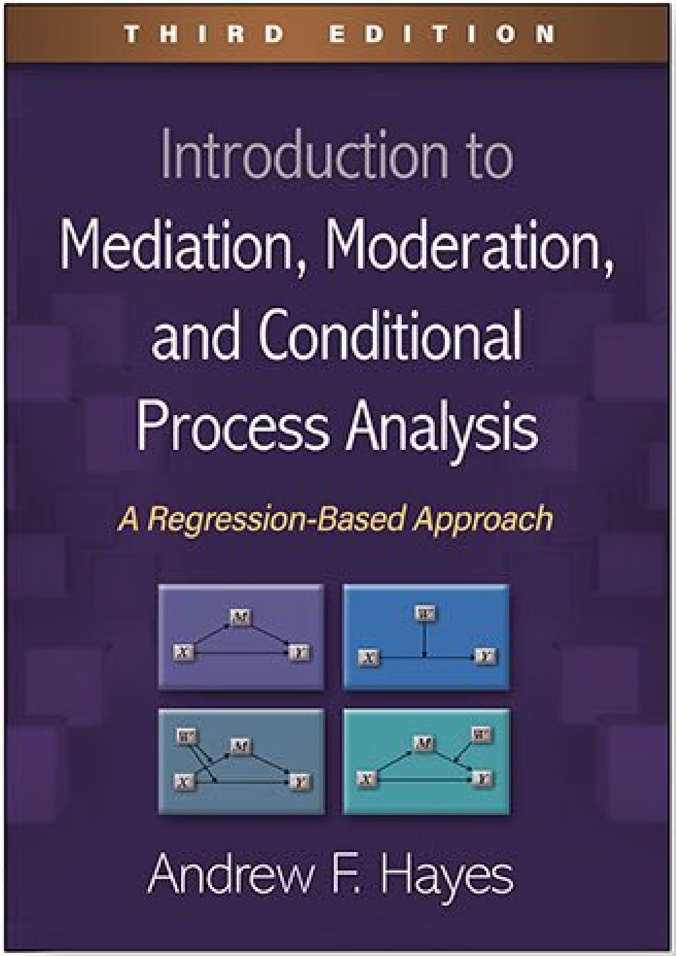
Monte Carlo empirically estimate the sampling distribution of the indirect effect and generate a confidence interval (CI) for estimation and hypothesis testing. This simulation based method assumes each individual path (*a* and *b*) are normally distributed.

(1) Generate *k* samples from a normal distribution with mean *a* and standard deviation s_a
(2) Generate *k* samples from a normal distribution with mean *b* and standard deviation s_b
(3) Multiply samples together to get a distribution of *k* estimates of *ab.*
(4) Rank order estimates and select estimates which define the lower percentile of sorted *k* estimates and upper percentile of sorted estimates which define CI of interest.
(5) For 95% CI lower and upper bounds are 2.5th and 97.5th percentile in *k* bootstrap estimates of the indirect effect.

This method performs well (similarly to bootstrapping) in a variety of simulation studies, but is still less popular.

This method makes stronger assumptions than bootstrapping, but does not seem to result in greater power.

PROCESS Macro



Published in January 2022 and available through The Guilford Press, Amazon.com, and elsewhere.

- First released in beta form in March of 2012 and later documented in Hayes (2013, IMMCPA, published by The Guilford Press).
- Available for both SPSS (in macro and “custom dialog” form) and SAS.
- An integration of functions available in my other published macros for mediation and moderation analysis (SOBEL, INDIRECT, MODMED, MODPROBE, MED3C) and a whole lot more, all in one command.
- A handy tool for both “confirmatory” and “exploratory” approaches to data analysis.
- Freely available at www.processmacro.org. The current release is v4.3

For more on the use of PROCESS take my Statistical Horizons Course

Understanding Cause and Effect

As scientists we're often looking to support a claim that "X causes Y." Many of us are familiar with the phrase "correlation is not causation." But what then is needed to support a claim of cause?

Often we rely on experimentation to help us support the claim of cause. But what happens when we cannot (ethically or practically) manipulate our causal variable?

Consider the claim "Smoking tobacco causes lung cancer." Is it unethical to randomize people to smoke or not smoke. How then do we know this claim is true?

Necessary Conditions for Cause:

1. Covariation
2. Temporal Ordering
3. Elimination of competing explanations

Research on tobacco use easily found evidence for 1 & 2, and slowly over time accumulation of evidence supported 3.

Understanding Cause and Effect

Necessary Conditions for Cause:

1. Covariation
2. Temporal Ordering
3. Elimination of competing explanations

The following methods are frequently used to support the claim of cause. Which conditions for cause are supported by each method?

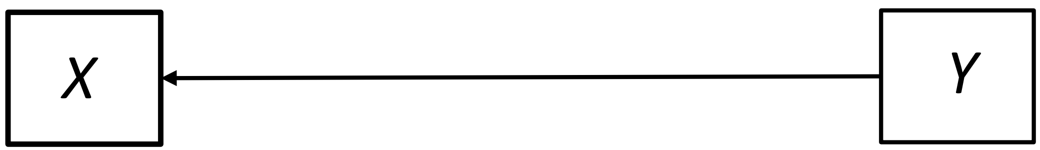
Experimental Manipulation:

Longitudinal Studies:

Cross-sectional Data Collection analyzed using Linear Regression:

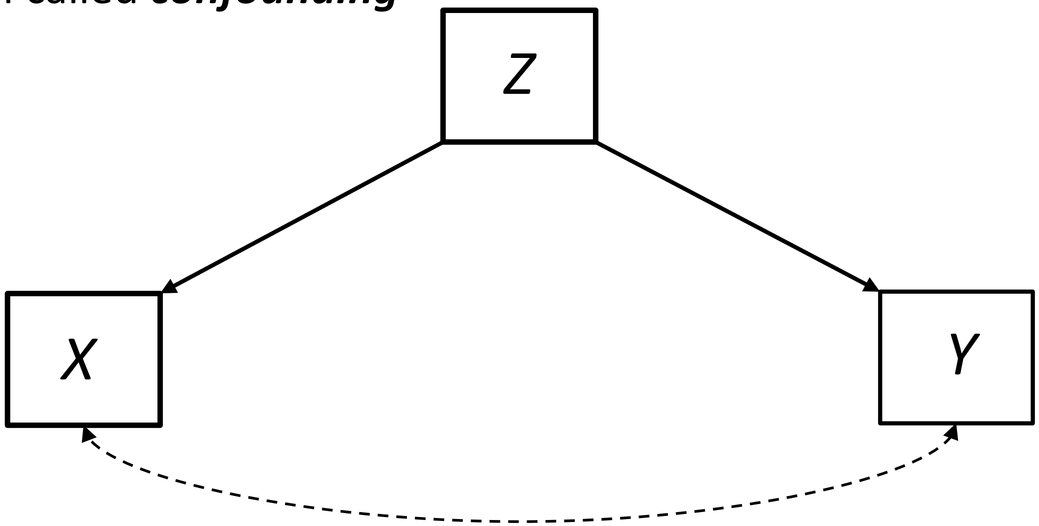
If it's not a cause, what is it?

Effect/Reverse Causation: It's possible that *X* is an effect of *Y*, rather than *Y* being an effect of *X*



If it's not a cause, what is it?

Spurious association: when relationship between X and Y is induced by a shared cause.
This is often called ***confounding***

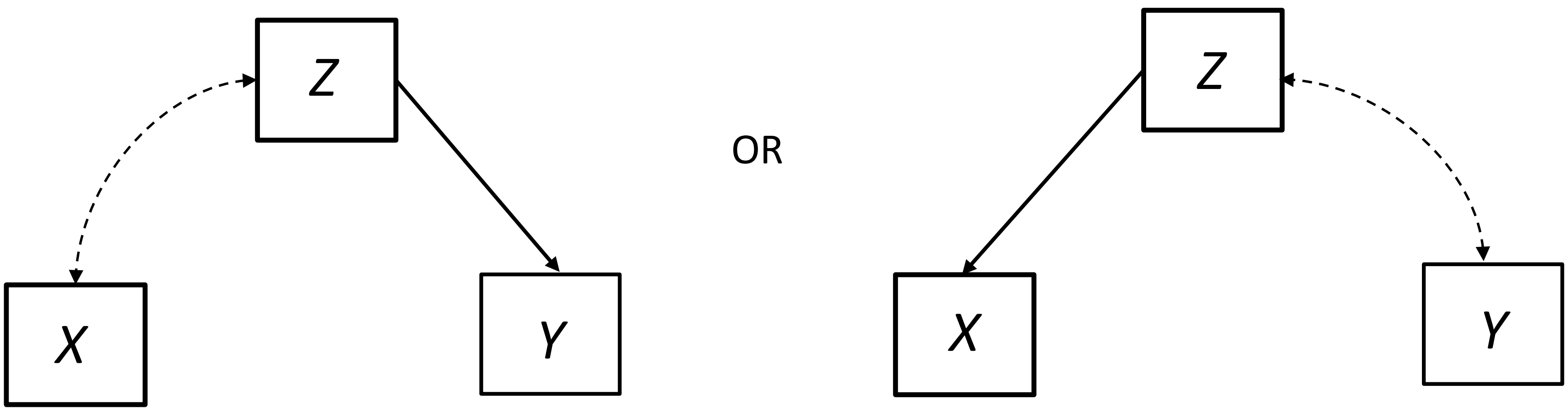


Example: Skirt length theory is one that suggests that skirt lengths predict the stock market (short skirt → market going up). Likely this is not a causal relationship but rather both skirt length and market trends are influenced by other larger cultural/economic trends.

Controlling for *Z* helps us eliminate spurious explanations.

If it’s not a cause, what is it?

Epiphenominality: X and Y are related because X is correlated with a cause of Y

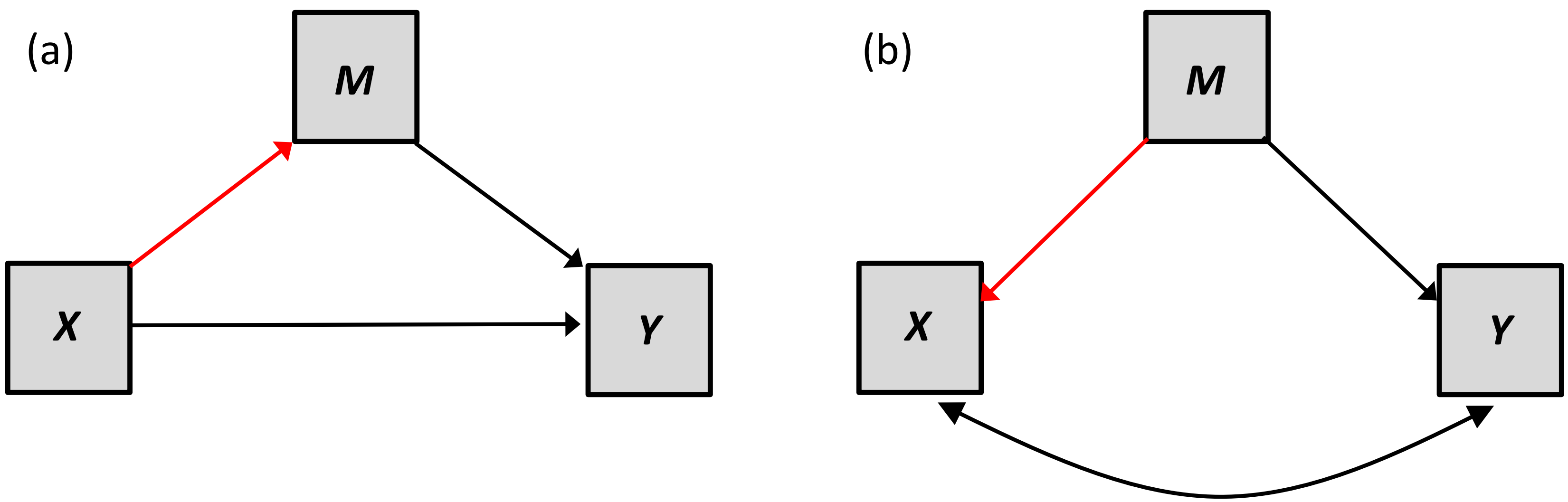


Example: Having an increased risk of breast cancer (Y) concurrent with taking an antibiotic is an **epiphenomenon**. It is not the antibiotic that is causing the increased risk, but the increased inflammation associated with the bacterial infection (Z) that prompted the taking of an antibiotic (X).

Controlling for *Z* helps us eliminate epiphenomenal explanations.

Mediation and spuriousness

Mediation analysis cannot distinguish between (a)mediation and (b)spuriousness. If (b) can be deemed plausible, that weakens the case for (a) regardless of what the data analysis tells you.

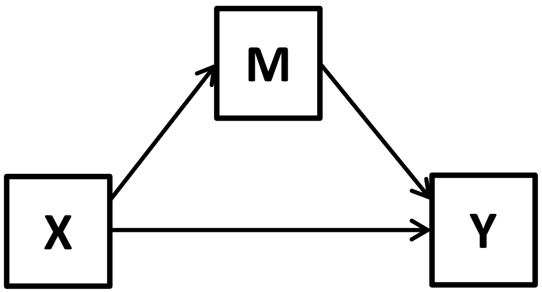


Inferences are always design-bound.

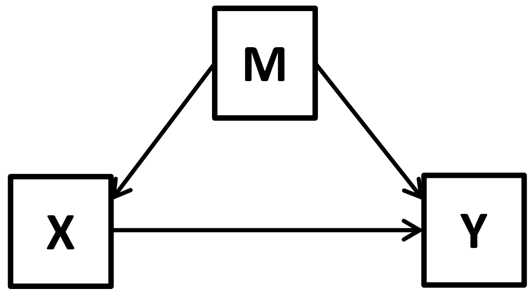
Mediation is a causal process, but causal claims are only justified if the design allows such claims, regardless of what the statistics say.

Mediation and Causality

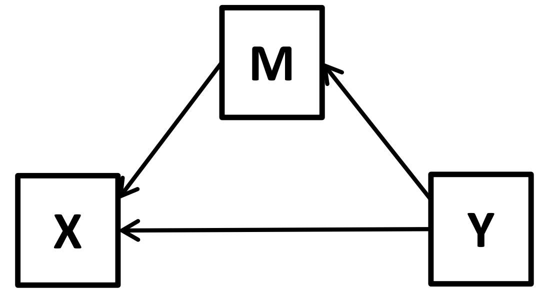
There are a number of alternative causal processes that may be occurring when a *statistical indirect effect* is present:



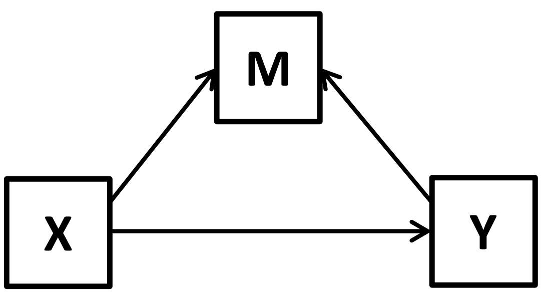
X affects *Y* through *M*



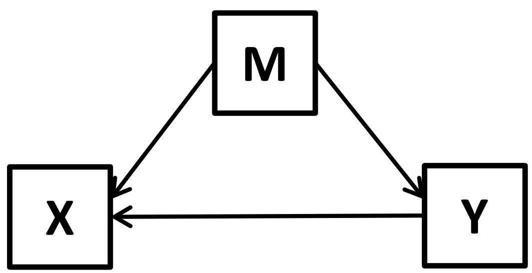
M affects *Y* through *X*



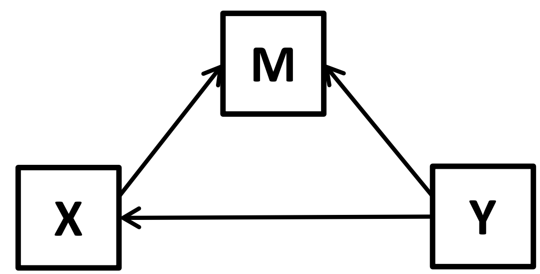
Y affects *X* through *M*



X affects *M* through *Y*



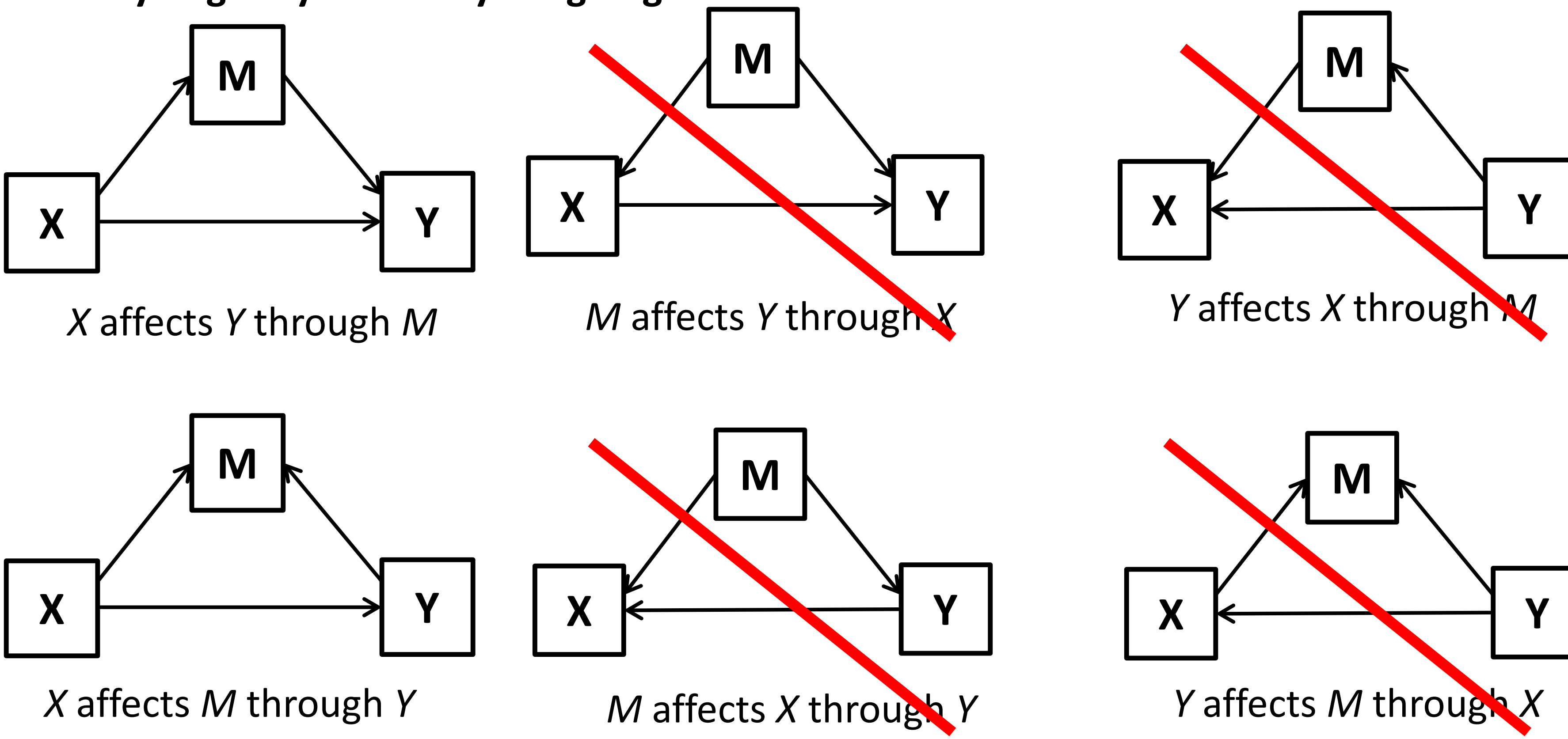
M affects *X* through *Y*



Y affects *M* through *X*

Mediation and Causality

What you get by randomly assigning *X*.

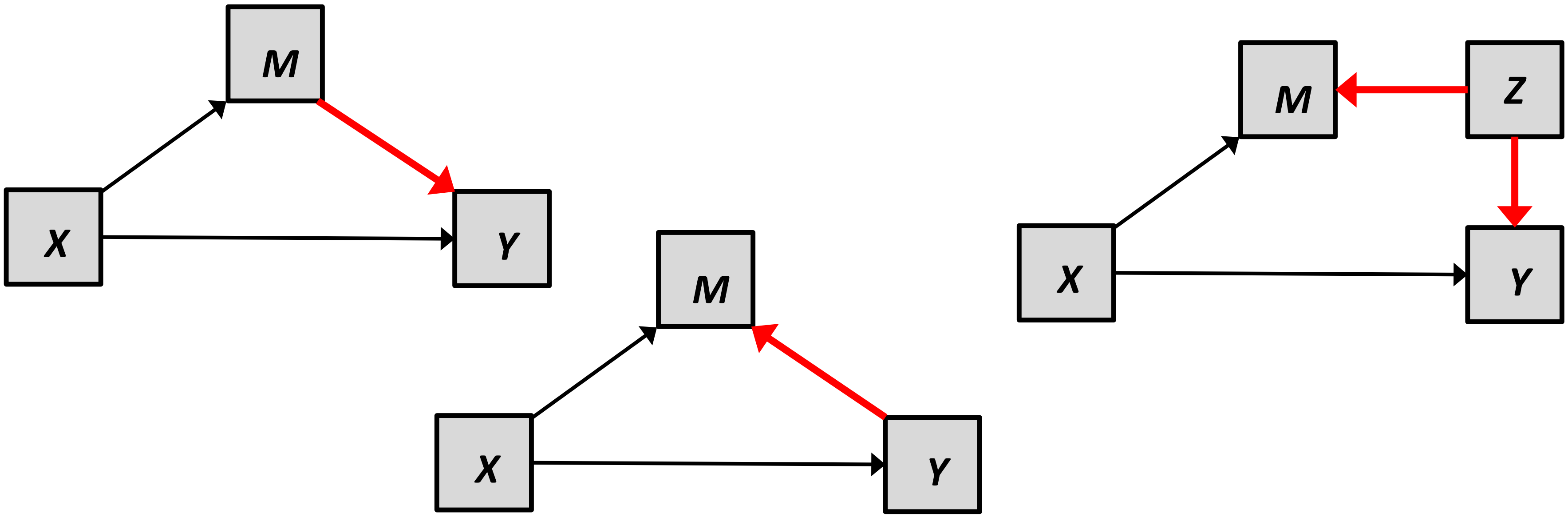


Even when *X* is randomly assigned, we can not provide evidence for the causal order between *M* and *Y*. This can only be supported using other experiments or previous research.

A statistically significant indirect effect does not lend credence to one model over another (Thoemmes, 2015, *Basic and Applied Social Psychology).*

Manipulation of X

Manipulation of and random assignment to *X* affords causal inference for the effect of *X* on *M* and *Y*, but not the effect of *M* on *Y*. We cannot establish causal order for the *M*-*Y* path using the methods that are the focus here. Theory is important. Multiple studies can help, one of which involves manipulation of *M*.



When *X* is not experimentally manipulated, all paths are subject to potential alternative causal orders or confounding.

MacKinnon, D. P. & Pirlott, A. G. (2015). Statistical Approaches for Enhancing Causal Interpretation of the M to Y Relation in Mediation Analysis. *Personality and Social Psychology Review, 19*(1), 30 – 43.

Understanding Cause and Effect

Consider two male, Caucasian politicians. One of these two has been convicted of political corruption. Which one is it?

Understanding causal effects

Never Convicted

Convicted

Understanding causal effects

Lin, C., Adolphs, R., & Alvarez, R. M. (2018). Inferring whether officials are corruptible from looking at their faces. *Psychological Science, 29*(11), 1807-1823.

Table 1. Results for Correctly Categorized Officials Based on Aggregate-Level Trait Inferences and Individual-Level Trait Inferences From Study 1

	Aggregate-level accuracy				Average individual-level accuracy[a]				
Trait	Percentage of correctly categorized officials (N = 72)	Lower bound of 95% CI	$\chi^2(1)$	p	Mean accuracy (N = 82)	SD	Lower bound of 95% CI	$t(81)$	Cohen's d
Corruptibility	69.44%	59.22%	10.13	< .001	55.73%	6.95%	54.46%	7.47	0.82
Dishonesty	70.83%	60.67%	11.68	< .001	54.82%	6.41%	53.64%	6.81	0.75
Selfishness	66.67%	56.36%	7.35	.003	55.10%	6.76%	53.86%	6.83	0.75
Trustworthiness	68.06%	57.79%	8.68	.002	55.03%	6.41%	53.85%	7.10	0.78
Generosity	63.89%	53.53%	5.01	.013	54.97%	5.99%	53.87%	7.51	0.83

Note: CI = confidence interval.
[a]All ps for this variable are less than .001.

Research Article

Psychological Science
2018, Vol. 29(11) 1807–1823

DOI: 10.1177/0956797618788882
www.psychologicalscience.org/PS
SAGE

Inferring Whether Officials Are Corruptible From Looking at Their Faces

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Division of Humanities and Social Sciences, California Institute of Technology

Abstract

While inferences of traits from unfamiliar faces prominently reveal stereotypes, some facial inferences also correlate with real-world outcomes. We investigated whether facial inferences are associated with an important real-world outcome closely linked to the face bearer's behavior: political corruption. In four preregistered studies (N = 325), participants made trait judgments of unfamiliar government officials on the basis of their photos. Relative to peers with clean records, federal and state officials convicted of political corruption (Study 1) and local officials who violated campaign finance laws (Study 2) were perceived as more corruptible, dishonest, selfish, and aggressive but similarly competent, ambitious, and masculine (Study 3). Mediation analyses and experiments in which the photos were digitally manipulated showed that participants' judgments of how corruptible an official looked were causally influenced by the face width of the stimuli (Study 4). The findings shed new light on the complex causal mechanisms linking facial appearances with social behavior.

Keywords

face perception, corruption, social attribution, stereotyping, political psychology, open data, open materials, preregistered

Received 9/25/17; Revision accepted 6/3/18

Faces are rich in information: They provide clues about gender, race, age, and trait attributes, which are inferred spontaneously and ubiquitously (Engell, Haxby, & Todorov, 2007; Todorov, 2017). Moreover, such inferences often guide our social behavior—for instance, we decide whom to trust on the basis of how trustworthy a face looks (Rezlescu, Duchaine, Olivola, & Chater, 2012; Van't Wout & Sanfey, 2008). Many trait judgments made by participants across generations and cultures show consensus (Cogsdill, Todorov, Spelke, & Banaji, 2014; Lin, Adolphs, & Alvarez, 2017; Rule et al., 2010). But are trait judgments from faces accurate?

Previous research has shown that trait judgments from faces can be associated with important real-world social outcomes, such as dating and mating (Olivola et al., 2014; Valentine, Li, Penke, & Perrett, 2014), earnings and fundraising (Genevsky & Knutson, 2015; Hamermesh, 2011; Ravina, 2012), science communication (Gheorghiu, Callan, & Skylark, 2017), sentencing decisions (Berry & Zebrowitz-McArthur, 1988; Blair, Judd, & Chapleau, 2004; Wilson & Rule, 2015; Zebrowitz & McDonald, 1991), and leader selection (Todorov, Mandisodza, Goren, & Hall, 2005; for reviews, see Antonakis & Eubanks, 2017; Todorov, Olivola, Dotsch, & Mende-Siedlecki, 2015). Yet this prior research on the association between trait judgments from faces and real-world outcomes leaves open two important questions. First, most associations have focused on prosocial outcomes (e.g., correlations between competence judgments and election success; Todorov et al., 2005). Second, most associations are plausibly driven not by the behavior of the targets whose face is being judged but by the interests of the perceivers who are making the judgments (e.g., correlations between interesting-looking scientists and the perceiver's interest in their work). Here, we investigated an antisocial judgment that

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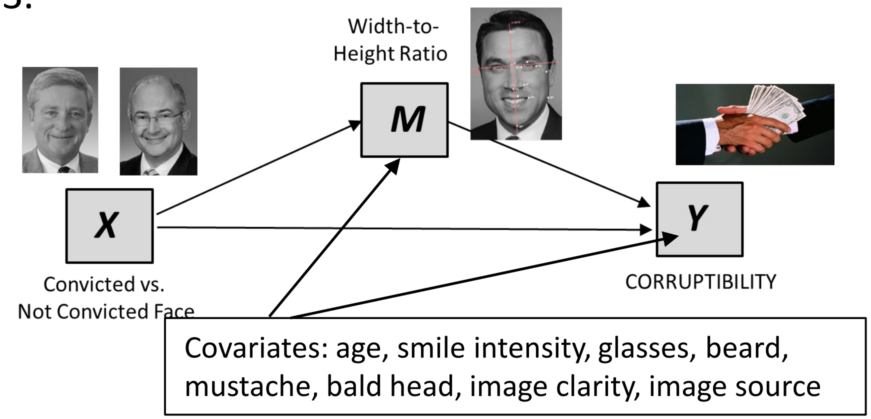
Understanding Cause and Effect

Lin, C., Adolphs, R., & Alvarez, R. M. (2018). Inferring whether officials are corruptible from looking at their faces. *Psychological Science, 29*(11), 1807-1823.

Necessary Conditions for Cause:

1. Covariation
2. Temporal Ordering
3. Elimination of competing explanations



1. Data from Studies 1 – 3 were combined to run the above mediation analysis. Have the necessary conditions for cause been met? Which necessary conditions does the mediation analysis support? Which require further support?

(Hint: Think about what would make you feel more confident in this causal order)

2. Study 4b experimentally manipulated the width-to-height ratios of faces to measure the impact on corruptibility. Which condition for cause does this help support? Are we now completely convinced about the causal claims of this mediation analysis?

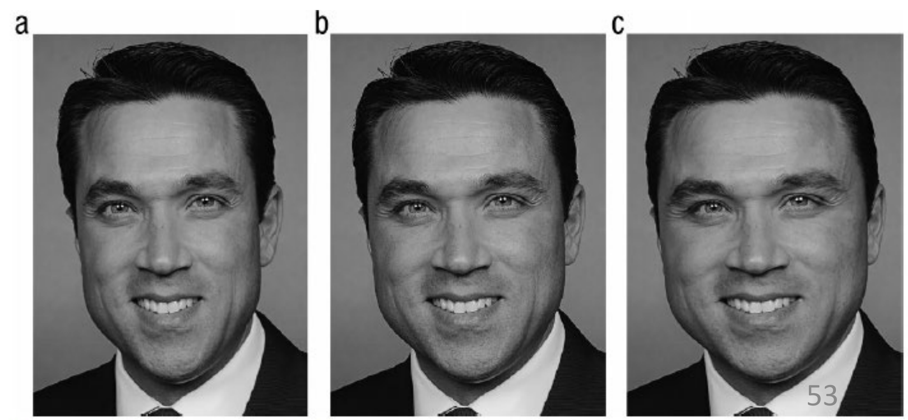


Fig. 6. Example of the same face in (a) slim, (b) original, and (c) fat versions.

Running Example: Group Work in Computer Science (WS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Honors Thesis).

Within-Subjects Version (CompSci_WS.sav, CompSci_WS.sas, CompSci_WS.csv) :
Female participants (N = 51) read <u>two syllabi</u> for a different computer science classes. One of the syllabi reported the class would have group projects throughout, and the other syllabi stated that individual project would be scheduled throughout.

- Syllabi also differed in professor's name (but not gender), and the primary programming language used in the class.

Measured Variables:

- Interest in each the class (same as BS version)
  - Two measures: `int_i int_g`
- Perceptions that the class has a communal environment.
  - Two measures: `comm_i comm_g`
  - Taking this class would assist me in _______________.
  - Helping others, serving the community, working with others, connecting with others, caring for others.
- How difficult would you rate the class you read about?
  - Two measures: `diff_i diff_g`

Judd, Kenny, and McClelland (2001)

Judd, C. M., Kenny, D. A., & McClelland, G. H. (2001). Estimating and testing mediation and moderation in within-subject designs. *Psychological Methods*, *6*, 115-134.

Psychological Methods
2001, Vol. 6, No. 2, 115–134

1082-989X/01/$5.00 DOI: 10.1037//1082-989X.6.2.115

Estimating and Testing Mediation and Moderation in Within-Subject Designs

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Analyses designed to detect mediation and moderation of treatment effects are increasingly prevalent in research in psychology. The mediation question concerns the processes that produce a treatment effect. The moderation question concerns factors that affect the magnitude of that effect. Although analytic procedures have been reasonably well worked out in the case in which the treatment varies between participants, no systematic procedures for examining mediation and moderation have been developed in the case in which the treatment varies within participants. The authors present an analytic approach to these issues using ordinary least squares estimation.

The issues of mediation and moderation have received considerable attention in recent years in both basic and applied research (Baron & Kenny, 1986; James & Brett, 1984; Judd & Kenny, 1981b; MacKinnon & Dwyer, 1993). In addition to knowing whether a particular intervention has an effect, the researcher typically wants to know about factors that affect the magnitude of that effect (i.e., moderation) and mechanisms that produce the effect (i.e., mediation). Such knowledge helps in both theory development and intervention application.

To illustrate the difference between mediation and moderation, consider a design in which a researcher is interested in whether students who are taught with a new curriculum (the treatment condition) show higher performance on a subsequent standardized test than students taught under the old curriculum (the control condition). Assuming that a performance difference is found, one might plausibly hypothesize different mediating mechanisms for this effect. The new curriculum might increase students' interest in the subject matter; it might cause students to study harder outside of class; or it might convey the material more clearly. These are alternative reasons why the performance difference is found, that is, alternative mediators of the treatment effect. The researcher might also be interested in factors that affect the magnitude of the difference between performance following the old curriculum and performance following the new one. That difference might be larger or smaller for different types of students or in different types of classrooms or when taught by different kinds of teachers. All of these then are potential moderators of the treatment effect.

It is possible that the same variable may serve as both a mediator and a moderator. For instance, study time might serve both roles. First, as a mediator, the new curriculum might lead to higher performance because it causes students to study more. Second, as a moderator, the treatment might be especially effective for students who spend more time studying.

Procedures for assessing mediation and moderation have been relatively well worked out through ordinary least squares regression and analysis of variance procedures. Mediation is assessed through a four-step procedure (Baron & Kenny, 1986; Judd & Kenny,

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One of the few treatments of mediation analysis in this common research design.

A "causal steps", Baron and Kenny type logic to determining whether *M* is functioning as a mediator of *X*'s effect on *Y* when both *M* and *Y* are measured twice in difference circumstances but on the same people.

1. On average, does *Y* differ by condition?
2. On average, does *M* differ by condition?
3. Does difference in *M* predict a difference in *Y*?
4. Does the difference in *M* account for all the difference in *Y*?

Computer Science Within-Subjects Data Example

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Thesis).

Research Question: Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?

Data is in *wide form*: repeated measurements of the same variables are saved as separate variables (one row per participant). *Long form* is when there is a variable coding instance of repeated measurements (multiple rows per participant, one for each instance).

CompSci_WS.sav

int_I	int_G	comm_I	comm_G
1.50	4.00	1.00	6.80
2.75	3.25	2.00	5.40
5.75	2.50	3.20	3.60
3.50	5.75	1.60	5.20
2.25	2.00	4.40	4.60
1.50	1.75	3.00	5.00
2.50	4.25	4.20	4.40
6.00	1.75	4.80	2.40
3.00	2.00	2.60	5.80
4.00	5.25	1.60	5.00
5.00	5.00	4.60	6.20
2.00	1.75	3.80	4.20
1.00	1.75	2.60	3.20
1.25	4.50	1.00	6.00
5.75	4.50	2.60	6.00
3.25	4.75	3.00	6.20
2.75	2.25	4.80	4.60
5.50	2.00	4.00	7.00
1.75	5.25	1.60	5.60
4.00	5.50	1.80	5.40
2.25	4.00	2.20	4.80
4.00	6.50	2.00	6.80
5.00	4.50	3.20	6.00

Analysis using Judd et al. (2001)

1. On average, does *Y* differ by condition?

Setup a model of the outcome in each condition:

$$Y_{1i} = c_1 + \epsilon_{Y^*_{1i}}$$
$$Y_{2i} = c_2 + \epsilon_{Y^*_{2i}}$$

Is c_1 different from c_2?

Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $c_2 - c_1$):

$$Y_{2i} - Y_{1i} = (c_2 - c_1) + \left(\epsilon_{Y^*_{2i}} - \epsilon_{Y^*_{1i}}\right) = c + \epsilon_{Y^*_i}$$

Use intercept only regression analysis, or a paired sample t-test, or a one sample t-test on the differences to conduct inference on $c_2 - c_1$

With the data: On average, is class interest higher in the group work condition?

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	95% Confidence Interval of the Difference: Upper			
Pair 1	int_G - int_I	.37255	1.99585	.27948	-.18879	.93389	1.333	50	.189

```
T-TEST PAIRS = int_G with int_I (PAIRED).
```

```
PROC TTEST DATA=CompSci_WS; PAIRED int_G*int_I; RUN;
```

```
t.test(Pair(int_G, int_I)~1, data = CompSci_WS)
```

Analysis using Judd et al. (2001)

2. On average, does *M* differ by condition?

Setup a model of the mediator in each condition:

$$M_{1\mathrm{i}} = a_1 + \epsilon_{M_{1i}}$$
$$M_{2\mathrm{i}} = a_2 + \epsilon_{M_{2i}}$$

Is a_1 different from a_2?

Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $a_2 - a_1$):

$$M_{2i} - M_{1i} = (a_2 - a_1) + \left(\epsilon_{M_{2i}} - \epsilon_{M_{1i}}\right) = a + \epsilon_{M_i}$$

Use intercept only regression analysis, or a paired sample t-test, or a one sample t-test on the differences to conduct inference on $a_2 - a_1$

With the data: On average, is communal goal affordance higher in the group work condition?

Paired Samples Test

		Paired Differences					t	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	comm_G - comm_I	2.29412	1.77870	.24907	1.79385	2.79438	9.211	50	.000

```
T-TEST PAIRS = comm_G with comm_I (PAIRED).
```

```
PROC TTEST DATA=CompSci_WS; PAIRED comm_G*comm_I; RUN;
```

```
t.test(Pair(comm_G, comm_I)~1, data = CompSci_WS)
```

Analysis using Judd et al. (2001)

3. Does difference in *M* predict a difference in *Y*? / Does *M* predict *Y* controlling for condition?

Setup a model of the outcome in each condition:

$$Y_{1\mathrm{i}} = g_{10} + g_{11}M_{1i} + \epsilon_{Y_{1i}}$$
$$Y_{2\mathrm{i}} = g_{20} + g_{21}M_{2i} + \epsilon_{Y_{2i}}$$

Note that there are **two estimates** of the effect of *M* on *Y*. Let's average them to estimate an average effect of *M* on *Y*. Setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $\frac{1}{2}(g_{21} + g_{11})$):

Optional board work

$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + g_{21}M_{2i} - g_{11}M_{1i} + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$
$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + \underbrace{\frac{g_{21}+g_{11}}{2}}_{b}(M_{2i} - M_{1i}) + \underbrace{\frac{(g_{21}-g_{11})}{2}}_{d}(M_{2i} + M_{1i}) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

Analysis using Judd et al. (2001)

3. Does *M* predict *Y* controlling for condition?

With the data: Does communal goal affordance predict interest in the class?

```
compute int_diff = int_G - int_I.
compute comm_diff = comm_G - comm_I.
compute comm_sum = comm_G+comm_I.
EXECUTE.
regression dep = int_diff /method = enter comm_diff comm_sum.
```

```
data CompSci_WS;set CompSci_WS;int_diff=int_g-int_I;
comm_diff = comm_G - comm_I; comm_sum = comm_G+comm_I;run;
proc reg data=CompSci_WS;model int_diff=comm_diff comm_sum;run;
```

```
CompSci_WS <- transform(CompSci_WS,
                        int_diff = int_G - int_I,
                        comm_diff = comm_G - comm_I,
                        comm_sum = comm_G +comm_I)
summary(lm(int_diff~comm_diff+comm_sum, data = CompSci_WS))
```

Analysis using Judd et al. (2001)

3. Does *M* predict *Y* controlling for condition?

With the data: Does communal goal affordance predict interest in the class?

```
compute int_diff = int_G - int_I.
compute comm_diff = comm_G - comm_I.
compute comm_sum = comm_G+comm_I.
EXECUTE.
regression dep = int_diff /method = enter comm_diff comm_sum.
```

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.310	1.877		.698	.489
	comm_diff	.590	.135	.526	4.385	.000
	comm_sum	-.275	.216	-.153	-1.272	.210

a. Dependent Variable: int_diff

Analysis using Judd et al. (2001)

4. Does the difference in communal goal affordance account for all the difference in interest?

$$Y_{2i} - Y_{1i} = (g_{20} - g_{10}) + \underbrace{\frac{g_{21}+g_{11}}{2}}_{b}(M_{2i} - M_{1i}) + \underbrace{\frac{(g_{21}-g_{11})}{2}}_{d}(M_{2i} + M_{1i}) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

Next we center the sum term, so the intercept has the interpretation of the predicted difference in *Y* for someone with no difference in *M*'s but is average on *M*'s.

$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} - (\overline{M_2 + M_1})) + (\epsilon_{Y_{2i}} - \epsilon_{Y_{1i}})$$

where $\quad c' = \left(g_{20} - g_{10} + d(\overline{M_2 + M_1})\right)$

Intercept is predicted *outcome* when all regressors are zero. This means predicted difference in *Y* when there is no difference in *M* and a person is average on the sum of *M*.

Analysis using Judd et al. (2001)

4. Does the difference in communal goal affordance account for all the difference in interest?

With the data: Is there a significance difference in interest predicted when there is no difference in communal goals?

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	-.981	.388		-2.527	.015
	comm_diff	.590	.135	.526	4.385	.000
	comm_sum	-.275	.216	-.153	-1.272	.210

a. Dependent Variable: int_diff

```
compute comm_sumc = comm_sum- 8.325490.
EXECUTE.
regression dep = int_diff /method = enter comm_diff comm_sumc.
```

```
data CompSci_WS;set CompSci_WS;comm_sumc = comm_sum-8.325490;run;
proc reg data=CompSci_WS;model int_diff=comm_diff comm_sumc;run;
```

```
CompSci_WS <- transform(CompSci_WS,
                        comm_sumc = comm_sum-8.325490)
summary(lm(int_diff~comm_diff+comm_sumc, data = CompSci_WS))
```

Analysis using Judd et al. (2001)

1. On average, is interest higher in the group work condition?
2. On average, is communal goal affordance higher in the group work condition?
3. Does difference in communal affordance predict a difference in interest?
4. Does the difference in communal goal affordance account for all the difference in interest?

According to Judd, Kenny, and McClelland we do not have a mediated effect!

Because there is no evidence that interest is higher in the group work condition, the Judd et al. (2001) method would conclude there is not mediation.

Judd et al. Criticisms and Misuses

All criticisms of the causal steps approach apply to this approach:

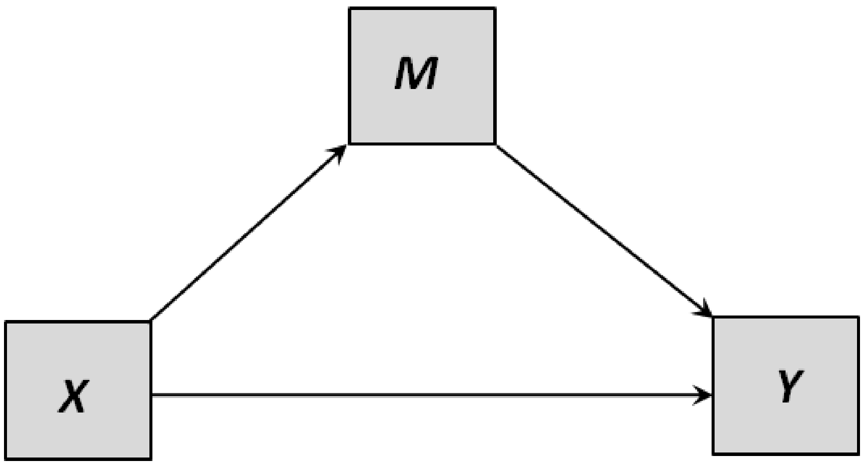
- There is no explicit quantification of the indirect effect
    - Inference about an indirect effect should be the result of a test on a *quantification* of the indirect effect
- Requiring that there must be a total effect is too restrictive
    - The direct and indirect effect could be of opposite sign
    - There is greater power to detect the indirect effect than total effect *(Judd, Kenny, 2014, Psych Science)*

This method has been used by a variety of researchers:

- Approximately 1150 citing papers, with around 1/3 using this method
- Many researchers do not report or estimate the partial regression coefficient for the sum of the mediators
- Because the estimate of the indirect effect is not made explicit, researchers often misinterpret the coefficients
    - *b* path is often interpreted as indirect effect
- Extensions to more complicated models have been poorly implemented until recently

Can we think about it like a path analysis?

Analytic Goal: **Can <u>group work</u> in computer science classes increase women's <u>interest</u> by increasing their perception that computer science is <u>communal</u>?**



Where is *X* in the data?

