An Index and Test of Linear Moderated Mediation

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I describe a test of linear moderated mediation in path analysis based on an interval estimate of the parameter of a function linking the indirect effect to values of a moderator—a parameter that I call the *index of moderated mediation*. This test can be used for models that integrate moderation and mediation in which the relationship between the indirect effect and the moderator is estimated as linear, including many of the models described by Edwards and Lambert (2007) and Preacher, Rucker, and Hayes (2007) as well as extensions of these models to processes involving multiple mediators operating in parallel or in serial. Generalization of the method to latent variable models is straightforward. Three empirical examples describe the computation of the index and the test, and its implementation is illustrated using Mplus and the PROCESS macro for SPSS and SAS.

AN INDEX AND TEST OF LINEAR MODERATED MEDIATION

Empirically substantiating the boundary conditions of one variable's causal effect on another and the mechanism(s) by which that effect operates are recognized as markers of deeper understanding than merely establishing that X affects Y. Successfully answering such questions as "Under what circumstance does X affect Y?" and "How does X affect Y?" adds much merit to one's science and can enhance its impact on a field.

Questions about the contingencies of an effect are often answered statistically through moderation analysis. Assuming continuous Y and dichotomous and/or continuous X and W (the only case considered in this article), moderation of the effect of X on Y by W is popularly tested by estimating a linear model of the form

$$Y = i_Y + b_1 X + b_2 W + b_3 XW + e_Y$$

where b_1, b_2, and b_3 are estimated regression coefficients, e_Y is an error in estimation, and i_Y is a regression intercept. X's effect on Y is linearly moderated by W if the regression coefficient for XW is different from zero by an inferential test or confidence interval.[1]

Questions about the mechanism by which an effect operates are frequently answered with mediation analysis. In a mediation analysis, focus is on the estimation of the indirect effect of X on Y through an intermediary *mediator* variable M causally located between X and Y (i.e., a model of the form $X \rightarrow M \rightarrow Y$). Assuming continuous Y and M, the indirect effect of X on Y through M can be derived using two linear models:

$$M = i_M + aX + e_M$$
$$Y = i_Y + c'X + bM + e_Y$$

where a, b, and c' are estimated regression coefficients, i_M and i_Y are regression intercepts, and e_M and e_Y are errors in estimation. The product of a and b quantifies the indirect effect of X on Y and estimates how much two cases that differ by one unit on X are estimated to differ on Y through the effect of X on M which in turn influences Y. Evidence that the indirect effect is different from zero by an inferential test or confidence interval bolsters a claim that the effect of X on Y is mediated at least in part by M.

Mediation and moderation analysis can be analytically integrated into a unified statistical model. Although not a new idea by any means—such terms as "mediated moderation" and "moderated mediation" appeared in the literature decades ago (e.g., Baron & Kenny, 1986; James & Brett, 1984; Judd & Kenny, 1981)—it is only recently that a handful of articles in the methodology literature has provided researchers the tools and systematic procedures for answering questions focused on the "when of the how" or the "how of the when." Hayes (2013) introduces the term

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[1]Unless otherwise stated, assume that all regression coefficients and functions thereof in my notation are estimates based on the data available.

conditional process analysis (also see Hayes & Preacher, 2013) to refer to this collection of methods and procedures described by such teams as Edwards and Lambert (2007), Fairchild and MacKinnon (2009), Morgan-Lopez and MacKinnon (2006), Muller, Judd, and Yzerbyt (2005), and Preacher et al. (2007). Differing more in interpretative emphasis than in mathematical approach, these articles all focus on describing statistical methods using rudimentary principles of path analysis that offer researchers a means of understanding the boundary conditions of the mechanism(s) by which X influences Y through a mediator M—the moderation of mediation.

In this article, I describe an approach to testing a moderated mediation hypothesis that builds on and extends these methods. It can be applied to any model in which the indirect effect of X on Y through M is estimated as linearly related to a moderator. Although the method is discussed in the context of continuous moderators, it generalizes to models with a dichotomous moderator. The heart of the test is a quantification of the association between an indirect effect and a moderator—an "index of moderated mediation"—followed by an inference as to whether this index is different from zero. I show that if the outcome of this test results in a claim that an indirect effect is moderated, this means that any two conditional indirect effects estimated at different values of the moderator are significantly different from each other. Furthermore, a claim that the indirect effect is not moderated as a result of this test implies that no two conditional indirect effects are statistically different.

After introducing the logic and derivation of the method, emphasis shifts to applying the test to data from three studies based on models that differ with respect to the number of mediators and which part of the mediation process is moderated. To facilitate implementation, example code is provided in the appendix showing how to apply the test using Mplus (Muthén & Muthén, 2011) and the PROCESS macro for SPSS and SAS (Hayes, 2013). With its emphasis on application, this article targets the substantive researcher, yet methodologists will likely find something here that is thought-provoking, and I expect it will inspire further research as well as debate.

Though mediation is a causal process, the method described here is not a formal test of causality. In the examples presented here, there are plausible noncausal interpretations of the results. Inferences are generated not by the mathematics of the statistical procedures we employ to understand our data, but by our interpretation of the results generated by those procedures. Although the outcome of the application of this method may support a claim that the size, sign, or strength of an indirect effect depends on a moderator, users of this approach should be careful to condition their claims on the strength of the causal argument that can be made given limitations of the data available and how they were collected.

THE MODERATION OF A MECHANISM

Edwards and Lambert (2007), Muller et al. (2005), and Preacher et al. (2007) provide nearly identical definitions of moderated mediation:

> " ... moderated mediation refers to a mediated effect that varies across levels of a moderator variable" (Edwards & Lambert, 2007, pp. 6–7).

> " ... [moderated mediation] ... happens if the mediating process that is responsible for producing the effect of the treatment on the outcome depends on the value of a moderator" (Muller et al., 2005, p. 854).

> " ... moderated mediation occurs when the strength of an indirect effect depends on the level of some variable, or in other words, when mediation relations are contingent on the level of a moderator" (Preacher et al., 2007, p. 193).

In other words, if the mechanism linking X to Y through a mediator M is somehow related to (i.e., *a function of*) another variable, then it can be said to be moderated by that variable. The notion that a mechanism (an indirect effect) can be expressed as a function of a moderator is important in the definition and test of moderated mediation I describe here.

In addition to sharing a definition, the authors who introduced these methods seem to share the perspective, either explicitly or implicitly, that in order to claim that mediation is moderated, one should (if not also *must*) have evidence that at least one of the paths in the $X \rightarrow M \rightarrow Y$ system is moderated, with some kind of inferential statistical procedure used as the litmus test. That is, because an indirect effect is a product of two effects (the effect of X on M and the effect of M on Y controlling for X), if one of these effects is moderated, then by definition so too is the indirect effect. But if one cannot establish evidence that one of the paths is moderated, then the indirect effect cannot be construed as moderated as proposed.

Muller et al. (2005) are the most explicit of the three in this respect. By their approach, a claim of moderated mediation is substantiated by evidence of statistically significant moderation of at least one path in the causal system linking X to Y through M *and*, for any path not proposed to be moderated, evidence that this unmoderated path is statistically different from zero. So much like Baron and Kenny (1986) did for mediation analysis, Muller et al. (2005) lay out criteria that need to be met in order to claim that mediation is moderated.

Preacher et al. (2007) do not provide a formal test of moderated mediation but, rather, offer a means of probing moderated mediation by quantifying an indirect effect conditioned on a value of a moderator and conducting an inference about this *conditional indirect effect*. Using their procedure, the cleanest evidence of moderated mediation is evidence

of moderation of one of the paths in the causal system combined with evidence that the conditional indirect effect of X on Y through M is statistically different from zero at some value(s) of the moderator but not at another value or values. Implied through their example (see Preacher et al., 2007, pp. 211–212) is that at least one of the paths in a causal sequence (either the $X \to M$ or $M \to Y$ path) should be moderated by the standards of some kind of inferential test in order to claim that an indirect effect is moderated, though they do not state this as a formal requirement as Muller et al. (2005) do.

Like Preacher et al. (2007), Edwards and Lambert (2007) describe a means of quantifying indirect effects at different levels of a moderator. Furthermore, they get closer than both Preacher et al. (2007) and Muller et al. (2005) to offering a formal test of moderated mediation similar to the one that is the focus of this article by describing a bootstrap-based procedure for comparing two indirect effects conditioned on different values of the moderator. Nevertheless, as at least one of the component paths of the indirect effect in their examples is moderated based on a test of significance, the reader could easily go away with the impression that evidence of moderation of at least one path is a requirement in their framework in order to claim that a mediated effect is moderated. Furthermore, their example is based on a model in which the indirect effect is a *nonlinear* function of a continuous moderator, meaning that the outcome of their test will be dependent on the two values of the moderator chosen. The test described here does not apply to such a model, for reasons I discuss later.

The definition of moderated mediation I use in this article is mathematically more formal but is otherwise similar to the definitions provided above. A mediation process can be said to be moderated if the proposed moderator variable has a nonzero weight in the function linking the indirect effect of X on Y through M to the moderator. As will be shown, this weight is a product of at least two regression coefficients. A test as to whether this weight—what I call the *index of moderated mediation*—is different from zero serves as a formal test of moderated mediation. An important feature of this test is that evidence of statistically significant interaction between any variable in the model and a putative moderator is *not* a requirement of establishing moderation of a mechanism. As Fairchild and MacKinnon (2009) first described and I elaborate upon here, an indirect effect could be moderated even if one cannot substantiate moderation of one of the components of the indirect effect by an inferential test. By the same token, establishing that a component of an indirect effect is moderated does not necessarily establish that the indirect effect is.

Motivating Principles

The method that is the focus of the rest of this article has three desirable qualities of an inferential method. First, a claim about an effect or lack thereof should be based on a

quantification of the effect that is most directly pertinent to that claim. The index of moderated mediation is a direct quantification of the linear association between the indirect effect and the putative moderator of that effect. Second, the number of inferential statistical tests employed used to substantiate a claim should be kept as small as possible. Using the method I describe, only a single inferential test is needed to determine whether a hypothesis of moderated mediation is supported. The inference is based on the size of the index of moderated mediation rather than a set of two or more inferential tests about components of the model. Third, ideally, a claim should include some acknowledgment of the uncertainty inherent in our estimation of quantities used to support that claim. The methods currently in use result in dichotomous yes/no claims of whether a mechanism is moderated. By contrast, inference using the method described here is based on an interval estimate of the index of moderated mediation, which conveys the inevitable uncertainty in the size of the association between the moderator and the indirect effect.

AN INDEX OF MODERATED MEDIATION

In addition to a definition of moderated mediation, Edwards and Lambert (2007), Muller et al. (2005), and Preacher et al. (2007) share a path analytic framework for estimating effects in models that include both a moderation and a mediation component, and this is the framework I use in this article.

Models with One Mediator

Consider the *first stage moderation model*, so-named by Edwards and Lambert (2007), that allows the effect of X on M in a mediation model (sometimes called the "action theory" component of the model; e.g., Fairchild & MacKinnon, 2009) to be moderated by W. Some examples in the substantive literature include Martel, Nikolas, Jernigan, Friderici, and Nigg (2012), Wang, Stroebe, and Dovidio (2012), and Zhou, Hirst, and Shipton (2012). This model is diagrammed in conceptual form in Figure 1, panel A (left) and requires two equations to estimate the indirect effect of X. Typically M is estimated as a linear function of X, with the effect of X on M modeled as linearly related to W, and Y estimated as a linear function of both M and X. In equation form:

$$M = i_M + a_1 X + a_2 W + a_3 XW + e_M \qquad (1)$$

$$Y = i_Y + c'X + bM + e_Y \qquad (2)$$

These equations are represented in the form of a path diagram in Figure 1, panel A (right). The coefficients for each predictor in the model can be estimated using either ordinary least squares regression or a structural equation modeling program.