Y_1	Y_2	M_1	M_2
int_I	int_G	comm_I	comm_G
1.50	4.00	1.00	6.80
2.75	3.25	2.00	5.40
5.75	2.50	3.20	3.60
3.50	5.75	1.60	5.20
2.25	2.00	4.40	4.60
1.50	1.75	3.00	5.00
2.50	4.25	4.20	4.40
6.00	1.75	4.80	2.40
3.00	2.00	2.60	5.80
4.00	5.25	1.60	5.00
5.00	5.00	4.60	6.20
2.00	1.75	3.80	4.20
1.00	1.75	2.60	3.20
1.25	4.50	1.00	6.00
5.75	4.50	2.60	6.00
3.25	4.75	3.00	6.20
2.75	2.25	4.80	4.60
5.50	2.00	4.00	7.00
1.75	5.25	1.60	5.60
4.00	5.50	1.80	5.40
2.25	4.00	2.20	4.80
4.00	6.50	2.00	6.80
5.00	4.50	3.20	6.00
5.00	3.75	4.00	4.80
4.75	5.25	1.20	6.60

Advantages of a path analytic approach

Provides an estimate of the indirect, total, and direct effects

- Allows us to conduct inferential tests directly on an estimate of the indirect effect

Connects researchers understanding of between-subjects mediation to within-subjects mediation

- Reduce misinterpretation of regression coefficients

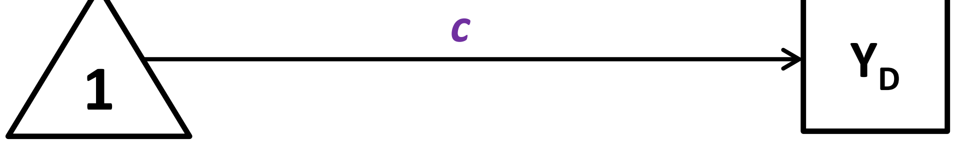
Using a path analytic framework will help extend the simple mediation model to more complicated questions

- Multiple mediators
- Moderated mediation
- Integration of between and within-subjects designs

Path-Analytic Approach

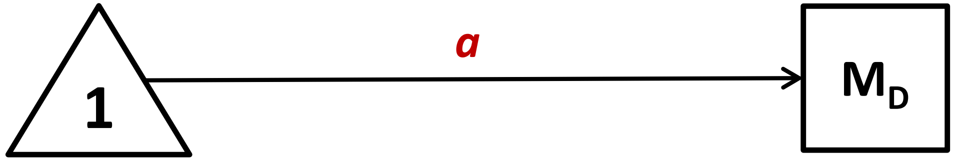
Total Effect (*c*): The effect of our presumed cause (*X*) on our outcome (*Y*), without controlling for any other variables. (i.e. mean difference in outcome between the two conditions).

$$Y_{2i} - Y_{1i} = c + \epsilon_{Y_i^*}$$



***a*-path:** The effect of our presumed cause (*X*) on our mediator *(M).* (i.e. mean difference in mediator between the two conditions).

$$M_{2i} - M_{1i} = a + \epsilon_{M_i}$$

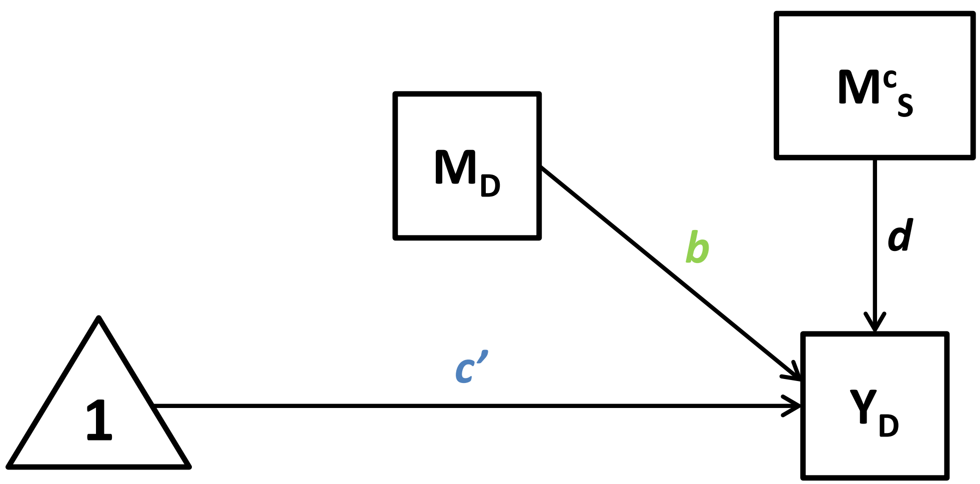


Path-Analytic Approach

***b*-path:** The effect of our mediator (*M*) on the outcome (*Y*) while controlling for *X*. (i.e. predicted difference in *Y* for two people with the <u>same score on *X*</u> but who differ on *M* by one unit).

Direct effect (*c'*): The effect of our presumed cause (*X*) on *Y* while controlling for *M*. (i.e. predicted difference in *Y* for two people who differ by one unit on *X* but with the same score on *M*)

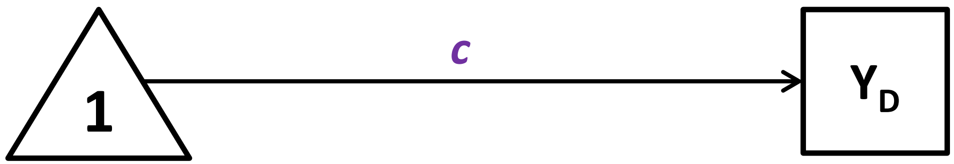
$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} - (\overline{M_2 + M_1})) + \epsilon_{Y_i}$$

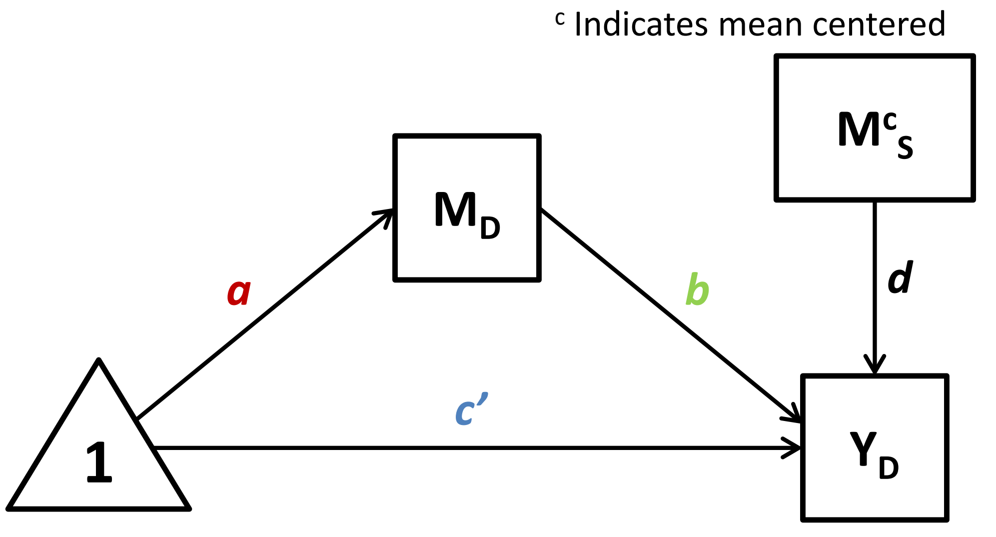


[c] Indicates mean centered

Path-Analytic Approach

Indirect Effect (*ab*): Product of effect of *X* on *M*, and effect of *M* on *Y* controlling for *X*. The effect of *X* on *Y* through *M*.

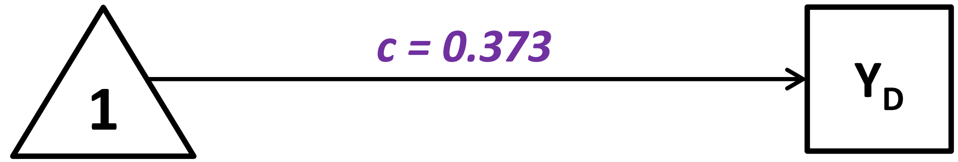




Within Subjects: Path Estimates

Total Effect c: (Regress Y_D on a constant)

$$\widehat{Y_D} = c$$
$$\widehat{Y_D} = .373$$



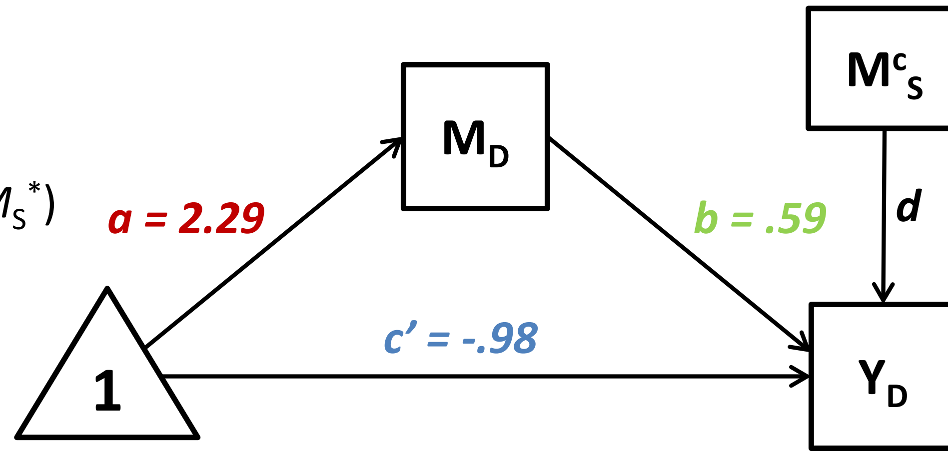
a path: (Regress M_D on a constant)

$$\widehat{M_D} = a$$
$$\widehat{M_D} = 2.29$$

^{c} Indicates mean centered

b path and c' path: (Regress Y_D on M_D and M_S^*)

$$\widehat{Y_D} = c' + b_1 M_D + d M_S^c$$
$$\widehat{Y_D} = -.98 + .59 M_D - .28 M_S^c$$



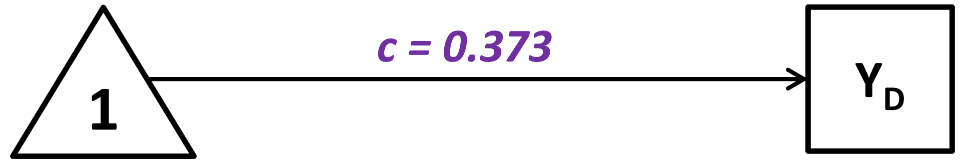
A one unit increase in the difference in communal goal affordance is expected to result in a .59 unit increase in the difference in interest.

People with no difference in communal goal affordance perceptions are expected to be .98 units more interested in the individual class than the group work class .

Note: M_s must be mean centered for c' to have intended interpretation

Data Example: Partitioning effect of X on Y

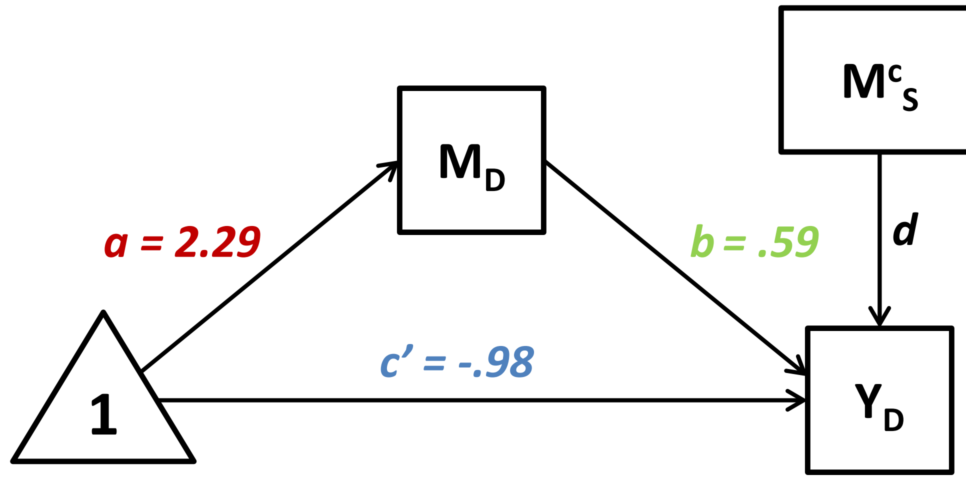
The effect of *X on Y* partitions into two components: direct and indirect, in the usual way.



$$c = c' + a \times b$$

$$.373 = -.98 + 2.29 \times .59$$

$$.373 = -.98 + 1.35$$

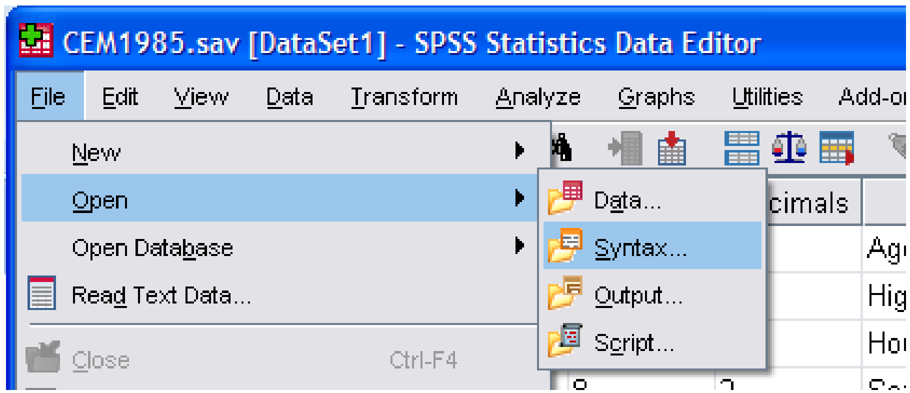


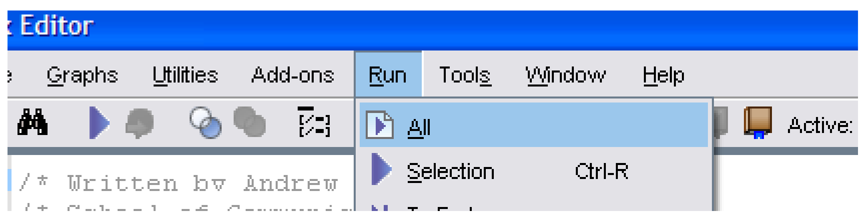
We can conduct inferential tests on the estimate of the indirect effect as in any other mediation analysis.

MEMORE has three methods of inference for the indirect effect available: bootstrapping, Monte Carlo confidence intervals, Sobel Tests

Teaching your package MEMORE (SPSS)

MEMORE is a command which must be taught and re-taught to your statistical package (SPSS) every time you open the package. To teach your program the MEMORE command, open the memore.sps file and run the script exactly as is.

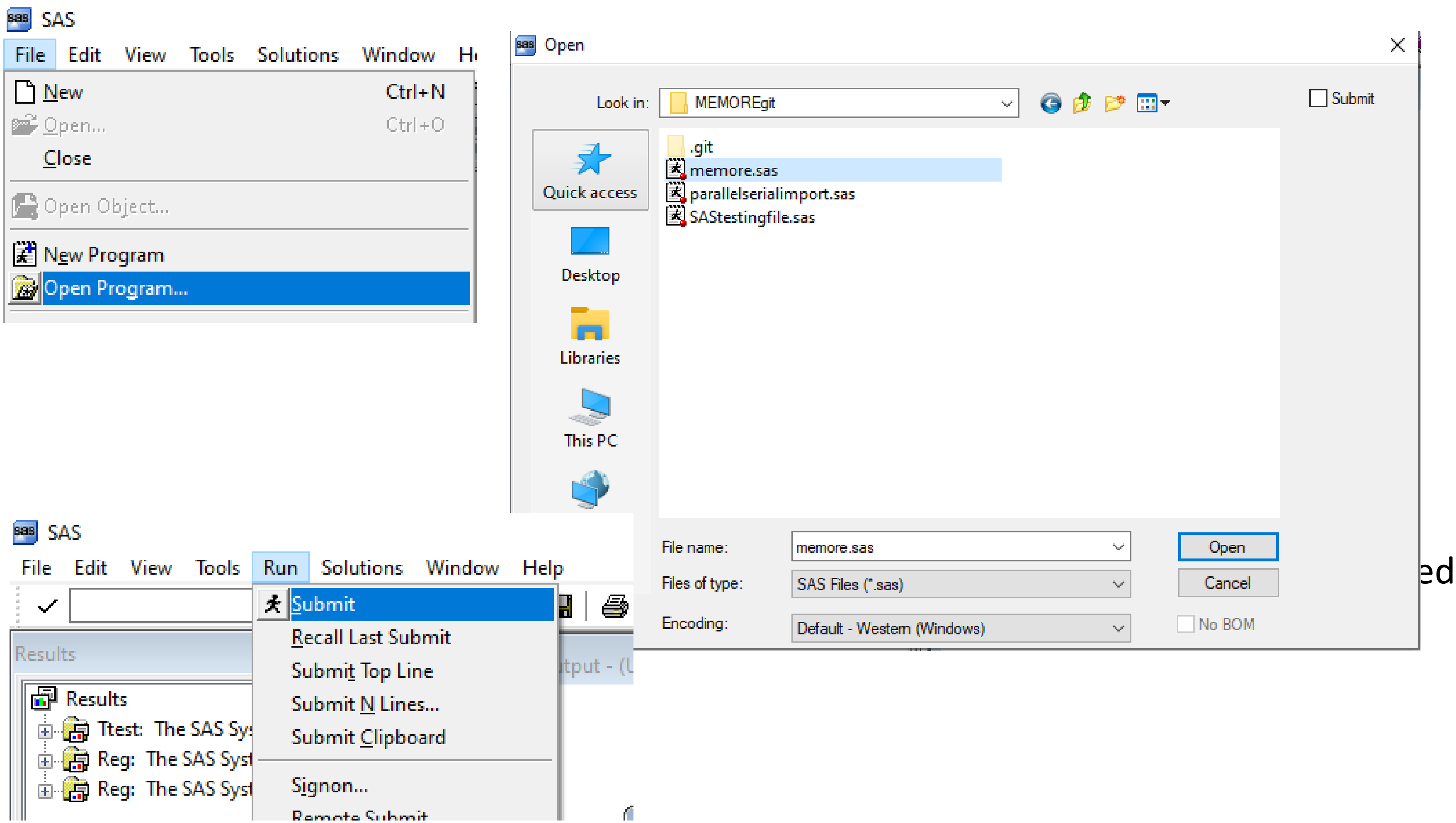




SPSS now knows a new command called MEMORE

Teaching your package MEMORE (SAS)

MEMORE is a command which must be taught and re-taught to your statistical package (SAS) every time you open the package. To teach your program the MEMORE command, open the memore.sas file and run the script exactly as is.



Writing MEMORE Syntax

MEMORE has 2 required arguments: `Y` and `M`, `data` is required for SAS

```
MEMORE m= comm_G comm_I /y = int_G int_I /normal=1/samples=10000
/conf = 90 /model = 1.
```

```
%memore(m = comm_G comm_I, y = int_G int_I, normal = 1, samples =
10000, conf = 90, model = 1, data = CompSci_WS);
```

M is your list of mediators (order matters)
Y is you list of outcomes (order should be matched to the order in the M list)

Arguments:

model specifies the model you are interested. The default is 1, mediation. Moderation models are 2 and 3, 4+ are moderation.

normal = 1 asks for Sobel test

samples corresponds to the number of bootstrap/MC samples you would like

conf specifies level of confidence you want (default is 95)

mc = 1 asks for Monte Carlo confidence intervals

bc = 1 asks for bias corrected bootstrap confidence intervals

Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I /model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, data=CompSci_WS);
```

```
********************** MEMORE Procedure for SPSS Version 2.Beta ************************

                           Written by Amanda Montoya

                    Documentation available at akmontoya.com

**************************************************************************************

Model:
  1

Variables:
Y = int_G    int_I
M = comm_G   comm_I

Computed Variables:
Ydiff =          int_G     -       int_I
Mdiff =          comm_G    -       comm_I
Mavg =  (        comm_G    +       comm_I    )        /2       Centered

Sample Size:
  51

**************************************************************************************
```

First part of output repeats what you told MEMORE to do. Always double check that this is correct!

Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I /model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, data=CompSci_WS);
```

```
*****************************************************************************
Outcome: Ydiff =  int_G     -       int_I

Model
         Effect         SE          t          p       LLCI       ULCI
'X'       .3725      .2795     1.3330      .1886     -.1888      .9339

Degrees of freedom for all regression coefficient estimates:
  50

*****************************************************************************
Outcome: Mdiff =  comm_G    -       comm_I

Model
         Effect         SE          t          p       LLCI       ULCI
'X'      2.2941      .2491     9.2108      .0000     1.7938     2.7944

Degrees of freedom for all regression coefficient estimates:
  50

*****************************************************************************
```

Outcome variable

c = .37

a = 2.29

First few sections are regression models involved in the mediation analysis. This is the model of *Y* from *X*, therefore this is the model which produces the estimate of *c*

Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I /model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, data=CompSci_WS);
```

```
**************************************************************************************
Outcome: Ydiff =  int_G      -      int_I

Model Summary
          R        R-sq         MSE           F         df1         df2           p
      .5639       .3180      2.8299     11.1909      2.0000     48.0000       .0001

Model
              coeff          SE           t           p        LLCI        ULCI
'X'          -.9814       .3884     -2.5269       .0149     -1.7623     -.2005
Mdiff         .5902       .1346      4.3845       .0001       .3195       .8608
Mavg         -.5505       .4328     -1.2718       .2096     -1.4208       .3198

Degrees of freedom for all regression coefficient estimates:
  48
```

This is the model predicting Y_D from a constant, M_D, and M^c_{avg} therefore this model gives us an estimate of *b* and *c′*

***c′* = -.98**

***b* = .590**

Using MEMORE for CompSci WS data

```
MEMORE m= comm_G comm_I /y = int_G int_I /model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, data=CompSci_WS);
```

```
********************** TOTAL, DIRECT, AND INDIRECT EFFECTS **********************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
      .3725      .2795     1.3330    50.0000      .1886    -.1888      .9339

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     -.9814      .3884    -2.5269    48.0000      .0149   -1.7623     -.2005

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1      1.3540      .3260      .6827     1.9653

Indirect Key
Ind1   X        ->       M1diff    ->       Ydiff
```

Important effects for mediation and inference about these effects

Based on a 95% bootstrap confidence interval we have evidence of mediation!

Turning off the *XM* interaction

$$Y_{2i} - Y_{1i} = c' + b(M_{2i} - M_{1i}) + d(M_{2i} + M_{1i} \; - (\overline{M_2 + M_1})) + \epsilon_{Y_i}$$

When we estimate this regression model, we allow the relationship between *M* and *Y* to differ by instance (*X*). This is like allowing for an interaction between *X* and *M* when estimating *Y*.

We do this by including the sum term in the regression model.

d estimates the difference in the relationship between M_1 → Y_1 and M_2 → Y_2. If we fix this coefficient to zero (do not include the sum term in the model) we fix the interaction to zero.

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0/model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, xmint = 0,
data=CompSci_WS);
```

Turning off the XM interaction

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0/model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, xmint = 0,
data=CompSci_WS);
```

No interaction **Interaction**

```
Outcome: Ydiff =  int_G     -       int_I

Model
         Effect         SE          t          p       LLCI       ULCI
'X'       .3725      .2795     1.3330      .1886     -.1888      .9339

Degrees of freedom for all regression coefficient estimates:
  50

*************************************************************************************
Outcome: Mdiff =  comm_G    -       comm_I

Model
         Effect         SE          t          p       LLCI       ULCI
'X'      2.2941      .2491     9.2108      .0000     1.7938     2.7944

Degrees of freedom for all regression coefficient estimates:
  50

*************************************************************************************
```

c = .3725 (.2795)

a = 2.2941 (.2491)

Turning off the XM interaction

```
MEMORE m= comm_G comm_I /y = int_G int_I /xmint = 0/model = 1.
```

```
%memore(m=comm_G comm_I, y=int_G int_I, model=1, xmint = 0,
data=CompSci_WS);
```

No interaction | **Interaction**

```
**************************************************************************************
Outcome: Ydiff =  int_G     -       int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .5432      .2950     2.8655    20.5060     1.0000    49.0000      .0000

Model
          coeff         SE          t          p       LLCI       ULCI
'X'     -1.0257      .3893    -2.6349      .0112    -1.8079    -.2434
Mdiff     .6095      .1346     4.5284      .0000      .3390      .8799


 Indirect Effect of X on Y through M
           Effect     BootSE   BootLLCI   BootULCI
 Ind1      1.3982      .3082      .8034     2.0156
```

c = -.9814(.2795)

b = .5902 (.1346)

d = -.5505 (.4328)

ab = 1.3540 [.6827,1.9653]

Ultimately results are mostly unchanged, but that is not always the case.

Testing the *XM* interaction

There are three potential approaches to testing the *XM* interaction that you might take:

1. **Include the *XM* interaction by default**
   - This is what is recommended in causal mediation analysis (Vo et al., 2020)
   - Use the xmint=1 option in MEMORE always (this is the default)
2. **Test/Evaluation *XM* interaction as a pre-step**
   - Estimate coefficient and use a hypothesis test or effect size measure to evaluate whether it should be included
   - Include your threshold in your preregistration
   - Final model will depend on whether *XM* is included or not
3. **Robustness Check**
   - Fit the model as hypothesized (*XM* in or out)
   - Afterwards fit the other model and evaluate whether the results are sensitive to the choice (change in effect size or significance level)

Writing up a Repeated Measures Mediation Analysis

Tips:

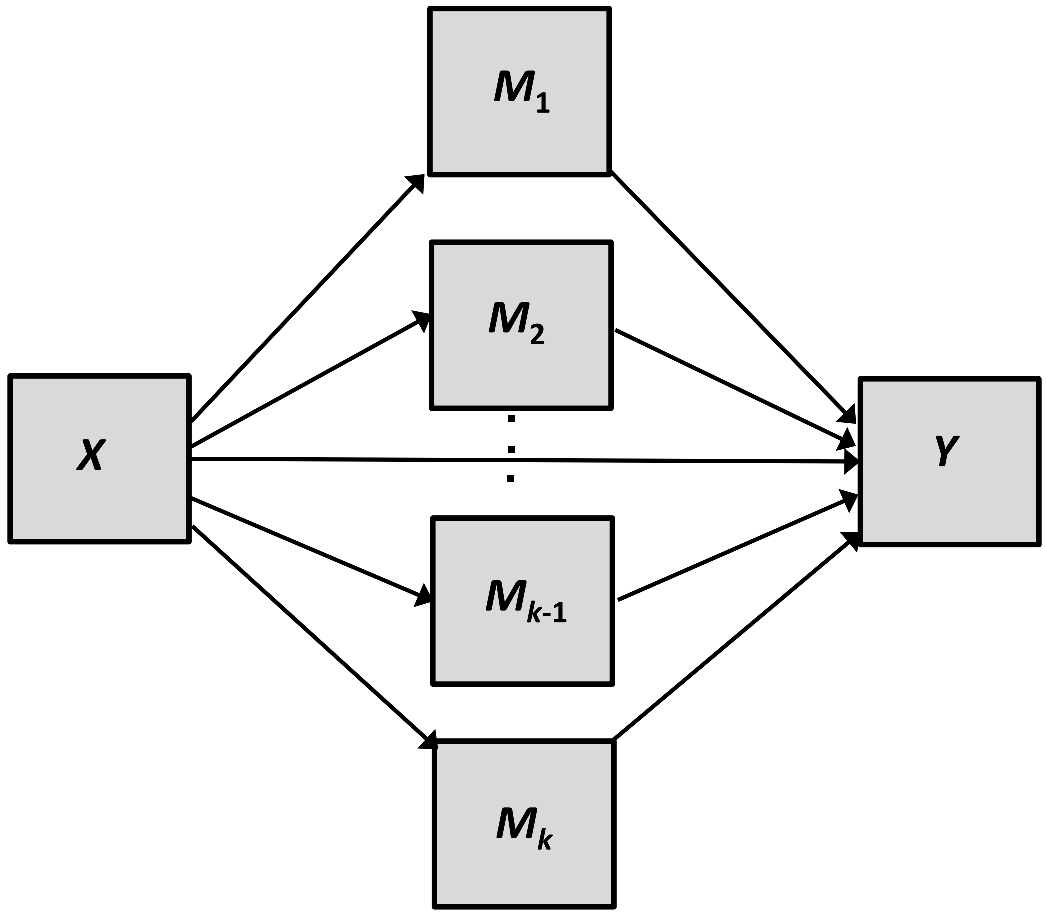
- Walk the reader through the steps of the mediation in a way that is intuitive.
    - Include interpretations of the results: b.e.g. "The total effect was significant, $p < .05$"
- Use equations and numbers *where helpful*.
- Avoid using computational variable names (e.g. RESPAPPR)
- Avoid causal language if it is not supported by your research design.
- Pick one inferential method and report it
- Read the write ups of other's mediation analyses

Is the effect of group work on class interest mediated by communal goal affordance of the class?

Overall there was no evidence of a total effect of group work on interest in computer science classes, we estimate that individuals were .37 units higher on interest in group work than individual work classes ($p = .19$). The class with group work was rated 2.29 units higher on communal goal affordance than the class with individual work ($p < .001$). A one unit increase in perception of communal goal affordance increased interest in the class by .59 units ($p = .0001$), and the relationship between communal goal affordance and interest in a class did not depend on condition ($p = .21$). The effect of group work on interest through communal goal fulfillment was different from zero ($ab = 1.35$, 95% Bootstrap CI [.68, 1.96]). This means that we expect women to be 1.35 units more interested in a computer science class with group work compared to one without group work, through the effect of group work on communal goal affordance, and the subsequent effect of communal goal affordance on interest. There was a significant direct effect between group work and interest ($c' = -.98$, $p = .01$). This indicates that there may be some other process, separate from communal goal affordance, which is actually deterring women from computer science classes with group work.

Models with More Than One Mediator

A parallel multiple mediator model



Why estimate such a model?

- Many causal effects probably operate through multiple mechanisms simultaneously. Better to estimate a model **consistent with such real-world** complexities.

- If your proposed mediator is correlated with the real mediator but not caused by the independent variable, a model with only your proposed mediator in it will be a **misspecification** and will potentially misattribute the process to your proposed mediator rather than the real mediator—"epiphenomenality."

- Different theories may postulate different mediators as mechanisms. Including them all in a model simultaneously allows for a formal statistical comparison of indirect effects **representing different theoretical mechanisms**.

Path Analysis: Total, Direct, and Indirect Effects

$$\hat{Y} = c_0 + cX$$

$$\widehat{M_j} = a_{0j} + a_j X$$

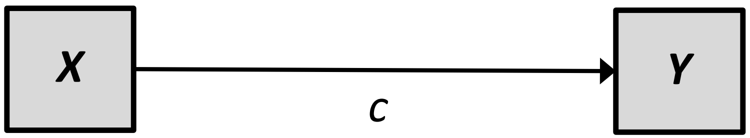
$$\hat{Y} = c'_0 + c'X + \sum_{j=1}^{K} b_j M_j$$

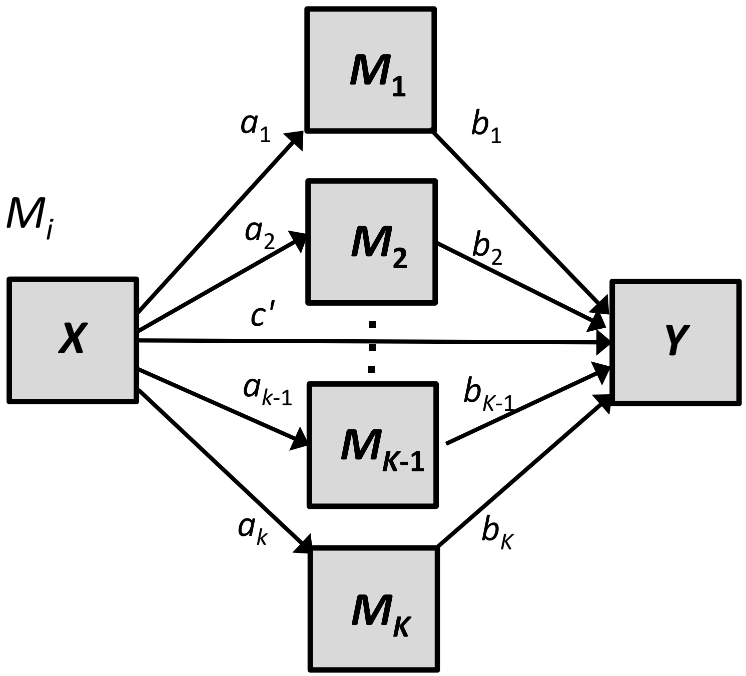
c = "total effect" of X on Y

$a_j \times b_j$ = "specific indirect effect" of X on Y through M_i

$\Sigma (a_j \times b_j)$= "total indirect effect" of X on Y

c' = "direct effect" of X on Y





total effect = direct effect + total indirect effect
$c \quad = \quad c' \quad + \quad \Sigma (a_j \times b_j)$

total indirect effect = total effect – direct effect
$\Sigma (a_j \times b_j) \quad = \quad c \quad - \quad c'$

Example: Group work in Computer Science

Participants read two syllabi for computer science classes. The syllabi had of two policies: **procollaboration** or **no collaboration**. Participants were randomly assigned read each syllabus in a random order

Participants completed questionnaire (Higher = greater):

(1) interest in the class (this is the primary DV).
(2) how much they felt the class would help them in achieving **communal goals** (helping others, working with others)
(3) how **difficult** they expected the class to be.

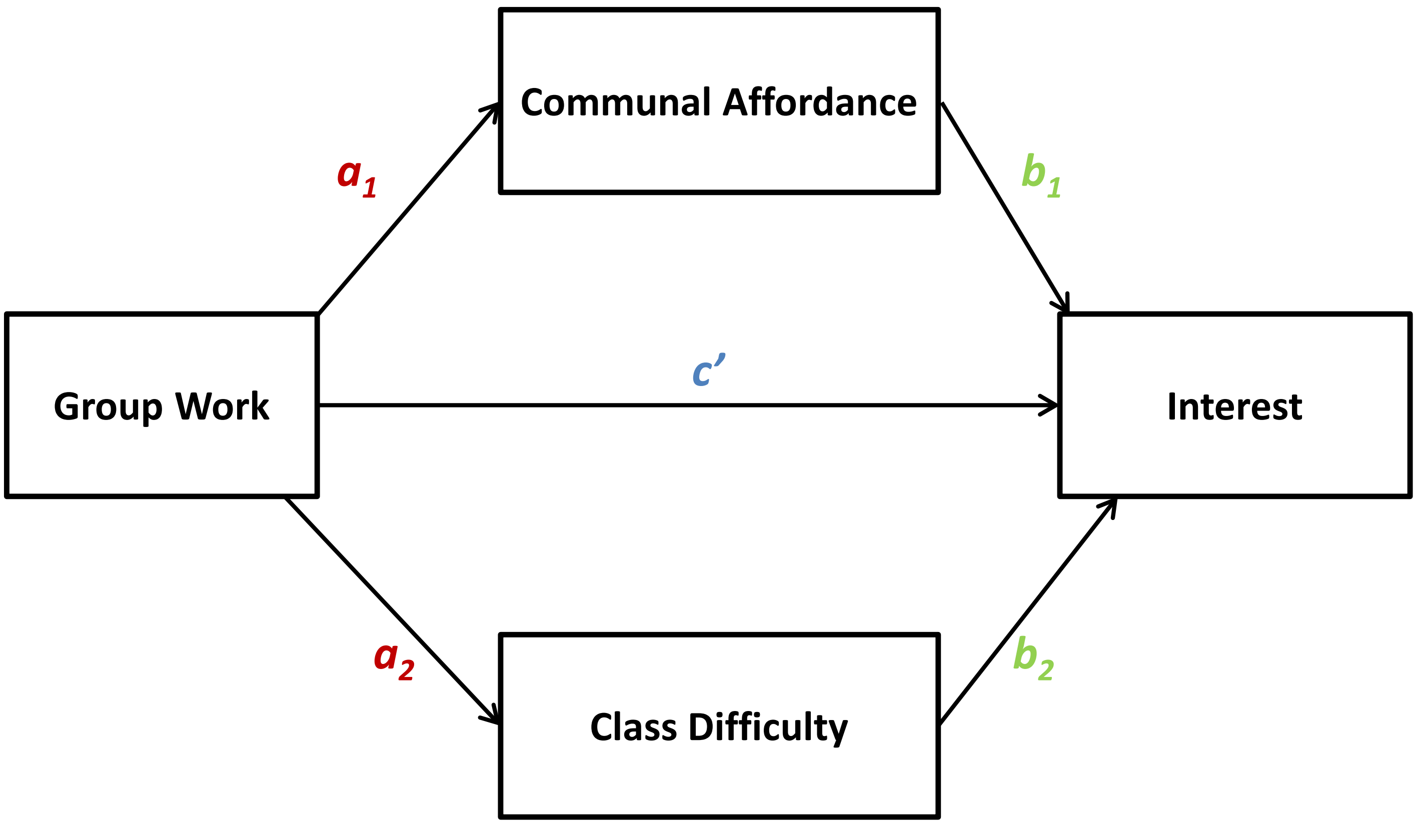

Girls who Code

Question: Does group work in computer science classes increase interest in the class indirectly through perceived communal goal fulfillment, through class difficulty, or both?

Would people who read about the procollaboration policy think the class is more communal and would that communality then predict greater interest? Would the procollaboration policy make students think the course is easier, and this would increase interest?

Using MEMORE for CompSci WS data

Do people just like group work classes because they are easier?



Using MEMORE for CompSci WS data

Do people just like group work classes because they are easier?

```
MEMORE m = comm_I comm_G diff_I diff_G /y = int_I int_G /model = 1.
```

```
%memore(m = comm_G comm_I diff_G diff_I, y = int_G int_I, model = 1,
data = CompSci_WS);
```

```
**************************************************************************
Outcome: Ydiff =  int_I      -       int_G

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .6307      .3978     2.6073     7.5978     4.0000    46.0000      .0001

Model
           coeff         SE          t         df          p       LLCI       ULCI
'X'        .9172      .3815     2.4042    46.0000      .0203      .1493     1.6851
M1diff     .4847      .1448     3.3460    46.0000      .0016      .1931      .7762
M2diff    -.4123      .1878    -2.1952    46.0000      .0332     -.7904     -.0342
M1avg      .5160      .4157     1.2411    46.0000      .2209     -.3209     1.3528
M2avg     -.3781      .2879    -1.3133    46.0000      .1956     -.9577      .2014
```

Notice that we are now **controlling** for difficulty of the class when estimating the effect of communal goal affordance on interest!

Using MEMORE for CompSci WS data

Do people just like group work classes because they are easier?

```
MEMORE m = comm_I comm_G diff_I diff_G /y = int_I int_G /model = 1.
```

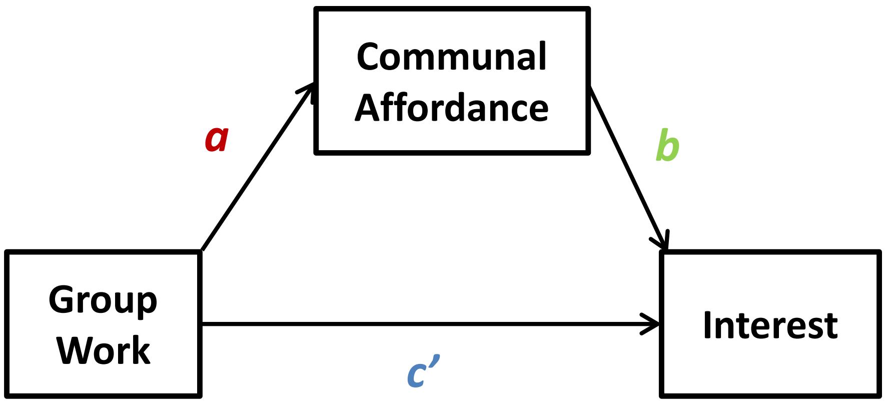
```
%memore(m = comm_G comm_I diff_G diff_I, y = int_G int_I, model = 1,
data = CompSci_WS);
```

```
********************** TOTAL, DIRECT, AND INDIRECT EFFECTS **********************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     -.3725      .2795    -1.3330    50.0000      .1886     -.9339      .1888

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
      .9172      .3815     2.4042    46.0000      .0203      .1493     1.6851

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1     -1.1119      .3812    -1.8531     -.3522
Ind2      -.1779      .1160     -.4465      .0000
Total    -1.2897      .3507    -1.9566     -.5612