Additional predictors could be included in the models of M and Y to account for their potential effects as confounders of the relationships between X, M, and Y. Furthermore, one

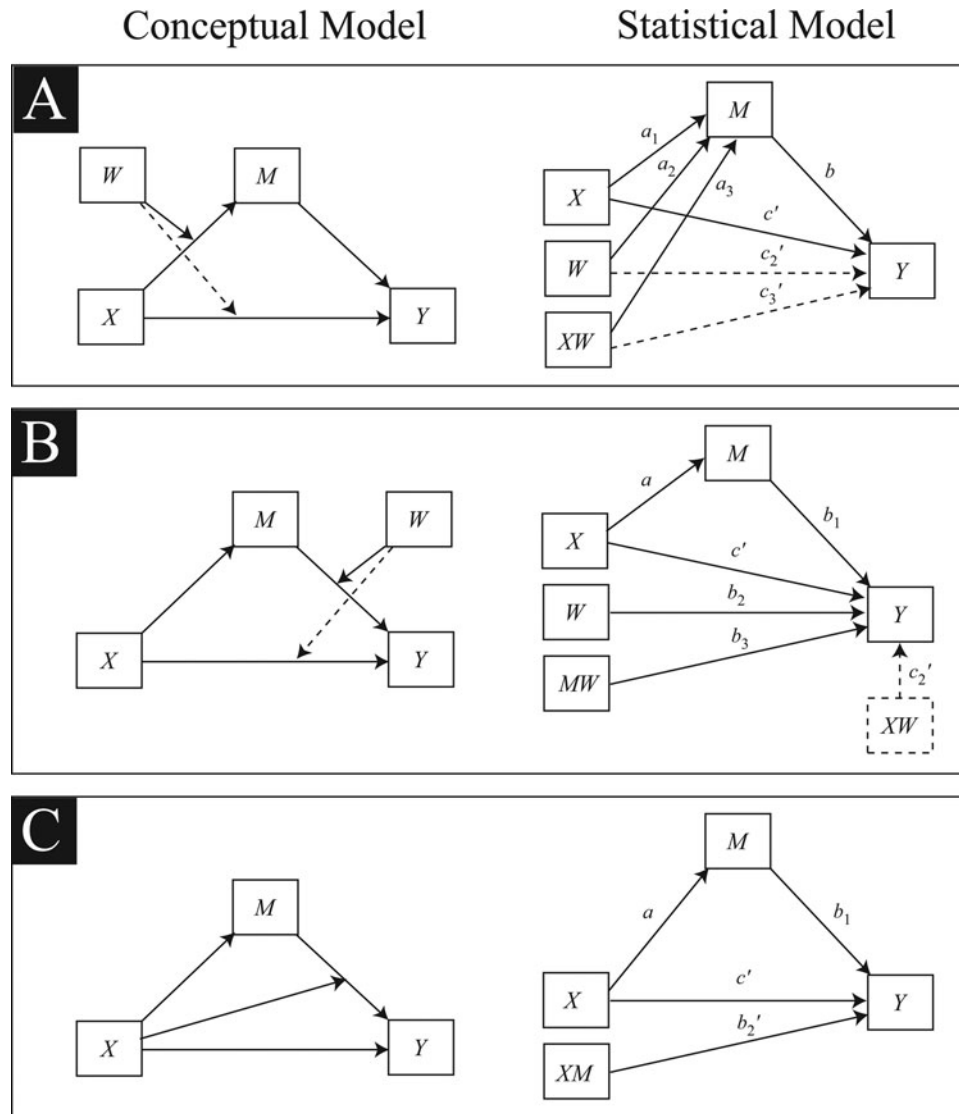## Conceptual Model   Statistical Model

FIGURE 1   Some forms of a moderated mediation model in which the indirect effect of *X* on *Y* through *M* is moderated. Dotted lines represent optional moderator effects which can be included without disrupting the derivation of the index of moderated mediation provided in the text.

could also allow *W* to moderate the direct effect of *X* on *Y* (path c') by adding *W* and *XW* to Equation (2) (see the dotted paths in Figure 1, panel A). Doing so yields the *first stage and direct effect moderation* model described by Edwards and Lambert (2007), or "Model 2" in Preacher et al. (2007). The mathematics and argument below are unaffected by the inclusion of covariates or moderation of the direct effect of *X* by *W*.

As Edwards and Lambert (2007) and Preacher et al. (2007) show, the indirect effect of *X* on *Y* through *M* (ω in the notation below) is the product of the conditional effect of *X* on *M* from Equation (1) and the effect of *M* on *Y* controlling for *X* in Equation (2):

$$\omega = (a_1 + a_3 W)b \qquad (3)$$

Equation (3) can be rewritten in an equivalent form

$$\omega = a_1 b + a_3 b W \qquad (4)$$

which is a line with intercept $a_1 b$ and slope $a_3 b$. As pointed out by Morgan-Lopez and MacKinnon (2006), $a_3 b$ is a quantification of the effect of *W* on the indirect effect of *X* on *Y* through *M* in such a model. Thus, in the first stage (and, by extension, the first stage and direct effect) moderation model, the indirect effect of *X* on *Y* through *M* is a linear function of *W*. The weight for *W* in this function, $a_3 b$, I henceforth call the *index of moderated mediation* for this model. To foreshadow the discussion of inference below, if the indirect effect is systematically larger or smaller for some values of *W* than others (and assuming the model is correctly specified) the expectation is that $a_3 b$ is different from zero. But if

the indirect effect is linearly unrelated to W, this leads to the expectation that a_3b is equal to zero.

Another popular model that combines moderation and mediation is the *second stage moderation model* (Edwards & Lambert, 2007, "Model 3" in Preacher et al., 2007), which allows the effect of M on Y in a mediation model (the "conceptual theory" component of the model; Fairchild & MacKinnon, 2009) to be moderated by W while fixing the effect of X on M to be unmoderated. See Green and Auer (2013), Suveg, Shaffer, Morelen, and Thomassin (2011), and Warner, Schwarzer, Schüz, Wurm, and Tesch-Römer (2012) for examples. Diagrammed in conceptual form in Figure 1, panel B (left), this model requires the estimation of the coefficients in two regression equations:

$$M = i_M + aX + e_M \tag{5}$$

$$Y = i_Y + c'X + b_1M + b_2W + b_3MW + e_Y \tag{6}$$

This model is represented in the form of a path diagram in Figure 1, panel B (right). As in the first stage model, potential confounders could be included in the models of M and Y, or one could allow W to moderate the direct effect of X on Y by adding XW to Equation (6) (see the dotted paths in Figure 1, panel B) yielding the *second stage and direct effect moderation model* described by Edwards and Lambert (2007). The mathematics below are unaffected.

Edwards and Lambert (2007) and Preacher et al. (2007) show that the indirect effect of X on Y through M is the product of the effect of X on M from Equation (5) and the conditional effect of M on Y from Equation (6):

$$\omega = a(b_1 + b_3W) \tag{7}$$

Equation (7) can be rewritten in equivalent form as

$$\omega = ab_1 + ab_3W,$$

which is a line with intercept ab_1 and slope ab_3. So as in the first stage model, in the second stage moderation model (and, by extension, the second stage and direct effect moderation model), the indirect effect of X on Y through M is a linear function of W. The weight for W in this function, ab_3, is the index of moderated mediation for this model. It quantifies the effect of W on the indirect effect of X on Y through M.

An intriguing model discussed by Kraemer, Wilson, Fairburn, and Agras (2002), Preacher et al. (2007), and Valeri and VanderWeele (2013) allows X to moderate its own indirect effect on Y through M via the moderation of the effect of M on Y by X, as in Figure 1, panel C (left). Examples in the literature of this model include D'Lima, Pearson, and Kelley (2012), MacNeil et al. (2010), Moneta (2011), and Oei, Tollenaar, Elzinga, and Spinhoven (2010). In equation form,

$$M = i_M + aX + e_M \tag{8}$$

$$Y = i_Y + c'X + b_1M + b_2XM + e_Y \tag{9}$$

with additional variables added to the models of M and Y if desired. In this *independent variable as moderator* model,

the indirect effect of X on Y through M is the product of the effect of X on M from Equation (8) and the conditional effect of M on Y from Equation (9):

$$\omega = a(b_1 + b_2X)$$

(see Preacher et al., 2007) which is the equation for a line

$$\omega = ab_1 + ab_2X$$

with intercept ab_1 and slope ab_2. In this model, ab_2 is the index of moderated mediation. It quantifies the relationship between X and the indirect effect of X on Y through M. X is functioning as a linear moderator of its own indirect effect.

Multiple Mediator Models

In all examples thus far, X's effect on Y is modeled as operating through a single mediator. But models with more than one mediator are commonly hypothesized and estimated. For some forms of multiple mediator models that include moderation, one or more of the specific indirect effects can be expressed as a linear function of a moderator. As an example, consider a *first stage moderated parallel multiple mediation model* (for an example, see Duffy, Bott, Allen, Torrey, & Dik, 2012). This model is depicted in Figure 2, panel A (left). In equation form, this model translates to

$$M_1 = i_{M_1} + a_{11}X + a_{21}W + a_{31}XW + e_{M_1}$$

$$M_2 = i_{M_2} + a_{12}X + a_{22}W + a_{32}XW + e_{M_2}$$

$$Y = i_Y + c'X + b_1M_1 + b_2M_2 + e_Y \tag{10}$$

and is represented in path diagram form in Figure 2, panel A (right). This model has two specific indirect effects of X on Y, one through M_1 and one through M_2. In these equations, the first stage of the specific indirect effects are both modeled as functions of W. Moderation of the direct effect would be accomplished by including W and XW as predictors in Equation (10), but doing so does not affect the derivation of the index of moderated mediation. In this model, the specific indirect effects of X on Y through M_1 and M_2, respectively, are

$$\omega_{M_1} = (a_{11} + a_{31}W)b_1 = a_{11}b_1 + a_{31}b_1W$$

$$\omega_{M_2} = (a_{12} + a_{32}W)b_2 = a_{12}b_2 + a_{32}b_2W$$

These are both linear functions of W, with the index of moderated mediation through M_1 and M_2 quantified as $a_{31}b_1$ and $a_{32}b_2$, respectively. This same logic can be applied to a second stage parallel multiple moderated mediation model comparable to the second example earlier but with multiple mediators.

Next, consider the *serial* or *three path* multiple mediation model, in which X is modeled to exert an effect on Y indirectly through two mediators linked in a causal chain. One or more of the paths in such a model could be moderated such that the indirect effect of X on Y is a linear function of the moderator.