Indirect Key
Ind1  X        ->       M1diff    ->       Ydiff
Ind2  X        ->       M2diff    ->       Ydiff
```

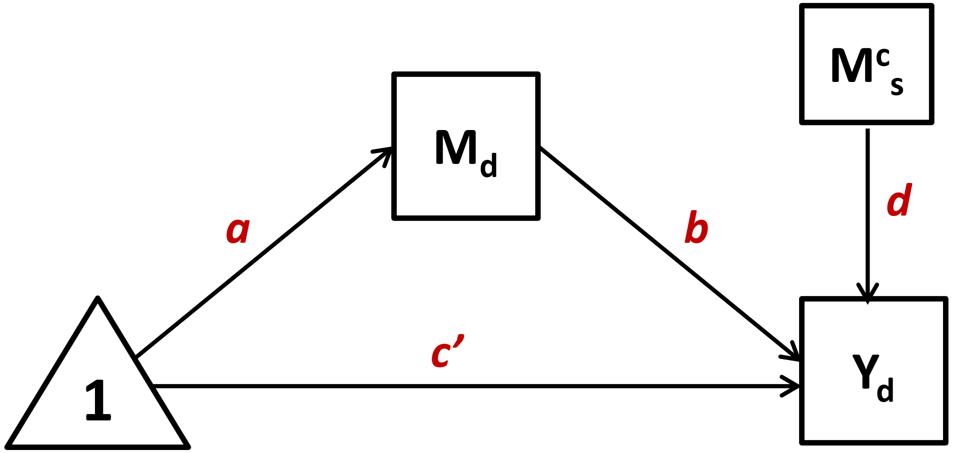
Controlling for difficulty, there is still a significant indirect effect through communal affordance!

Visualizations

I suggest using both a conceptual and statistical visualization in order to help the reader understand the process you are testing.



Conceptual



Statistical

Tips:

- Providing a conceptual diagram helps the readers understand the process you are interested in.
- Providing a statistical diagram helps readers understand how you estimated the model, and that you did it correctly.
- Provide path estimates on statistical diagram or in a table.
- Don't forget to report the path estimates and statistics for the *d* path. It's important!

Visualization Examples

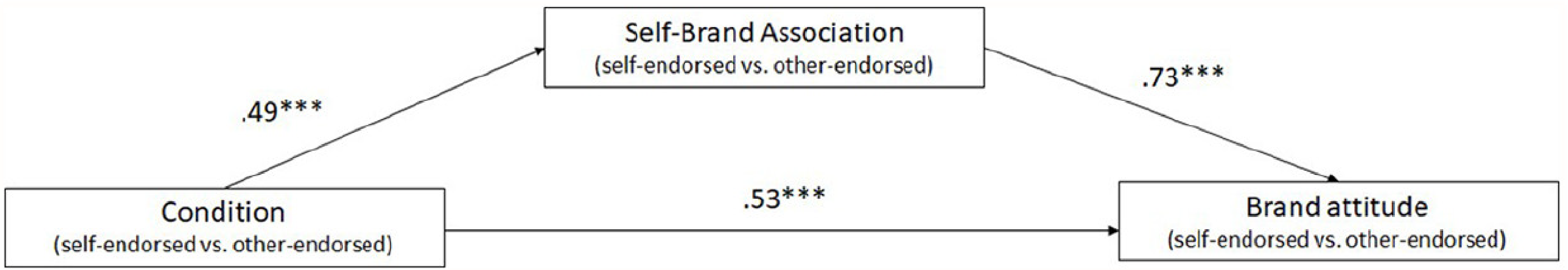
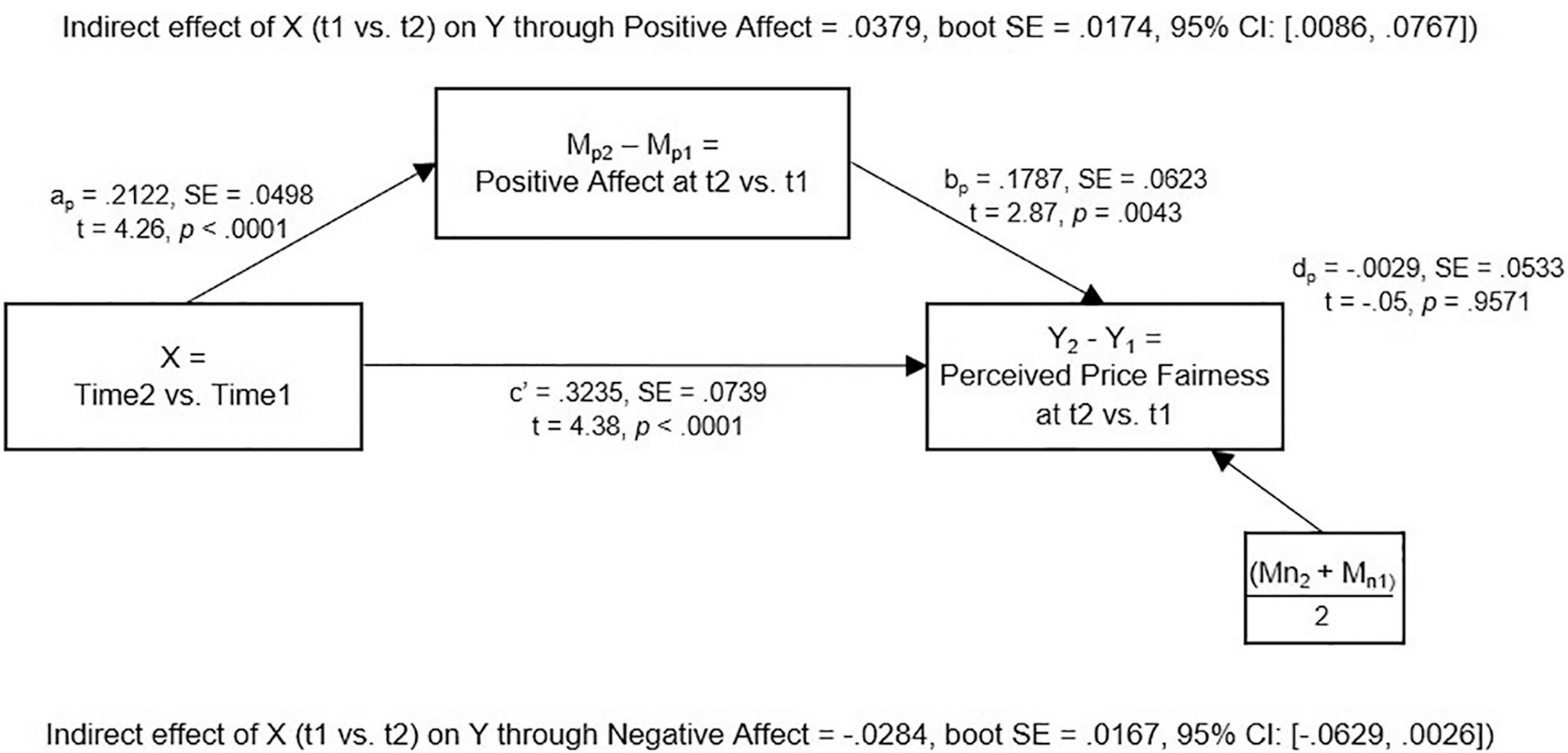


Figure 1. Self-brand association as a mediator of the relationship between condition (self-endorsed vs other-endorsed) and brand attitude (n = 119). *Indirect Effect* = 0.35 CI = [0.23, 0.51]. Used 5000 sample bootstraps to calculate indirect effect. ***p < .001.

https://journals.sagepub.com/doi/10.1177/1461444820912197

Visualization Examples



https://onlinelibrary.wiley.com/doi/10.1002/cb.1934

Visualization Examples

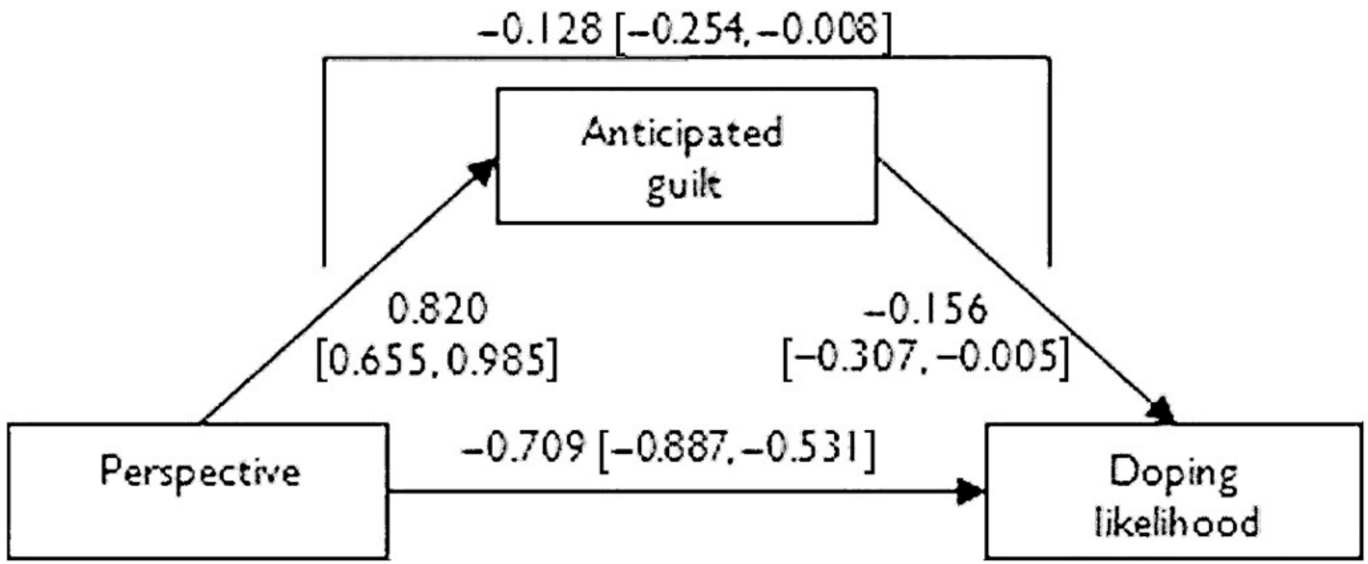


Figure 1—The direct effects of perspective (self minus other) on doping likelihood and guilt, and the indirect effect of perspective on doping likelihood via guilt. Unstandardized coefficients are reported, with 95% confidence intervals in brackets. Solid lines indicate significant paths.

https://journals.humankinetics.com/view/journals/jsep/42/5/article-p417.xml

Visualization Examples

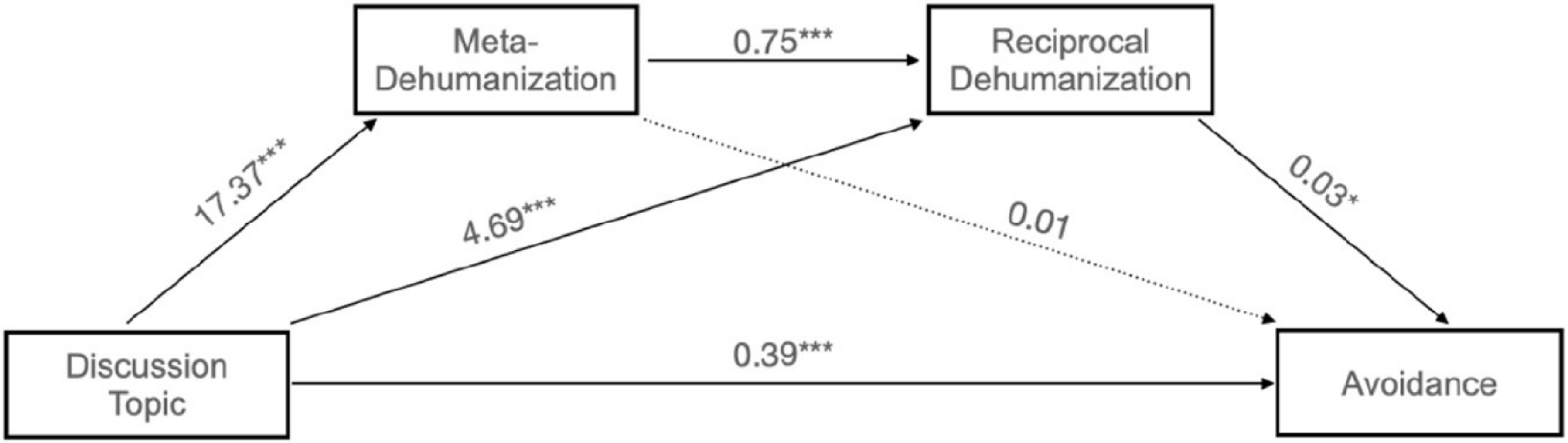
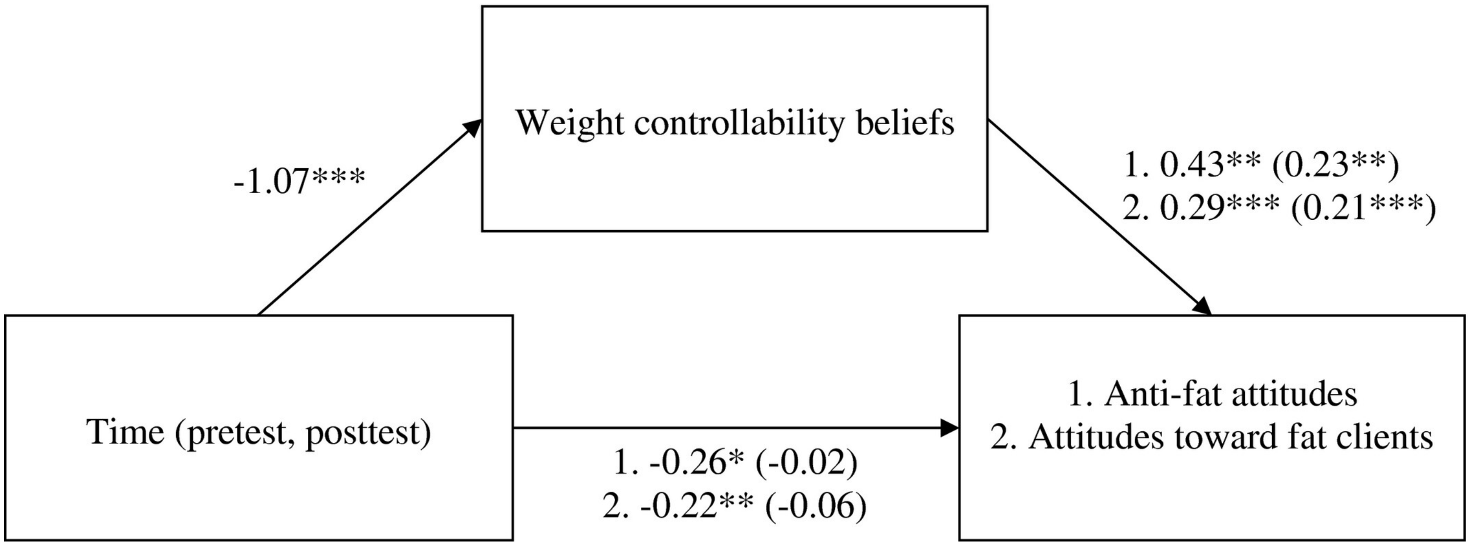


Figure 6. Within-person serial mediation: Study 3.
Note. All reported coefficients are unstandardized effect sizes because standardized betas could not be computed using this analysis.

https://journals.sagepub.com/doi/10.1177/13684302221099449

Visualization Examples



Weight controllability beliefs mediate the effect of the weight bias seminar (time: pretest, posttest) on (1) anti-fat attitudes and (2) attitudes toward fat clients. *Note*. Unstandardized betas are presented on figure paths. Numbers in parentheses represent betas when the mediator and all model variables were included. *p < .05. **p < .01. ***p < .001

https://onlinelibrary.wiley.com/doi/10.1111/jasp.12653

Common Questions

Can this method be used for more than two conditions?

YES! Judd, Kenny, and McClelland (2001) describe a system for setting up contrasts among conditions, and testing the indirect effects of those contrasts.

I recommend reading Hayes & Preacher (2014) on mediation analysis with a multicategorical IV if you want to try this out. I am happy to give instructions on how to trick MEMORE into doing this. There will be functionality (soonish) for MEMORE to do this.

ALTERNATIVES: Some of the other repeated-measures mediation options are more appropriate if you have more than two conditions (especially longitudinal), so take a look at those when thinking about these options.

Can I use multiple mediators?

YES! MEMORE is already set up to do parallel mediation with up to 10 sets of mediators and serial mediation with up to **five** sets of mediators (See Montoya & Hayes 2017 for instructions).

Can we do conditional process models?

YES! We'll review this briefly on the second day, after covering moderation

How do I control for covariates?

All of MEMORE's mediation analyses are within-person models, so you do not need to control for any between subjects variables such as age, gender, big-5.

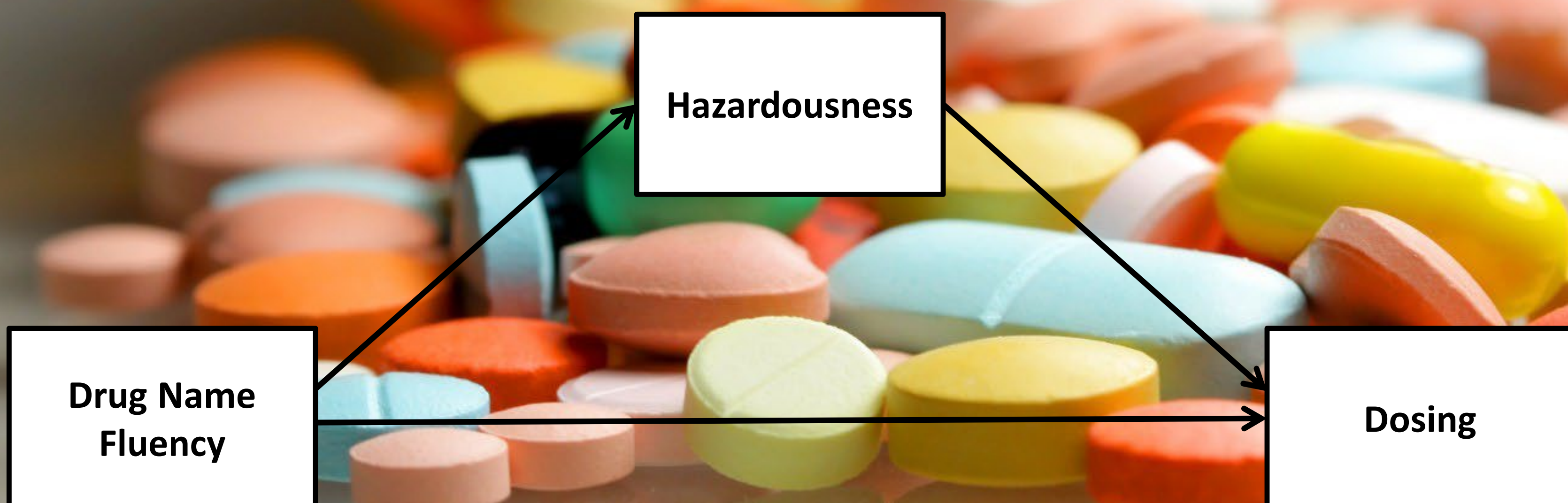
Sometimes there are covariates which change within a person across conditions that you want to account for, this can be done by treating this additional variable as another set of mediators.

Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

Research Question: Can the name of drugs impact how hazardous they seem and how much people are willing to dose the drugs?

Imagine you have a cold, and there are a variety of medications available including (a) Fastinorbine and (b) Cytrigmcmium. Which seems more hazardous? Which are you willing to dose more of?



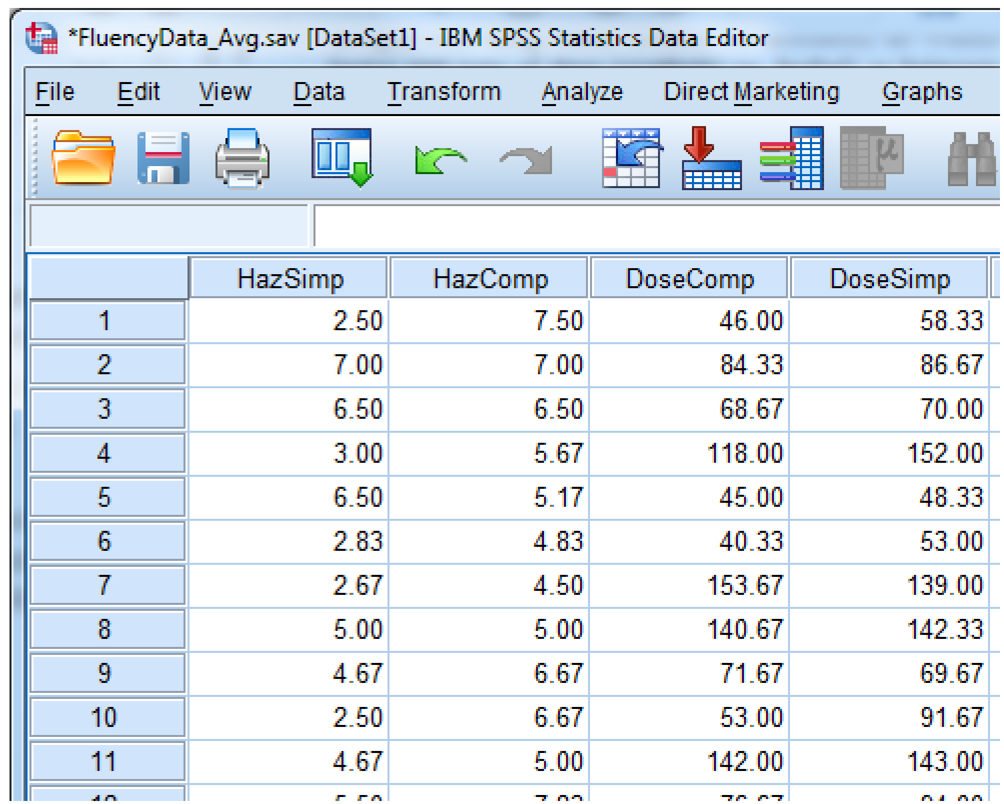
Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

Participants (N = 70) were asked to imagine they had the flu, and 6 different drugs were provided to treat the drug. Participants poured the dose they would feel comfortable taking at maximum into a plastic cup. Each person judged drugs with simple or complex names (3 of each). Responses on the measured variables were averaged across the 3 drugs (but later we'll look at what happens when we treat these separately).

Measured Variables:

- Dosage in mL
  - Variable name: `Dose`
  - 0 mL – 200mL
- Hazardousness of drug
  - Variable name: `Haz`
  - Average of two questions:
    - Hazardousness (1-7)
    - Dangerousness (1-7)



	HazSimp	HazComp	DoseComp	DoseSimp
1	2.50	7.50	46.00	58.33
2	7.00	7.00	84.33	86.67
3	6.50	6.50	68.67	70.00
4	3.00	5.67	118.00	152.00
5	6.50	5.17	45.00	48.33
6	2.83	4.83	40.33	53.00
7	2.67	4.50	153.67	139.00
8	5.00	5.00	140.67	142.33
9	4.67	6.67	71.67	69.67
10	2.50	6.67	53.00	91.67
11	4.67	5.00	142.00	143.00

Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

1. Estimate the proposed model (Fluency -> Hazardousness -> Dosage) using MEMORE
2. Turn off the *XM* interaction
3. Find estimates of the following paths: *a*, *b*, *c*, *c'*
4. Of the following inferential methods, which support the hypothesized mediation model (use $\alpha = 0.05$ or 95% confidence intervals):
   Percentile bootstrap CIs, Monte Carlo CIs, Sobel Test / Normal Theory

5. Practice writing up some of the results explored above.

Activity for Break

Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /model = 1.
```

```
%memore(m=HazSimp HazComp,y=DoseSimp DoseComp,xmint=0,
         model=1,data=Fluency);
```

```
**************************************************************************
Outcome: Ydiff =  DoseSimp  -       DoseComp

Model
         Effect         SE          t          p       LLCI       ULCI
'X'     11.0476     1.5770     7.0055      .0000     7.9016    14.1937

Degrees of freedom for all regression coefficient estimates:
  69

**************************************************************************
Outcome: Mdiff =  HazSimp   -       HazComp

Model
         Effect         SE          t          p       LLCI       ULCI
'X'     -2.1048      .1848   -11.3893      .0000    -2.4734    -1.7361

Degrees of freedom for all regression coefficient estimates:
  69

**************************************************************************
```

c = 11.05

Interpretation?

a = -2.10

Interpretation?

On average, participants dosed 11.05 mL more of the simply named drug than the complex named drug ($t(69) = 7.06$, $p < .001$).

Participants thought the complex drug was 2.10 points more hazardous than the simply named drug, on average ($t(69) = 11.39$, $p < .001$).

Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /model = 1.
```

```
%memore(m=HazSimp HazComp,y=DoseSimp DoseComp,xmint=0,
        model=1,data=Fluency);
```

```
**************************************************************************
Outcome: Ydiff =  DoseSimp  -      DoseComp

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4020      .1616   148.1029    13.1047     1.0000    68.0000      .0006

Model
           coeff         SE          t          p       LLCI       ULCI
'X'       3.8280     2.4684     1.5508      .1256    -1.0978     8.7537
Mdiff    -3.4302      .9475    -3.6200      .0006    -5.3210    -1.5393

Degrees of freedom for all regression coefficient estimates:
  68
```

c′ = 3.83 *Interpretation?*

b = -3.43 *Interpretation?*

After controlling for hazardousness, participants were expected to dose 3.8 mL more of the simple drug. This effect was not significantly different than zero ($t(68) = 1.55$, $p = .13$).

A one unit increase in the difference in perceived hazardousness between conditions results in a 3.43 unit decrease in the difference in dosage ($t(68) = 3.62$, $p < .001$).

A one unit increase in perceived hazardousness results in a 3.43 unit decrease in dosage, averaged across fluency conditions ($t(68) = 3.62$, $p < .001$).

Using MEMORE for Drug Fluency data

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /model = 1.
```

```
%memore(m=HazSimp HazComp,y=DoseSimp DoseComp,xmint=0,
          model=1,data=Fluency);
```

```
************************* TOTAL, DIRECT, AND INDIRECT EFFECTS *************************

Total effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
    11.0476     1.5770     7.0055    69.0000      .0000     7.9016    14.1937

Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     3.8280     2.4684     1.5508    68.0000      .1256    -1.0978     8.7537

Indirect Effect of X on Y through M
          Effect     BootSE   BootLLCI   BootULCI
Ind1      7.2197     1.8940     3.8590    11.1609

Indirect Key
Ind1   'X'      ->       M1diff   ->       Ydiff
```

ab = 7.22 *Interpretation?*

Participants were dosed simple drugs 7.22 mL more, through the effect of simple drugs on hazardousness and the subsequent effect of hazardousness on dosage (Percentile CI = [3.86, 11.16]).

Drug name fluency increased dosage indirectly effect through hazardousness by 7.22 mL (Percentile CI = [3.86, 11.16]).

Simple names were perceived as less hazardous, which then increased dosage, resulting in an indirect effect of 7.22 mL on dosage (Percentile CI = [3.86, 11.16]).

Using MEMORE for Drug Fluency data

Methods of Inference

Percentile Bootstrap CI

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0.
```

Indirect Effect of X on Y through M

	Effect	BootSE	BootLLCI	BootULCI
Ind1	7.2197	1.8940	3.8590	11.1609

Monte Carlo CI

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /mc = 1.
```

Indirect Effect of X on Y through M

	Effect	MCSE	MCLLCI	MCULCI
Ind1	7.2197	2.0916	3.2369	11.3834

Sobel Test / Normal Theory

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0 /normal = 1.
```

Normal Theory Tests for Indirect Effect

	Effect	SE	Z	p
Ind1	7.2197	2.0927	3.4500	.0006

Preregistration

Preregistration: A process where you create a time-stamped, publicly accessible record of your plan for a specific study.

Current preregistrations of mediation analysis are light on the details:
"We plan to conduct mediation tests (including multiple mediation) using the MEMORE macro (Montoya & Hayes, 2017) and the PROCESS macro (Hayes, 2017)."

- ❑ Planned sample size
- ❑ α-level/CI-level for each test
- ❑ Role of different variables in the analysis (e.g., independent variable, mediator, outcome, covariate), and how they are computed
- ❑ Estimation Method
- ❑ Inferential method and any important specifications for that method (e.g., how many bootstraps, seed number)
- ❑ Plans for sensitivity analysis: Benchmarks for acceptable correlations for evaluating confounding
- ❑ *Plans for XM interaction*
- ❑ **Tools and specifications**

Example Preregistration

We will estimate a simple mediation analysis where protest condition (X) affects liking (Y) through response appropriateness (M) using a sample of 500 participants. Protest condition is assigned in a random order where 50% will be assigned to the protest condition first and 50% to the no protest condition first. Liking is measured using 4 Likert type items evaluating the lawyers positive characteristics. We will check the reliability of our scale prior to analysis and if the reliability (alpha) is greater than 0.7, we will proceed. If it is not we may consider dropping items. Similarly, response appropriateness is measured using 5 Likert type items. We will use the same procedure for measurement evaluation as with liking. In our analysis we will not control for any covariates.

We will use the **MEMORE for SPSS (version 3.0)** to estimate our model, which uses ordinary least squares regression and generates percentile bootstrap confidence intervals for the indirect effect. We will use 5,000 bootstraps. *We plan to test whether the XM interaction is statistically significant at alpha = 0.10, if it is, we will use the xmint=1 option in MEMORE to estimate the indirect effect (using 232714). If the XM interaction is not statistically significant, we will use xmint = 0, to estimate the indirect effect. We will test hypothesis 1 by examining the 95% bootstrap confidence interval for the indirect effect includes zero.*

After our analysis we will use the mediation package in R to conduct sensitivity analysis, specifically for the relationship between response appropriateness and liking. Correlations greater than .5 would suggest robustness to reasonable levels of confounding.

Finding Examples

DataStudio for finding examples

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Publication Year | Research Areas | ModelNumb... (1) | Xtype | levelsX | numYvars

Journal Title | Covariates | Sample Size

Article Count
50

Article Title (link)	Authors	Journal Title	Publication...	Research Areas	Study Num...	SampleSize	ModelNumb...	numXvars	Xtype	levelsX	numYvars
When should retail...	Jeong, H; Ye,...	JOURNAL O...	2021	Business & Economics	1	271	1 (Mediation)	1	Pre-post	2	1
What hinders resi...	Dong, XJ	TOURISM A...	2022	Social Sciences - Other...	2	130	1 (Mediation)	1	Experimental	2	1
The role of metad...	Hartley, S; P...	GROUP PRO...	2022	Psychology	3	239	1 (Mediation)	1	Experimental	2	2
The connotative ...	Motoki, K; P...	JOURNAL O...	2022	Business & Economics	1	154	1 (Mediation)	1	Experimental	2	3
The Self-Other Div...	Ring, C; Kav...	JOURNAL O...	2020	Social Sciences - Other...	1	100	1 (Mediation)	1	Experimental	2	1
The Impact of Mix...	Huang, XZ; Z...	INTERNATIO...	2022	Environmental Science...	2	434	1 (Mediation)	1	Experimental	2	1
The Hypoalgesic E...	Song, JS; Ka...	RESEARCH ...	2022	Social Sciences - Other...	1	40	1 (Mediation)	1	Experimental	2	1
Testing the effecti...	Brochu, PM	JOURNAL O...	2023	Psychology	1	45	1 (Mediation)	1	Pre-post	2	2
Suicidality and so...	Breitborde, ...	EARLY INTE...	2021	Psychiatry	1	38	1 (Mediation)	1	Pre-post	2	1
Putting the Me in ...	Hamilton, K...	NEW MEDIA...	2021	Communication	1	119	1 (Mediation)	1	Experimental	2	1
Positive reputatio...	Inoue, Y; Mif...	FRONTIERS ...	2023	Psychology	2	293	1 (Mediation)	1	Experimental	2	1
Perceptions of a P...	Pals, AM; Go...	JOURNAL O...	2022	Psychology; Family Stu...	1	52	1 (Mediation)	3	Experimental	2	1
Pandemic Pedago...	Armstrong, ...	SOUTHERN ...	2022	Communication	1	163	1 (Mediation)	1	Pre-post	2	4
Mind the ad: How ...	Kocak, A; Ro...	JOURNAL O...	2022	Psychology; Business ...	1	123	1 (Mediation)	1	Experimental	2	1
Mind the ad: How ...	Kocak, A; Ro...	JOURNAL O...	2022	Psychology; Business ...	2	151	1 (Mediation)	1	Experimental	2	1
Mediation and Mo...	Wong, CL; C...	JOURNAL O...	2020	Public, Environmental ...	1	1001	1 (Mediation)	1	Pre-post	2	1

Mediation

Introduction

- Within- and between-subject designs
- Overview of between-subject mediation

Within-Subject Mediation

- Estimation & Inference
- MEMORE Macro
- Issues of Causality
- Power Analysis
- Preregistration and writing guides
- Visuals for Publication

If time (Multilevel Mediation)

New York Elegance

Mediation and Multilevel Modeling

One of the primary assumptions of Ordinary least squares (OLS) regression is that each observation is independent of all other observations.

Ordinary least squares (OLS) regression is not directly applicable when data are nested.

- Students nested within classrooms
- Employees nested within companies
- Repeated measurements nested within individuals

Responses from employees within the same company tend to be more related to each other than responses from employees in different companies.

This violates the assumption of independence.

Several methods are available for accounting for this dependence, but today we will focus on multilevel/mixed modeling.

Multilevel Modeling

What's a level?

Students (Level 1) within classrooms (Level 2)
Employees (Level 1) within companies (Level 2)
Repeated measurements (Level 1) within individuals (Level 2)

Multilevel models are often expressed either as separate equations for the different levels of the model, or as one combined model.

Let *i* denote Level 1 units and *j* denote Level 2 units

X_{ij}: Person *i* in group *j*'s observation on *X*
Y_{ij}: Person *i* in group *j*'s observation on *Y*
W_j: Group *j*'s observation on *W* (Level 2 characteristics (e.g., Company size))

Two-Level Unconditional Model

Let's predict an outcome at Level 1 using a predictor from Level 1

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

b_{0j}: The expected value of *Y* for someone in group *j* with $X_{ij} = 0$. Notice this is allowed to vary by group! This is the **intercept** for group *j*.

b_{1j}: The expected difference in *Y* for two people in the same group *j* that differ by 1 unit on X_{ij}. This is the **slope** for group *j.*

e_{ij}: The error in estimating Y_{ij}. $e_{ij} \sim N(0, \sigma^2)$

Even if you're not familiar with multilevel models, this should look familiar to what we do in regression. Except the intercept and slope are allowed to randomly vary across groups. We call these *random effects*.

Two-Level Unconditional Model

We also create a Level 2 Model, for the intercept and slope:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$b_{0j} = b_0 + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

Notice that it's the *u*'s that make the "random effects" random. By allowing the intercept and slope to vary across groups, we soak up all the "dependence" in the data.

b_0 is the grand-mean intercept (i.e., the average intercept across groups)

b_1 is the grand-mean slope (i.e., the average slope across groups)

We assume that $(u_{0j}, u_{1j}) \sim MVN(0, T)$ where $T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{01} & \tau_{11} \end{bmatrix}$

The random effects are not individuals estimated, but rather we estimate their covariance matrix as well as the grand-mean intercept and slope.

Simplifying the Model

Not all coefficients need to be random. For example the intercept could be random but the slope could vary across groups:

$$Y_{ij} = b_{0j} + b_1 X_{ij} + e_{ij}$$

$b_{0j} = b_0 + u_{0j}$

$b_1 = b_1$

b_0 is the grand-mean intercept (i.e., the average intercept across groups)

b_1 is the slope (assumed to be the same for all groups)

This is like a special case where we assume $\tau_{11} = 0$

We will mostly use the case where we have random-slopes as this is what adds complexity to mediation in multilevel models.

The Combined Model

Sometimes it's clearer to represent both the Level 1 and Level 2 equations together in a *combined model*. We plug in the Level 2 equations in their spots in the Level 1 model

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = (b_0 + u_{0j}) + (b_1 + u_{1j})X_{ij} + e_{ij}$$

We can combine and rearrange terms to separate the parts of the model which are random and those which are not random (i.e., fixed).

$$Y_{ij} = \underbrace{\left(b_0 + b_1X_{ij}\right)}_{\text{Fixed: does not vary by group}} + \underbrace{u_{0j} + u_{1j}X_{ij} + e_{ij}}_{\text{Random: varies by group}}$$

You can see how each individual's response is a function of the **overall intercept** the **overall slope** as well as their **group's deviations** from the overall intercept and slope and a **individual-specific error.**

Adding Level 2 Predictors

We can explain variability in the group intercept or slope using characteristics of the Level 2 units.

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$b_{0j} = b_0 + g_{01}W_j + u_{0j}$

$b_{1j} = b_1 + g_{11}W_j + u_{1j}$

b_0 is the expected group intercept when W_j is zero.

b_1 is the expected group slope when W_j is zero.

g_{01} is how much we expect the intercept to change with a one unit change in W_j

g_{11} is how much we expect the slope (relationship between X and Y) to change with a one unit change in W_j

Adding Level 2 Predictors

We can rewrite the model as a *combined* model, by combining Level 1 and Level 2 equations:

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = (b_0 + g_{01}W_j + u_{0j}) + (b_1 + g_{11}W_j + u_{1j})X_{ij} + e_{ij}$$

$$Y_{ij} = \underbrace{b_0 + g_{01}W_j + b_1X_{ij} + g_{11}W_jX_{ij}}_{\text{Fixed: does not vary by group}} + \underbrace{u_{1j}X_{ij} + u_{0j} + e_{ij}}_{\text{Random: varies by group}}$$

You can see in the combined equation that by including W_j as a predictor of the *slope* we include an <u>interaction</u> between W_j and X_{ij}.

This means the effect of *X* on *Y* depends on the value of *W*.

Fluency Data

The Fluency data we used for within-subjects mediation (FluencyData_Avg.sav) is in wide form and we must convert it to long-form for multilevel modeling (FluencyData_Avg_long.sav).

```
VARSTOCASES
  /ID=id
  /MAKE Hazard FROM HazSimp HazComp
  /MAKE Dose FROM DoseSimp DoseComp
  /INDEX=Simple(2)
  /KEEP=
  /NULL=KEEP.

RECODE Simple (2=0) (1=1).
EXECUTE.
```

Fluency Data

We can use the SPSS MIXED procedure to fit a multilevel model.

Let’s look at the relationship between Dosage and Hazardousness using a model with a random intercept and a random slope.

```
MIXED Dose WITH Hazard
    /Fixed = Hazard | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard | Subject(id) COVTYPE(UN).
```

$$Y_{ij} = \left(b_0 + b_1 X_{ij}\right) + u_{0j} + u_{1j} X_{ij} + e_{ij}$$

Y_{ij}: Dosage for observation *i* for person *j*

X_{ij}: Hazardousness for observation *i* for person *j*

Give it a try!

Fluency Data: Fixed Effects

```
MIXED Dose WITH Hazard
    /Fixed = Hazard | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard | Subject(id) COVTYPE(UN).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	109.484321	4.932038	52.923	22.199	.000	99.591570	119.377072
Hazard	-4.838863	.615111	33.198	-7.867	.000	-6.090033	-3.587694

a. Dependent Variable: Dosing Simple.

The expected dose administration of drugs is 109.48 mL given a hazardousness rating of zero ($X_{ij} = 0$). But remember this is an average across all people.

For each one unit increase in harazardousness, dose administration of drugs is expected to decrease by 4.84 mL. Remember this is an average across all people.

Fluency Data: Random Effects

$(u_{0j}, u_{1j}) \sim MVN(0, T)$ where $T = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{01} & \tau_{11} \end{bmatrix}$

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		58.509962	18.167383	3.221	.001	31.836928	107.529709
Intercept + Hazard [subject = id]	UN (1,1)	1062.426086	309.657876	3.431	.001	600.074242	1881.015899
	UN (2,1)	-30.092244	35.454856	-.849	.396	-99.582485	39.397997
	UN (2,2)	5.018740	5.501233	.912	.362	.585545	43.015932

a. Dependent Variable: Dosing Simple.

There is substantial between-person variability ($\tau_{00} = 1062.43$) in dosage of drugs with a hazardousness rating of zero.

The relationship between hazardousness and dosage varies across individuals ($\tau_{11} = 5.02$)

Those with higher-than-average dose values at $X_{ij} = 0$ (hazardousness is zero) have lower-than-average slopes for the relationship between hazardousness and dosage ($\tau_{01} = -30.09$)

Centering Variables

There is substantial between-person variability ($\tau_{00} = 1062.43$) in dosage of drugs with a hazardousness rating of zero.

When we interpret τ_{00} we condition on the predictor being zero (i.e., Hazardousness is zero).

In this data a score of zero is impossible for hazardousness because it's the average of two items scored 1 – 7. So the intercept and it's variance are not interpretable.

For multilevel models, there are two common centering options (grand mean centering and **group mean centering**).

The choice of centering has a big impact on the parameter estimates and their substantive meaning.

Enders, C. K. & Tofighi, D. (2007). Centering predictor variables in cross-sectional multilevel models: a new look at an old issue. *Psychological Methods, 12*(2), 121-138.

Within-Group Centering

Typically variables at Level 1 contain information about Level 1 and Level 2.

Consider the hazardousness ratings: X_{ij}

Part of X_{ij} has to do with how hazardous **that specific drug** is compared to other drugs. (Level 1)

But another part has to do with how hazardous the person sees **drugs in general.** (Level 2)

$$X_{ij} = \underbrace{X_{ij} - \bar{X}_{.j}}_{\text{Within-group/ Level 1}} + \underbrace{\bar{X}_{.j}}_{\text{Between-group/ Level 2}}$$

$\bar{X}_{.j}$ is the group *j*'s mean of X_{ij}

Within-group centering divides these two pieces of information, so we can see what is Level 1 variance and what is Level 2 variance, separately.

The within and between group pieces are uncorrelated.

Within-Group Centering

To group mean center we subtract the group's mean of *X* from each observation on that predictor.

Person 1

Simple	Hazard	Hazard_Centered
0	7.50	2.50
1	2.50	-2.50
Group mean->	5	

Person 34

Simple	Hazard	Hazard_Centered
0	3.83	.58
1	2.67	-.58
Group mean->	3.25	

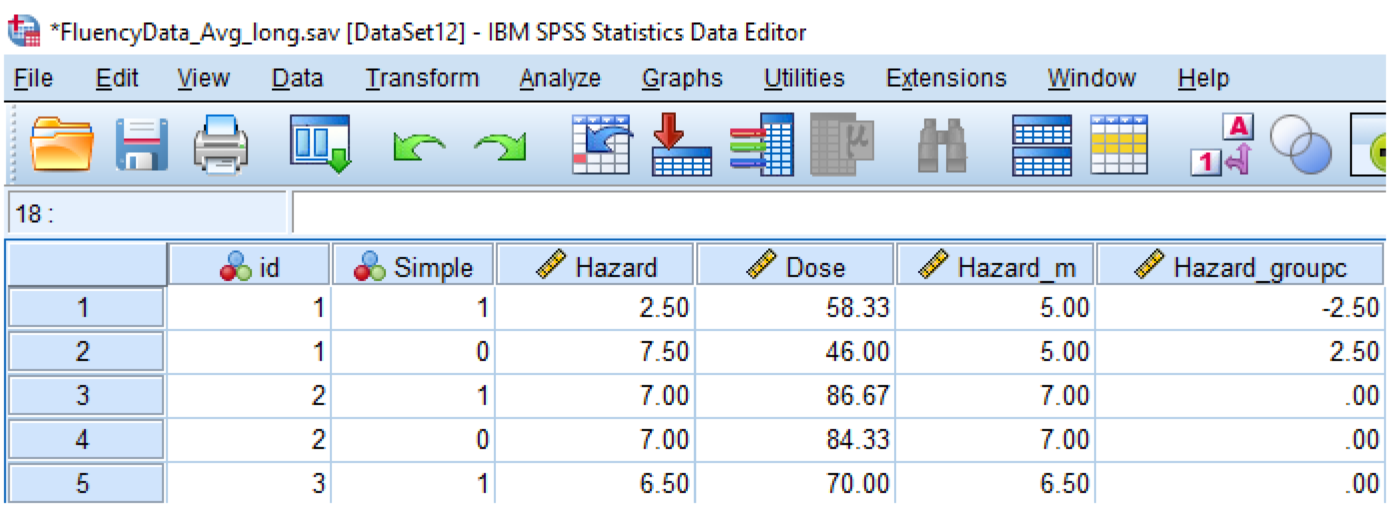
Within-Group Centering

```
AGGREGATE
   /OUTFILE = * MODE = ADDVARIABLES
   /BREAK = id
   /Hazard_m = MEAN(Hazard).

COMPUTE Hazard_groupc = Hazard - Hazard_m.
Execute.
```

Compute a new variable called Hazard_m, which will be the group mean of hazard.

Next we compute the group-mean centered hazard ratings, and call these Hazard_groupc.



	id	Simple	Hazard	Dose	Hazard_m	Hazard_groupc
1	1	1	2.50	58.33	5.00	-2.50
2	1	0	7.50	46.00	5.00	2.50
3	2	1	7.00	86.67	7.00	.00
4	2	0	7.00	84.33	7.00	.00
5	3	1	6.50	70.00	6.50	.00

Within-Group Centering

Thinking about within and between group variance, we can see how there may be **two relationships** of interest:

(1) How does within-group variance in X predict variance in Y?

(2) How does between-group variance in X predict variance in Y?

$$Y_{ij} = b_{0j} + b_{1j}X_{ij} + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j} + \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + b_{1j}\bar{X}_{.j} + e_{ij}$$

When we don't use any centering (or use grand mean centering) we're fixing the relationship between the within-group part of *X* and *Y* to be equal to the relationship between the between-group part of *X* and *Y*.

Ultimately this makes these coefficients difficult to interpret because they're a blend of these two relationship (Raudenbush & Bryk, 2002).

Contextual Effects

Sometimes we are interested in the within-group relationship between a Level 1 predictor and an outcome as well as the between-group relationship.

When the between-group effect is different from the within-group effect, we call this a **contextual effect** (Raudenbush & Bryk, 2002).

The within-group relationship is tested by including the group-mean centered Level 1 predictor.

The between-group relationship can be tested by adding the group mean of the Level 1 predictor as a Level 2 predictor for the random intercept.

$$Y_{ij} = b_{0j} + b_{1j}(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$b_{0j} = b_0 + g_{01}\bar{X}_{.j} + u_{0j}$$

$$b_{1j} = b_1 + u_{1j}$$

Contextual Effects

The combined contextual effects model:

$$Y_{ij} = (b_0 + g_{01}\bar{X}_{.j} + u_{0j}) + (b_1 + u_{1j})(X_{ij} - \bar{X}_{.j}) + e_{ij}$$

$$Y_{ij} = \underbrace{b_0 + g_{01}\bar{X}_{.j} + b_1\left(X_{ij} - \bar{X}_{.j}\right)}_{\text{Fixed}} + \underbrace{u_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + u_{0j} + e_{ij}}_{\text{Random}}$$

b_1 represents the **average within-group effect** of X_{ij} on Y_{ij}
The variance in the **within-group effect** is $Var\left(b_{1j}\right) = Var\left(u_{1j}\right) = \tau_{11}$

g_{01} represents the **between group effect** of X_{ij} on Y_{ij}.

When b_1 and g_{01} differ from each other, this means there is a contextual effect.

Contextual Effects

```
MIXED Dose WITH Hazard_groupc Hazard_m
    /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard_groupc |
Subject(id) COVTYPE(UN).
```

Var: DOSE
Dose for drug *i* for person *j*

Var: Hazard_groupc
Drug *i*'s deviation from Person *j*'s average hazardousness rating

$$Y_{ij} = b_0 + g_{01}\bar{X}_{.j} + b_1\left(X_{ij} - \bar{X}_{.j}\right) + u_{1j}\left(X_{ij} - \bar{X}_{.j}\right) + u_{0j} + e_{ij}$$

Var: Hazard_m
Person *j*'s average hazardousness rating

Contextual Effects

$$Y_{ij} = \boxed{b_0} + \boxed{g_{01}}\bar{X}_{.j} + \boxed{b_1}(X_{ij} - \bar{X}_{.j}) + u_{1j}(X_{ij} - \bar{X}_{.j}) + u_{0j} + e_{ij}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m
    /Fixed = Hazard_groupc Hazard_m | SSTYPE(3)
    /Method = REML
    /Print = G Solution Testcov
    /Random = INTERCEPT Hazard_groupc | Subject(id)
COVTYPE(UN).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	115.262328	19.454675	68.013	5.925	.000	76.441268	154.083388
Hazard_groupc	-4.827233	.669329	22.424	-7.212	.000	-6.213817	-3.440649
Hazard_m	-5.939038	3.603082	68.013	-1.648	.104	-13.128853	1.250776

a. Dependent Variable: Dosing Simple.

For drug's at <u>each individual's group mean</u> the expected dosage is 115.26 mL.

For two drugs that differ by 1 unit on hazardousness, the more hazardous drug is expected to be dosed 4.83 mL less, controlling for average hazardousness rating.

Individuals 1 unit higher on average rating of hazardousness, are expected to dose drugs 5.94 units less, controlling for deviation of the drug from the individual's average.

Mediation Modeling with Multilevel Data

Multilevel mediation processes are often labeled by the level at which each variable varies.

- 1-1-1 implies that *X*, *M*, and *Y* are all measured at Level 1.
  Example: Measuring individuals on a variety of days, we may wonder if number of calories eaten before noon each day (*X*) predicts daily stress (Y) through improved productivity in the afternoon (*M*).

- 2-1-1 implies that *X* is measured at Level 2, but *M* and *Y* are at Level 1
  Example: Individuals are randomly assigned to either a healthy breakfast supplement (*X*) and tracked over a variety of days to see if their daily stress (*Y*) is improved through improved through afternoon productivity (*M*).

- 2-2-1 implies *X* and *M* are Level 2, but *Y* is measured at Level 1.
  Example: Perhaps individuals were randomly assigned to either a healthy breakfast supplement (*X*) and asked at the end of the week whether they felt able accomplish what they needed to in the afternoon this week (*M*) and we track their daily stress (*Y*).

Mediation Modeling with Multilevel Data

You may notice that when some variables are at Level 2, they should not be able to predict Level 1 variability.

For example, knowing whether someone is in the breakfast supplement condition, should help us predict their average daily stress, but none of the deviation of day-to-day stress from the person's average.

As such we'll focus on the 1-1-1 model, as it is the most general multilevel mediation model that exists when all variables are measured at Level 1.

We can explore between-group and within-group variability. **Indirect effects** can occur at both levels!

General 1-1-1 Mediation Model

Recall the single-level (between-subjects) mediation model:

$$M_i = a_0 + a_1 X_i + e_{M_i}$$

$$Y_i = b_0 + c' X_i + b_1 M_i + e_{Y_i}$$

Let's make it multilevel, where *X*, *M*, and *Y* are Level 1 variables:

$$M_{ij} = a_{0j} + a_{1j} X_{ij} + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j X_{ij} + b_{1j} M_{ij} + e_{Y_{ij}}$$

Let's use group-mean centering, because we're interested in differentiating within and between effects:

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

1-1-1 Mediation Model: Within-Group Effects

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

$a_{1j} = a_W + u_{a_1 j}$ This is the within-group effect of *X* on *M* for group *j*

$b_{1j} = b_W + u_{b_1 j}$ This is the within-group effect of *M* on *Y*, controlling for *X* in group *j*

$c'_j = c'_W + u_{c' j}$ This is the within-group effect of *X* on *Y*, controlling for *M* in group *j*

a_W, b_W, and c'_W are the **average within-group effects** of *X* and *M, M* on *Y* controlling for *X*, and *X* and *Y* controlling for *M* respectively.

1-1-1 Mediation Model: Between-Group Effects

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

BUT WAIT! We haven't included between-group effects. How did we do that before?

Include $\bar{X}_{.j}$ as a Level 2 predictor of the intercept.

$$a_{0j} = a_M + a_B \, \bar{X}_{.j} + u_{a_0 j}$$

$$b_{0j} = b_Y + c_B' \, \bar{X}_{.j} + b_B \, \bar{M}_{.j} + \; u_{b_0 j}$$

a_B, b_B, and c_B' are the **between-group effects** of *X* and *M, M* on *Y* controlling for *X*, and *X* and *Y* controlling for *M* respectively.