Conceptual Model   Statistical Model
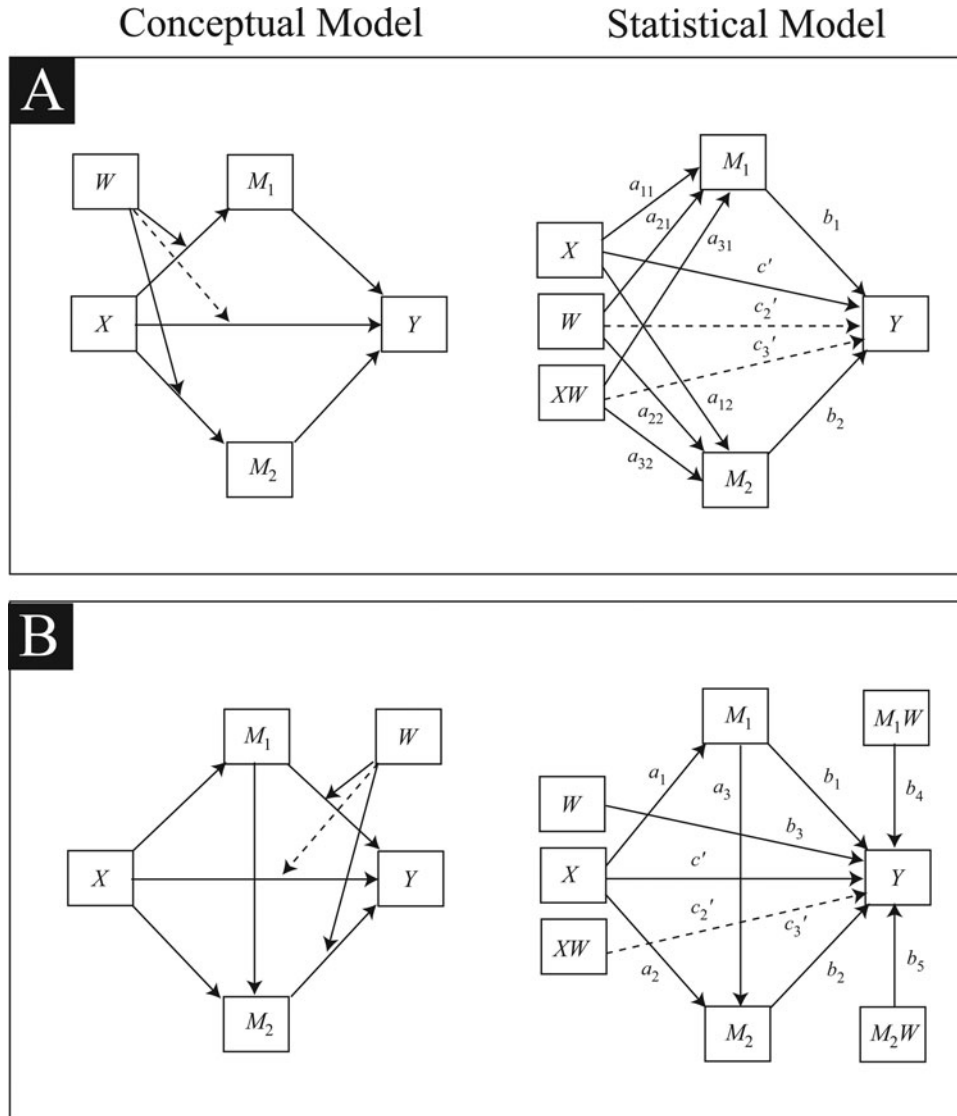
FIGURE 2   Some forms of a moderated multiple mediation model in which the indirect effects of X on Y through M are linear functions of W. Dotted lines represent optional moderator effects which can be included without disrupting the derivation of the index of moderated mediation provided in the text.

The model in Figure 2, panel B (left) is an example that allows M_1 to affect M_2, with the effects of M_1 and M_2 on Y moderated by W. This model is expressed in path diagram form in Figure 2, panel B (right), which visually represents three equations:

$$M_1 = i_{M_1} + a_1 X + e_{M_1}$$
$$M_2 = i_{M_2} + a_2 X + a_3 M_1 + e_{M_2}$$
$$Y = i_Y + c'X + b_1 M_1 + b_2 M_2 + b_3 W + b_4 M_1 W$$
$$+ b_5 M_2 W + e_Y$$

The specific indirect effect through M_1 is

$$\omega_{M_1} = a_1(b_1 + b_4 W) = a_1 b_1 + a_1 b_4 W$$

and through M_2, the specific indirect effect is

$$\omega_{M_2} = a_2(b_2 + b_5 W) = a_2 b_2 + a_2 b_5 W$$

Both of these specific indirect effects are functions of W, and the indices of moderated mediation are $a_1 b_4$ and $a_2 b_5$, respectively.

The specific indirect through M_1 and M_2 in serial is the product of the effect of X on M_1, the effect of M_1 on M_2, and the conditional effect of M_2 on Y:

$$\omega_{M_1 M_2} = a_1 a_3 (b_2 + b_5 W) = a_1 a_3 b_2 + a_1 a_3 b_5 W \qquad (11)$$

Equation (11) is a linear function of W with intercept $a_1 b_3 b_2$ and slope $a_1 a_3 b_5$. The slope of this line—$a_1 a_3 b_5$—is the index of moderated mediation for the serial, three-path specific indirect effect in this model.

Dichotomous Moderators

The derivations above do not require that the moderator be a quantitative variable. Oftentimes, a moderator in a model that integrates moderation and mediation is dichotomous, such as a variable coding experimental conditions or a naturally-existing category such as males and females. Moderated mediation, by any of the definitions provided earlier, implies that the indirect effect differs between the two groups coded by the moderator variable.

If the two numbers used to code groups on a dichotomous moderator differ by only a single unit (i.e., one), the index of moderated mediation equates to the difference between the two conditional indirect effects. Otherwise, the index will differ from the difference between conditional indirect effects by a factor of $1/\delta$, where δ is the difference between the two values used to code the groups. It is convenient (though not necessary) for description and interpretation purposes to transform the index when the moderator is dichotomous so that it corresponds to the difference between the two conditional indirect effects. This transformation is accomplished by multiplying the index by δ. For instance, if moderator W is a dichotomous variable coded -1 and 1 for the two groups, then $\delta = 2$.

First and Second Stage Moderated Mediation

Investigators typically posit based on theory or other guidance that a specific path (i.e., $X \rightarrow M$ or $M \rightarrow Y$) in a mediation process is moderated. But sometimes theory predicts that both paths are moderated, or an investigator may not have specific predictions as to which path in a mediation process is moderated or is otherwise agnostic. In that case one could, for example, allow the effect of X on M *and* the effect of M on Y to depend on a common moderator W. Examples in the substantive literature include Belogolovsky, Bamberger, and Bacharach (2012), Huang, Zhang, and Broniarczyk (2012), Kim and Labroo (2011), Malouf, Stuewig, and Tangney (2012), and Silton et al. (2011). Assuming a single mediator, such a model is represented with two equations

$$M = i_M + a_1 X + a_2 W + a_3 XW + e_M$$
$$Y = i_Y + c'X + b_1 M + b_2 W + b_3 MW + e_Y$$

with the equation for Y including XW as well if moderation of the direct effect is desired. The indirect effect of X on Y is

$$\omega = (a_1 + a_3 W)(b_1 + b_3 W)$$

(see Edwards & Lambert, 2007; Preacher et al., 2007), which can be written in equivalent form as

$$\omega = a_1 b_1 + (a_1 b_3 + a_3 b_1)W + a_3 b_3 W^2 \tag{12}$$

Equation (12) is a nonlinear function of W with four regression coefficients required to estimate the relationship between the moderator and the indirect effect. Although this

is an important model, the test of moderated mediation described in this article *cannot* be applied to such a model when W is continuous. This model is the focus of the method described in Muller et al. (2005) and some of the models discussed by Edwards and Lambert (2007) and Preacher et al. (2007). Furthermore, Fairchild and MacKinnon (2009) discuss a variation of this model that allows X to moderate the $M \rightarrow Y$ path. The special case of the first and second stage model when W is dichotomous is discussed in the final section of this article.