1-1-1 Mediation Full Model

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c'_j(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

$$a_{0j} = a_M + a_B\, \bar{X}_{.j} + u_{a_0 j}$$

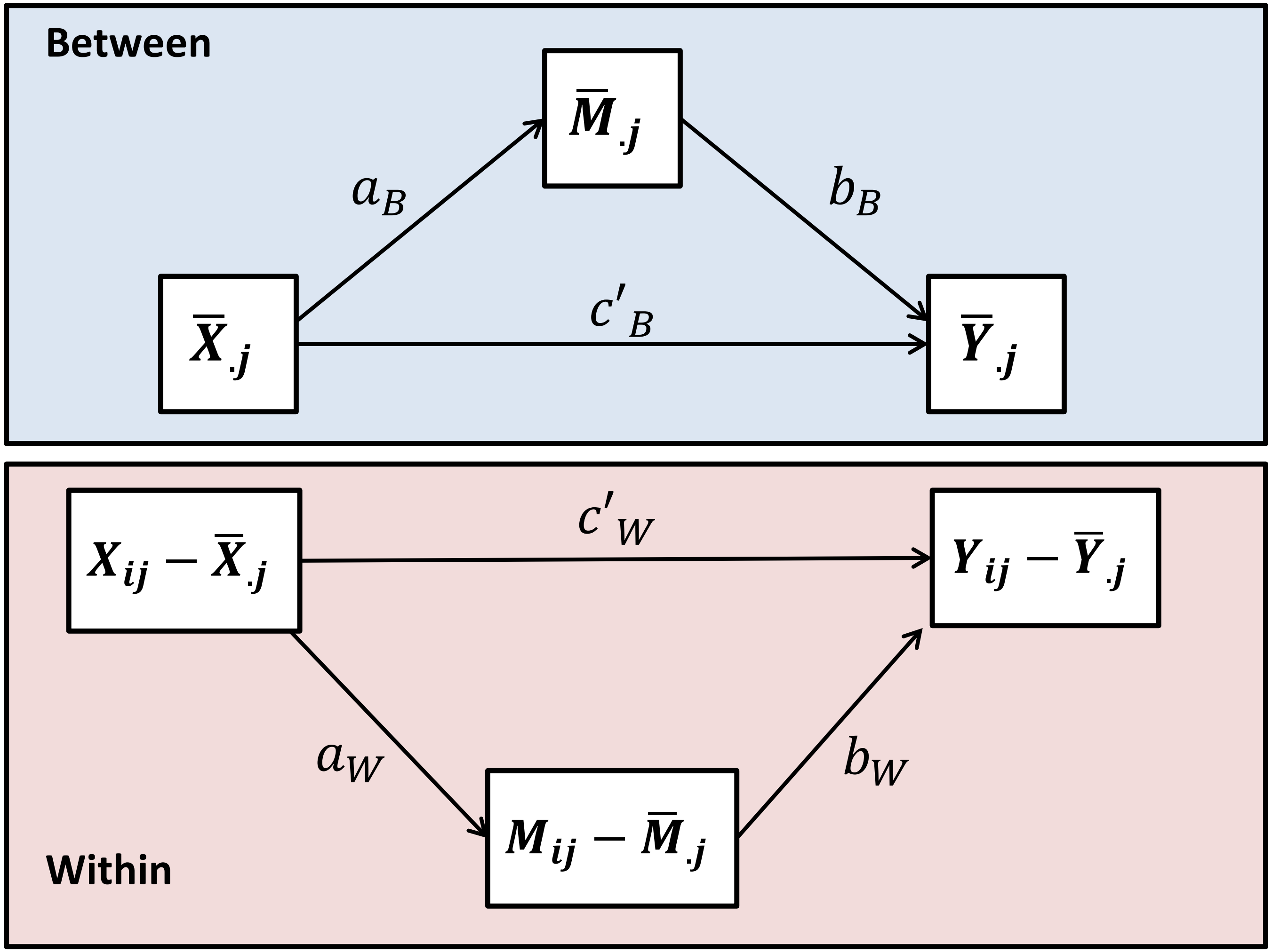
$$b_{0j} = b_Y + c'_B\, \bar{X}_{.j} + b_B\, \bar{M}_{.j} + u_{b_0 j}$$

$$a_{1j} = a_W + u_{a_1 j}$$

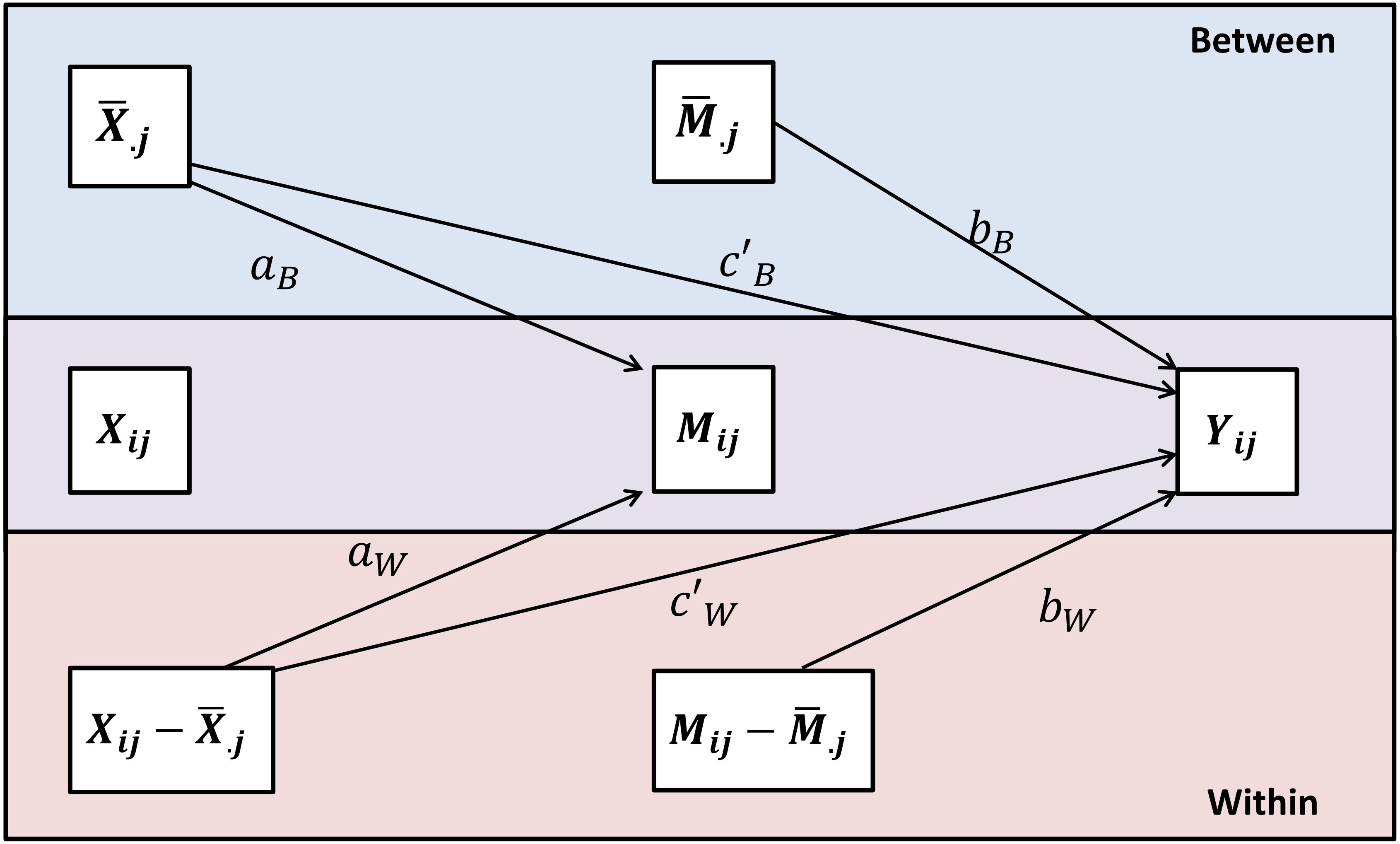
$$b_{1j} = b_W + u_{b_1 j}$$

$$c'_j = c'_W + u_{c'j}$$

1-1-1 Mediation Model: Conceptual Diagram



1-1-1 Mediation Model: Statistical Diagram

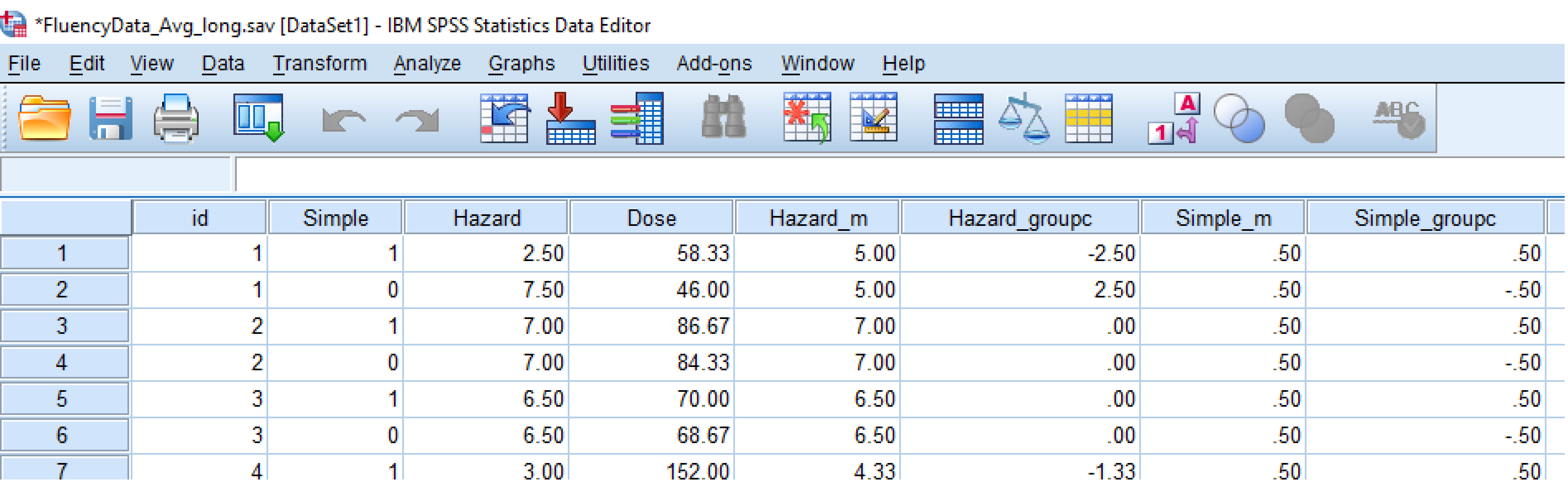


Estimating the *M* Equation

First we have to group-mean center `Simple` and create a variable which is the group mean of `Simple`.

```
AGGREGATE
/OUTFILE = * MODE = ADDVARIABLES
/BREAK = id
/Simple_m = MEAN(Simple).

COMPUTE Simple_groupc = Simple - Simple_m.
EXECUTE.
```



	id	Simple	Hazard	Dose	Hazard_m	Hazard_groupc	Simple_m	Simple_groupc
1	1	1	2.50	58.33	5.00	-2.50	.50	.50
2	1	0	7.50	46.00	5.00	2.50	.50	-.50
3	2	1	7.00	86.67	7.00	.00	.50	.50
4	2	0	7.00	84.33	7.00	.00	.50	-.50
5	3	1	6.50	70.00	6.50	.00	.50	.50
6	3	0	6.50	68.67	6.50	.00	.50	-.50
7	4	1	3.00	152.00	4.33	-1.33	.50	.50

Estimating the *M* Equation

Next we predict `Hazard` from the group-mean centered `Simple` (`Simple_groupc`) and the group means of `Simple` (`Simple_m`)

```
MIXED Hazard WITH Simple_groupc Simple_m
/FIXED = Simple_groupc Simple_m | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept Simple_groupc| Subject(id) COVTYPE(UN).
```

Estimates of Fixed Effects[b]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	5.304762	.121182	69.000	43.775	.000	5.063010	5.546513
Simple_groupc	-2.104762	.184802	69.000	-11.389	.000	-2.473433	-1.736091
Simple_m	0[a]	0	.	.	.	.	.

a. This parameter is set to zero because it is redundant.
b. Dependent Variable: Hazardousness Simple.

UH OH! Something went wrong? What happened?

Estimating the *M* Equation

"Redundant" group-mean for simple

Person 1

Simple	Simple_Centered
0	-.50
1	.50
Group mean-> 0.5	

Person 34

Simple	Hazard_Centered
0	-.50
1	.50
Group mean-> 0.5	

Estimating the *M* Equation

“Redundant” group-mean for simple!

Because each participant complete both levels of `Simple` (0 and 1) one time each, all participants have a person-level mean of 0.5 (see `Simple_m`)

This means there is no between-person variability on `Simple`, so the group means and the intercept are linear combinations of one another (`Intercept = 2*Simple_m`; ., the model is not identified).

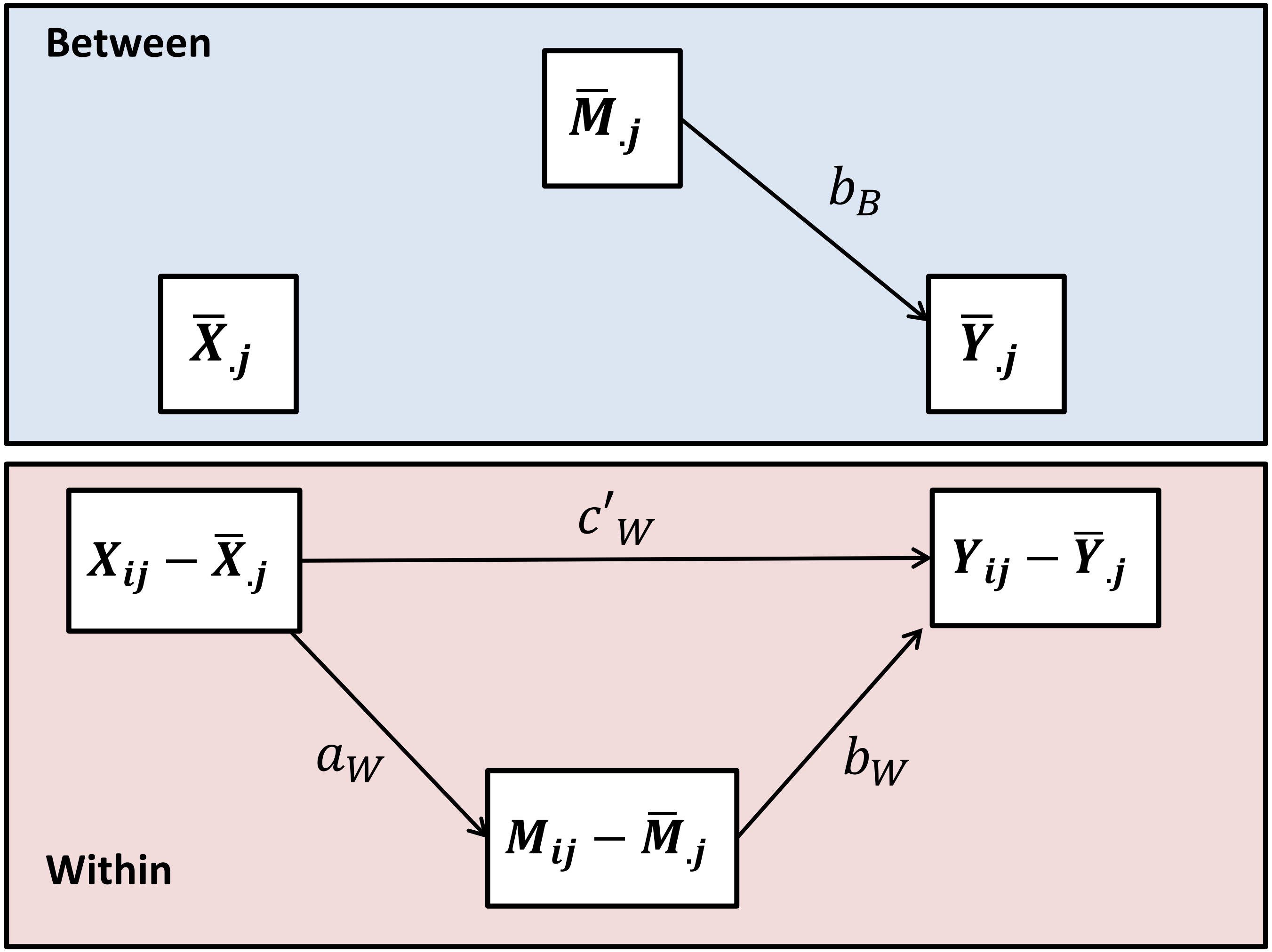
We can remove the between-person effect of *X* from the model, meaning there will not be a between-person indirect effect.

$$a_{0j} = a_M + \cancel{a_B\,\bar{X}_{.j}} + u_{a_0 j} = a_M + u_{a_0 j}$$

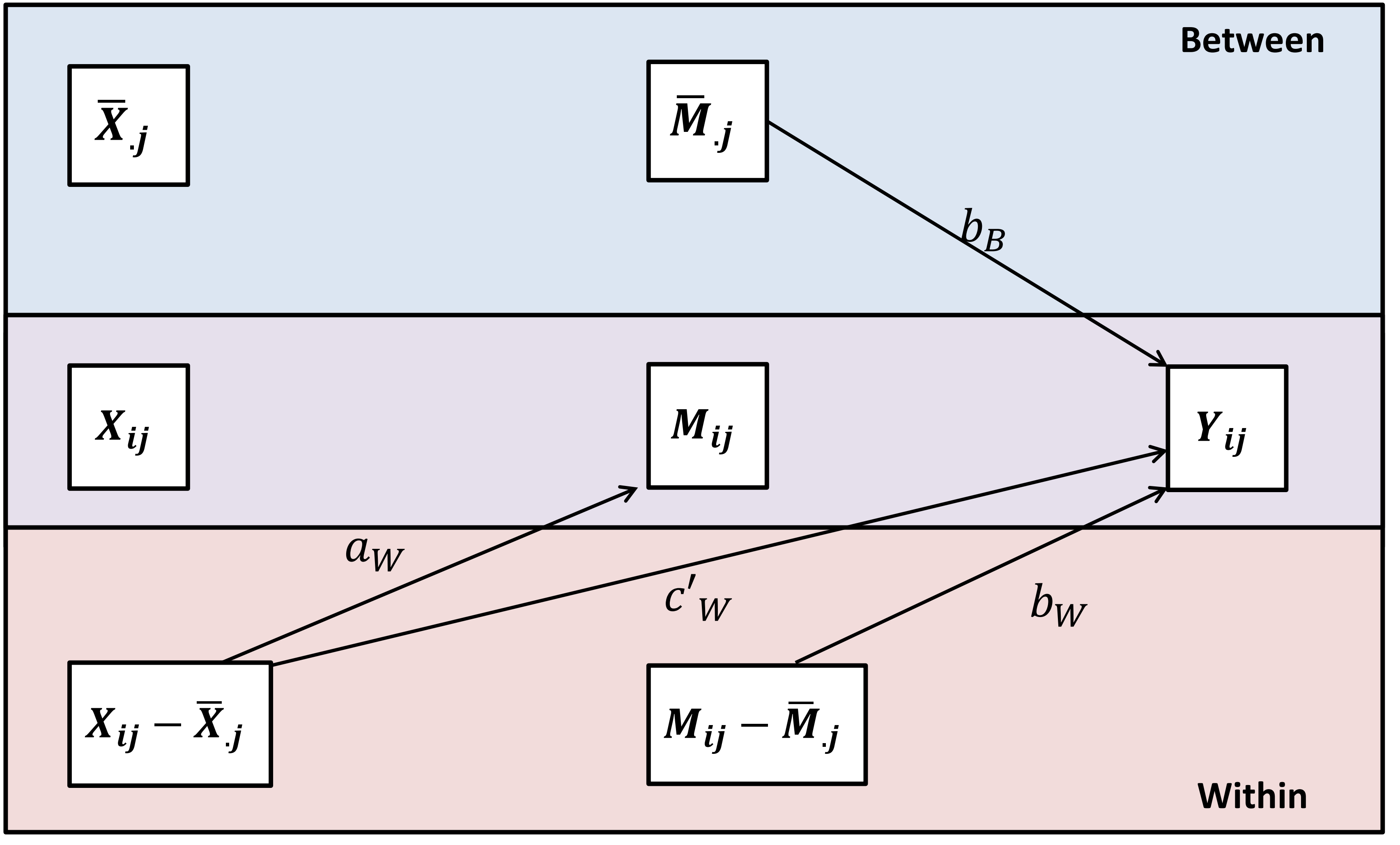
$$b_{0j} = b_Y + \cancel{c'_B\,\bar{X}_{.j}} + b_B\,\bar{M}_{.j} + u_{b_0 j} = b_Y + b_B\,\bar{M}_{.j} + u_{b_0 j}$$

Simple_m
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50
.50

Revised: Conceptual Diagram



Revised: Statistical Diagram



Estimating the *M* Equation (Again)

```
MIXED Hazard WITH Simple_groupc
/FIXED = Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept Simple_groupc| Subject(id)
COVTYPE(UN).
```

Estimates of Covariance Parameters[b]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		.653447	.203505	3.211	.001	.354910	1.203101
Intercept + Simple_groupc [subject = id]	UN (1,1)	.701233	.202258	3.467	.001	.398426	1.234173
	UN (2,1)	.071440	.188917	.378	.705	-.298830	.441710
	UN (2,2)	1.083744[a]	.000000	.	.	.	.

a. This covariance parameter is redundant. The test statistic and confidence interval cannot be computed.
b. Dependent Variable: Hazardousness Simple.

Something has gone wrong with the variance for the slope!

1-1-1 Mediation Full Model

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$a_{0j} = a_M + u_{a_0 j}$$

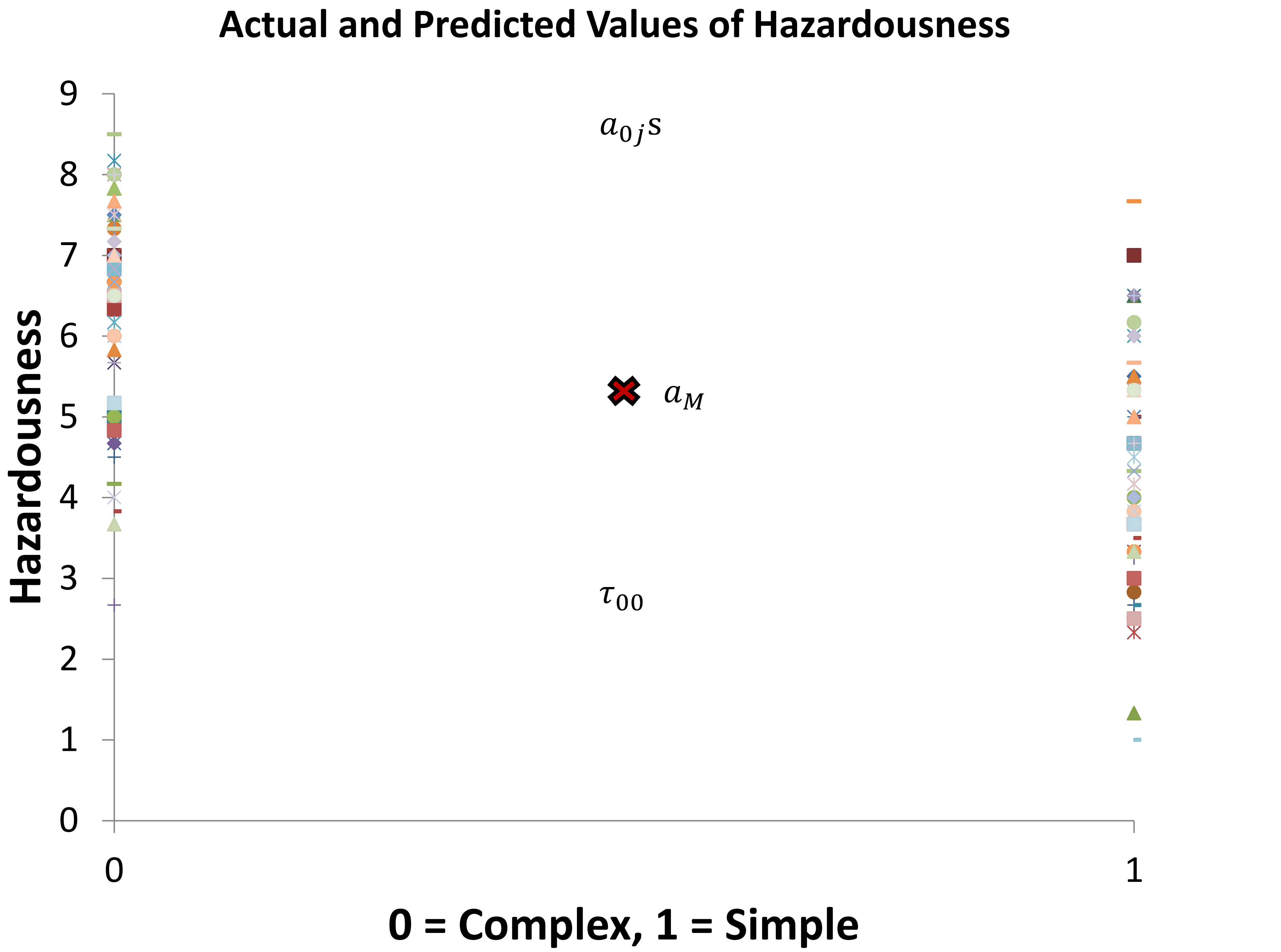
$$a_{1j} = a_W + u_{a_1 j}$$

$$M_{ij} = (a_M + u_{a_0 j}) + (a_W + u_{a_1 j})(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

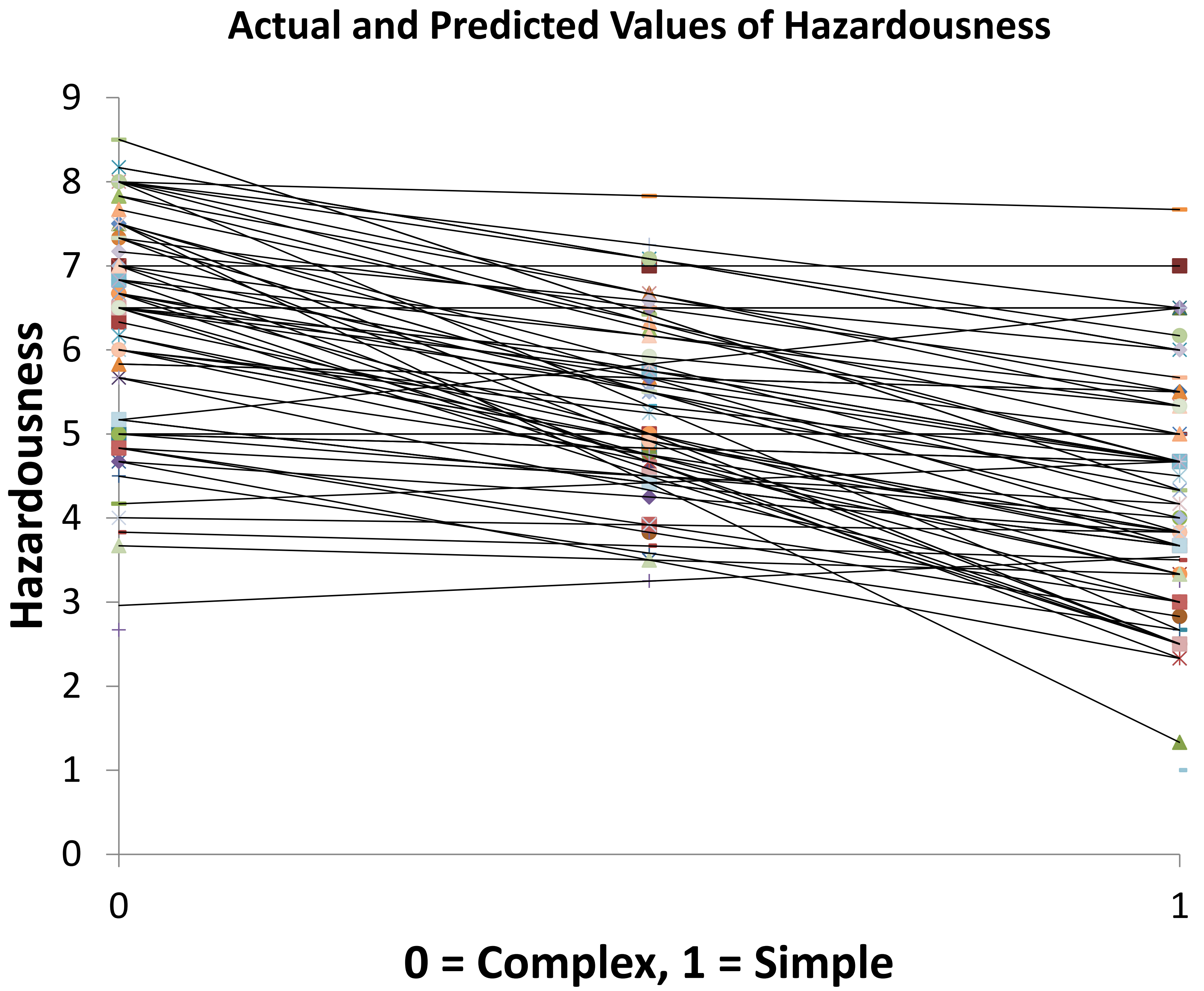
Each person get's their own intercept

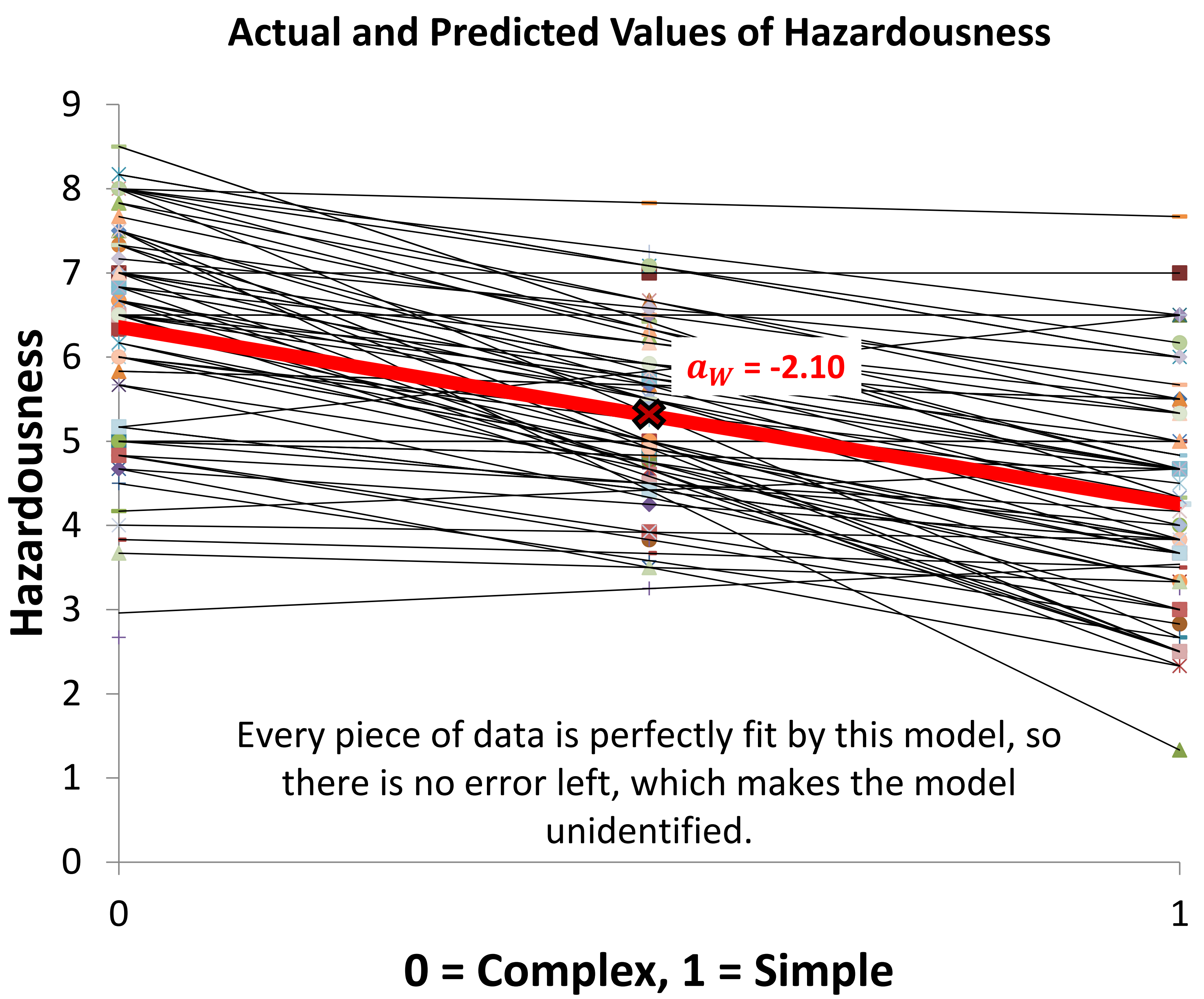
Each person get's their own slope

We only have two observations per person, so giving each person their own intercept and their own slope would perfectly fit the data, and there will be no error left over!



Actual and Predicted Values of Hazardousness
$a_{0j}s$
a_M
τ_{00}
Hazardousness
0
1
2
3
4
5
6
7
8
9
0
1
0 = Complex, 1 = Simple


Actual and Predicted Values of Hazardousness
Hazardousness
0 = Complex, 1 = Simple
0
1
2
3
4
5
6
7
8
9


Actual and Predicted Values of Hazardousness
Hazardousness
9
8
7
6
5
4
3
2
1
0
a_W = -2.10
Every piece of data is perfectly fit by this model, so there is no error left, which makes the model unidentified.
0
1
0 = Complex, 1 = Simple

Estimating the *M* Equation (Again, Again)

Get rid of the random slope, assuming there is no variance in a_W

```
MIXED Hazard WITH Simple_groupc
/FIXED = Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	5.304762	.121182	69.000	43.775	.000	5.063010	5.546513
Simple_groupc	$\boxed{-2.104762}$	.184802	69.000	-11.389	.000	-2.473433	-1.736091

a. Dependent Variable: Hazardousness Simple.

An one unit increase in `Simple_groupc` (i.e., moving from the complex to simple condition) predicts a 2.10 unit decrease in perceptions of hazardousness averaged across individuals. $a_W = -2.10$

Estimating the Y Equation

$$Y_{ij} = b_{0j} + c'_j(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

$$b_{0j} = b_Y + \cancel{c'_B\,\bar{X}_{.j}} + b_B\,\bar{M}_{.j} + u_{b_0 j}$$

$$c'_j = c'_W \cancel{+ u_{c'j}}$$

$$b_{1j} = b_W \cancel{+ u_{b_1 j}}$$

Cut out terms involving group mean of *X*, remove random slopes

Why do we keep the term involving group mean of *M*?

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

Estimating the *Y* Equation

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + \boxed{b_W}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	115.420197	19.457732	68	5.932	.000	76.592897	154.247498
Hazard_groupc	-3.430154	.947546	68.000	-3.620	.001	-5.320953	-1.539354
Hazard_m	-5.968798	3.603669	68	-1.656	.102	-13.159807	1.222211
Simple_groupc	3.827962	2.468446	68.000	1.551	.126	-1.097745	8.753670

a. Dependent Variable: Dosing Simple.

A one unit increase in deviation from the group mean on hazardousness, predicts a 3.43 mL decrease in dosage, controlling for group mean hazardousness and name complexity.

$b_W = -3.43$

Estimating the *Y* Equation

$$Y_{ij} = b_Y + \boxed{b_B} \bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	115.420197	19.457732	68	5.932	.000	76.592897	154.247498
Hazard_groupc	-3.430154	.947546	68.000	-3.620	.001	-5.320953	-1.539354
Hazard_m	-5.968798	3.603669	68	-1.656	.102	-13.159807	1.222211
Simple_groupc	3.827962	2.468446	68.000	1.551	.126	-1.097745	8.753670

a. Dependent Variable: Dosing Simple.

A one unit increase in the group-mean hazard rating predicts a 5.97 mL decrease in dosage, controlling for deviation from the group-mean in hazard rating and name complexity. $b_B = -5.97$

Estimating the Y Equation

$$Y_{ij} = b_Y + b_B \bar{M}_{.j} + u_{b_0 j} + \boxed{c'_W}(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

```
MIXED Dose WITH Hazard_groupc Hazard_m Simple_groupc
/FIXED = Hazard_groupc Hazard_m Simple_groupc | SSTYPE(3)
/METHOD = REML
/PRINT = G SOLUTION TESTCOV
/RANDOM = Intercept | Subject(id) COVTYPE(VC).
```

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	115.420197	19.457732	68	5.932	.000	76.592897	154.247498
Hazard_groupc	-3.430154	.947546	68.000	-3.620	.001	-5.320953	-1.539354
Hazard_m	-5.968798	3.603669	68	-1.656	.102	-13.159807	1.222211
Simple_groupc	3.827962	2.468446	68.000	1.551	.126	-1.097745	8.753670

a. Dependent Variable: Dosing Simple.

A one unit increase simplicity rating (i.e., going from a complex to simple name) increases dosage by 3.83 mL, controlling for hazardousness ratings $c'_W = 3.83$

Indirect Effects in Multilevel Modeling

$$M_{ij} = a_M + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + \ u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j})\ + e_{Y_{ij}}$$

Now we have estimates of everything needed for a mediation model.

There's a lot more coefficients here than when we did between-subjects or two instance repeated-measures.

Generally there are going to be two types of indirect effects in MLMs:

Within-Indirect Effects
Between-Indirect Effects

Because there is no group-mean variation in *X* in this data, we'll only look at the within-indirect effect.

Within-Indirect Effects

$$M_{ij} = a_M + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + b_B\,\bar{M}_{.j} + \ u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) \ + e_{Y_{ij}}$$

A within-group indirect effect quantifies the expected difference in *Y* through *M* for two Level 1 units in the **same Level 2 unit** who differ by one unit on *X*.

When a_{1j} and b_{1j} don't randomly vary the indirect effect is: $a_W b_W$

From our data $a_W = -2.1048$, $b_W = -3.4301$,

$$a_W b_W = (2.1048)(-3.4301) = 7.2197$$

Within a given Level 2 unit (within a specific person), we expect dosage to be 7.22 mL higher in the simple name condition as compared to the complex name condition, through the specific mechanism where name complexity influences perceived hazardousness which then in turn affects dosage.

Between-Indirect Effects

$$M_{ij} = a_M + a_B\,\bar{X}_{.j} + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + c'_B\,\bar{X}_{.j} + b_B\,\bar{M}_{.j} + u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

We don't have a between indirect effect in the model that we estimated.

But we could think of a similar study where some people saw lots of complex drugs and a few simple drugs and others saw lots of simple drugs and a few complex ones.

The between-indirect effect quantifies the expected difference in the group-mean of *Y* through the group-mean of *M* for two Level 2 units that differ by 1 unit on the average of *X*.

In the above example this would be the expected difference in average dosage through average hazardousness for two individuals two differ by 1 on the average simple exposure.

The estimate of the between indirect effect is always $a_B b_B$

Between-Indirect Effects

$$M_{ij} = a_M + a_B\,\bar{X}_{.j} + u_{a_0 j} + a_W(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_Y + c'_B\,\bar{X}_{.j} + b_B\,\bar{M}_{.j} + \; u_{b_0 j} + c'_W(X_{ij} - \bar{X}_{.j}) + b_W(M_{ij} - \bar{M}_{.j}) \; + e_{Y_{ij}}$$

Often the between-indirect effect can be difficult to interpret, as the meaning of the group-aggregate of a variable may differ from the meaning of the variable at the individual level.

Example by Preacher et al. (2010) on differentiating individual efficacy and collective efficacy of a group:

- The aggregate individual efficacy for a given group is a group-level variable (in that it only varies between groups).
- But the focus is still at the individual level and the meaning of such a variable is likely to differ from the meaning of a variable characterizing the dynamics of the self efficacy of the group as a collective.

It’s okay not to estimate or be interested in the between-indirect effect, often times in psychology we’re interested is within-individual change.

Inference about Indirect Effects

As with single-level mediation models, the Sobel/normal theory methods are not appropriate due to the non-normal sampling distribution of the indirect effect.

Bootstrapping in multilevel models can be very difficult, as we want to bootstrapping to mimic the way data is collected from the population. It’s unclear if we should be resampling at the group level, or resampling groups and then sample Level 1 units from the group.

For inference in multilevel models, we’ll rely on **Monte Carlo Confidence Intervals**

Monte Carlo Confidence Intervals (MCCIs) are constructed by simulating data from the estimated sampling distribution of the model parameters and constructing an estimate of the sampling distribution of the indirect effect(s) using the simulated distribution of each part of the indirect effect.

Inference about Indirect Effects

Monte Carlo Confidence Intervals (MCCIs) are constructed by simulating data from the estimated sampling distribution of the model parameters and constructing an estimate of the sampling distribution of the indirect effect(s) using the simulated distribution of each part of the indirect effect.

$$\begin{bmatrix} \hat{\boldsymbol{f}} \\ \hat{\boldsymbol{r}} \end{bmatrix}$$

- Consider two vectors: $\hat{\boldsymbol{f}}$ is a vector containing all of the **FIXED** effect estimates. $\hat{\boldsymbol{r}}$ is a vector containing all of the **RANDOM** effect estimates.
- If we did the study again we would get different estimates for $\hat{\boldsymbol{f}}$ and $\hat{\boldsymbol{r}}$ so let's represent their sampling covariance matrices as $\widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{f}}}}$ (estimated sampling variances and covariances among fixed effects) and $\widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{r}}}}$ (estimated sampling variances and covariances among random effects)
- We know that both random and fixed effects are *normally distributed* and we know they are independent of each other.
- We generate $\boldsymbol{f}^*$ **and** $\boldsymbol{r}^*$ to have a multivariate normal distribution with means, variances, and covariances set by the estimates from the model.

Inference about Indirect Effects

$$\begin{bmatrix} \boldsymbol{f}^* \\ \boldsymbol{r}^* \end{bmatrix} \sim MVN\left(\begin{bmatrix} \hat{\boldsymbol{f}} \\ \hat{\boldsymbol{r}} \end{bmatrix}, \begin{bmatrix} \widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{f}}}} & \boldsymbol{0} \\ \boldsymbol{0} & \widehat{\boldsymbol{\Sigma}_{\hat{\boldsymbol{r}}}} \end{bmatrix}\right)$$

We generate a large number of samples of $\boldsymbol{f}^*$ and $\boldsymbol{r}^*$ (e.g., 10,000)

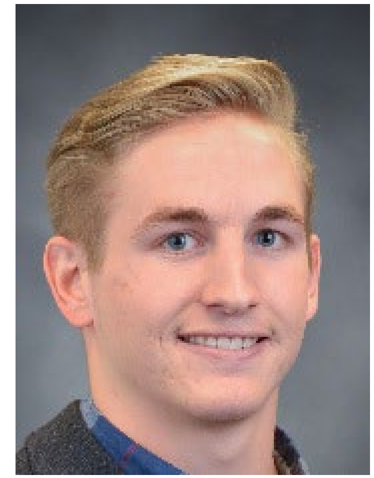
For each sample we calculate the within-indirect effect (and/or between-indirect effect), giving us 10,000 estimates of the indirect effect, which approximates the sampling distribution of the indirect effect.

A $100(1-\alpha)\%$ confidence interval is obtained by using the $100\left(\frac{\alpha}{2}\right)$and $100\left(1-\frac{\alpha}{2}\right)$ percentiles of the simulated sampling distribution.

This method has some similarities to bootstrapping, and is sometimes called the **parametric bootstrap.**

Application in SPSS: MLmed

`Mlmed` is a package for SPSS which can do all of the analysis for you. It does all the recentering, estimates the indirect effects, and does the MCCI on your behalf.



MLmed is written and maintained by Nick Rockwood (PhD Candidate at Ohio State working with Dr. Andrew Hayes). It can be found at njrockwood.com. You also have a copy in your folder.

Just like MEMORE, you need to open the MLmed.sps file, select run all, and now SPSS knows what to do when you use an MLmed command.

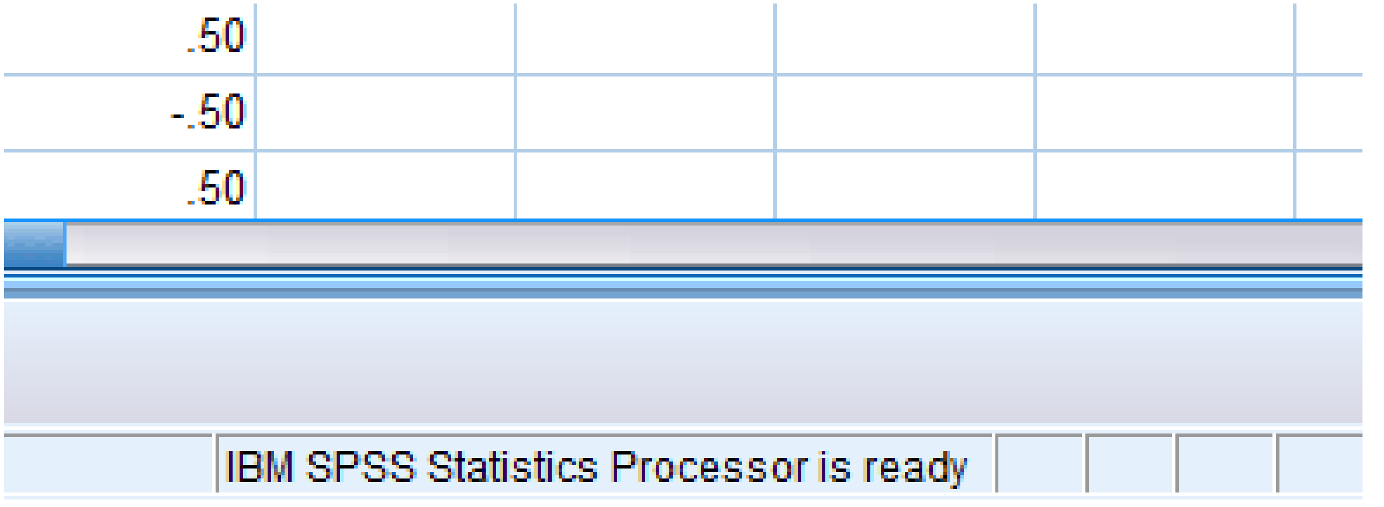
The macro does much more than what I describe here today. Check out the User Guide as well as Rockwood & Hayes (2018).

I'll explain the syntax as we go along.

Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = Simple
/xB = 0
/m1 = Hazard
/y = Dose
/cluster = id
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

WARNING: When you run the code, some windows may pop up on your screen. Let everything resolve, and don't try to interact with those windows.



Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
**********************  MLMED -  BETA VERSION ***********************

                    Written by Nicholas J. Rockwood

             Documentation available at www.njrockwood.com

             Please report any bugs to rockwood.19@osu.edu

********************************************************************

Model Specification
N             140
Fixed           6
Rand(L1)        2
Rand(L2)        2
Total          10

Model Fit Statistics
        Value
-2LL 1669.977
AIC  1677.977
AICC 1678.126
CAIC 1696.430
BIC  1692.430
```

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
***************************  FIXED EFFECTS  ***************************

**********************************************************************
 Outcome: Hazard

Within- Effects
         Estimate     S.E.        df         t         p        LL        UL
constant   5.3048    .1212   69.0000   43.7751     .0000    5.0630    5.5465
Simple    -2.1048    .1848   69.0000  -11.3893     .0000   -2.4734   -1.7361

Note: No Between- Effect(s) Specified.

**********************************************************************
 Outcome: Dose

Within- Effects
         Estimate     S.E.        df         t         p        LL        UL
constant 115.4202  19.4577   68.0000    5.9318     .0000   76.5929  154.2475
Simple     3.8280   2.4684   68.0000    1.5508     .1256   -1.0977    8.7537
Hazard    -3.4302    .9475   68.0000   -3.6200     .0006   -5.3210   -1.5394

Between- Effects
       Estimate     S.E.        df         t         p        LL        UL
Hazard  -5.9688   3.6037   68.0000   -1.6563     .1023 -13.1598    1.2222
```

a_W

c'_W

b_W

b_B

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
**************************  RANDOM EFFECTS  ***************************

Level-1 Residual Estimates
       Estimate      S.E.    Wald Z         p        LL        UL
Dose    74.0515   12.6997    5.8310     .0000   52.9120  103.6367
Hazard   1.1953     .2035    5.8737     .0000     .8562    1.6688

Random Effect Estimates
   Estimate      S.E.    Wald Z         p        LL        UL
1      .4303     .2024    2.1255     .0335     .1711    1.0820
2  884.0882  158.0973    5.5921     .0000  622.7006  1255.197

Random Effect Key
1     Int          Hazard
2     Int          Dose
```

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  ************************

Within- Indirect Effect(s)
            E(ab)    Var(ab)    SD(ab)
Hazard     7.2197     .0000      .0000

Within- Indirect Effect(s)
           Effect         SE          Z          p       MCLL       MCUL
Hazard     7.2197     2.1000     3.4379      .0006     3.2952    11.4551

Note: No Between- Indirect Effect(s) Specified.

------ END MATRIX -----
```

$a_W b_W$

$a_B b_B$

A Comparison: MEMORE vs. MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /m1 = Hazard /y = Dose
/cluster = id /covmat = UN /folder = /Users/Akmontoya/Desktop/
```

MLmed

a_W	Simple	-2.1048	.1848	69.0000	-11.3893	.0000	-2.4734	-1.7361
c'_W	Simple	3.8280	2.4684	68.0000	1.5508	.1256	-1.0977	8.7537
b_W	Hazard	-3.4302	.9475	68.0000	-3.6200	.0006	-5.3210	-1.5394

Within- Indirect Effect(s)

		Effect	SE	Z	p	MCLL	MCUL
$a_W b_W$	Hazard	7.2197	2.1000	3.4379	.0006	3.2952	11.4551

```
MEMORE m= HazSimp HazComp /y = DoseSimp DoseComp /xmint = 0.
```

MEMORE

		Effect	SE	t	p	LLCI	ULCI
a	'X'	-2.1048	.1848	-11.3893	.0000	-2.4734	-1.7361
c'	'X'	3.8280	2.4684	1.5508	.1256	-1.0978	8.7537
b	Mdiff	-3.4302	.9475	-3.6200	.0006	-5.3210	-1.5393

Indirect Effect of X on Y through M

		Effect	BootSE	BootLLCI	BootULCI
ab	Ind1	7.2197	1.8940	3.8590	11.1609

A Comparison: MEMORE vs. MLmed

The model MEMORE fits is equivalent to a random intercept only 1-1-1 mediation model:

- when we have 2 observations per person
- *X* is dichotomous
- each person is observed once for each level of *X*

MLmed is a more general multilevel mediation tool

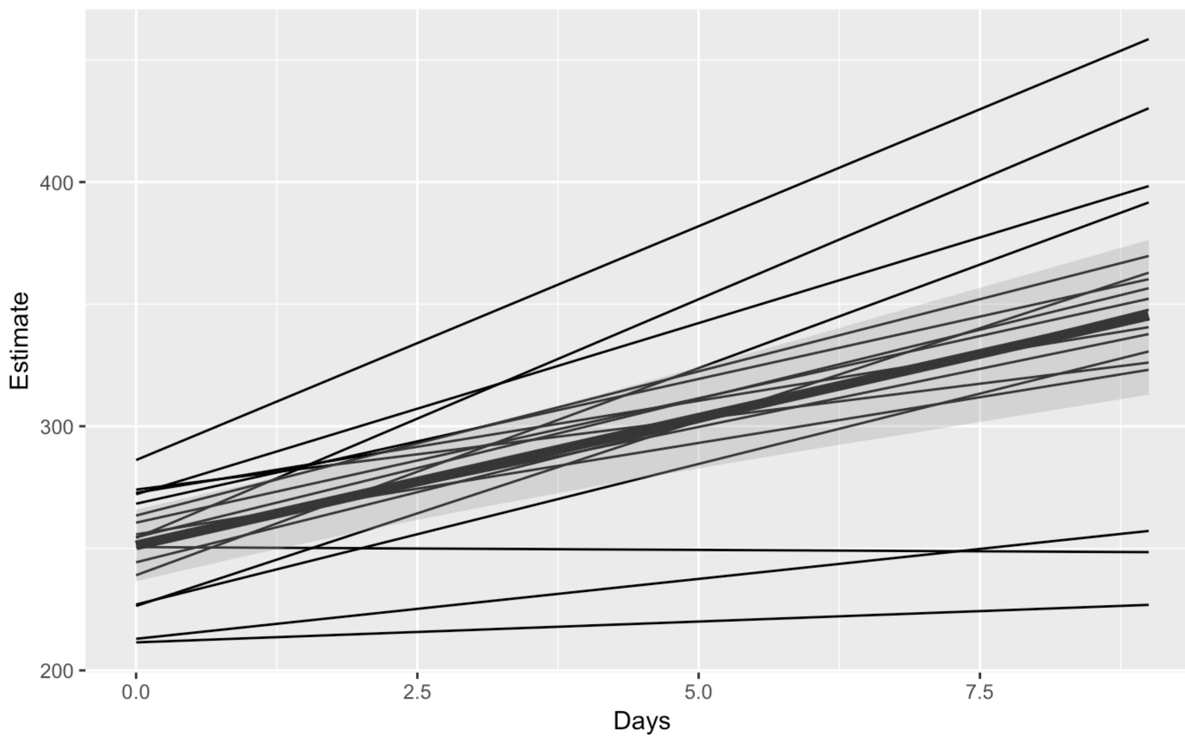
- Syntax is more verbose
- Much more flexible
- Can fit 1-1-1 or 2-1-1 mediations
- Can include covariates, multiple mediators, Level 2 moderators
- Can include random slopes

Adding random slopes

One of the major benefits of multilevel modeling is the ability to incorporate **random slopes**

We can allow the relationship between two variables to vary across groups.

This often more closely resembles the reality of the world as we understand it, where a relationship is not constant but rather has some variance around a mean slope.



Random slopes: Indirect Effect

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

When the slopes have random variance what does this do to the indirect effect?

Between Indirect Effect: unchanged
Within Indirect Effect

When a_{1j} and b_{1j} vary across groups, we may want to estimate the **average within-group indirect effect** and it's variance.

Expected Value (i.e. average) of group j's indirect effect

$$E(a_j b_j) = a_W b_W + \sigma_{a_j,b_j}$$

Even when both a_W and b_W are zero, the average within-group indirect effect can be non-zero if the two random effects covary.

This is also true when one or more slope is fixed, but in that case the covariance is zero, so the equation simplifies to $a_W b_W$

Random slopes: Indirect Effect

$$M_{ij} = a_{0j} + a_{1j}(X_{ij} - \bar{X}_{.j}) + e_{M_{ij}}$$

$$Y_{ij} = b_{0j} + c_j'(X_{ij} - \bar{X}_{.j}) + b_{1j}(M_{ij} - \bar{M}_{.j}) + e_{Y_{ij}}$$

If a_j and b_j is random, the within-group indirect effect is also random.

We can calculate the variance of the within-group indirect effect across groups as:

$$Var(a_j b_j) = b^2\sigma^2_{a_j} + a^2\sigma^2_{b_j} + \sigma^2_{a_j}\sigma^2_{b_j} + 2ab\sigma_{a_j,b_j} + \sigma^2_{a_j,b_j}$$

This tells us how much we can expect the within-group indirect effect to vary across groups.

When a_j or b_j is fixed, this variance is zero.

The way we do inference for the indirect effect is unchanged, we continue to use the MCCI and MLmed will include the relevant factors.

Random slopes: Indirect Effect

$$E(a_j b_j) = a_W b_W + \boxed{\sigma_{a_j,b_j}}$$

$$Var(a_j b_j) = b^2\sigma^2_{a_j} + a^2\sigma^2_{b_j} + \sigma^2_{a_j}\sigma^2_{b_j} + 2ab\sigma_{a_j,b_j} + \boxed{\sigma^2_{a_j,b_j}}$$

When we estimate the *M* equation and *Y* equation separately we do not estimate the covariance σ_{a_j,b_j}

In these circumstances, it is not possible to estimate the average within group indirect effect.

Instead the equations need to be estimated simultaneously.

Some SEM packages (e.g., Mplus) can estimate multilevel models simultaneously.

Bauer, Preacher, and Gil (2006) demonstrate how the equations can be estimated simultaneously using traditional (univariate) multilevel modeling software.

MLmed utilizes this method when estimating mediation models with random slopes

Example: Drug Name Fluency

Dohle, S., & Montoya, A. K. (2017). The dark side of fluency: Fluent names increase drug dosing. *Journal of Experimental Psychology: Applied*, *23*(3), 231 – 239.

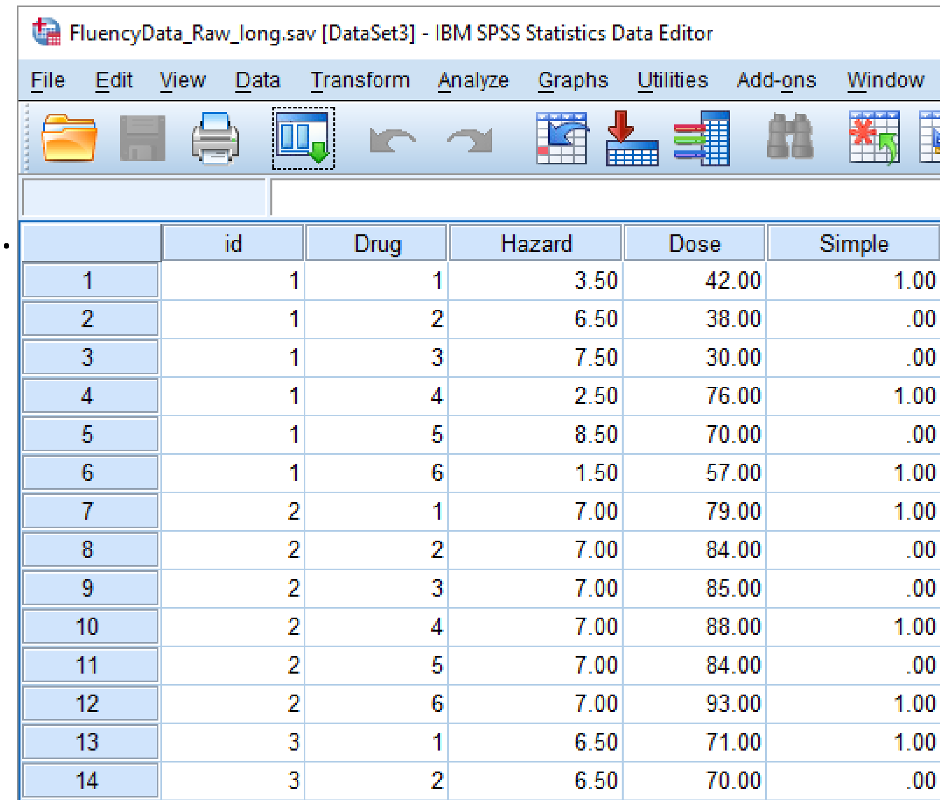
Participants (N = 70) were asked to imagine they had the flu, and 6 different drugs were provided to treat the drug. Participants poured the dose they would feel comfortable taking at maximum into a plastic cup. Each person judged drugs with simple or complex names (**3 of each**).

Open the dataset FluencyData_Raw_long.sav

There are six drugs (Drug = 1 – 6)

Each person saw 3 simple and 3 complex drugs.

We will treat each of these as repeated observations of the same person (6 instead of 2).



	id	Drug	Hazard	Dose	Simple
1	1	1	3.50	42.00	1.00
2	1	2	6.50	38.00	.00
3	1	3	7.50	30.00	.00
4	1	4	2.50	76.00	1.00
5	1	5	8.50	70.00	.00
6	1	6	1.50	57.00	1.00
7	2	1	7.00	79.00	1.00
8	2	2	7.00	84.00	.00
9	2	3	7.00	85.00	.00
10	2	4	7.00	88.00	1.00
11	2	5	7.00	84.00	.00
12	2	6	7.00	93.00	1.00
13	3	1	6.50	71.00	1.00
14	3	2	6.50	70.00	.00

Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = Simple
/xB = 0
/randx = 01
/m1 = Hazard
/randm = 1
/y = Dose
/cluster = id
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

First number is random effect of *X* on *Y* (c_j'), second number is random effect of *X* on *M* (a_j)

Random effect of *M* on *Y* (b_j). When you have *k* mediators this list should be *k* long (e.g., 3 mediators 010)

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx =
01 /m1 = Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
***************************  FIXED EFFECTS  ***************************

**********************************************************************
 Outcome: Hazard

Within- Effects
          Estimate      S.E.        df         t         p        LL        UL
constant    5.3048     .1212   69.0000   43.7751     .0000    5.0630    5.5465
Simple     -2.1048     .1848   69.0000  -11.3893     .0000   -2.4734   -1.7361

Note: No Between- Effect(s) Specified.

**********************************************************************
 Outcome: Dose

Within- Effects
          Estimate      S.E.        df         t         p        LL        UL
constant  115.4202   19.4577   68.0000    5.9318     .0000   76.5929  154.2475
Simple      3.8946    2.0466  342.6421    1.9029     .0579    -.1309    7.9201
Hazard     -3.0849     .6996   76.2642   -4.4094     .0000   -4.4782   -1.6916

Between- Effects
        Estimate      S.E.        df         t         p        LL        UL
Hazard   -5.9688    3.6037   68.0000   -1.6563     .1023  -13.1598    1.2222
```

Fixed effects are not different from when we used the averaged data.

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx = 01 /m1 =
Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
**************************  RANDOM EFFECTS  ***************************

Level-1 Residual Estimates
       Estimate     S.E.    Wald Z        p        LL        UL
Dose   245.4574  20.2443   12.1247    .0000  208.8202  288.5225
Hazard   1.3619    .1151   11.8322    .0000    1.1540    1.6073

Random Effect Estimates
      Estimate     S.E.    Wald Z        p        LL        UL
(1,1)     .8010    .1761    4.5494    .0000     .5206    1.2323
(2,2)  880.2044 158.0058    5.5707    .0000  619.1332  1251.362
(3,3)    1.4827    .4142    3.5799    .0003     .8576    2.5635
(4,3)     .6696    .8673     .7720    .4401  -1.0303    2.3695
(4,4)    3.9093   3.5234    1.1095    .2672     .6682   22.8701

Random Effect Covariance Matrix
            1         2         3         4
1       .8010     .0000     .0000     .0000
2       .0000  880.2044     .0000     .0000
3       .0000     .0000    1.4827     .6696
4       .0000     .0000     .6696    3.9093

Random Effect Correlation Matrix
            1         2         3         4
1      1.0000     .0000     .0000     .0000
2       .0000    1.0000     .0000     .0000
3       .0000     .0000    1.0000     .2781
4       .0000     .0000     .2781    1.0000
```

```
Random Effect Key
1     Int        Hazard
2     Int        Dose
3     Slope      Simple        ->      Hazard
4     Slope      Hazard        ->      Dose
```

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = Simple /xB = 0 /randx = 01 /m1 =
Hazard /randm = 1 /y = Dose /cluster = id
/covmat = UN /folder = /Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  *************************

 Within- Indirect Effect(s)
           E(ab)   Var(ab)    SD(ab)
 Hazard   7.1626   46.3690    6.8095

 Within- Indirect Effect(s)
          Effect        SE         Z         p      MCLL      MCUL
 Hazard   7.1626    1.8403    3.8921     .0001    3.6753   10.8879

 Note: No Between- Indirect Effect(s) Specified.
```

On average, within an individual, the difference in dosage between sample drugs and complex drugs that operates indirect through perceived hazardousness is estimated to be 7.16 (MCCI = [3.68, 10.89]), where simple drugs are administered at higher dosages than complex drugs. However there is substantial between-person variability in this indirect effect (SD = 6.81).

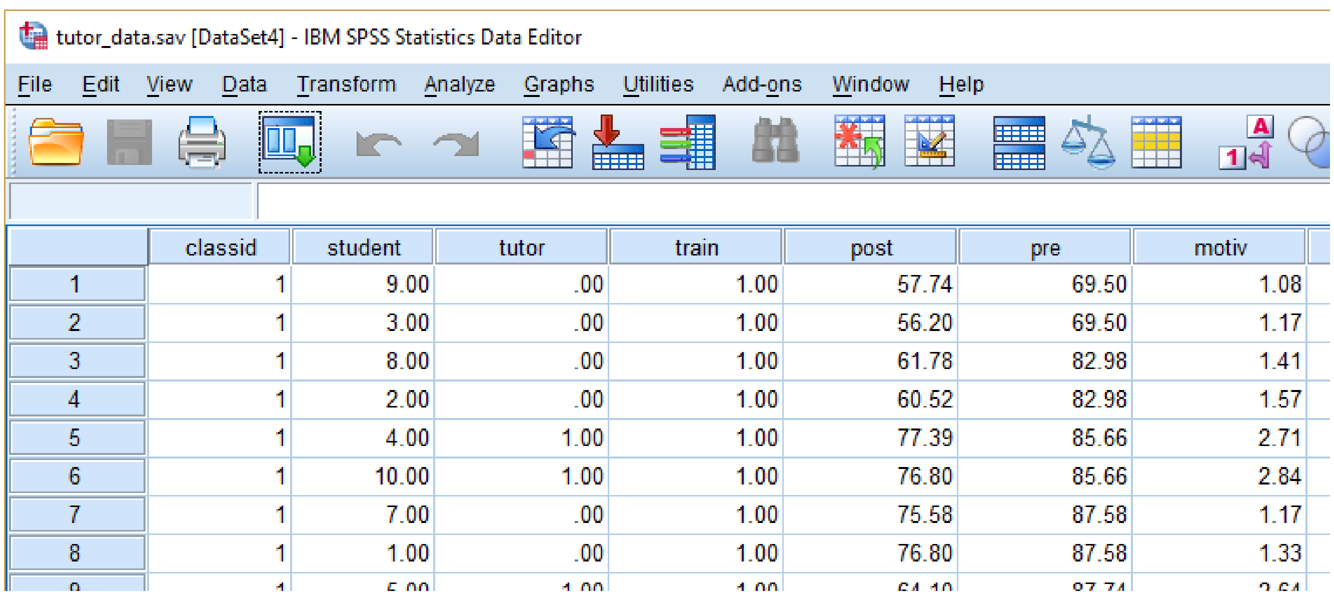
Tutor Data

This example uses a simulated dataset (tutor_data.sav) based on an educational experiment.

Suppose 48 classrooms were randomly sampled where no classrooms were in the same school.

Next, students within each classroom were randomly sampled to participate in an after-school tutoring program throughout the school year.

The total number of students is 450, where 223 students are assigned to tutoring and 227 are assigned to control (no tutoring program).



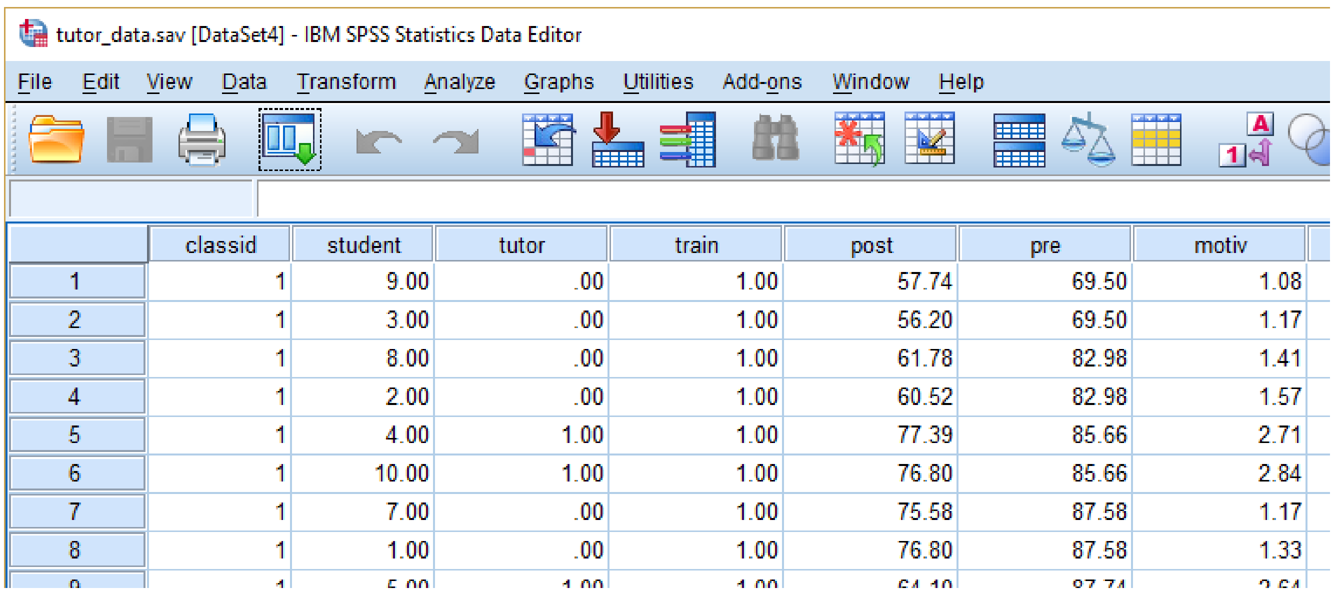
	classid	student	tutor	train	post	pre	motiv
1	1	9.00	.00	1.00	57.74	69.50	1.08
2	1	3.00	.00	1.00	56.20	69.50	1.17
3	1	8.00	.00	1.00	61.78	82.98	1.41
4	1	2.00	.00	1.00	60.52	82.98	1.57
5	1	4.00	1.00	1.00	77.39	85.66	2.71
6	1	10.00	1.00	1.00	76.80	85.66	2.84
7	1	7.00	.00	1.00	75.58	87.58	1.17
8	1	1.00	.00	1.00	76.80	87.58	1.33

Tutor Data

The **tutor** variable in the dataset codes the assignment of each student
(0 = control, 1 = tutoring)

Before completing an end of year mathematics exam (**post**), the students' academic motivation was measured (**motiv**)

There is also data on the students' test scores from the previous year (**pre**).



	classid	student	tutor	train	post	pre	motiv
1	1	9.00	.00	1.00	57.74	69.50	1.08
2	1	3.00	.00	1.00	56.20	69.50	1.17
3	1	8.00	.00	1.00	61.78	82.98	1.41
4	1	2.00	.00	1.00	60.52	82.98	1.57
5	1	4.00	1.00	1.00	77.39	85.66	2.71
6	1	10.00	1.00	1.00	76.80	85.66	2.84
7	1	7.00	.00	1.00	75.58	87.58	1.17
8	1	1.00	.00	1.00	76.80	87.58	1.33

Tutor Data

We are interested in testing whether there is evidence that the participation in after-school tutoring program (X = tutor) results in higher mathematics post-test scores (Y = post), on average, due to an increase in student motivation (M = motiv).

Further we are interested in whether this effect is consistent across classrooms, or whether there is between-classroom variability in the effect.

Throughout, we will use the previous year's math test score (Q = pre) as a covariate.

All variables are level-1 (student level).

The proportion of students assigned to tutoring in each class is not constant, so there is between-class variability in X. Additionally there will be between class variability in M, Q, and Y.

We will use group-mean centering to remove between-class variability and add this back into the model using the classroom means as predictors of the random intercepts to that within-class and between-class effect can be estimated separately.

Doing it in SPSS: MLmed

```
Mlmed data = dataset1
/x = tutor
/m1 = motiv
/y = post
/cov1 = pre
/cluster = classid
/covmat = UN
/folder = /Users/Akmontoya/Desktop/
```

Add in a Level 1 covariate

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = tutor /m1 = motiv /y = post /cov1 = pre
/cluster = classid /covmat = UN /folder = Users/Akmontoya/Desktop/
```

```
 Outcome: motiv

Within- Effects
          Estimate      S.E.         df          t          p         LL         UL
constant    1.2844     .5128    46.8208     2.5049      .0158      .2528     2.3160
tutor       1.3517     .0462   398.0015    29.2783      .0000     1.2610     1.4425
pre          .0194     .0022   398.0015     8.7609      .0000      .0151      .0238

Between- Effects
       Estimate      S.E.         df          t          p         LL         UL
tutor     .8443     .5654    47.1328     1.4933      .1420     -.2930     1.9817
pre       .0039     .0058    45.9489      .6771      .5017     -.0077      .0156

***********************************************************************
 Outcome: post

Within- Effects
          Estimate      S.E.         df          t          p         LL         UL
constant   50.3689    7.2512    43.9803     6.9463      .0000    35.7548    64.9830
tutor      -3.2913    1.4885   394.8792    -2.2112      .0276    -6.2176     -.3649
motiv       4.4675     .9097   394.8792     4.9112      .0000     2.6791     6.2559
pre          .3746     .0440   394.8792     8.5215      .0000      .2882      .4611

Between- Effects
       Estimate      S.E.         df          t          p         LL         UL
tutor  -24.6103    7.7505    48.0227    -3.1753      .0026   -40.1935    -9.0271
motiv   12.3961    1.9524    43.5166     6.3490      .0000     8.4600    16.3323
pre       .0119     .0775    44.9123      .1532      .8789     -.1443      .1680
```

Doing it in SPSS: MLmed

```
Mlmed data = dataset1 /x = tutor /m1 = motiv /y = post /cov1 = pre
/cluster = classid /covmat = UN /folder = Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  ****************

Within- Indirect Effect(s)
          E(ab)   Var(ab)    SD(ab)
motiv    6.0388     .0000     .0000

Within- Indirect Effect(s)
         Effect        SE         Z         p      MCLL      MCUL
motiv    6.0388    1.2475    4.8408     .0000    3.6273    8.5073

Between- Indirect Effect(s)
         Effect        SE         Z         p      MCLL      MCUL
motiv   10.4665    7.2843    1.4368     .1508   -3.3701   25.5751

Test of Indirect Contextual Effect(s): Between - Within
            Dif      MCLL      MCUL
motiv    4.4276   -9.5305   19.8116
```

Doing it in SPSS: MLmed

```
************************  INDIRECT EFFECT(S)  ****************

Within- Indirect Effect(s)
         E(ab)  Var(ab)   SD(ab)
motiv   6.0388    .0000    .0000

Within- Indirect Effect(s)
        Effect       SE        Z        p     MCLL     MCUL
motiv   6.0388   1.2475   4.8408    .0000   3.6273   8.5073

Between- Indirect Effect(s)
        Effect       SE        Z        p     MCLL     MCUL
motiv  10.4665   7.2843   1.4368    .1508  -3.3701  25.5751

Test of Indirect Contextual Effect(s): Between - Within
           Dif     MCLL     MCUL
motiv   4.4276  -9.5305  19.8116
```

Within a given classroom, there is a significant indirect effect of tutoring on posttest through motivation controlling for pretest ($E\left(a_j b_j\right) = 6.04,\ MCCI = [3.53{,}8.50]$), where students who participated in tutoring performed better on the post test.

There was not significant evidence that between classroom variability in proportion of students assigned to tutoring influenced average classroom performance through average motivation ($a_B b_B = 10.47\ \mathrm{MCCI} = [-3.42\ 25.32]$). There's not significant evidence that the within and between indirect effects significantly different.

Exercise: Adding random slopes

In addition to being interested in the average within-class indirect effect, we are also interested in determining if that within-class indirect effect varies across classrooms.

- Using MLmed, expand the model to include a random a_j and b_j, as well as the covariance between these paths
- Interpret the individual coefficients and their variances making up the mediation model.
- Interpret the average and variance of the within-group indirect effect in the context of the specific example.

2-1-1 Models

The 1-1-1 model is for the general data design where *X*-*M*-*Y* all contain within and between group variability.

In the dosage data, the model we fit only had within group variability in *X*.

MLmed can also be used to fit models where *X* only contains information about between-group variability (2-1-1 models). This type of model is useful for cluster-randomized designs (each group assigned to a condition).

MLmed does not fit 2-2-1 models but these can be fit piecewise, where $X \rightarrow M$ is an OLS regression and the *Y* equation is fit using MLM.

Models with “upward effects” (e.g., 1-2-1) cannot be fit in MLM software and require multilevel structural equation modeling.

2-1-1 Example

Suppose that rather than students assigned to tutoring, teachers completed a training program deisgned to teach a number of skills focused on engaging their students through the use of interactive real-world applications.

It is thought that students who are exposed to the interactive real-world applications will see the utility of the content being taught and they will be more motivatted, leading to an increase in their post-test scores.

The variable **train** is a teacher-level (Level 2) training identifier (1 = completed training, 0 = control)

2-1-1 Example

In the tutor dataset we may be interested in testing if the average amount of student motivation mediates the relationship between the teacher's completion of a training program and the average post-test score of their students.

This is a 2-1-1 model since training (*X*) is a Level 2 variable (classroom level), motivation and posttest scores are both at Level 1 (student level).

There can only be a between-group indirect effect.

```
MLmed data = dataset4
/x = train
/xW = 0
/m1 = motiv
/y = post
/cov1 = pre
/cluster = classid
/folder = /Users/Akmontoya/Desktop/
```

Set the within effect of X to zero

2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
**************************************************************************
 Outcome: motiv

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant   2.2615     .4840   56.7024    4.6724     .0000    1.2921    3.2308
pre         .0295     .0039  398.6255    7.6001     .0000     .0219     .0372

Between- Effects
      Estimate      S.E.        df         t         p        LL        UL
train    .3489     .1743   46.2330    2.0014     .0512    -.0019     .6997
pre     -.0065     .0078   55.2377    -.8368     .4063    -.0220     .0090

**************************************************************************
 Outcome: post

Within- Effects
         Estimate      S.E.        df         t         p        LL        UL
constant  48.8441    8.4186   45.1925    5.8019     .0000   31.8902   65.7980
motiv      2.8059     .5150  396.2348    5.4481     .0000    1.7934    3.8185
pre         .3991     .0427  396.2348    9.3352     .0000     .3150     .4831

Between- Effects
      Estimate      S.E.        df         t         p        LL        UL
train   5.2720    2.6375   43.0894    1.9989     .0520    -.0467   10.5907
motiv   9.8785    2.1110   43.3758    4.6796     .0000    5.6224   14.1346
pre     -.0992     .1109   47.9342    -.8953     .3751    -.3221     .1236

**************************************************************************
```

2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
**************************  RANDOM EFFECTS  ***************************

Level-1 Residual Estimates
      Estimate     S.E.   Wald Z        p        LL        UL
post   75.1677   5.3404  14.0754    .0000  65.3968   86.3983
motiv    .7104    .0503  14.1178    .0000    .6184     .8162

Random Effect Estimates
   Estimate     S.E.   Wald Z        p        LL        UL
1      .0818    .0358   2.2833    .0224    .0347     .1931
2    26.8186   7.9874   3.3576    .0008  14.9596   48.0784

Random Effect Key
1     Int          motiv
2     Int          post
```

2-1-1 Example

```
MLmed data = dataset4 /x = train /xW = 0 /m1 = motiv /y = post
/cov1 = pre /cluster = classid /folder = /Users/Akmontoya/Desktop/
```

```
************************  INDIRECT EFFECT(S)  *************************

Note: No Within- Indirect Effect(s) Specified.

Between- Indirect Effect(s)
          Effect        SE         Z         p      MCLL      MCUL
motiv     3.4462    1.9085    1.8057     .0710     .1184    7.4554
```

There is a significant between-group indirect effect of teacher training on student posttest, by way of student motivation ($a_B b_B = 3.45, MCCI = [0.05, 7.47]$). Specifically, the students of teachers who participated in the training had higher motivation on average, than students of teachers who did not participate in the training, and higher average motivation led to higher average posttest scores.

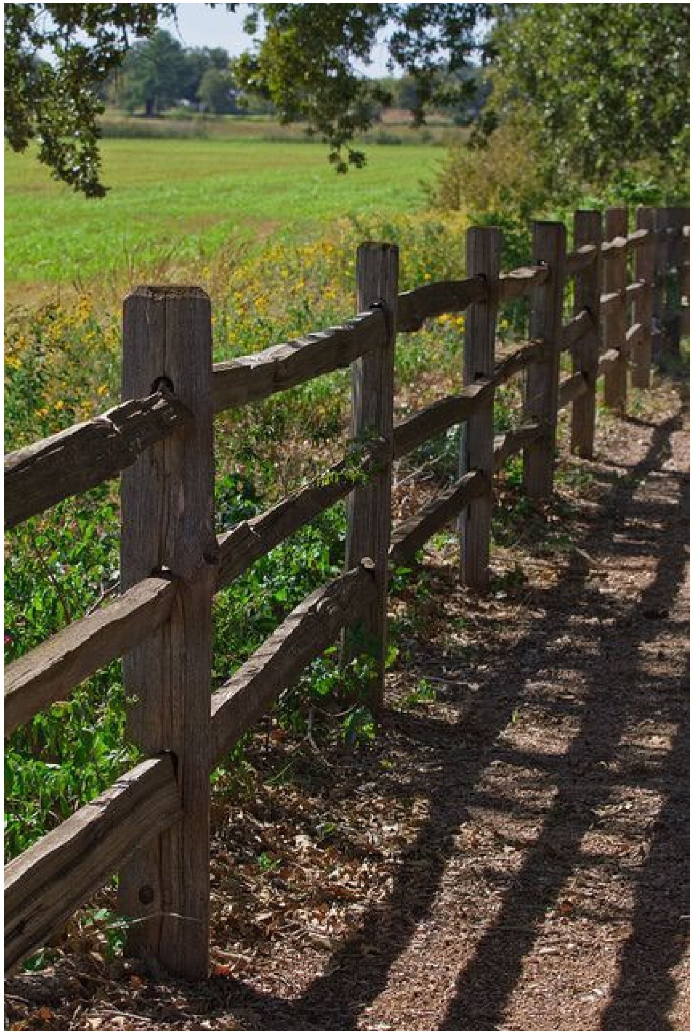
Resources for Repeated Measures Mediation

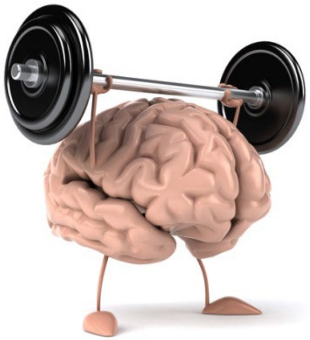
- Multilevel Models
  - Bauer, Preacher, Gil (2006) *Psychological Methods*
    Covers Mediation and Moderated Mediation for 1-1-1 multilevel mediation
  - Kenny, Korchmaros, Bolger (2003) *Psychological Methods*
    Covers mediation for 1-1-1 multilevel models
  - **COMING SOON**: Nick Rockwood's MLMediation Macro (see afhayes.com for updates)
- Latent Growth Curve Models (Longitudinal Processes M-Y measured over time)
  - Choeng, MacKinnon, Khoo (2003) *Structural Equation Modeling*
- Structural Equation Modeling (Can be used for a variety of data types)
  - Cole & Maxwell (2003) *Journal of Abnormal Psychology*
    X, M, and Y all measured over time
  - Newsom (2009) *Structural Equation Modeling*
    Dyadic data using LGMs
  - Selig & Little (2012) *Handbook of Developmental Research Methods*
    Autoregressive models and cross-lagged panel models for longitudinal data X, M, and Y all measured over time.
- **Selig & Preacher (2009) *Research in Human Development***
  - **Longitudinal Models X, M, and Y measured across time. Cross-lagged panel models, latent growth models, latent difference score models**
- Multilevel SEM
  - Preacher, Zyphyr, Zhang, 2010
  - Preacher, Zhang, Zyphur, 2011

Moderation

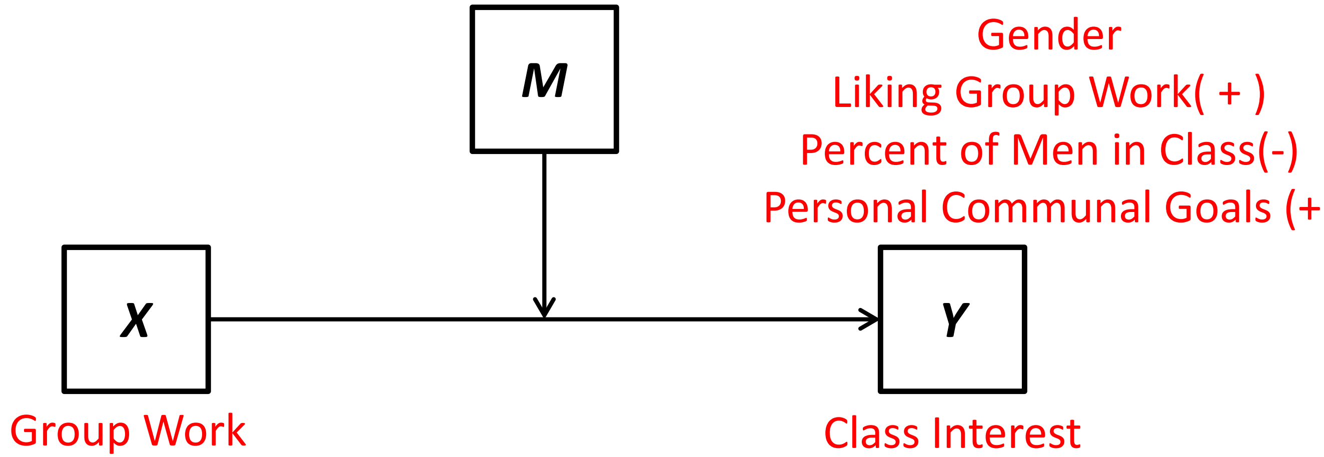
- Two-Condition Within Subjects Moderation
  - Judd Kenny and McClelland (2001, 1996)
  - Interpretations
  - Probing
  - MEMORE
  - Reporting (Writing and Figures)
  - Common Questions

- Other Types of Repeated Measures Moderation
  - Multilevel
  - Longitudinal
  - Multilevel SEM





Moderation



The relationship between the focal predictor (*X*) and an outcome (*Y*) is said to be moderated when the size or direction depends on *M*. Moderation helps us understand boundary conditions of effect: for whom on when is the effect large or small, present or absent, positive or negative.

X and *M* are frequently described as “interacting” in their prediction of *Y*.

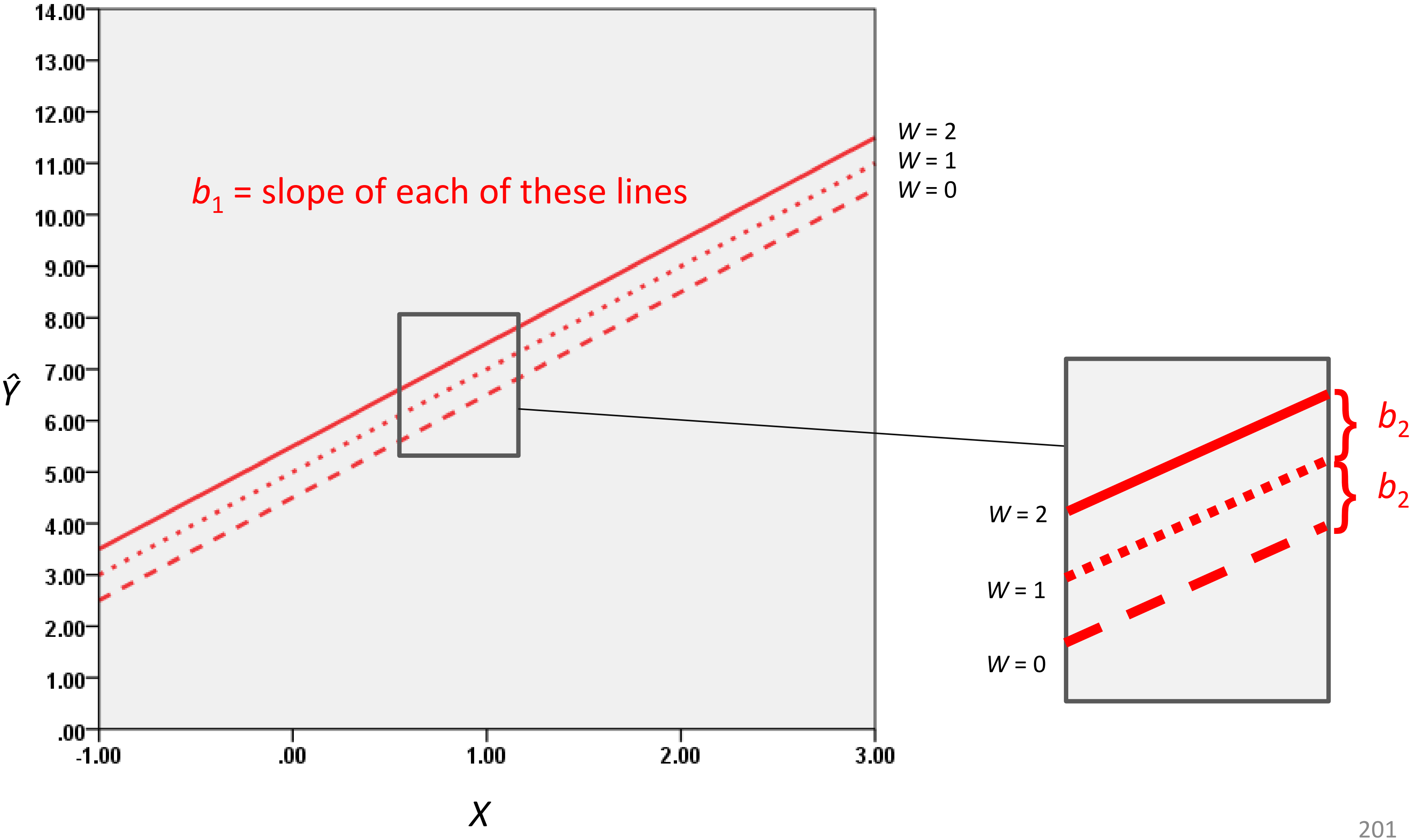
Many different kind of variables may act as moderators. Emotional variables, situational, individual level variables, cognitive variables, environmental variables, etc.

A quick example: Name some possible moderators!

Moderation in Between-Subject Designs

Partial regression coefficients as unconditional effects

$$\widehat{Y}_i = 4.50 + 2.00X_i + 0.50W_i$$



Releasing this constraint on the model

Suppose we let *X*'s effect be a function of *W*, *f*(*W*), as in

$$\hat{Y}_i = b_0 + f(W_i)X_i + b_2W_i$$

For instance, let *f*(*W*) be a linear function of *W*, $b_1 + b_3W$. Thus,

$$\hat{Y}_i = b_0 + (b_1 + b_3W_i)X_i + b_2W_i$$

This can be rewritten in an equivalent form as

$$\hat{Y}_i = b_0 + b_1X_i + b_2W_i + b_3X_iW_i$$

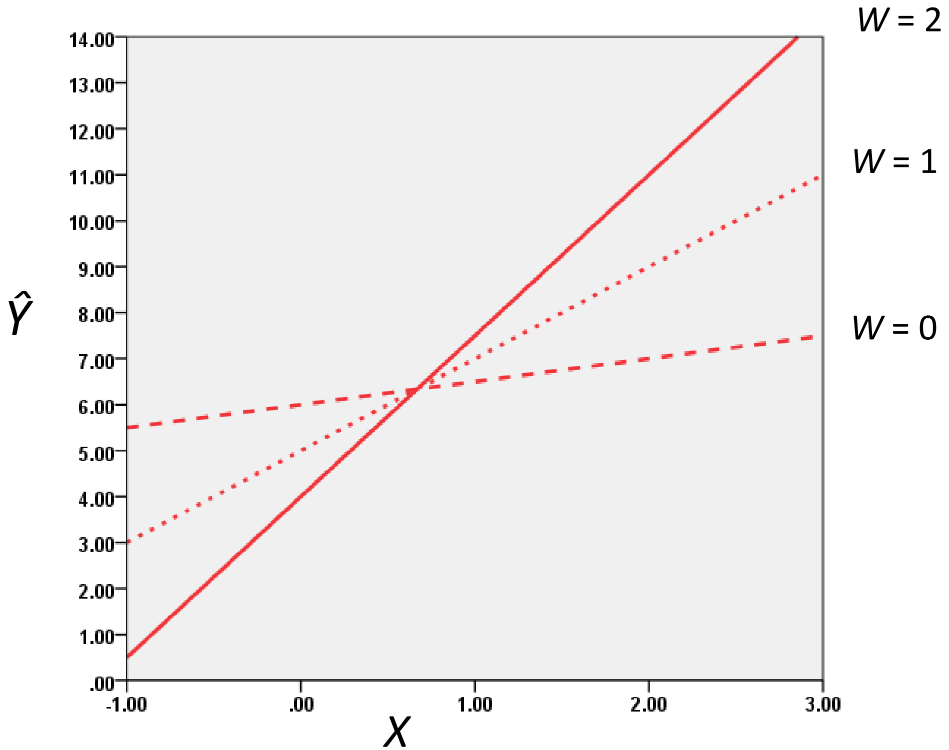
This model, the "simple moderation model," allows *X*'s effect on *Y* to depend linearly on *W*. Other forms of moderation are possible, but this form is the one most frequently estimated.