STATISTICAL INFERENCE

The computation of the index of moderated mediation is simple and requires only a few regression coefficients from the full model integrating the mediation and moderation components of the process being modeled. But this index is estimated using the data available and is subject to sampling variability. Answering the question as to whether an indirect effect is linearly moderated involves both description of the relationship between the moderator and the size of the indirect effect in the sample as well as inference about its size in the population being investigated.

The index of moderated mediation in all examples thus far is a product of two or more regression coefficients. With an estimate of the standard error of this product and a assuming a normal sampling distribution of the product, a Sobel-type test (Sobel, 1982, 1986) could be conducted to test the null hypothesis that the corresponding population value of the index is zero. Alternatively, a symmetric confidence interval for the index could be constructed as approximately the point estimate plus and minus two standard errors. Morgan-Lopez and MacKinnon (2006) and Wang and Preacher (in press) provide estimators of the standard error of $a_3 b$ in the first stage model, and Wang and Preacher (in press) derive the standard error for $a b_3$ in the second stage model. Their formulas generalize to inference about moderated mediation using these indices.[2]

But it is already well established that the sampling distribution of the product of regression coefficients is not normal (e.g., Bollen & Stine, 1990; Craig, 1936; Stone & Sobel, 1990), meaning the normal distribution is not the proper reference distribution for inference about the index of moderated mediation. There are several alternatives for inference about the product of regression coefficients that could be applied here (see, e.g., Biesanz, Falk, & Savalei, 2010; Fritz, Taylor, & MacKinnon, 2012; Hayes & Scharkow, 2013; MacKinnon, Fairchild, & Fritz, 2007; Preacher & Selig, 2012; Shrout & Bolger, 2002; Taylor & MacKinnon, 2012; Yuan & MacKinnon, 2009). I recommend the bootstrap confidence

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[2]Wang and Preacher (in press) recommend Bayesian credible intervals for inference rather than using the standard error and assuming normality of the sampling distribution of the product.

interval because bootstrapping is already widely used in statistical mediation analysis, its performance has been extensively studied and shown to be superior to the Sobel test, it is simple to understand, and it is already implemented in software that substantive researchers are using for mediation analysis. But it is not the only option available.

To generate a bootstrap confidence interval for the index of moderated mediation, a bootstrap sample of the original data is generated, the regression coefficients for the statistical model are estimated in this bootstrap sample, and the index of moderated mediation is calculated. Repeated k times, where k is preferably at least $1,000$ (but more is better. In the examples later in this article, I use $k = 10,000$), the end points of a 95% bootstrap confidence interval are the two values of the index in the distribution of k values that define the 2.5th and 97.5th percentiles of the distribution. If desired, the endpoints of the confidence interval could be adjusted using bias-correction or bias-correction and acceleration (see, e.g., Efron, 1987; Hayes, 2013; MacKinnon, 2008). If the confidence interval includes zero, then one cannot exclude no relationship between the moderator and the indirect effect from the realm of plausibility, meaning no definitive evidence of moderation of the mediation of X's effect on Y through M. But if the confidence interval does not include zero, this leads to the inference that the relationship between the indirect effect and the moderator is not zero—moderated mediation.

Any statistics program capable of estimating the coefficients in a linear model and that can bootstrap functions of model coefficients could be programmed to calculate the index of moderated mediation and produce a bootstrap confidence interval for this index. In the appendix, I provide code for Mplus for the three empirical examples described in the next section. Researchers more comfortable with the SPSS or SAS computing environment will find this method implemented in the freely-available PROCESS macro described and documented in Hayes (2013), and example PROCESS commands can also be found in the appendix.

THREE EMPIRICAL EXAMPLES

I next illustrate the application of this index and test of moderated mediation using three data sets. Two are models with a single mediator that differ with respect to whether moderation is estimated in the first stage ($X \rightarrow M$) or second stage ($M \rightarrow Y$) of the mechanism. The third is a moderated serial mediation model with two mediators and all paths from X to mediators and outcome estimated as moderated by a common variable.

An Illustration with First Stage Moderation of the Effect of X on M

The first illustration is based on a mediation model that includes moderation of the effect of X on mediator M by moderator variable W. The data come from Goodin et al.

(2009), who conducted a study examining "pain catastrophizing" as a psychological mechanism by which strenuous physical activity has analgesic effects on people's perceptions of pain. According to some existing research, people who engage in intense physical exertion such as strenuous exercise develop a reduced inclination to worry about or internally exaggerate the noxious physical experiences that can result relative to those who do not engage in such physical exertion. This reduced contemplation about the negative experience of pain in turn translates into a bona-fide reduction in the subjective experience of pain relative to those higher in the tendency to engage in pain catastrophizing. Thus, the indirect effect of strenuous activity on perceptions of pain through pain catastrophizing is negative according to this process. However, this process was hypothesized by Goodin et al. (2009) to be at work more so among those high in anxiety. By their argument, individual differences in strenuous activity frequency would be more strongly related to pain catastrophizing among those who are relatively more anxious because people who are lower in anxiety are less inclined to catastrophize in response to the potential for pain in the first place.

The 79 participants in the study came to a laboratory and were first measured with respect to how much intense, strenuous exercise they had engaged in during the last week (X), and they also filled out two standardized inventories measuring pain catastrophizing (M) and anxiety (W). Participants were subsequently instructed to immerse a hand in near-freezing ($4\,^{\circ}$C) water for at least two minutes, after which they responded to a series of questions gauging their experience of pain (Y) during the cold water hand immersion task.