X's effect as a function of W

b_0 = 6.00
b_1 = 0.50
b_2 = -1.00
b_3 = 1.50

$$\hat{Y}_i = 6.00 + 0.50X_i - 1.00W_i + 1.50X_iW_i$$

Observe that the amount by which two cases that differ by one unit on *X* are estimated to differ on *Y* **depends on** *W*.



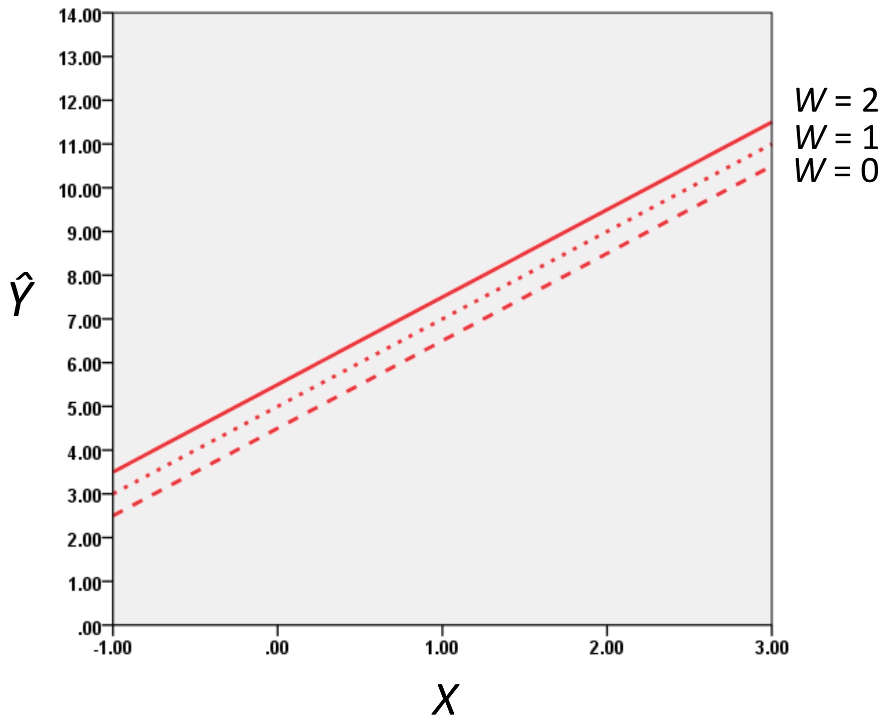
X	W	$\hat{Y}$
-1	0	5.50
-1	1	3.00
-1	2	0.50
0	0	6.00
0	1	5.00
0	2	4.00
1	0	6.50
1	1	7.00
1	2	7.50
2	0	7.00
2	1	9.00
2	2	11.00

Differences in interpretation

	$\hat{Y}_i = b_0 + b_1X_i + b_2W_i$	$\hat{Y}_i = b_0 + b_1X_i + b_2W_i + b_3X_iW_i$
b_0	The estimated value of *Y* when *X* and *W* = 0.	The estimated value of *Y* when *X* and *W* = 0.
b_1	The effect of *X* on *Y* holding *W* constant. This is a *partial* effect.	The effect of *X* on *Y* when *W* = 0. This is a *conditional* effect. It is Not a "main effect" or "average effect" of *X*.
b_2	The effect of *W* on *Y* holding *X* constant. This is a *partial* effect.	The effect of *W* on *Y* when *X* = 0. This is a *conditional* effect. It is not a "main effect" or "average effect" of *W*.
b_3		How much the effect of *X* on *Y* changes as *W* changes by 1 unit.

The importance of b_3 when testing a moderation hypothesis

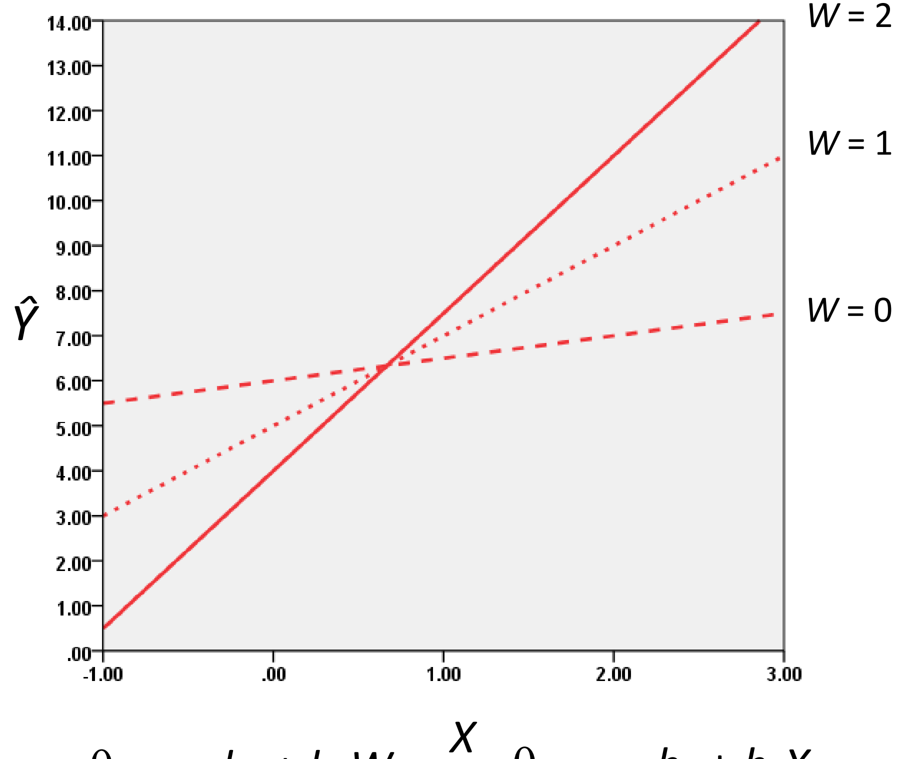
$$\hat{Y}_i = 4.50 + 2.00X_i + 0.50W_i + 0X_iW_i$$



$$\theta_{X \to Y} = b_1 + b_3W = 2.00 + 0W$$

$$\theta_{W \to Y} = b_2 + b_3X = 0.50 + 0X$$

$$\hat{Y}_i = 6.00 + 0.50X_i - 1.00W_i + 1.50X_iW_i$$



$$\theta_{X \to Y} = b_1 + b_3W = 0.50 + 1.50W$$

$$\theta_{W \to Y} = b_2 + b_3X = -1.00 + 1.50X$$

When $b_3 = 0$, a one unit change in X has the same effect on Y regardless of W, and a one unit change in W has the same effect on Y regardless of X. When $b_3 \neq 0$, the effect of a change in X on Y depends on W, and the effect of a change in W on Y depends on X. So we test a moderation hypothesis by testing whether b_3 is different from zero.

Probing an interaction

The coefficient for the product term carries information about how changes in one variable are related to changes in the effect of the other. A picture helps to understand how the focal variable's effect changes as a function of the moderator variable.

It is typically desirable to conduct statistical tests of the focal predictor variable's effect at values of the moderator. This allows you to make more definitive claims about where the focal predictor variables effect is zero versus where it is not.

"Pick-a-Point" Approach

Select values of the moderator and estimate the conditional effect of the focal predictor at those values of the moderator, along with a hypothesis test or confidence interval.

Johnson-Neyman Technique

Derive mathematically where on the moderator variable continuum the focal variable's effect transitions between statistically significant and nonsignificant.

Pick-a-point approach

$$\widehat{Y}_i = b_0 + b_1X_i + b_2W_i + b_3X_iW_i$$

Select a value of the moderator (*W*) at which you'd like to have an estimate of $\theta_{X\rightarrow Y}$, the focal predictor variable's (*X*) effect. Then derive its standard error. The ratio of the effect to its standard error is distributed as $t(df_{residual})$ under the null hypothesis that the effect of the focal predictor is zero at that moderator value, where df_{residual} is the residual degrees of freedom from the regression model.

We already know that $\theta_{X\rightarrow Y} = b_1 + b_3W$

The estimated standard error of $\theta_{X\rightarrow Y}$ is

$$s_{\theta_{X\rightarrow Y}} = \sqrt{s_{b_1}^2 + 2W s_{b_1b_3} + W^2 s_{b_3}^2}$$

Squared standard error of b_1

Covariance of b_1 and b_3

Squared standard error of b_3

You could do this by hand, and instructions are available in various books on regression analysis (e.g., Aiken and West, 1991; Cohen et al., 2003). But there is no reason to, and the potential for mistakes is high. It is made easier using "**regression centering**."

The Johnson-Neyman technique

The Johnson-Neyman technique seeks to find the value or values of the moderator *(W)* within the data, if they exist, such that the *p*-value for the ratio of the conditional effect of the focal predictor at that value or values of *W* is exactly equal to some chosen level of significance α

To do so, we ask what value of *W* produces a ratio exactly equal to the critical *t* value (t_{crit}) required to reject the null hypothesis that the conditional effect of *X* is equal to zero?

$$t_{crit} = \frac{b_1 + b_3 W}{\sqrt{s_{b_1}^2 + 2W s_{b_1 b_3}^2 + W^2 s_{b_3}^2}}$$

Isolate *W* and solve the polynomial that results. The quadratic formula finds the solutions:

$$W = \frac{-2(t_{crit}^2 s^{-}_{b_1 b_3} - b_1 b_3) \pm \sqrt{(2t_{crit}^2 s_{b_1 b_3} - 2b_1 b_3)^2 - 4(t_{crit}^2 s_{b_3}^2 - b_3^2)(t_{crit}^2 s_{b_1}^2 - b_1^2)}}{2(t_{crit}^2 s_{b_3}^2 - b_3^2)}$$

The Johnson-Neyman technique

This will produce no values, one value, or two values of *W* that are within the range of the moderator variable data.

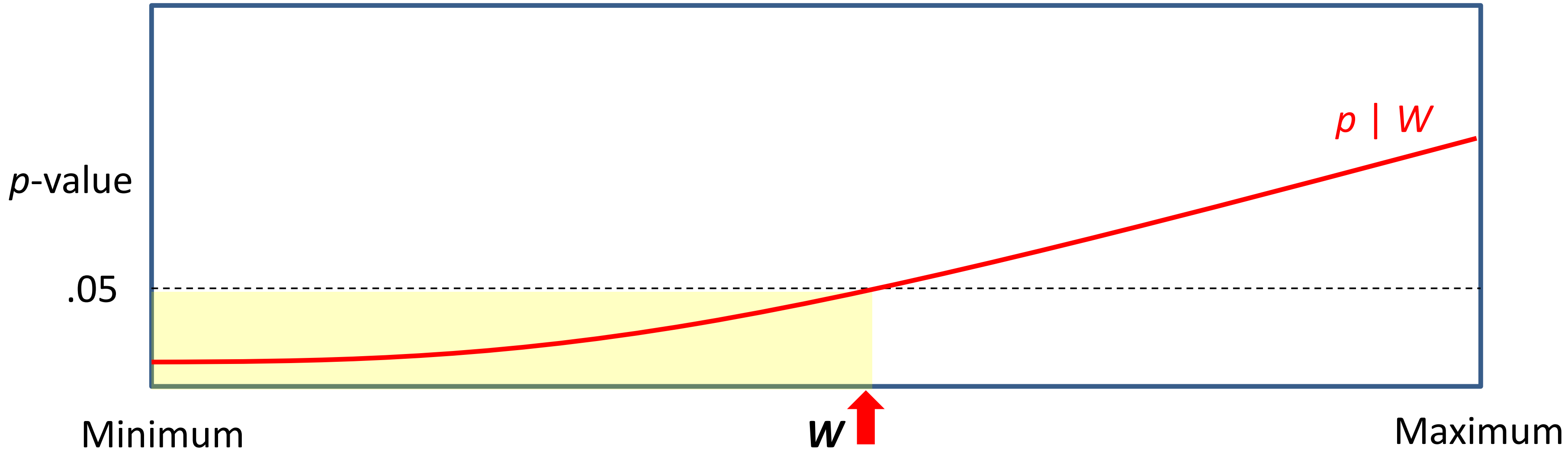
- If one value, this defines a single point of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that $p \leq .05$ for either values of the moderator (1) equal to above *W* or (2) equal to and below *W*.

- If two values, this defines the two points of transition between a statistically significant and a statistically nonsignificant conditional effect of the focal predictor, such that the conditional effect is statistically significant for either (1) values of the moderator between the two values of *W*, or (2) values of the moderator at least as large as the larger *W* and at least as small as the smaller *W*.

- If no values, that means the conditional effect is statistically significant for ALL values of the moderator within the range of the data, or it NEVER is.

We would not attempt to do this by hand

Examples of one solution



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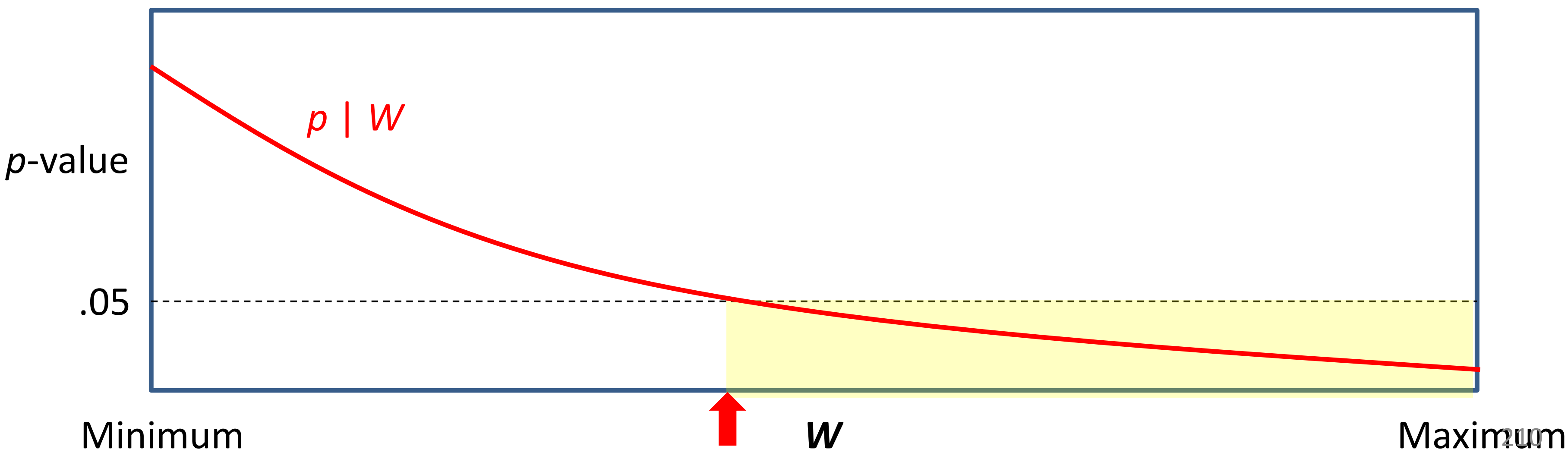
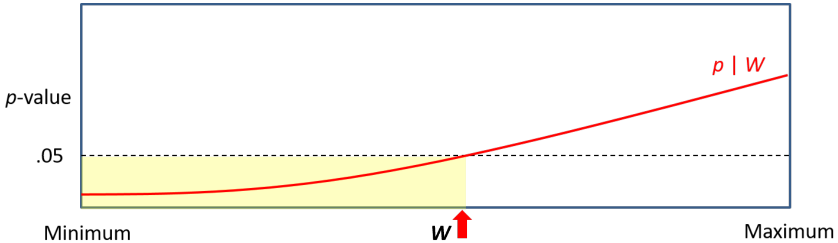
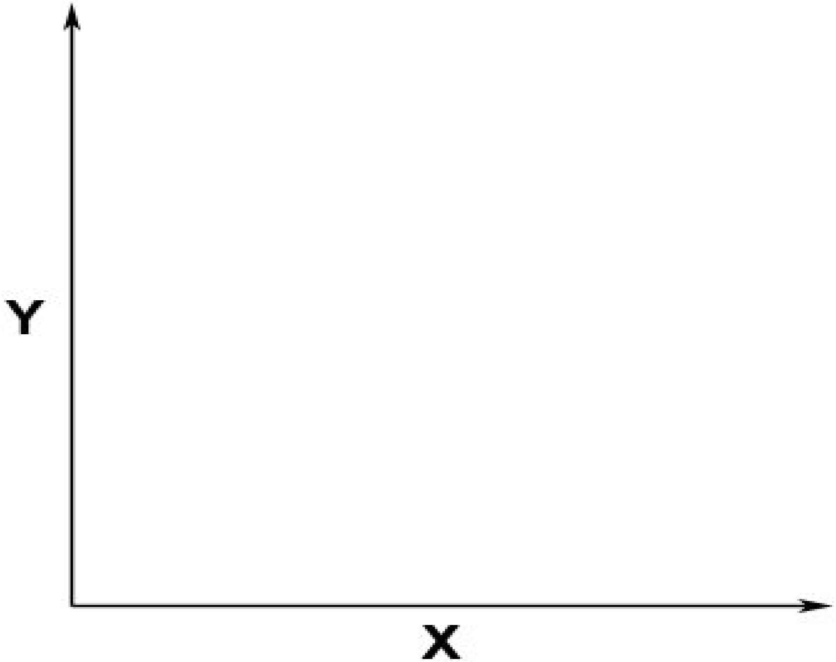
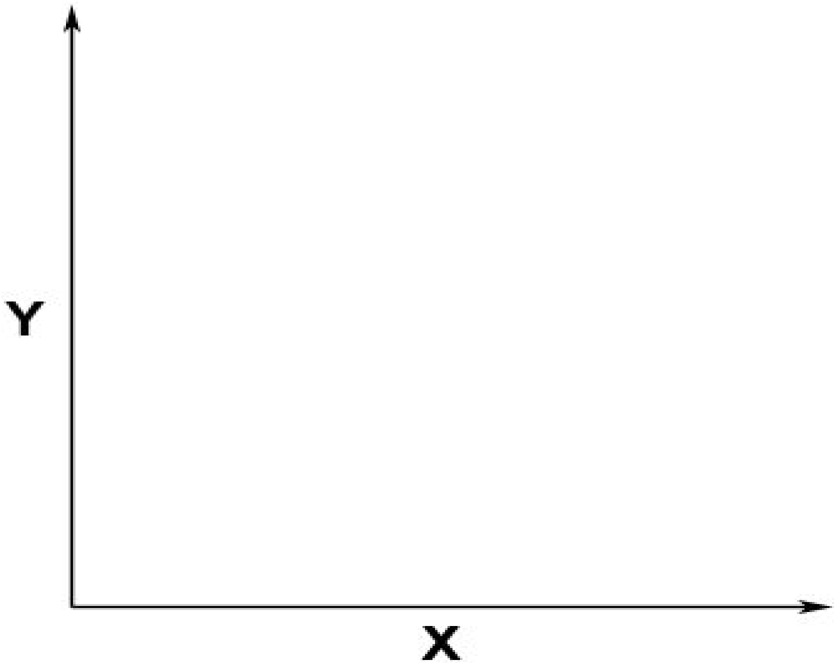
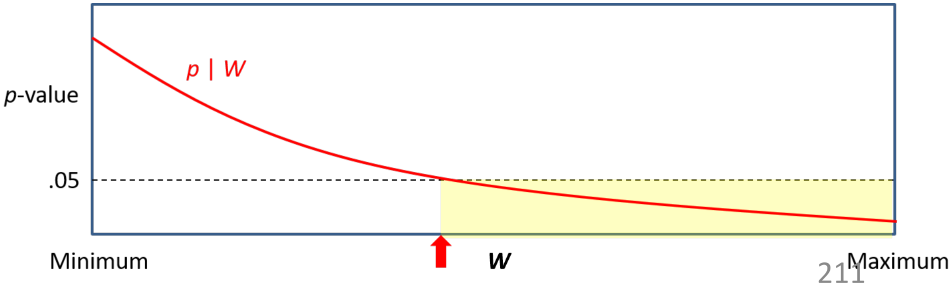


One Solution: What could graphs look like?

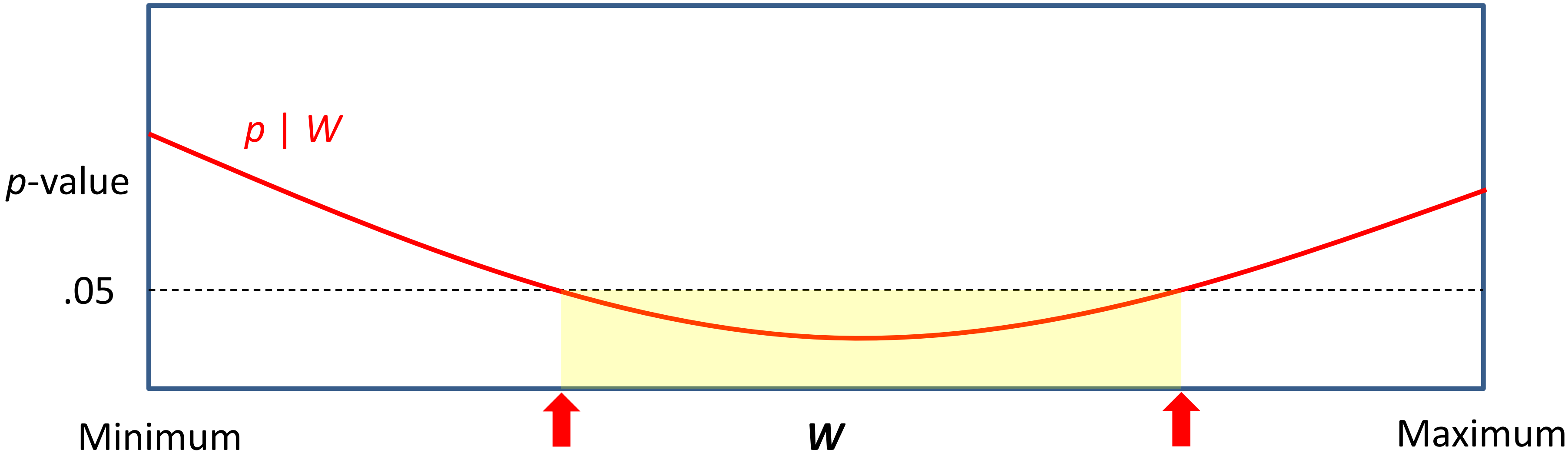








Examples of two solutions



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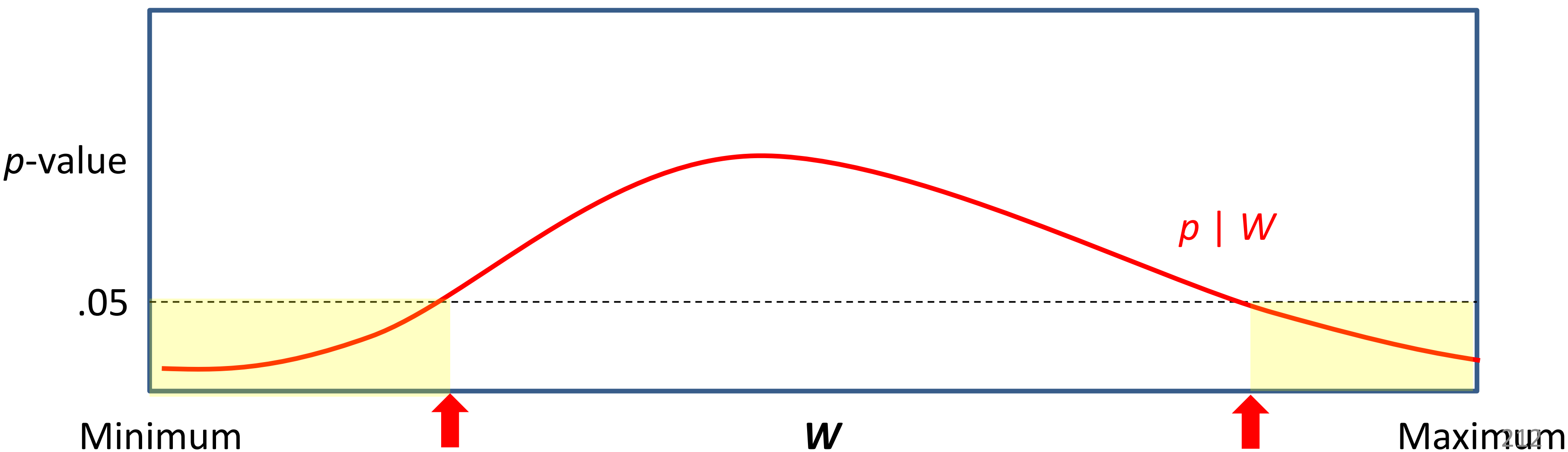
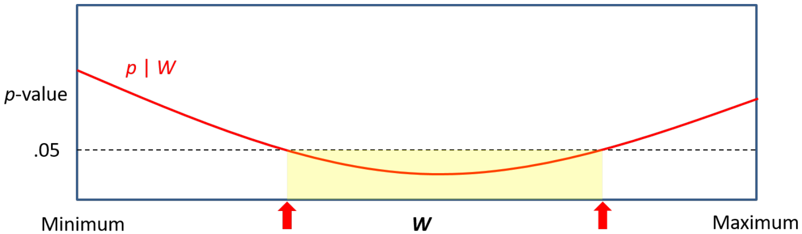
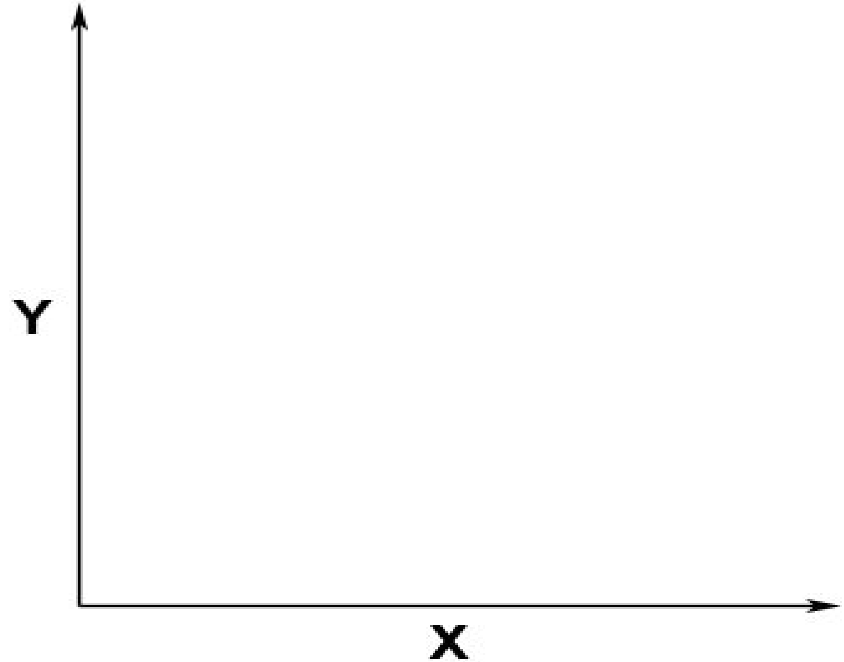
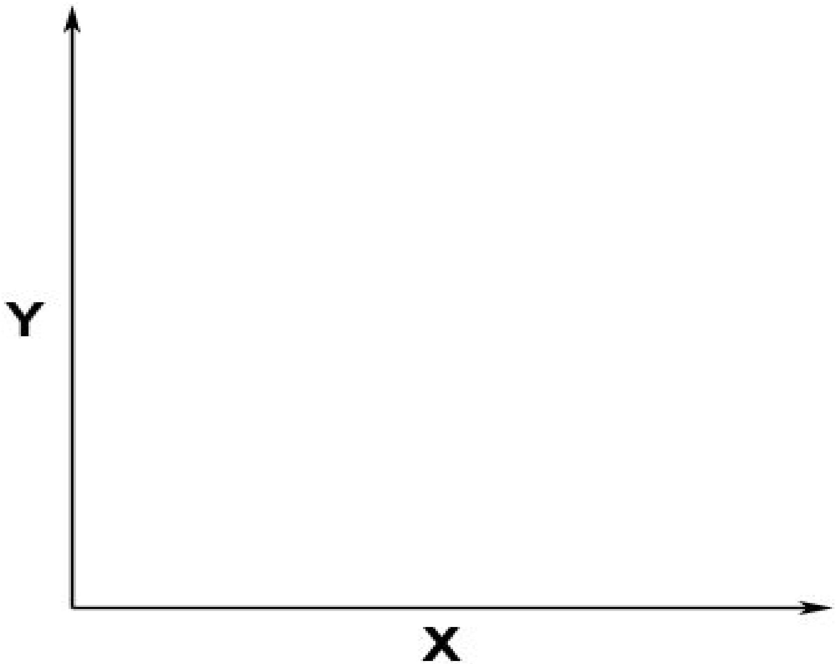
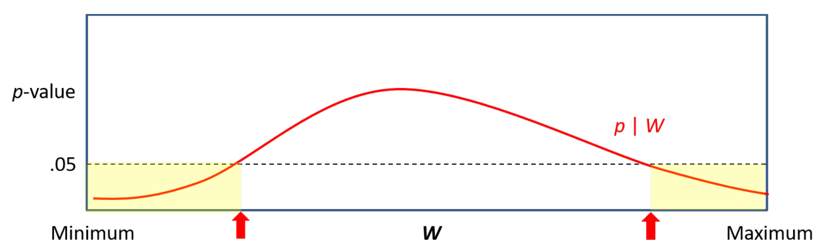


Two solution graphs

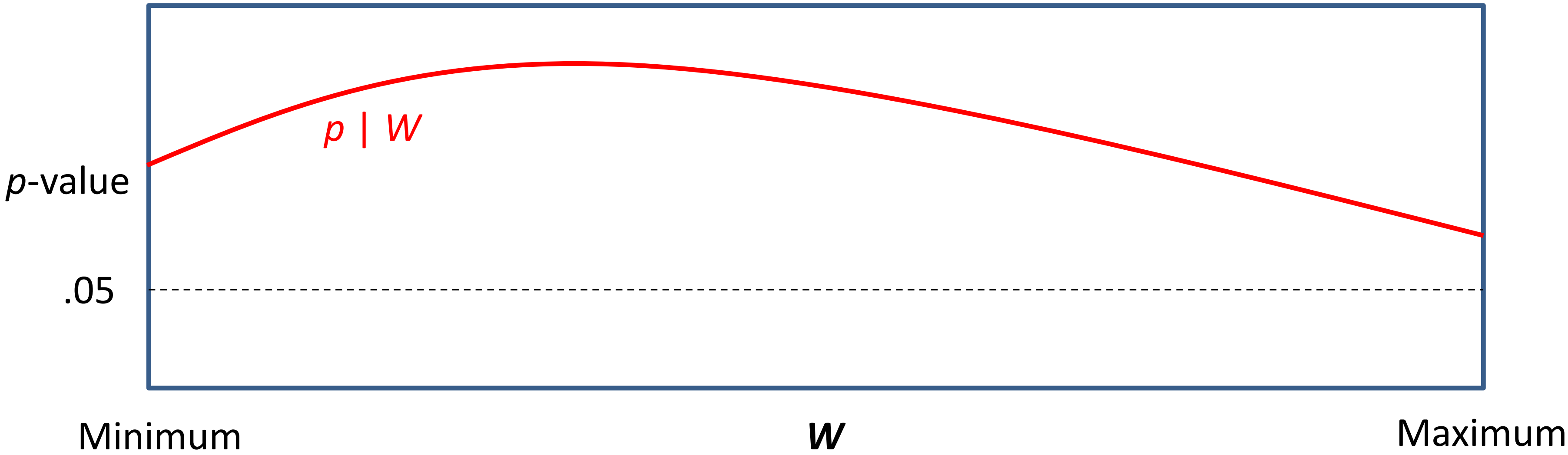








Examples of no solutions



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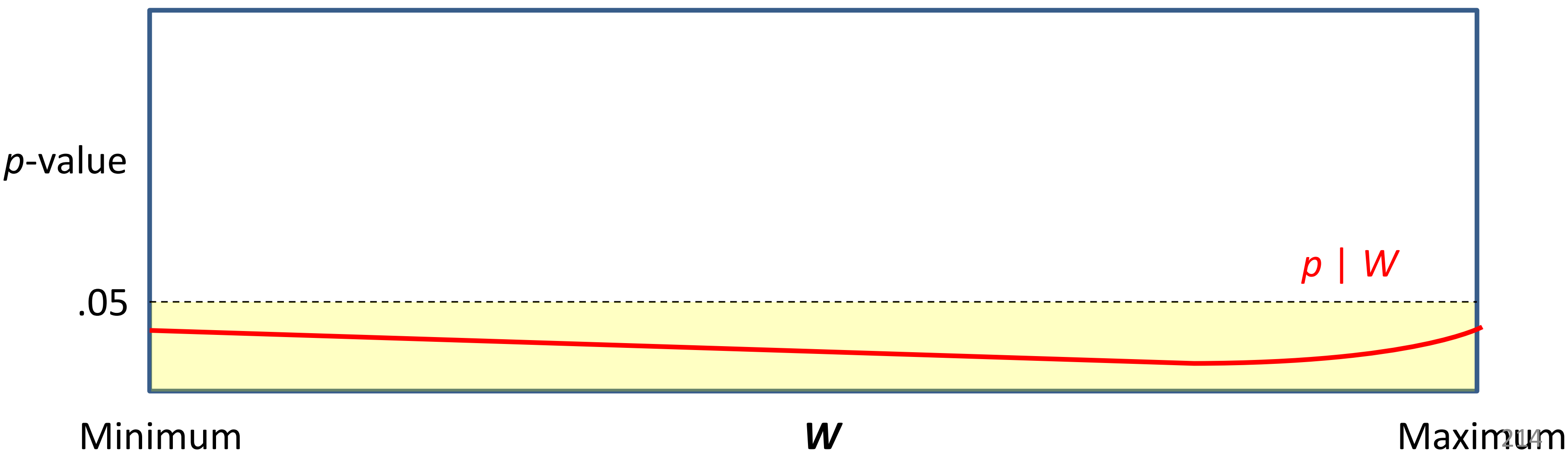
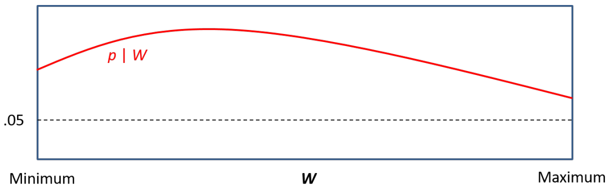
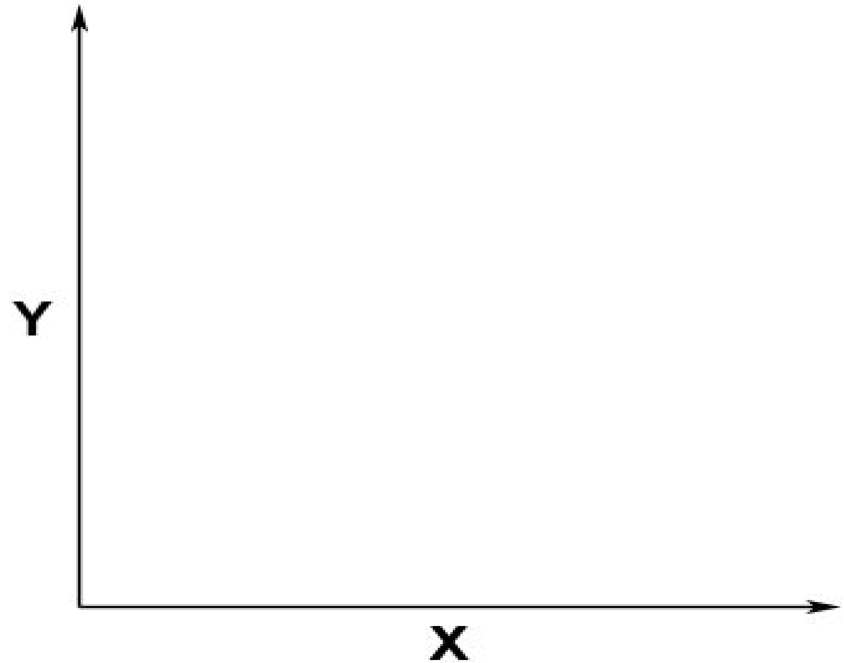
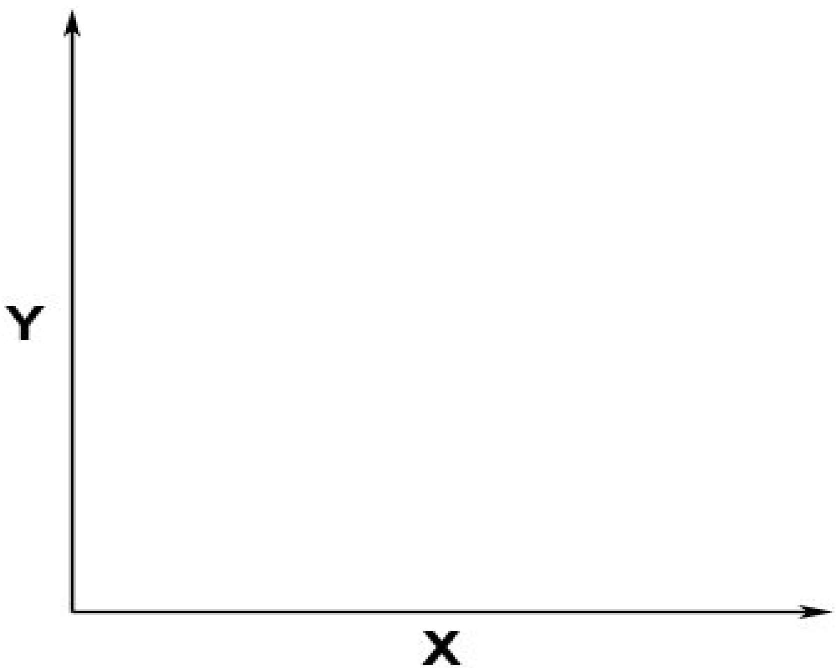
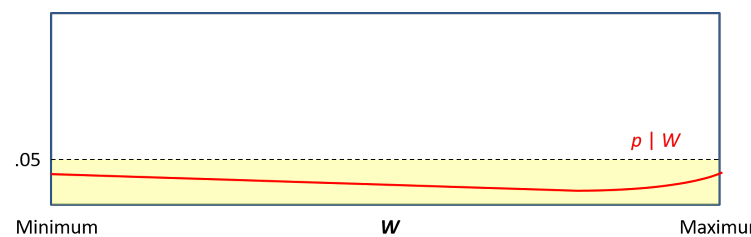


No solution graphs









MODERATION IN WITHIN-SUBJECT DESIGNS

Running Example: Group Work in Computer Science (WS)

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Honors Thesis).

Within-Subjects Version (CompSci_WS.sav, CompSci_WS.sas) :
Female participants (N = 51) read two syllabi for a different computer science classes. One of the syllabi reported the class would have group projects throughout, and the other syllabi stated that individual project would be scheduled throughout.

- Syllabi also differed in professor's name (but not gender), and the primary programming language used in the class.

Measured Variables:

- Interest in each the class `int_i` `int_g`
- `Perscom` Personal Communal Goals ($\alpha = .87$)
- `Order`
  - 1 = Group First; 2 = Individual First

Modeling Non-Contingent Relationships

When we consider non-contingent relationships in a repeated-measures design, this means the relationship between a variable (*W*) and the outcome (*Y*) is the same across conditions.

$$Y_{1i} = b_{10} + b_1 W_i + \epsilon_{1i}$$

$$Y_{2i} = b_{20} + b_1 W_i + \epsilon_{2i}$$

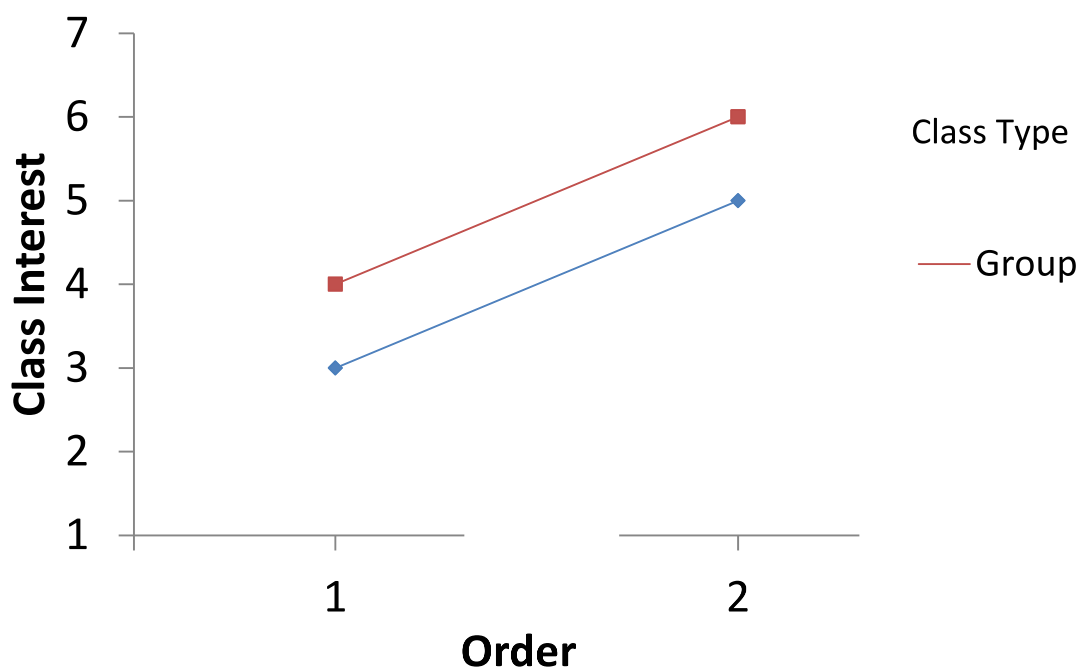
Example:

Y_1: Interest in Individual Work Class (1-7)

Y_2: Interest in Group Work Class

W: Order (1 = Group First, 2 = Individual First)

$\hat{Y}_1$	$\hat{Y}_2$	**W**
3	4	1
5	6	2



A one unit increase in order results in a 2 unit increase in interest, regardless of condition.

Modeling Contingent Relationships

What if instead we felt that the relationship between Order and Interest depends on condition? Thus the relationship between Order and Interest *differs* across the two conditions

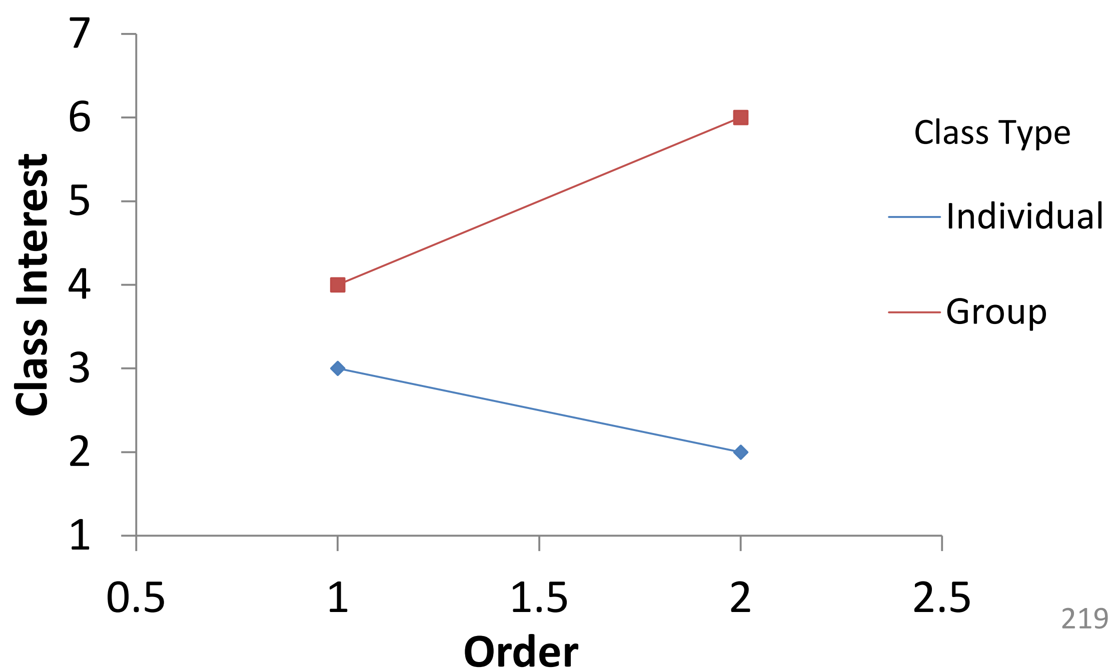
$$Y_{1\mathrm{i}} = b_{10} + b_{11}W_i + e_{1i}$$

$$Y_{2\mathrm{i}} = b_{20} + b_{21}W_i + e_{2i}$$

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})W_i + (e_{1i} - e_{2i}) = b_0 + b_1W_i + e_i$$

The difference between b_{11} and b_{21} tells us how much the relationship between *W* and *Y* differs across conditions. So b_1 tells us if there is moderation.

$\widehat{Y}_1$	$\widehat{Y}_2$	**W**
3	4	1
2	6	2



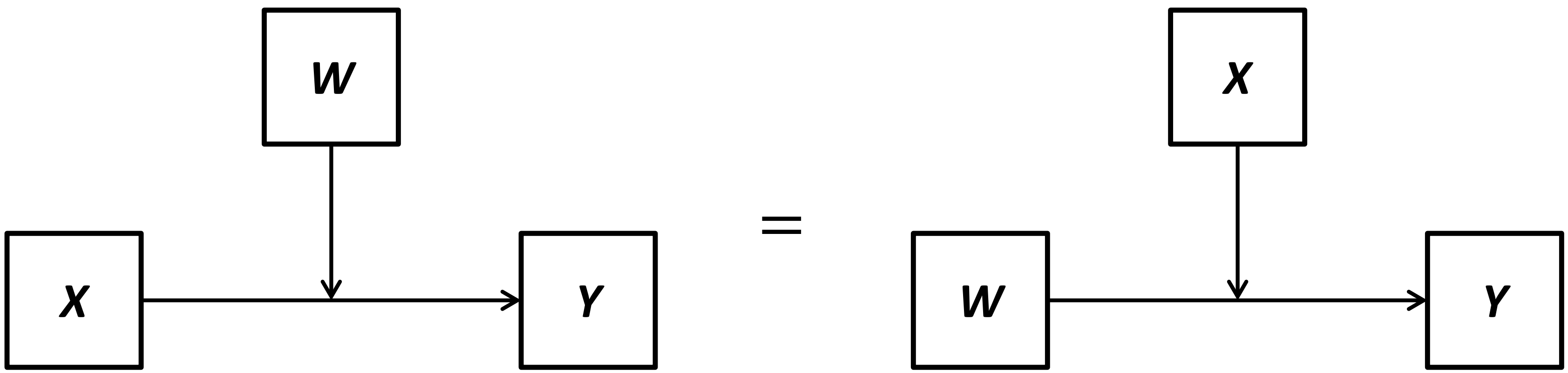
Symmetry in Within-Subjects Moderation

Does the effect of condition depend on *W*?

$$Y_{2i} - Y_{1i} = (b_{10} - b_{20}) + (b_{11} - b_{21})W_i + \ (e_{1i} - e_{2i}) = b_0 + b_1 M_i + \epsilon_i$$

$Y_{2i} - Y_{1i}$ is a quantification of the effect of condition, which means that if W predicts $Y_{2i} - Y_{1i}$ then the effect of condition depends on W.

$\boldsymbol{b_1}$ is a test of exactly that!



Judd, McClelland, and Smith (1996)

Judd, C. M., McClelland, G. H., and Smith, E. R. (1996). Testing Treatment by Covariate Interactions When Treatment Varies Within Subjects. *Psychological Methods*, *1*(4), 366-378.

Psychological Methods
1996, Vol. 1, No. 4, 366–378

1082-989X/96/$3.00

Testing Treatment by Covariate Interactions When Treatment Varies Within Subjects

Charles M. Judd and Gary H. McClelland
University of Colorado at Boulder

Eliot R. Smith
Purdue University

In contrast to the situation when an independent or treatment variable varies between subjects, procedures for testing treatment by covariate interactions are not commonly understood when the treatment varies within subjects. The purpose of this article is to identify analytic approaches that test such interactions. Two design scenarios are discussed, one in which the covariate is measured only a single time for each subject and hence varies only between subjects, and the other in which the covariate is measured at each level of the treatment variable and hence varies both within and between subjects. In each case, alternative analyses are identified and their assumptions and relative efficiencies compared.

An issue that arises with some frequency in data analysis in psychological research concerns the relationship between some measured variable and the dependent variable and whether that relationship depends on or varies across levels of a manipulated or experimental independent variable. For instance, in a clinical intervention study, we might randomly assign patients to one of two conditions, either a treatment intervention or a placebo intervention control condition. Prior to treatment, we measure a characteristic of the patients, probably focusing on the prior course and severity of their illness. Following the treatment, we assess the outcome variable of symptom severity. The primary question of interest, of course, is whether the outcome variable is affected by the manipulated treatment: Did the treatment make a difference on subsequent symptom severity? Additionally, however, we may well want to know whether the relationship between the treatment and posttreatment symptom severity depends on the patient's pretreatment course of illness. It may be, for instance, that the treatment's effect is greater for patients whose pretreatment symptoms were relatively severe. Equivalently, it may be that posttreatment symptom severity is less well predicted by pretreatment course of illness in the case of patients in the intervention condition than in the case of patients in the control condition.

The pretreatment measure of illness course is typically called a *covariate*. The analysis that is of interest is an analysis of covariance (ANCOVA), including the treatment by covariate interaction (Judd & McClelland, 1989). The two questions of interest are (a) Is there an overall treatment main effect? and (b) Is there a Treatment × Covariate interaction? If the interaction is significant, it indicates that the covariate:outcome variable relationship depends on the treatment variable. Equivalently, it suggests that the effect of the treatment on the outcome variable depends on the level of the covariate.

The analysis is readily conducted using multiple regression, making the standard assumption that errors or residuals are independently sampled from a single normally distributed population. Assume that Y_i is the outcome variable, Z_i is the covariate, and X_i is the contrast-coded (Judd & McClelland, 1989; Rosenthal & Rosnow, 1985) treatment variable. One estimates two least squares regression models:

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \varepsilon_i$$

and

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 X_i Z_i + \varepsilon_i.$$

In the first equation, β_1 represents the magnitude of

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A regression approach to considering a "cross level" interactions.

Approach is very simple:

1. Data should be a two-condition within-subjects design with a person level covariate.
2. Setup two regression equations, one for each condition
3. Take the difference between those two regression equations
4. Regression weight for person level covariate in Step 3 tests moderation.

Computer Science Within-Subjects Data Example

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Thesis).

1. Data should be a two-condition within-subjects design with a person level covariate.

Research Question: Does the degree to which <u>class order</u> predicts <u>interest</u> in computer science depend on whether the <u>class has group work or not</u>?

Or

Does effect of <u>group work</u> on <u>interest</u> in computer science classes depend on an the <u>order</u> they read the syllabi?

CompSci_WS.sav

	Subject	int_I	int_G	Order	grppref
1	300	1.50	4.00	1	6.67
2	301	2.75	3.25	1	6.33
3	325	5.75	2.50	1	2.67
4	342	3.50	5.75	1	6.00
5	349	2.25	2.00	1	4.00
6	350	1.50	1.75	1	3.67
7	305	2.50	4.25	1	4.00
8	348	6.00	1.75	1	2.33
9	318	3.00	2.00	1	4.67
10	320	4.00	5.25	1	4.00
11	332	5.00	5.00	1	3.67
12	338	2.00	1.75	1	3.00
13	310	1.00	1.75	1	3.00
14	304	1.25	4.50	2	5.67
15	306	5.75	4.50	2	4.00
16	308	3.25	4.75	2	4.00
17	315	2.75	2.25	2	4.33
18	322	5.50	2.00	2	2.33
19	343	1.75	5.25	2	6.00
20	314	4.00	5.50	2	3.00
21	319	2.25	4.00	2	5.00
22	330	4.00	6.50	2	5.67
23	334	5.00	4.50	2	3.33
24	309	5.00	3.75	2	1.00
25	329	4.75	5.25	2	4.00
26	333	1.75	5.25	2	6.33
27	336	4.50	2.25	2	3.67
28	341	1.00	3.75	2	4.33
29	302	1.75	1.75	2	4.00

Analysis using Judd et al. (1996)

2. Setup two regression equations, one for each condition

Setup a model of the outcome in each condition:

$$Y_{1\mathrm{i}} = b_{10} + b_{11}W_i + e_{1i}$$
$$Y_{2\mathrm{i}} = b_{20} + b_{21}W_i + e_{2i}$$

Is b_{11} different from b_{21}?

3. Based on these models, setup a new model where you can directly estimate and conduct inference on what you are interested in (in this case $b_{11} - b_{21}$):

$$Y_{2i} - Y_{1i} = (b_{20} - b_{10}) + (b_{21} - b_{11})W_i + (e_{2i} - e_{1i}) = b_0 + b_1W_i + e_i$$

Use simple regression to conduct inference on $b_1 = b_{11} - b_{21}$

With the data: Does the relationship between order and interest depend on group work condition?

```
regression /dep = int_diff /method = enter order.
```

```
proc reg data=CompSci_WS;model int_diff=order;run;
```

```
summary(lm(int_diff~Order, data = CompSci_WS))
```

What sign do you expect b_1 to be? **Remember:** int_diff = int_G – int_i.

Analysis using Judd et al. (1996)

4. Regression weight for person level covariate in Step 3 tests moderation.

$$Y_{2\mathrm{i}} = b_{20} + b_{21}W_i + e_{2i}$$

$$Y_{1\mathrm{i}} = b_{10} + b_{11}W_i + e_{1i}$$

$$Y_{2i} - Y_{1i} = (b_{20} - b_{10}) + (b_{21} - b_{11})W_i + \ (e_{2i} - e_{1i}) = b_0 + b_1W_i + e_i$$

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-1.476	.880		-1.676	.100
	Order	1.193	.541	.300	2.205	.032

a. Dependent Variable: int_diff

What does it mean that b_1 is positive?

$$b_1 = b_{21} - b_{11} = 1.193$$

$$b_{21} > b_{11}$$

Practically, this means that the relationship between order and interest is significantly stronger (more positive) in the group work condition.

```
regression /dep = int_diff /method = enter order.
```

```
proc reg data=CompSci_WS;model int_diff=order;run;
```

```
summary(lm(int_diff~Order, data = CompSci_WS))
```

Interpreting the Coefficients

$$Y_{2i} - Y_{1i} = (b_{20} - b_{10}) + (b_{21} - b_{11})W_i + \ (e_{2i} - e_{1i}) = b_0 + b_1 W_i + e_i$$

b_0 is the expected difference in *Y* when *W* = 0

We can think of this as the effect of "condition" on *Y* when *W* is zero.

In the Computer Science example, W can only be 1 or 2, so we do not interpret this parameter in this case.

b_1 is the degree to which the relationship between *W* and *Y* differs by condition.

Alternatively: the degree to which the effect of condition on *Y* depends on *W*. *i.e.,* if W increases by one unit the effect of condition on *Y* will increase by b_1 units

Conditional Effects in Within-Subjects Moderation

$$Y_{2i} - Y_{1i} = (b_{20} - b_{10}) + (b_{21} - b_{11})W_i + \ (e_{2i} - e_{1i}) = b_0 + b_1 W_i + e_i$$

Given a value of *W* what is the effect of condition on the outcome?

$Y_{2i} - Y_{1i}$ is a quantification of the effect of condition, which means that the conditional effect of condition $\theta_{X \to Y}(W) = b_0 + b_1 W$

Given a specific condition what is the effect of *W* on the outcome?

$$Y_{1\text{i}} = b_{10} + b_{11} W_i + e_{1i}$$

$$Y_{2\text{i}} = b_{20} + b_{21} W_i + e_{2i}$$

$$\theta_{W \to Y}(X) = b_{x1}$$

Conditional effects will become important when it comes to probing

Probing an Effect of Condition on Outcome: The "Pick-a-Point" Approach

$$\theta_{X \rightarrow Y}(W) = b_0 + b_1 W$$

Select a value of the moderator (*W*) at which you'd like to have an estimate of the condition's effect on *Y*. Then derive its standard error. The ratio of the effect to its standard error is distributed as $t(df_{residual})$ under the null hypothesis that the effect of condition is zero at that moderator value.

The estimated standard error of $\theta_{X \rightarrow Y}(W)$ is

$$s_{\theta_{X \rightarrow Y}(W)} = \sqrt{(s_{b_0}^2 + 2W s_{b_0 b_1} + W^2 s_{b_1}^2)}$$

Squared standard error of b_0

Covariance of b_0 and b_1

Squared standard error of b_1

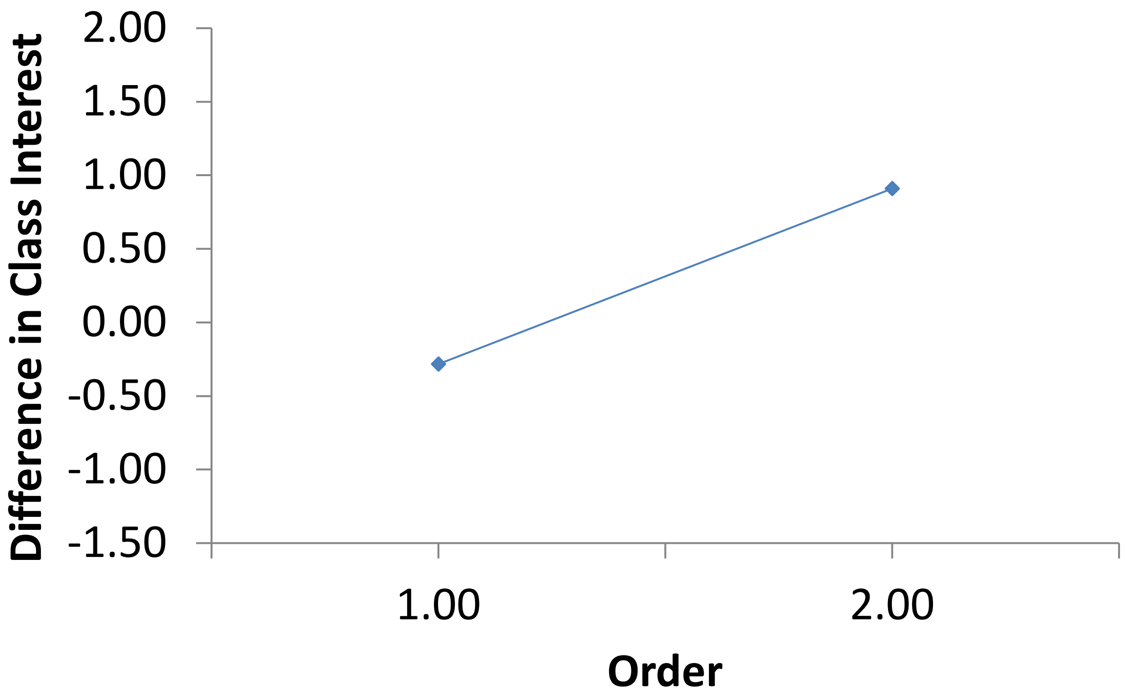
Probing an Effect of Condition on Outcome: The "Pick-a-Point" Approach

You must choose the points along the moderator to "probe" the effect of condition on Y.

Let's look at an example with our computer science data:

$$Y_{Di} = -1.476 + 1.193W_i$$

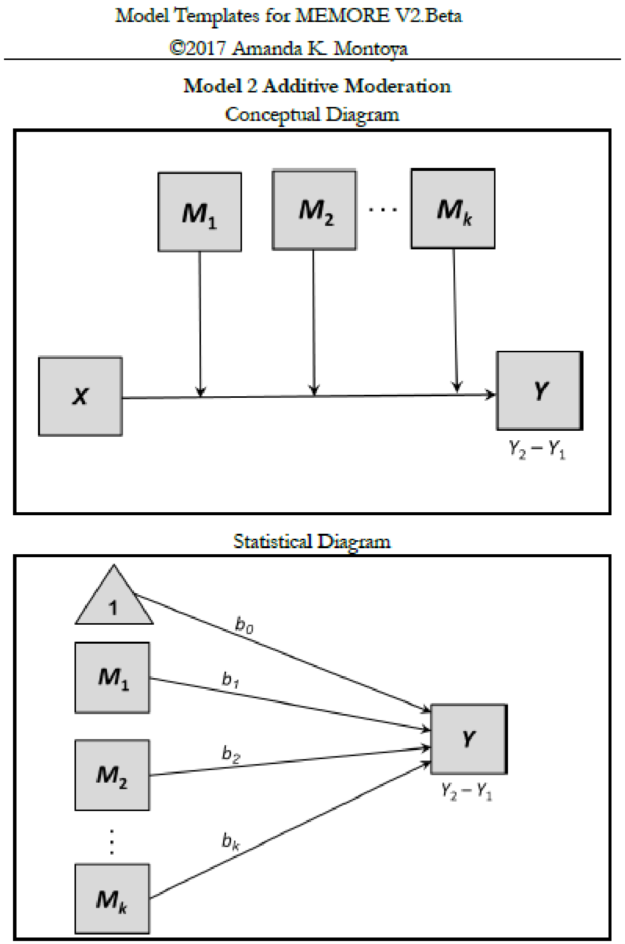
W	$\theta_{X \to Y \mid W}$	$s_{\theta_{X \to Y \mid W}}$	*p*
1	-.2826	0.4010	.4843
2	.9107	.3634	.0156



Participants who saw the group work class first did not show a difference in interest between the two classes. However, those who saw the individual work class first showed a larger effect of condition such students were significantly more interested in the group class.

MEMORE

We can use MEMORE to estimate and probe this model.




```
MEMORE w = order /y = int_G int_I
/model = 2 /plot = 1.
```

```
%memore(w=order,y = int_G int_I,
model = 2, plot = 1, data =
CompSci_WS);
```

- List moderator(s) in the `w` list
- List outcomes in the `y` list
- Can use `model 2` or `model 3` when you have 1 moderator there is no difference.
- `PLOT` option calls a table of values for making a nice plot.

Using MEMORE for CASC WS data

```
MEMORE w = order /y = int_G int_I /model = 2 /plot = 1.
```

```
%memore(w=order,y = int_G int_I, model = 2, plot = 1, data =
CompSci_WS);
```

```
********************* MEMORE Procedure for SPSS Version 2.1 ***********************

                         Written by Amanda Montoya

                  Documentation available at akmontoya.com

**************************************************************************************

Model:
  2

Variables:
Y = int_G    int_I
W = Order

Computed Variables:
Ydiff =        int_G      -       int_I

Sample Size:
  51
```

First part of output repeats what you told MEMORE to do. Always double check that this is correct!

I double checked to make sure the order of subtraction was the same as when we did this by hand.

Using MEMORE for CASC WS data

```
MEMORE w = order /y = int_G int_I /model = 2 /plot = 1.
```

```
%memore(w=order,y = int_G int_I, model = 2, plot = 1, data =
CompSci_WS);
```

```
Outcome: Ydiff =  int_G      -       int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3005      .0903     3.6978     4.8629     1.0000    49.0000      .0322

Model
              coeff         SE          t          p       LLCI       ULCI
constant    -1.4759      .8804    -1.6764      .1000    -3.2452      .2934
Order        1.1933      .5411     2.2052      .0322      .1058     2.2808

Degrees of freedom for all regression coefficient estimates:
  49
```

Regression results are the same as when we did this using `regression` command

Using MEMORE for CASC WS data

```
MEMORE w = order /y = int_G int_I /model = 2 /plot = 1.
```

```
%memore(w=order,y = int_G int_I, model = 2, plot = 1, data =
CompSci_WS);
```

Probing effect of condition on outcome at different values of the moderator

```
Conditional Effect of 'X' on Y at values of moderator(s)
      Order     Effect        SE         t         p      LLCI      ULCI
     1.0000     -.2826     .4010    -.7048     .4843   -1.0884     .5232
     2.0000      .9107     .3634    2.5061     .0156     .1804    1.6410

Degrees of freedom for all conditional effects:
  49

Values for quantitative moderators are the mean and plus/minus one SD from the mean.

Values for dichotomous moderators are the two values of the moderator.
```

Using MEMORE for CASC WS data

```
MEMORE w = order /y = int_G int_I /model = 2 /plot = 1.
```

```
%memore(w=order,y = int_G int_I, model = 2, plot = 1, data = CompSci_WS);
```

```
Conditional Effect of Moderator(s) on Y in each Condition

Condition 1 Outcome:
 int_G

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .3766      .1418     1.9306     8.0962     1.0000    49.0000      .0065

Model
              coeff         SE          t          p       LLCI       ULCI
constant     2.0070      .6362     3.1548      .0027      .7285     3.2854
Order        1.1126      .3910     2.8454      .0065      .3268     1.8984

Degrees of freedom for all conditional effects:
  49

----------------------------------------------------------------------------------

Condition 2 Outcome:
 int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .0281      .0008     2.1189      .0389     1.0000    49.0000      .8446

Model
              coeff         SE          t          p       LLCI       ULCI
constant     3.4829      .6665     5.2260      .0000     2.1436     4.8222
Order        -.0807      .4096     -.1971      .8446     -.9039      .7425

Degrees of freedom for all conditional effects:
  49

**********************************************************************************
```

Order positively predicts interest in **class with group work**

and does not significantly predict interest in **class with individual work**.

Using MEMORE for CASC WS data

```
MEMORE w = order /y = int_G int_I /model = 2 /plot = 1.
```

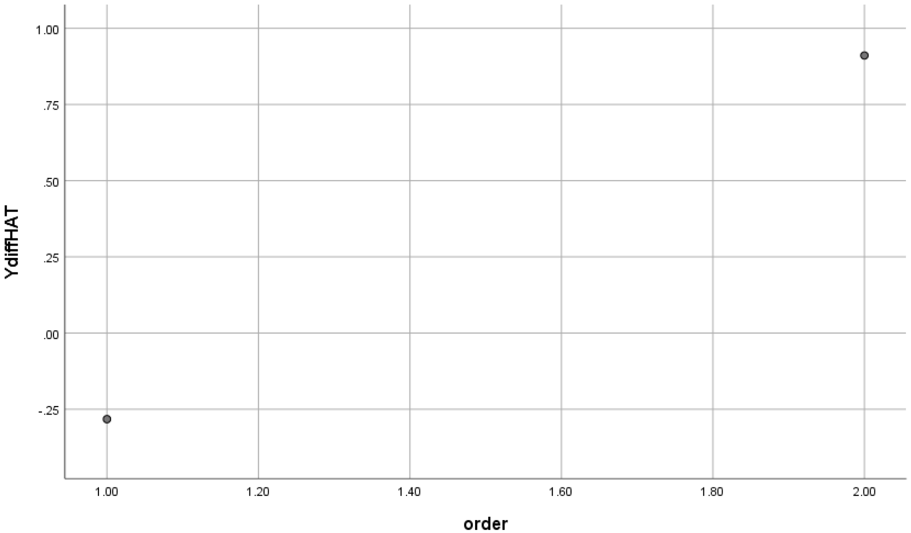
```
%memore(w=order,y = int_G int_I, model = 2, plot = 1, data =
CompSci_WS);
```

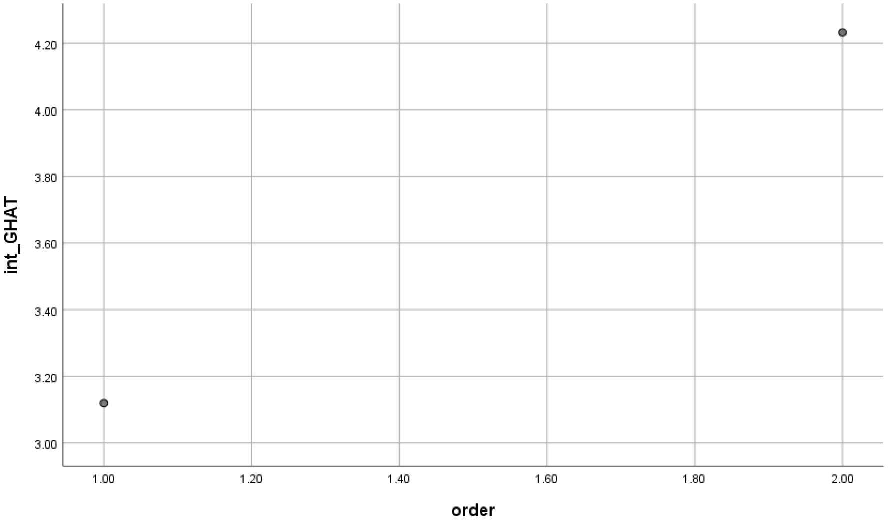
```
Data for visualizing conditional effect of X on Y.
Paste text below into a SPSS syntax window and execute to produce plot.

DATA LIST FREE/order YdiffHAT int_GHAT int_IHAT.

BEGIN DATA.

     1.0000     -.2826     3.1196     3.4022
     2.0000      .9107     4.2321     3.3214

END DATA.

GRAPH/SCATTERPLOT = order WITH YdiffHAT.
GRAPH/SCATTERPLOT = order WITH int_GHAT.
GRAPH/SCATTERPLOT = order WITH int_IHAT.
```

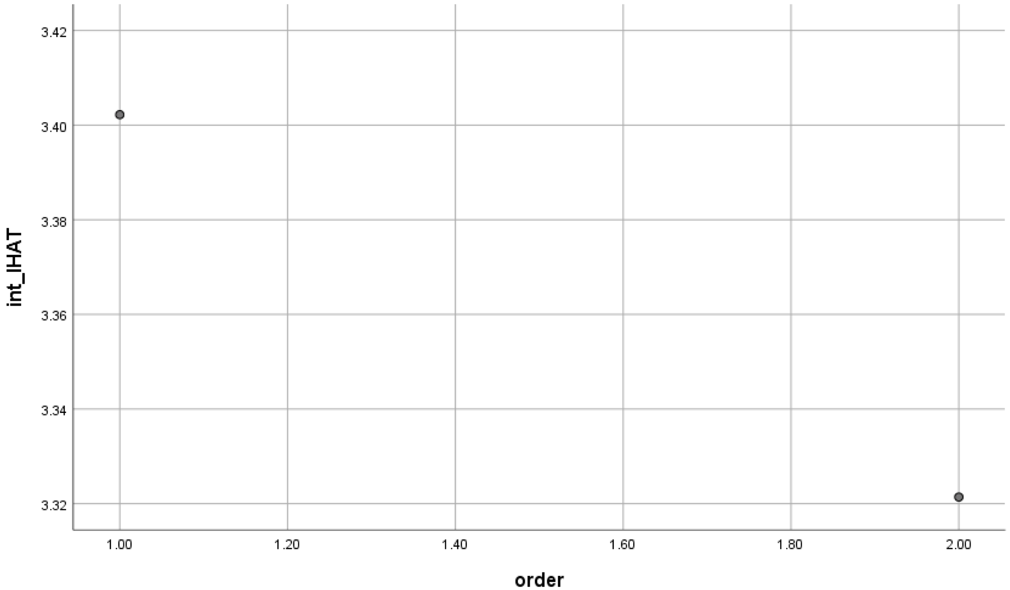
Code for plotting. You'll get three plots each with the moderator on the *X* axis and a different outcome on the *Y* axis.

1) Predicted Differences between *Y*'s
2) Predicted *Y* from first condition
3) Predicted *Y* from second condition

Graphs







You can snaz these up in SPSS or export the data to something more fit for preparing graphs (e.g., R or Excel)

Summarizing

Tips:

- Interpret the sign and the magnitude of the interaction coefficient with respect to *X*'s effect on *Y* (or *M*'s effect on *Y*; or both).
- Provide probing results with interpretations
- Read the write ups of other's moderation analyses
- Provide a graphical representation of the effect of interest (like the ones we've done)

Does the effect of group work on interest in a computer science class depend on order of syllabus presentation?

Overall, the impact of including group work in a computer science class on interest in the class depends on the order that students read the syllabus ($b_1 = 1.19$, $p = .001$). Among those who read the individual work syllabus first, we observed a 1.19 unit larger difference between interest in group work and interest in individual work classes. Among those who read the group work syllabus first, they did not significantly differ on their interest in the two classes ($\theta_{X\rightarrow Y|W} = -.283, p = .48$). But among those who read the individual work syllabus first, they were significantly more interested in the group work class ($\theta_{X\rightarrow Y|W} = .9107, p = .0156$). Considering the interaction another way, this result shows that order predicts interest differently across the conditions. Those who read the individual work syllabus first were significantly higher on interest in the group work class than those who read the group work syllabus first ($\theta_{W\rightarrow Y|X} = 1.1126, p = .0065$); whereas, order did not significantly predict interest in the individual work class ($\theta_{W\rightarrow Y|X} = -.0807, p = .8446$). Overall, this suggests that there may be some unique aspect of reading about the individual work class first, and then the group work class which is driving differences in interest between the two conditions. It is worth considering whether it is ecologically valid to rely on order of presentation occurring in one way versus another, and leads to many limitations of the utility of introducing group work into computer science classes as an effective method for recruiting and retaining women.

Computer Science Within-Subjects Data Example

Montoya, A. K. (2013) Increasing Interest in Computer Science thought Group Work: A Goal Congruity Approach (Undergraduate Thesis).

1. Data should be a two-condition within-subjects design with a person level covariate.

Research Question: Does the degree to which <u>preference for group work</u> predicts <u>interest</u> in computer science depend on whether or not the class has <u>group work</u>?

Or

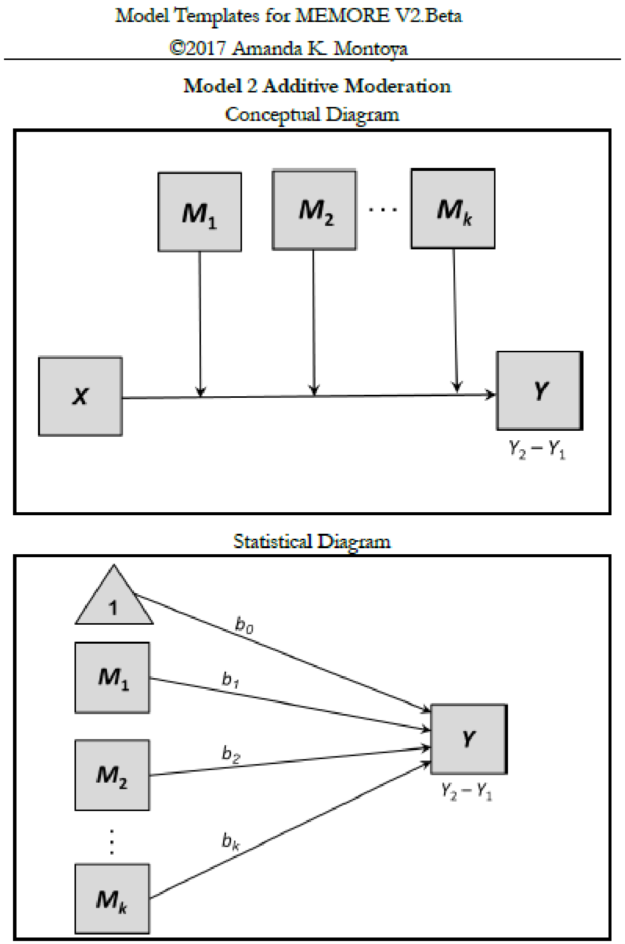
Does effect of <u>group work</u> on <u>interest</u> in computer science classes depend on an individual's <u>preference for group work</u>?

CompSci_WS.sav

Subject	int_I	int_G	grppref
300	1.50	4.00	6.67
301	2.75	3.25	6.33
325	5.75	2.50	2.67
342	3.50	5.75	6.00
349	2.25	2.00	4.00
350	1.50	1.75	3.67
305	2.50	4.25	4.00
348	6.00	1.75	2.33
318	3.00	2.00	4.67
320	4.00	5.25	4.00
332	5.00	5.00	3.67
338	2.00	1.75	3.00
310	1.00	1.75	3.00
304	1.25	4.50	5.67
306	5.75	4.50	4.00
308	3.25	4.75	4.00
315	2.75	2.25	4.33
322	5.50	2.00	2.33
343	1.75	5.25	6.00
314	4.00	5.50	3.00
319	2.25	4.00	5.00

MEMORE

We can use MEMORE to estimate and probe this model.




```
MEMORE w = grppref /y = int_G int_I
/model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model =
2, jn = 1, plot = 1, data = CompSci_WS);
```

- List moderator(s) in the `w` list
- List outcomes in the `y` list
- Can use `model 2` or `model 3` when you have 1 moderator there is no difference.
- `JN` option calls the Johnson-Neyman technique
- `PLOT` option calls a table of values for making a nice plot.

Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

```
********************* MEMORE Procedure for SPSS Version 2.1 ***********************

                           Written by Amanda Montoya

                    Documentation available at akmontoya.com

**************************************************************************************

Model:
  2

Variables:
Y = int_G    int_I
W = grppref

Computed Variables:
Ydiff =          int_G     -        int_I

Sample Size:
  51
```

First part of output repeats what you told MEMORE to do. Always double check that this is correct!

I double checked to make sure the order of subtraction was the same as when we did this by hand.

Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

```
**************************************************************************
Outcome: Ydiff =  int_G      -        int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .6741      .4544     2.2178    40.8067     1.0000    49.0000      .0000

Model
              coeff         SE          t          p       LLCI       ULCI
constant    -3.5500      .6485    -5.4742      .0000    -4.8532    -2.2468
grppref       .9936      .1555     6.3880      .0000      .6810     1.3062

Degrees of freedom for all regression coefficient estimates:
  49
```

Strong evidence for moderation, where as preference for group work increases, the difference between interest in the two classes increases.

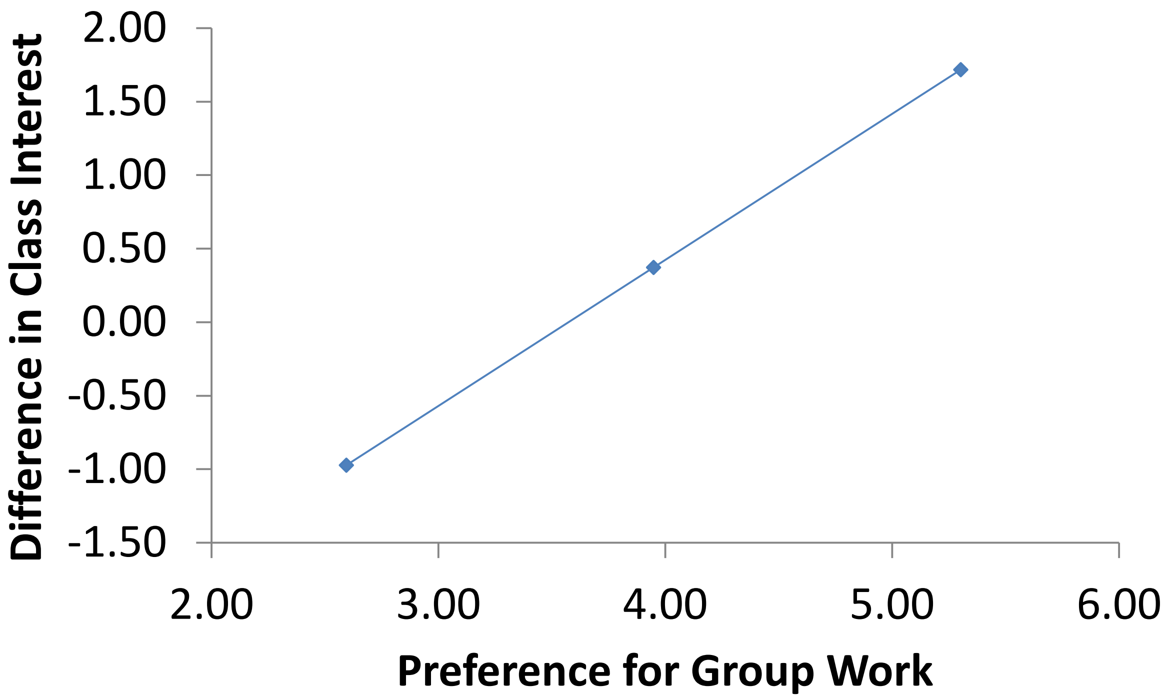
Probing an Effect of Condition on Outcome: The "Pick-a-Point" Approach

You must choose the points along the moderator to "probe" the effect of condition on *Y*.

Let's look at an example with our computer science data:

$$Y_{Di} = -3.55 + .99W_i$$

W	$\theta_{X \to Y \mid W}$	$s_{\theta_{X \to Y \mid W}}$	*p*
2.59	-0.97	0.30	0.00
3.95	0.37	0.21	0.08
5.30	1.72	0.30	0.00



Participants relatively low in preference for group work are more interested in the individual work class, and those high in preference for group work are more interested in the class with group work.

Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

Probing effect of condition on outcome at different values of the moderator

```
**************************************************************************************

Conditional Effect of 'X' on Y at values of moderator(s)
    grppref     Effect        SE         t         p      LLCI      ULCI
     2.5938     -.9728     .2964   -3.2823     .0019   -1.5684    -.3772
     3.9478      .3725     .2085    1.7865     .0802    -.0465     .7916
     5.3019     1.7179     .2964    5.7963     .0000    1.1223    2.3135

Degrees of freedom for all conditional effects:
  49

Values for quantitative moderators are the mean and plus/minus one SD from the mean.
```

This is the default. You can change this to the 10th, 25th, 50th, 75th, and 90th quantiles by adding `quantile =1` to the command line

The Johnson-Neyman Technique

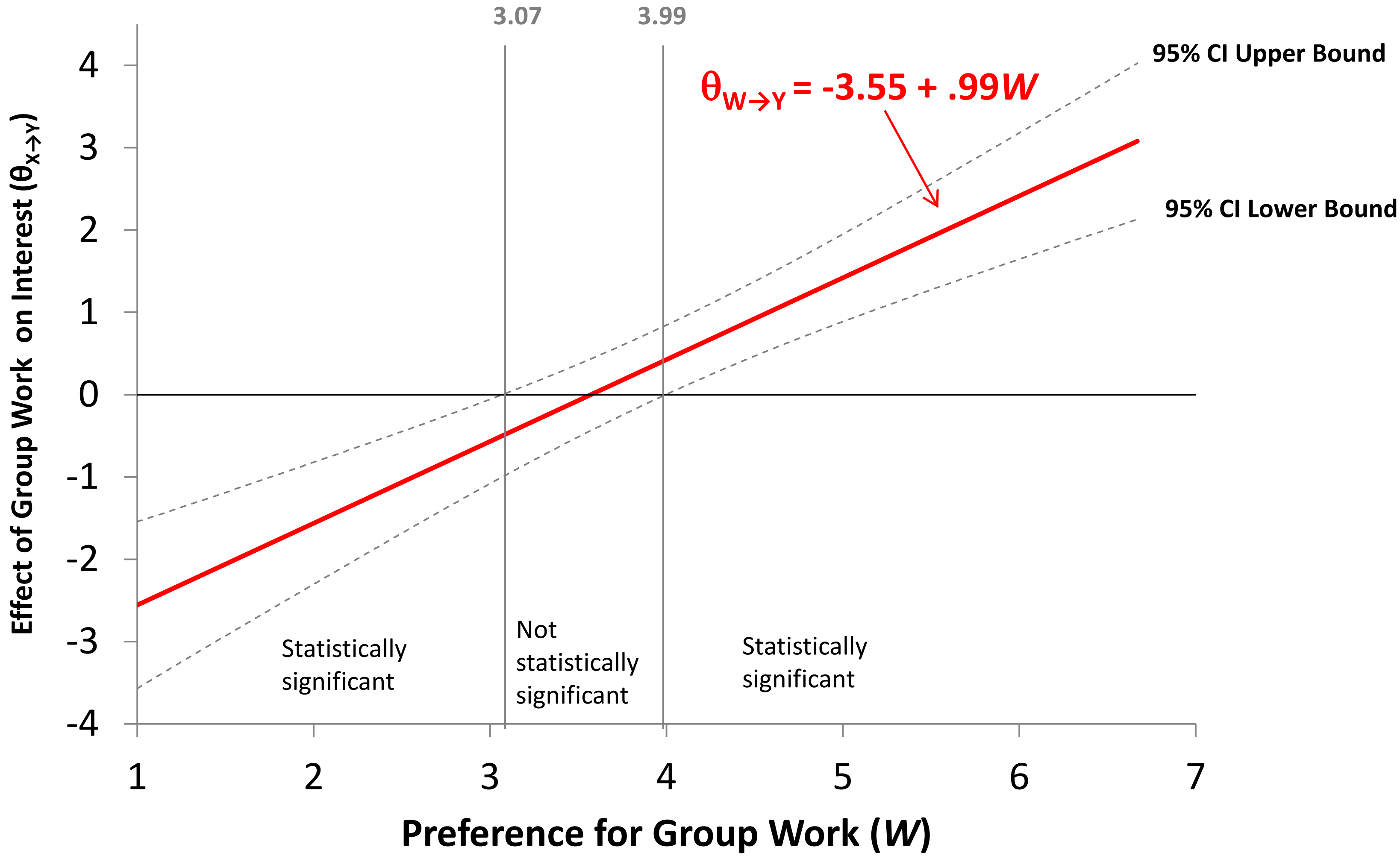
The Johnson-Neyman technique seeks to find the value or values of the moderator (*W*) within the data, if they exist, such that the *p*-value for the conditional effect of condition at that value or those values of *W* is exactly equal to some chosen level of significance α. Thus, no need to select values of *W* in advance.

To do so, we ask what value of *W* produces a ratio of $\theta_{X \rightarrow Y}(W)$ to its standard error exactly equal to the critical *t* value (t_{crit}) required to reject the null hypothesis that $\theta_{X \rightarrow Y}(W)$ is equal to zero at that value of *W*?

$$t_{crit} = \frac{b_0 + b_1 W}{\sqrt{s_{b_0}^2 + 2W s_{b_0 b_1} + W^2 s_{b_1}^2}}$$

Isolating *W* yields to the solution in the form of a quadratic equation which always has two roots, though not always two that are interpretable.

A Plot of the "Region of Significance"



Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

****************************** JOHNSON-NEYMAN PROCEDURE *******************************

Moderator value(s) defining Johnson-Neyman significance region(s) and percent of observed data above value:

Value	% Abv
3.0685	72.5490
3.9949	54.9020

Conditional Effect of 'X' on Y at values of moderator

grppref	Effect	SE	t	p	LLCI	ULCI
1.0000	-2.5564	.5037	-5.0752	.0000	-3.5687	-1.5442
1.2984	-2.2599	.4619	-4.8931	.0000	-3.1880	-1.3318
1.5968	-1.9634	.4210	-4.6641	.0000	-2.8094	-1.1174
1.8953	-1.6669	.3813	-4.3712	.0001	-2.4332	-.9006
2.1937	-1.3704	.3434	-3.9905	.0002	-2.0605	-.6803
2.4921	-1.0739	.3078	-3.4886	.0010	-1.6925	-.4553
2.7905	-.7774	.2755	-2.8218	.0069	-1.3310	-.2238
3.0685	-.5012	.2494	-2.0096	.0500	-1.0023	.0000
3.0889	-.4808	.2477	-1.9416	.0579	-.9785	.0168
3.3874	-.1843	.2260	-.8156	.4187	-.6385	.2699
3.6858	.1122	.2125	.5279	.5999	-.3148	.5392
3.9842	.4087	.2086	1.9591	.0558	-.0105	.8279
3.9949	.4193	.2087	2.0096	.0500	.0000	.8387
4.2826	.7052	.2149	3.2809	.0019	.2733	1.1371
4.5811	1.0017	.2306	4.3435	.0001	.5382	1.4652
4.8795	1.2982	.2539	5.1124	.0000	.7879	1.8085
5.1779	1.5947	.2830	5.6350	.0000	1.0260	2.1634
5.4763	1.8912	.3162	5.9804	.0000	1.2557	2.5267
5.7747	2.1877	.3525	6.2070	.0000	1.4794	2.8961
6.0732	2.4843	.3909	6.3560	.0000	1.6988	3.2697
6.3716	2.7808	.4308	6.4546	.0000	1.9150	3.6465
6.6700	3.0773	.4720	6.5200	.0000	2.1288	4.0258

Degrees of freedom for all conditional effects:
49

This will only print when we include jn =1 in the command line. JN technique does not work for multiple moderators.

Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

```
Conditional Effect of Moderator(s) on Y in each Condition

Condition 1 Outcome:
 int_G

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4488      .2014     1.7964    12.3612     1.0000    49.0000      .0010

Model
              coeff         SE          t          p       LLCI       ULCI
constant     1.7874      .5836     3.0624      .0036      .6145     2.9603
grppref       .4922      .1400     3.5158      .0010      .2109      .7735

Degrees of freedom for all conditional effects:
  49

------------------------------------------------------------------------------------

Condition 2 Outcome:
 int_I

Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4710      .2218     1.6502    13.9671     1.0000    49.0000      .0005

Model
              coeff         SE          t          p       LLCI       ULCI
constant     5.3374      .5594     9.5415      .0000     4.2132     6.4615
grppref      -.5014      .1342    -3.7373      .0005     -.7710     -.2318

Degrees of freedom for all conditional effects:
  49
```

Preference for group work positively predicts interest in **class with group work**

and negatively predicts interest in **class with individual work**.

Using MEMORE for CASC WS data

```
MEMORE w = grppref /y = int_G int_I /model = 2/jn = 1 /plot = 1.
```

```
%memore(w=grppref,y = int_G int_I, model = 2, jn = 1, plot = 1,
data = CompSci_WS);
```

```
*************************************************************************************

Data for visualizing conditional effect of X on Y.
Paste text below into a SPSS syntax window and execute to produce plot.

DATA LIST FREE/grppref YdiffHAT int_GHAT int_IHAT.

BEGIN DATA.

     2.5938     -.9728     3.0640     4.0368
     3.9478      .3725     3.7304     3.3578
     5.3019     1.7179     4.3968     2.6789

END DATA.

GRAPH/SCATTERPLOT = grppref WITH YdiffHAT.
GRAPH/SCATTERPLOT = grppref WITH int_GHAT.
GRAPH/SCATTERPLOT = grppref WITH int_IHAT.
```

Code for plotting. You'll get three plots each with the moderator on the *X* axis and a different outcome on the *Y* axis.

1) Predicted Differences between *Y*'s
2) Predicted *Y* from first condition
3) Predicted *Y* from second condition

Writing up a Moderation Analysis

Tips:

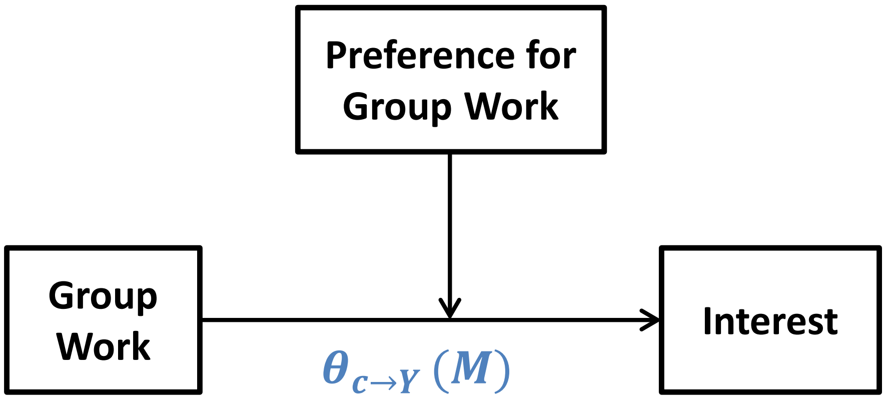
- Interpret the sign and the magnitude of the interaction coefficient with respect to *X*'s effect on *Y* (or *M*'s effect on *Y*; or both).
- Provide probing results with interpretations
- Read the write ups of other's moderation analyses
- Provide a graphical representation of the effect of interest (like the ones we've done)

Does the effect of group work on interest in a computer science class depend on preference for group work?