The model is represented in conceptual form in Figure 3, panel A and in the form of a path diagram in Figure 3, panel B. Not depicted in the conceptual model but a part of the statistical model are three covariates (U_1: sex, U_2: depressive symptoms, and U_3: hand immersion time in seconds) included in the analysis to statistically remove these potential confounding influences on the paths in the process model (see Hayes, 2013, pp. 172–183).

The path diagram represents two linear equations

$$M = i_M + a_1 X + a_2 W + a_3 XW + a_4 U_1 + a_5 U_2$$
$$+ a_6 U_3 + e_M \tag{13}$$
$$Y = i_Y + c'X + b_1 M + b_2 U_1 + b_3 U_2 + b_4 U_3 + e_Y \tag{14}$$

the parameters of which can be estimated using OLS regression or structural equation modeling. In this illustration and all that follow, variables that are used to construct products are first mean centered prior to constructing their product in order to render the regression coefficients for those variables in tables provided interpretable within the range of the data. The decision to center or not has no effect on the value of the index of moderated mediation or inference about its size.

The estimated regression coefficients are displayed in Table 1 (see the appendix for code using the PROCESS macro
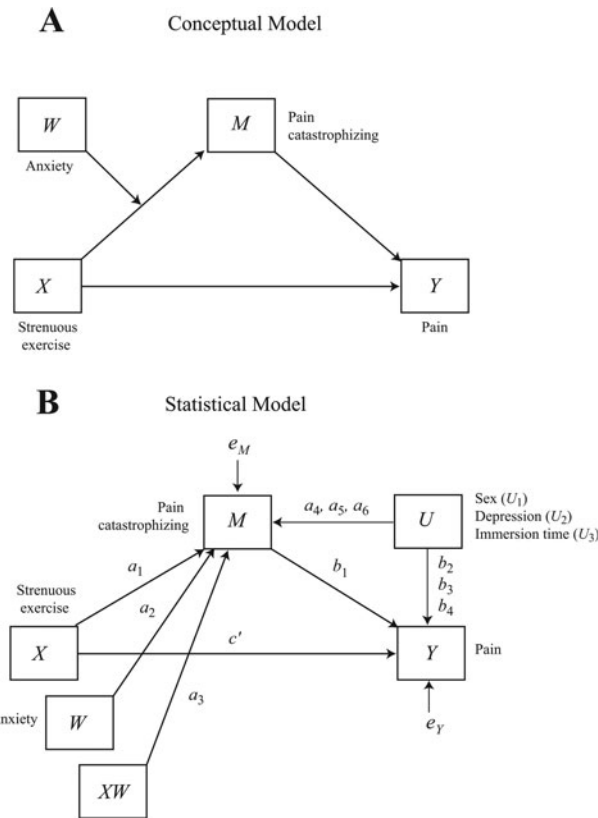
FIGURE 3 A first stage moderated mediation model in conceptual form (panel A) and in the form of a statistical model (panel B) as estimated in the pain catastrophizing example.

for SPSS and SAS as well as Mplus code that estimates the model). As can be seen, holding constant physical activity, sex, depression, and immersion time, participants relatively lower in pain catastrophizing report less intense experience of pain relative to those higher in pain catastrophizing ($b_1 = 0.328$, 95% CI = 0.219 to 0.437, $p < .001$). However, a test of moderation of the effect of strenuous physical activity on

pain catastrophizing by anxiety yields a nonsignificant result ($a_3 = -1.089$, 95% CI = -2.295 to 0.116, $p = 0.076$), although the bulk of the confidence interval in the predicted direction. As a moderated mediation analysis is most typically interpreted, this finding would yield the conclusion that the indirect effect of strenuous exercise on the experience of pain through the psychological mechanism of pain catastrophizing is *not* moderated by anxiety. That is, because the confidence interval for the regression coefficient of the product of X and W includes zero, one cannot definitely claim that anxiety is moderating any mediation of the effect of strenuous activity on the subjective experience of pain by pain catastrophizing.

But a nonsignificant interaction in this analysis (i.e., a confidence interval for the regression coefficient for XW that includes zero) does not imply the indirect effect is not moderated by W because a_3 does not quantify the relationship between the moderator and the indirect effect. In this model, a_3 estimates only moderation of the effect of X on M by W. A formal test of moderated mediation based on a quantification of the relationship between the proposed moderator and the size of the indirect effect is required to determine whether the indirect effect depends on the moderator.

In this model, the indirect effect of X on Y through M is a function, defined as the product of the conditional effect of X on M from Equation (13) and the effect of M on Y controlling for X from Equation (14):

$$\omega = (a_1 + a_3 W)b_1 = a_1 b_1 + a_3 b_1 W = -2.097 - 0.357W, \tag{15}$$

which is a linear of function of W with intercept $a_1 b_1 = -2.097$ and slope $a_3 b_1 = -0.357$. This function is depicted graphically in Figure 4. As can be seen, the indirect effect of strenuous physical activity on pain experience through pain catastrophizing seems to decrease with increasing anxiety, as the slope of the line—the index of moderated mediation—is negative.

TABLE 1
Unstandardized OLS Regression Coefficients With Confidence Intervals (Standard Errors in Parentheses) Estimating Pain Catastrophizing and Experience of Pain. Strenuous Exercise and Anxiety are Mean Centered

		Pain Catastrophizing (M)			Pain Experience (Y)	
		Coeff.	95% CI		Coeff.	95% CI
Physical Activity (X)	$a_1 \rightarrow$	-6.392 (4.481)	-15.324, 2.540	$c' \rightarrow$	-3.294 (2.158)	-7.594, 1.007
Pain Catastrophizing (M)				$b_1 \rightarrow$	0.328^* (0.055)	0.219, 0.437
Anxiety (W)	$a_2 \rightarrow$	0.328 (0.212)	-0.095, 0.751			
$X \times W$	$a_3 \rightarrow$	-1.089^+ (0.605)	-2.295, 0.116			
Sex (U_1)	$a_4 \rightarrow$	-3.901 (2.646)	-9.176, 1.375	$b_2 \rightarrow$	-2.733^* (1.272)	-5.268, -0.198
Depressive Symptoms (U_2)	$a_5 \rightarrow$	0.112 (0.298)	-0.481, 0.706	$b_3 \rightarrow$	-0.010 (0.129)	-0.267, 0.247
Immersion Time (U_3)	$a_6 \rightarrow$	-0.030^+ (0.017)	-0.065, 0.005	$b_4 \rightarrow$	-0.008 (0.008)	-0.024, 0.009
Constant	$i_M \rightarrow$	24.880^{***} (0.358)	17.766, 31.993	$i_Y \rightarrow$	13.209^{***} (2.145)	8.935, 17.483
		$R^2 = 0.224$			$R^2 = 0.487$	
		$F(6, 72) = 3.454, p = .005$			$F(5, 73) = 13.883, p < .001$	