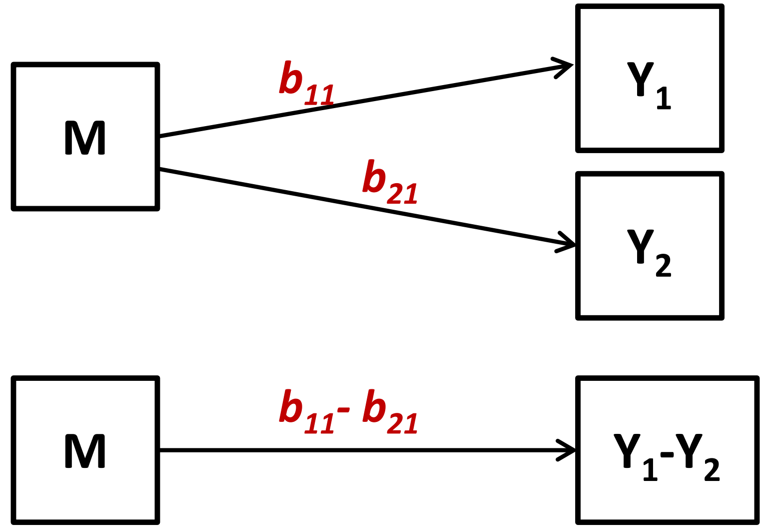
Overall, the impact of including group work in a computer science class on interest in the class depends on an individual's general preference for group work (b_1 = .49, p = .001). As preference for group work increases relative interest in the class with group work compared to the class with individual work increases as well. (i.e. the group work class is more preferred as general preference for group work increases). Indeed we found that those who were relatively low in preference for group work preferred the individual work class over the class with group work ($\theta_{X \rightarrow Y}(M = 2.59) = -.97$, p = .002). Whereas, those who were relatively moderate in preference for group work did not show a strong preference for one class over another, though they marginally preferred the class with group work ($\theta_{X \rightarrow Y}(M = 3.97) = .37$, p = .08). Finally, those who showed a strong general preference for group work, unsurprisingly preferred the class with group work over the class with individual work ($\theta_{X \rightarrow Y}(M = 5.30) = 1.72$, $p < .001$). The Johnson-Neyman procedure those whose preference for group work was less than 3.07 preferred the individual work class, and those who's preference for group work was greater than 3.99 preferred the group work class. Preference for group work was positively related to interest in the class with group work (b = .49, p = .001), and negatively related to interest in the class with individual work (b = -0.50, p = .001).

Visualizations

I recommend trying a number of different types of visualizations to decide what works best for your case.



Conceptual



Statistical

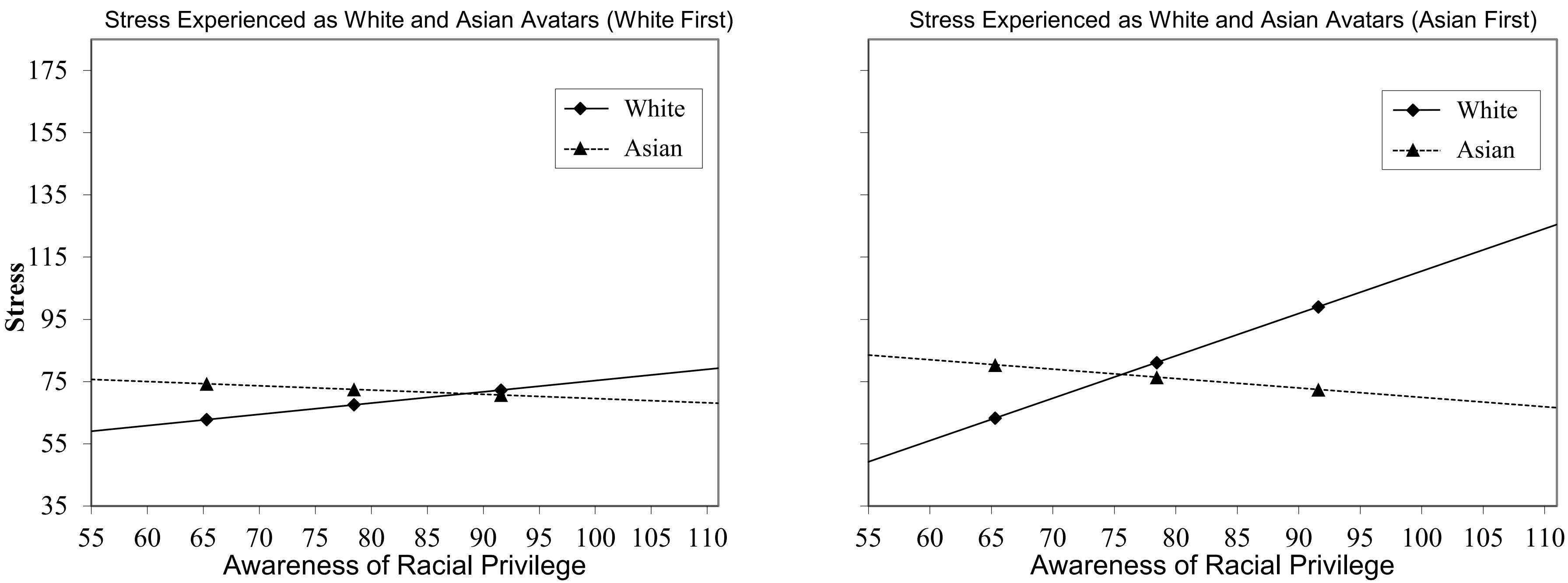
Tips:

- Try the different scales of the *Y* axis (difference vs. raw Y score with two lines for each condition)
- I do not like bar graphs with the effect of the moderator in each condition
- Provide path estimates on statistical diagram or in a table.

Visualizations: A Case Study

Tawa, J., & **Montoya, A. K.** (white paper) White students' physiological stress while operating non-White avatars and the moderating role of awareness of racial privilege.

White participants operated avatars of three difference races (White, Black, and Asian) and wrote heart monitors to measure their stress while operating each avatar. We found that individual's awareness of racial privilege moderated the effect of avatar race on stress, and that this effect depended on the order of operating the avatars.

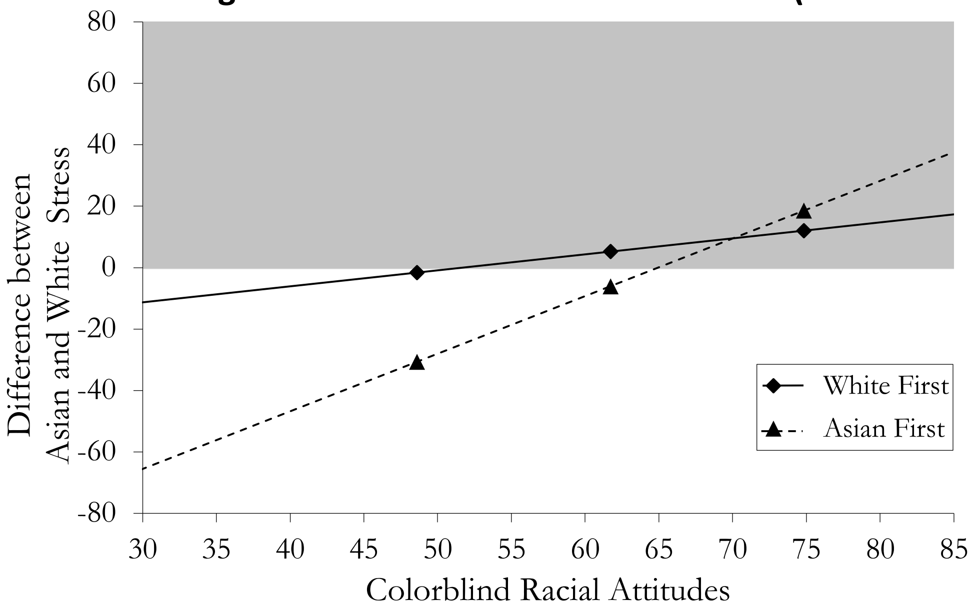


Visualizations: A Case Study

Tawa, J., & **Montoya, A. K.** (Under Review) White students' physiological stress while operating non-White avatars and the moderating role of awareness of racial privilege.

White participants operated avatars of three difference races (White, Black, and Asian) and wrote heart monitors to measure their stress while operating each avatar. We found that individual's awareness of racial privilege moderated the effect of avatar race on stress, and that this effect depended on the order of operating the avatars.

Figure 3. Predicted difference in Stress (Asian Stress – White Stress), split by order.



Note. Scores above zero on the Y-axis represent greater predicted stress while piloting the Asian avatar than while piloting the White avatar. Points marked by shapes indicate predicted stress differences at the mean plus/minus one standard deviation on CBA.

Common Questions

- Can this method be used for more than two conditions?
  YES! The same method for coming up with contrasts in Judd, Kenny, and McClelland (2001) describe a system for setting up contrasts among conditions can be used for moderation.

  I recommend reading Hayes & Montoya (*in press*) on moderation analysis with a multicategorical IV if you want to try this out. I am happy to give instructions on how to get MEMORE to doing this.

  ALTERNATIVES: Some of the other repeated-measures mediation options are more appropriate if you have more than two conditions (especially longitudinal), so take a look at those when thinking about these options.

- Can I use multiple moderators?
  YES! MEMORE models 2 and 3 accept up to 5 moderators. (See Documentation for instructions).
- How do I control for covariates?
  All of MEMORE's mediation analyses are within-person models, so you do not need to control for any between subjects variables such as age, gender, big-5. But you can include them as additional moderators (likely using model 2).

Multiple Moderator Models

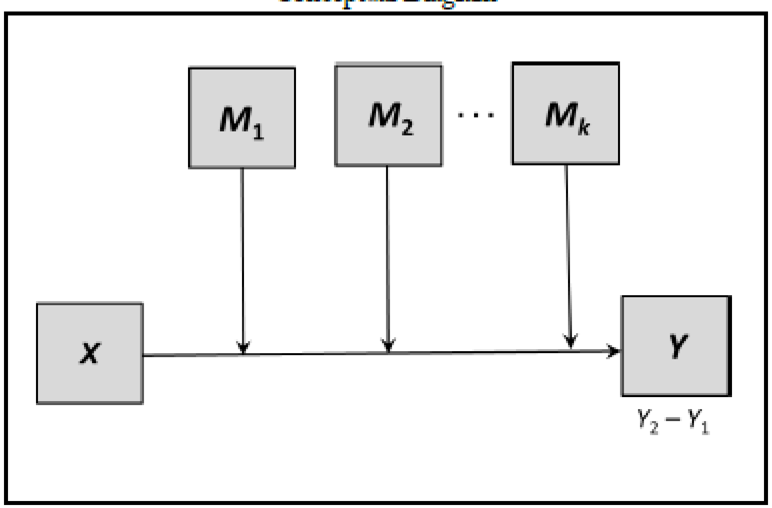
Model 2 vs. Model 3

When you have multiple moderators you are interested, consider whether you think those moderators will themselves interact or not.

If you believe the moderators will interact **with each other** → Model 3
If you believe the moderators will **only interact with condition** → Model 2

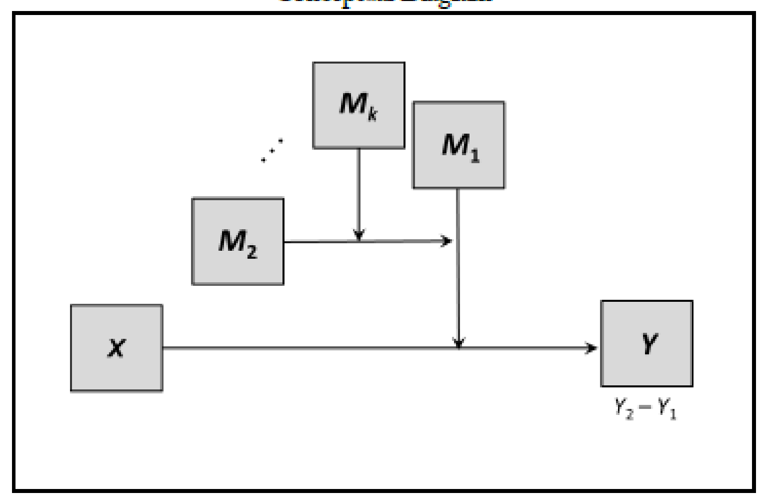
Model Templates for MEMORE V2.Beta
©2017 Amanda K. Montoya

Model 2 Additive Moderation
Conceptual Diagram



Model Templates for MEMORE V2.Beta
©2017 Amanda K. Montoya

Model 3 Multiplicative Moderation
Conceptual Diagram



Multiple Moderator Models

```
MEMORE m = grppref order/y = int_G int_I /model = 2.
```

```
%memore(w=grppref order,y = int_G int_I, model = 2,
        data = CompSci_WS);
```

```
Model:
  2

Variables:
Y =   int_G    int_I
M1 =  grppref
M2 =  Order

Computed Variables:
Ydiff =         int_G     -       int_I

Sample Size:
  51

**********************************************************************************
Outcome: Ydiff =  int_G     -       int_I

Model Summary
         R       R-sq        MSE          F        df1        df2          p
     .7113      .5059     2.0502    24.5734     2.0000    48.0000      .0000

Model
             coeff         SE          t          p       LLCI       ULCI
constant   -4.8074      .8394    -5.7269      .0000    -6.4952    -3.1196
grppref      .9562      .1505     6.3542      .0000      .6536     1.2588
Order        .9071      .4055     2.2372      .0300      .0918     1.7223

Degrees of freedom for all regression coefficient estimates:
  48
```

Think of it like two two-way interactions:
Condition x Group Preference
Condition x Order

Multiple Moderator Models

```
MEMORE m = grppref order/y = int_G int_I /model = 3.
```

```
%memore(w=grppref order,y = int_G int_I, model = 3,
        data = CompSci_WS);
```

```
Model:
  3

Variables:
Y =   int_G    int_I
M1 =  grppref
M2 =  Order

Computed Variables:
Ydiff =           int_G      -        int_I
Int1  =           grppref    x        Order

Sample Size:
  51

************************************************************************************
Outcome: Ydiff =  int_G      -       int_I

Model Summary
          R        R-sq        MSE          F        df1        df2          p
      .7125       .5077     2.0862    16.1569     3.0000    47.0000      .0000

Model
               coeff         SE          t          p       LLCI       ULCI
constant     -5.5239     1.9247    -2.8700      .0061    -9.3960    -1.6518
grppref       1.1401      .4690     2.4312      .0189      .1967     2.0836
Order         1.4057     1.2704     1.1065      .2742    -1.1501     3.9615
Int1          -.1263      .3048     -.4145      .6804     -.7395      .4868

Degrees of freedom for all regression coefficient estimates:
  47
```

Think of it like three-way interaction, and three two-way interactions:

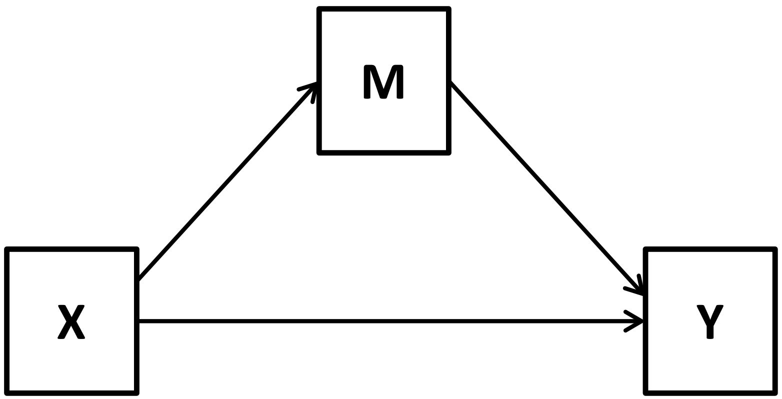
Condition x Group Preference

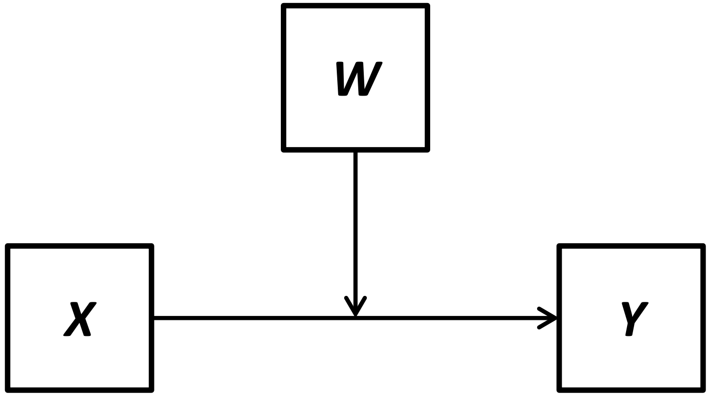
Condition x Order

Group Preference x Order

Condition x Group Preference x Order

Combining mediation and moderation





Research questions:

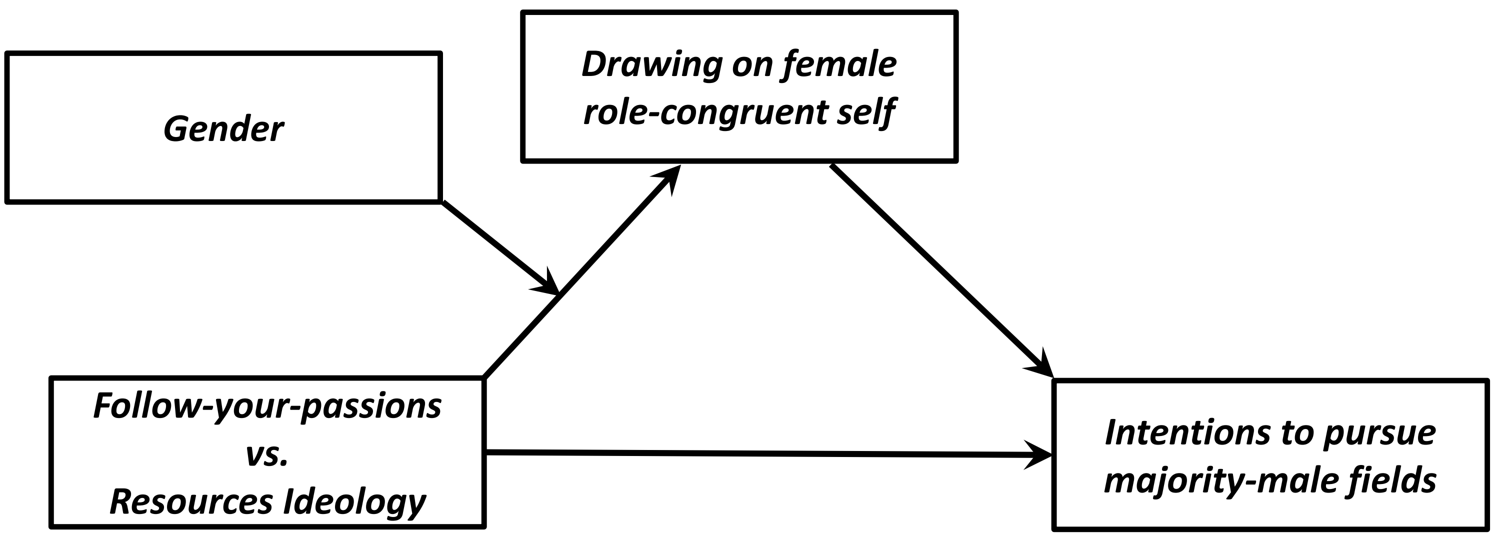
- Does the process through which *X* affects *Y* through *M* depend on *W*?
- Are there certain groups where *X* affects *Y* through *M* and certain groups where this process does not occur?

Conditional process analysis allows a mediated process to be moderated. Now the indirect effect can be defined as a *function of the moderator.*

CPA in Two-instance repeated-measures designs

Extending the path analytic from Montoya & Hayes (2017) we can now allow for moderation of a mediated pathway.

First stage moderated mediation allows *W* to moderate the path between the within-subjects factor and the mediator.



Montoya, *under review (https://psyarxiv.com/wrtb6)*

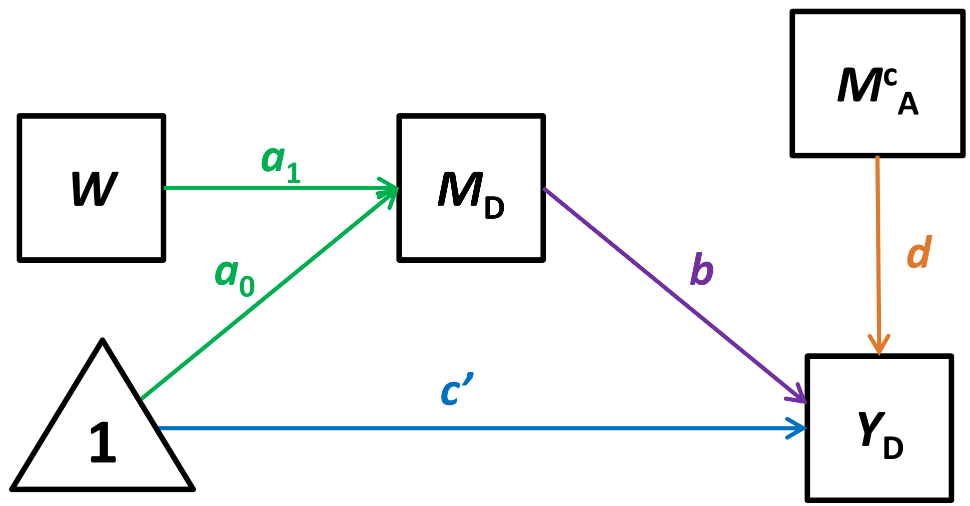
Equations and Path Diagram

First stage moderated mediation allows *W* to moderate the path between the within-subjects factor and the mediator.

$$M_{2i} - M_{1i} = a_0 + a_1 W_i + \epsilon_{Mi}$$

$$\theta_{X \to M}(W) = a_0 + a_1 W_i$$

$$Y_{Di} = c' + bM_{Di} + dM^c_{Ai} + \epsilon_{Yi}$$



What is the indirect effect?

$$\theta_{X \to M}(W) \times b = (a_0 + a_1 W)b = a_0 b + a_1 bW$$

Indirect effect is a *function* of the moderator

Montoya, *under review (https://psyarxiv.com/wrtb6)*

Inference

$$\theta_{X\rightarrow M}(W) \times b = (a_0 + a_1 W)b = a_0 b + a_1 bW$$

Conditional Indirect Effects
Select a value of *W*, plug that into the equation for the indirect effect, and use bootstrapping to make inference about the indirect effect at that value

Does the indirect effect *depend* on the moderator?

If $a_1 b = 0$ then the indirect effect *does not* depend on W

$$\theta_{X\rightarrow M}(W) \times b = a_0 b + 0 * W = a_0 b$$

$a_1 b$ can be called the **index of moderated mediation**

A test on the index will indicate if the indirect effect depends on *W.* We can do this formal test using bootstrapping.

Montoya, *under review (https://psyarxiv.com/wrtb6)*

```
Model:
  15
Variables:
Y = FMpas    FMinc
W = gendr
M = gendSpas gendSinc
Computed Variables:
Ydiff =          FMpas     -        FMinc
Mdiff =          gendSpas  -        gendSinc
Mavg =   (       gendSpas  +        gendSinc )        /2                          Centered
Sample Size:
  672
*************************************************************************************
Outcome: Ydiff =  FMpas     -       FMinc
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .2490      .0620     2.6562    44.2691     1.0000   670.0000      .0000
Model
             Effect         SE          t          p       LLCI       ULCI
constant      .0286      .2070      .1382      .8901     -.3779      .4351
W            -.8423      .1266    -6.6535      .0000    -1.0909     -.5937
Degrees of freedom for all regression coefficient estimates:
  670
Conditional Effect of Focal Predictor on Outcome at values of Moderator(s)
 Focal:   'X'       (X)
 Outcome: Ydiff     (Y)
 Mod:     gendr     (W)
      gendr     Effect         SE          t          p       LLCI       ULCI
     1.0000     -.8137      .0946    -8.6042      .0000     -.9994     -.6280
     2.0000    -1.6560      .0842   -19.6764      .0000    -1.8213    -1.4907
     Values for dichotomous moderators are the two values of the moderator.
Degrees of freedom for all conditional effects:
  670
*************************************************************************************
Outcome: Mdiff =  gendSpas  -       gendSinc
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .1513      .0229     2.4199    15.6875     1.0000   670.0000      .0001
Model
             Effect         SE          t          p       LLCI       ULCI
constant      .6179      .1976     3.1273      .0018      .2300     1.0059
W             .4786      .1208     3.9607      .0001      .2413      .7158
Degrees of freedom for all regression coefficient estimates:
  670
Conditional Effect of Focal Predictor on Outcome at values of Moderator(s)
 Focal:   'X'       (X)
 Outcome: Mdiff     (M)
 Mod:     gendr     (W)
      gendr     Effect         SE          t          p       LLCI       ULCI
     1.0000     1.0965      .0903    12.1478      .0000      .9193     1.2738
     2.0000     1.5751      .0803    19.6079      .0000     1.4174     1.7328
     Values for dichotomous moderators are the two values of the moderator.
Degrees of freedom for all conditional effects:
  670
*************************************************************************************
```

Model Information

Model for difference in outcomes (no mediators)

Conditional effects of *X* on *Y* at values of *W*

Model for the difference in mediators

Conditional Effect of *X* on *M* at values of *W*

Montoya, *under review (https://psyarxiv.com/wrtb6)*

```
Outcome: Ydiff =  FMpas     -       FMinc
Model Summary
          R       R-sq        MSE          F        df1        df2          p
      .4802      .2306     2.1819   100.2607     2.0000   669.0000      .0000
Model
              coeff         SE          t          p       LLCI       ULCI
constant     -.6056      .0754    -8.0268      .0000     -.7538     -.4575
Mdiff        -.4973      .0363   -13.7123      .0000     -.5685     -.4261
Mavg         -.2696      .0721    -3.7399      .0002     -.4111     -.1280
Degrees of freedom for all regression coefficient estimates:
  669
****************** CONDITIONAL TOTAL, DIRECT, AND INDIRECT EFFECTS ******************
Conditional Total Effect of X on Y at values of the Moderator(s)
      gendr     Effect         SE          t         df          p       LLCI       ULCI
     1.0000     -.8137      .0946    -8.6042   670.0000      .0000     -.9994     -.6280
     2.0000    -1.6560      .0842   -19.6764   670.0000      .0000    -1.8213    -1.4907
     Values for dichotomous moderators are the two values of the moderator.
Direct effect of X on Y
     Effect         SE          t         df          p       LLCI       ULCI
     -.6056      .0754    -8.0268   669.0000      .0000     -.7538     -.4575
Conditional Indirect Effect of X on Y through Mediator at values of the Moderator
  Ind:      Ind1
  Med:      Mdiff      (M)
      gendr     Effect     BootSE   BootLLCI   BootULCI
     1.0000     -.5453      .0549     -.6577     -.4411
     2.0000     -.7833      .0801     -.9449     -.6302
     Values for dichotomous moderators are the two values of the moderator.
Indirect Key
Ind1  'X'      ->       Mdiff    ->       Ydiff
******************************** INDICES OF MODERATION ********************************
Test of Moderation of the Total Effect
      Effect         SE          t         df          p       LLCI       ULCI
W     -.8423      .1266    -6.6535   670.0000      .0000    -1.0909     -.5937
Index of Moderated Mediation for each Indirect Effect.
          Effect     BootSE   BootLLCI   BootULCI
Ind1      -.2380      .0651     -.3701     -.1144
*************************** ANALYSIS NOTES AND WARNINGS ***************************
NOTE: Some cases were deleted due to missing data. The number of cases was:
  11
Bootstrap confidence interval method used: Percentile bootstrap.
Number of bootstrap samples for bootstrap confidence intervals:
  5000
The following variables were mean centered prior to analysis:
 (        gendSpas  +       gendSinc )        /2
Level of confidence for all confidence intervals in output:
      95.00
```

Model for difference in outcomes (including mediators)

Conditional total, direct, and indirect effects

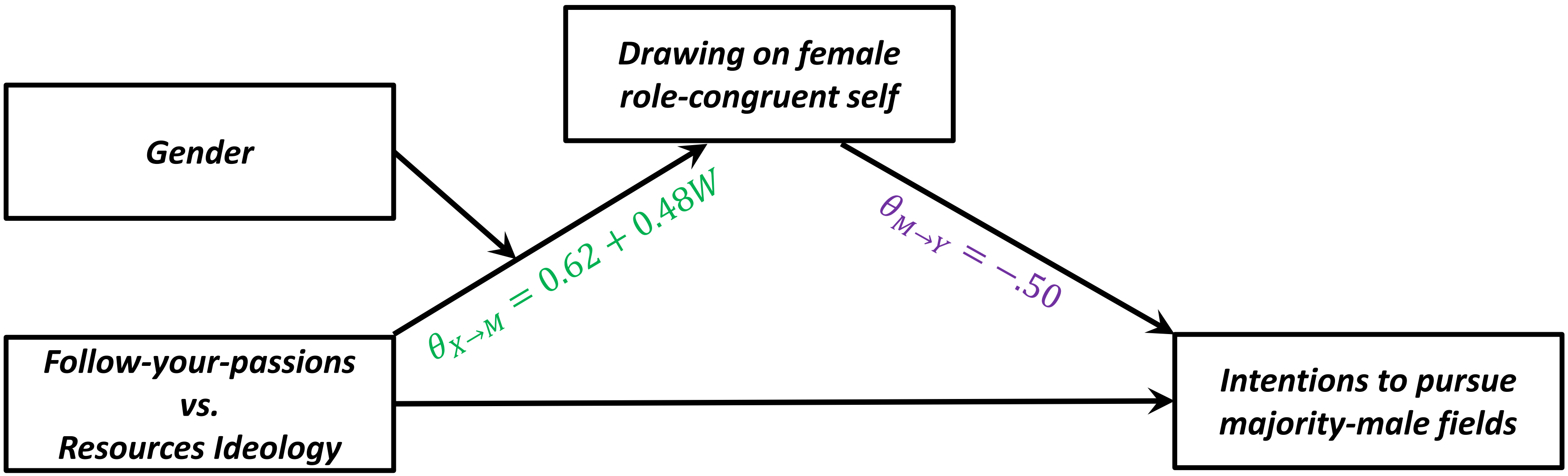
Tests of moderation for the total and indirect effects (direct effect not moderated in this model)

Errors, notes, etc

Montoya, *under review (https://psyarxiv.com/wrtb6)*

Follow your passions

```
memore y = FMpas FMinc /m = gendSpas gendSinc /w = gendr
/model = 15.
```



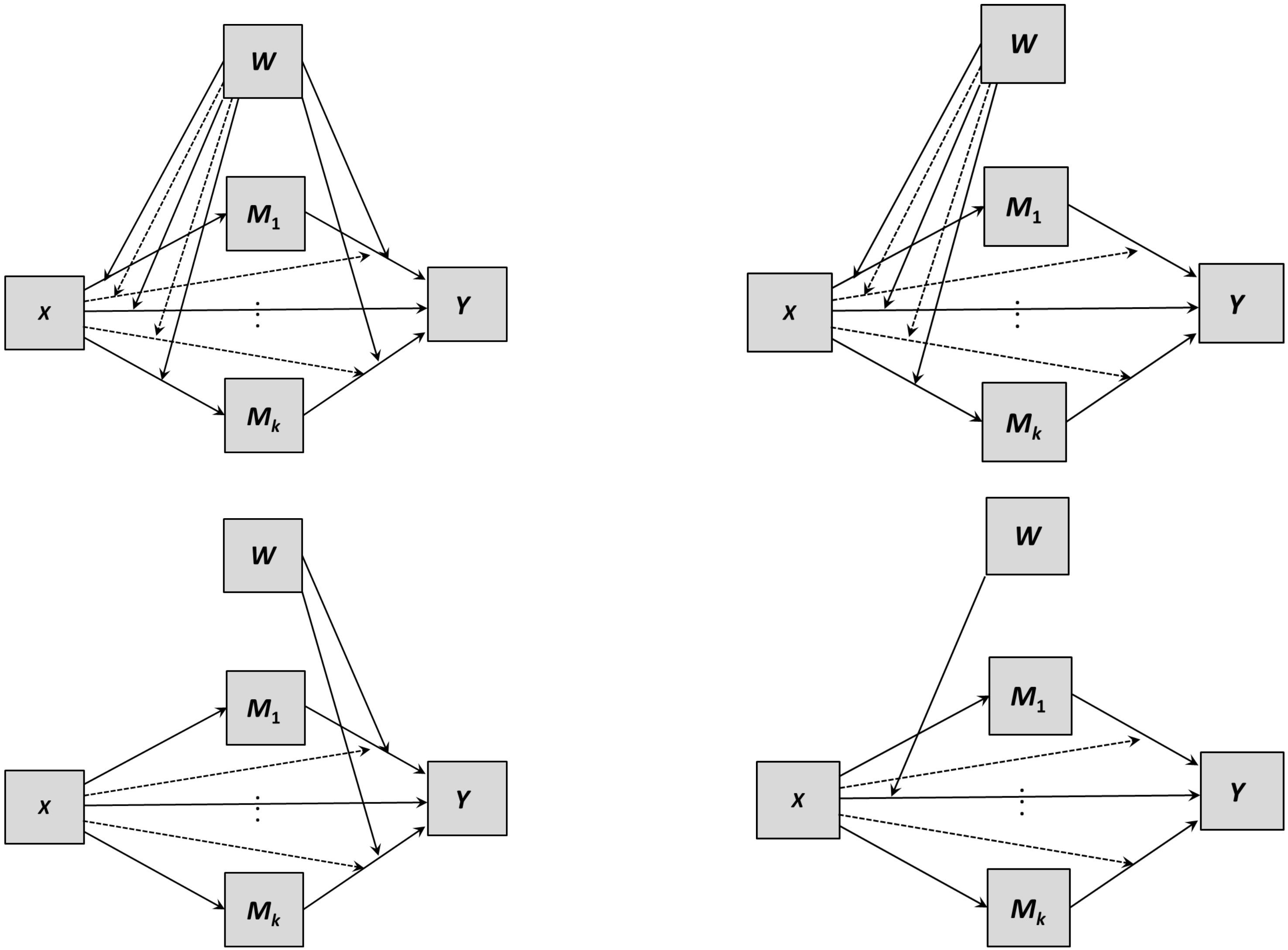
The indirect effect for both men and women was such that **the follow-your-passions ideology decreased interest through drawing on feminine self** (Men: -.55 [-.66, -.44], Women: -.78 [-.94, -.63]).

The *index of moderated mediation* was significantly different from zero (-.24 [-.37, -.12]), meaning the **indirect effect through drawing on the feminine self was stronger for women than for men**.

Siy, Germano, Vianna, Azpeitia, Yan, Montoya, Cheryan, in press, *JPSP*

MEMORE V3: Models 4 - 18

The latest version of MEMORE has expanded to models with a single moderator on any combination of paths in the mediation.

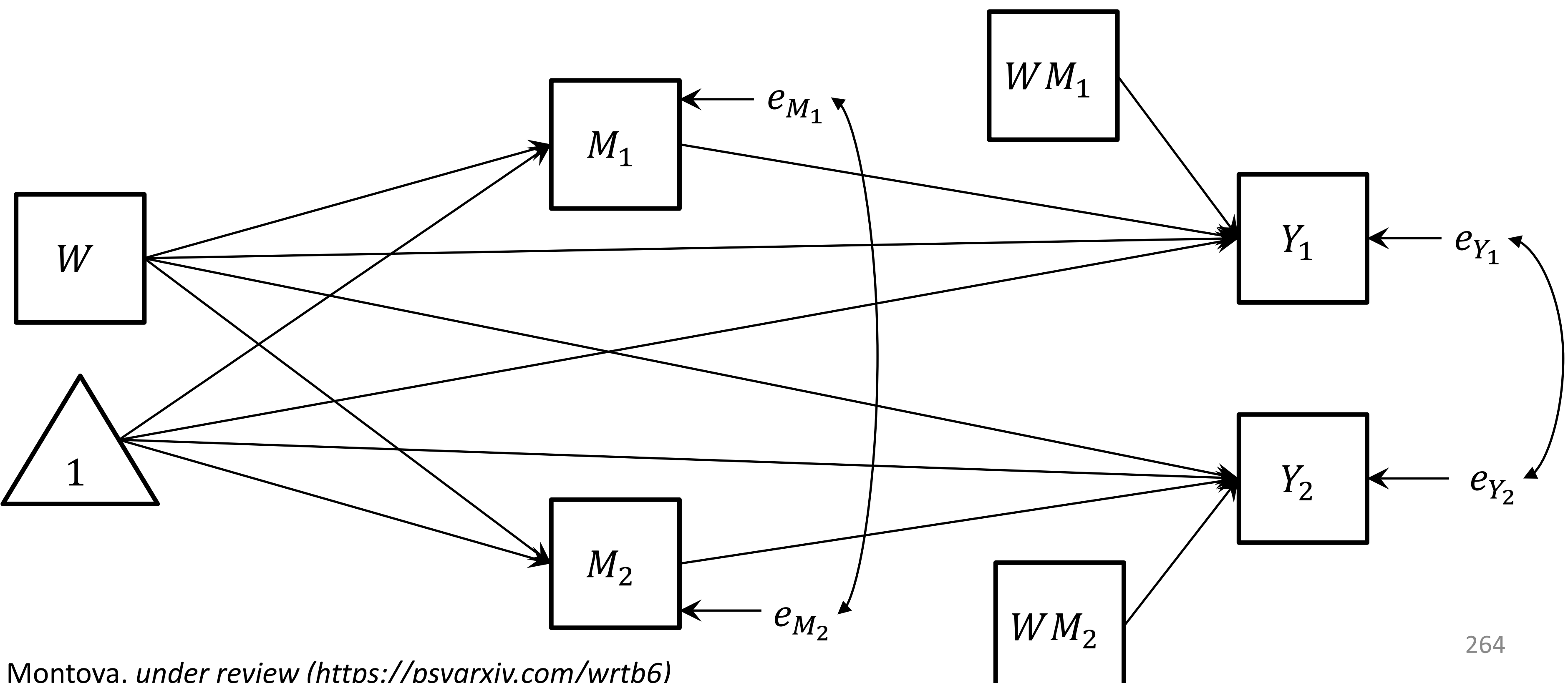


Montoya, *under review (https://psyarxiv.com/wrtb6)*

Structural equation models

- Three SEM approaches which are related to this model:

1. Simultaneous estimation of equations
   - Limited advantage over the OLS method
   - Does not scale up well
   - Could incorporate latent variables

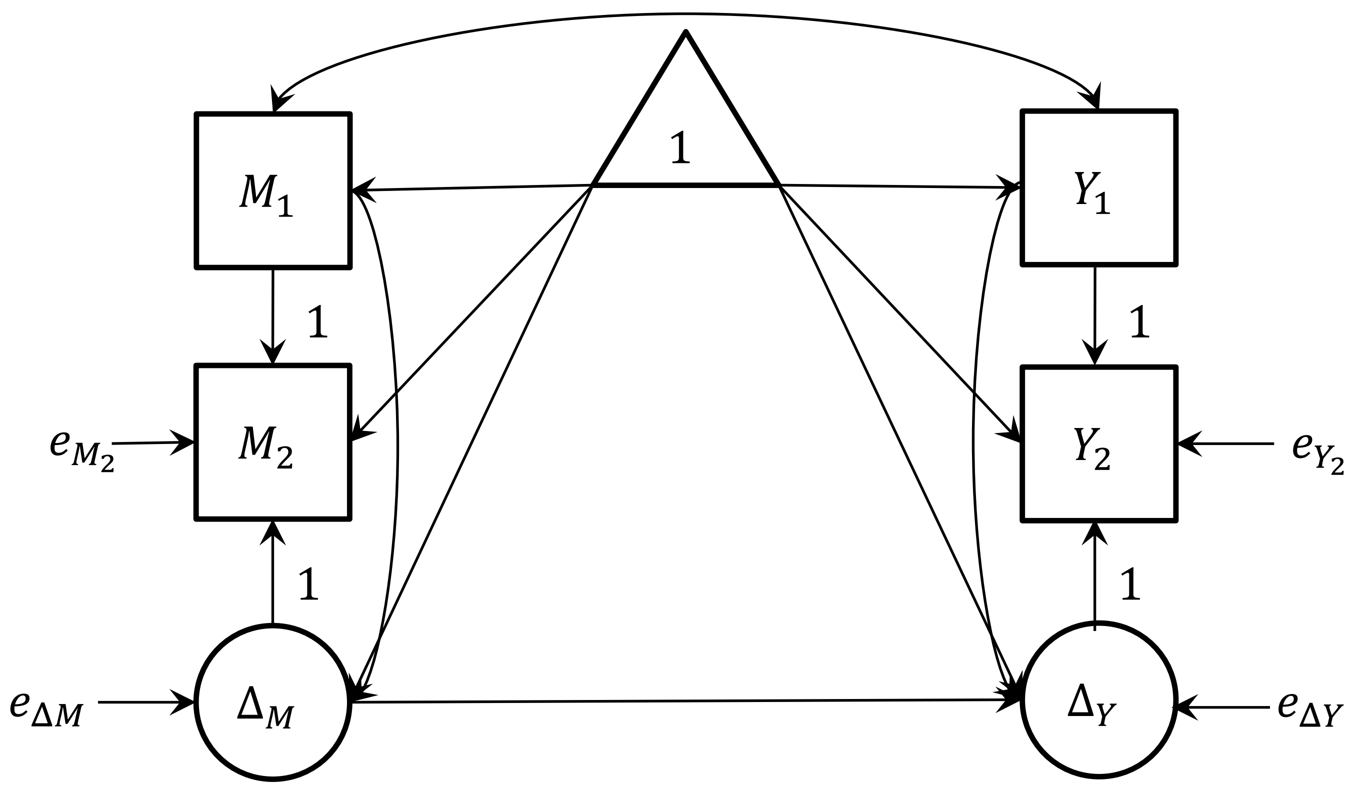


Montoya, *under review (https://psyarxiv.com/wrtb6)*

Structural equation models

- Three SEM approaches which are related to this model:

2. Latent difference score
   - Treats change as latent rather than observed
   - Generalizes beyond two-instances (especially repeated-replicates and more timepoints)
   - No generalization to conditional process model yet

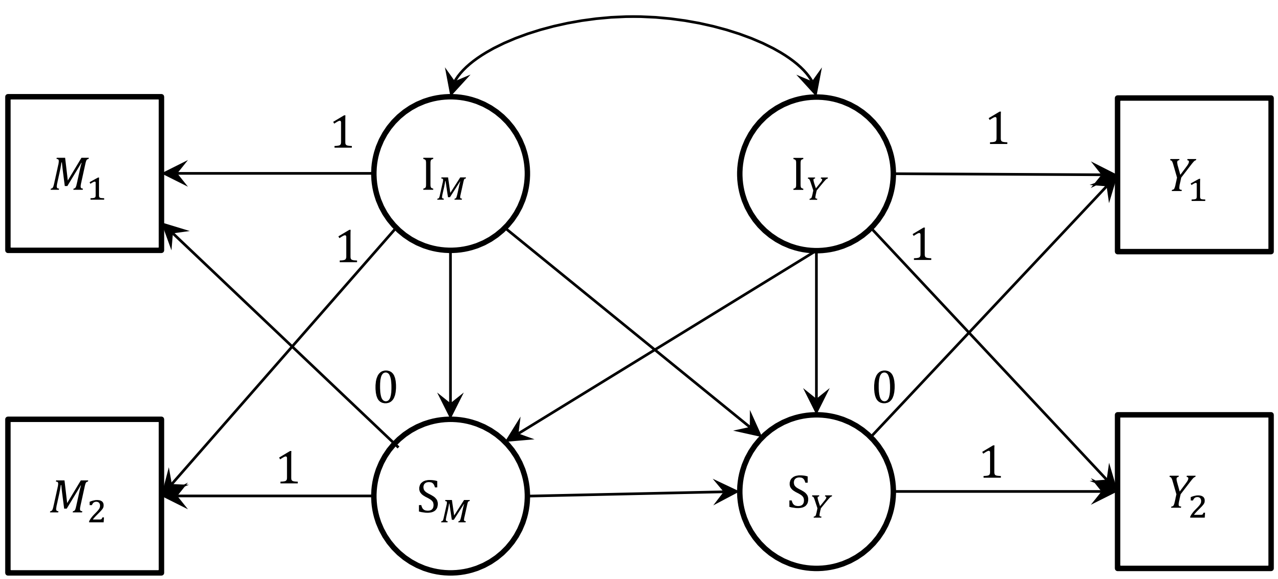


<u>Montoya</u>, *under review (https://psyarxiv.com/wrtb6)*

Structural equation models

- Three SEM approaches which are related to this model:

3. Latent growth curve model
   - Individual differences in intercepts and slopes as latent variables
   - Generalizes beyond two time-points (not more conditions)
   - No generalization to conditional process model yet



Montoya, *under review (https://psyarxiv.com/wrtb6)*

Wrapping Up

Where to learn more:

Amanda Kay Montoya	akmontoya.com	akmontoya@ucla.edu
Nick Rockwood	njrockwood.com	rockwood.19@osu.edu
@AmandaKayMontoya	@njrockwood	

MLMED and MEMORE both have many features not described here

MLMED does moderated mediation

MEMORE does moderation and (coming soon) moderated mediation

Mediation for Dyadic Data! http://afhayes.com/public/chj2019.pdf

Jacob Coutts: Poster Friday 2:30 – 3:30 V83

Andrew Hayes: Talk Saturday 1:30 – 1:55 Wilson AB

Github.com/akmontoya/APS2019