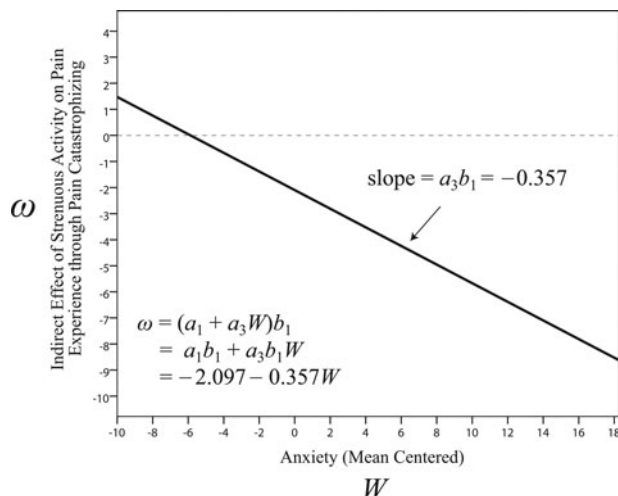$^+ p < .10, ^* p < .05, ^{**} p < .01, ^{***} p < .001.$

FIGURE 4   A visual representation of the linear function relating anxiety to the indirect effect of strenuous exercise on subjective experience of pain through pain catastrophizing.

But this index is estimated from the data and subject to sampling variation. Whether this index is different from zero most directly answers the question of interest—whether anxiety moderates the indirect effect. If so, then the indirect effect of X on Y through M is not independent of W but, rather, *depends* on W—the definition of moderated mediation given earlier.

The Mplus and PROCESS code in the appendix generates a bootstrap confidence interval for the index of moderated mediation. A 95% bootstrap confidence interval for this index—the slope of this function—is -0.759 to -0.025. As this confidence interval does not include zero, and with the upper bound negative, the conclusion is that the indirect effect of strenuous activity on pain experience through pain catastrophizing is negatively moderated by anxiety.

An Illustration with Second Stage Moderation of the Effect of M on Y

I next illustrate the application of this approach to a second stage moderated mediation model. The data come from a study conducted by Peltonen, Quota, Sarraj, and Punamäki (2010) of 227 Palestinian children living in Gaza during the Al-Aqsa Intifada. This analysis (which is not the same analysis Peltonin et al. conducted) is based on only the 113 boys who participated. The boys were asked a series of questions to gauge their exposure to various traumatic events (e.g., shelling of their home, witnessing killings) and were also administered standardized scales quantifying symptoms of posttraumatic stress disorder (PTSD) and depression. Finally, their loneliness with respect to peer relationships was also measured.

The model for this example is represented in conceptual form in Figure 5, panel A. This diagram represents a process in which the depression that results from exposure to
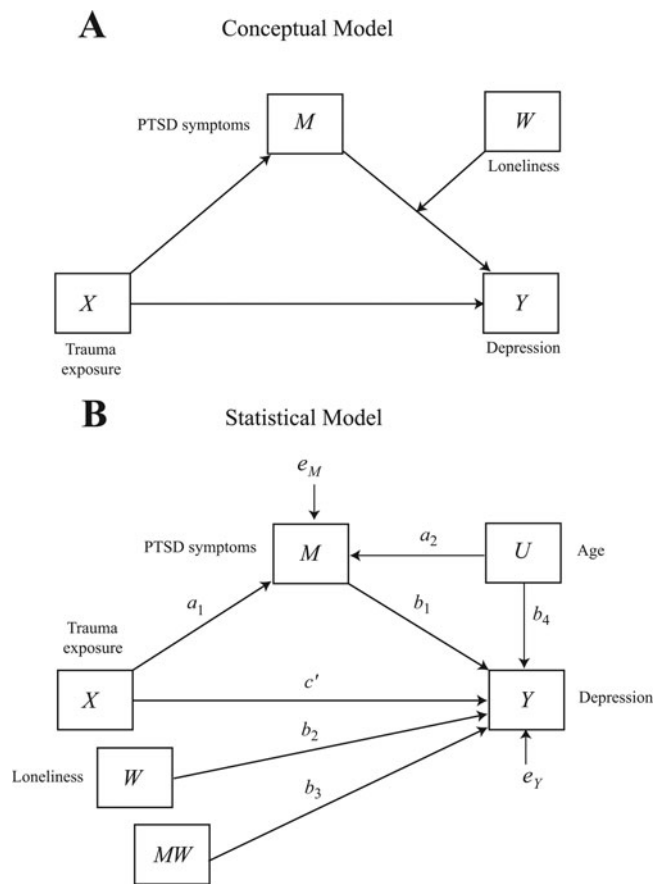
FIGURE 5   A second stage moderated mediation model in conceptual form (panel A) and in the form of a statistical model (panel B) as estimated in the trauma and mental health example.

traumatic events (X) occurs through the effect of trauma on the development of symptoms of PTSD (M), which in turn leads to depression (Y). However, the link between PTSD and depression—the second stage of the mechanism—is modeled as moderated by peer loneliness (W).

This process is modeled with two equations, one for PTSD symptoms and one for depression:

$$M = i_M + a_1 X + a_2 U + e_M \qquad (16)$$

$$Y = i_Y + c' X + b_1 M + b_2 W + b_3 MW + b_4 U + e_Y \quad (17)$$

where U is the age of the child and used as a covariate (not depicted in the conceptual diagram). This system of equations is represented visually in the form of a path diagram in Figure 5, panel B.

Table 2 presents the estimated regression coefficients. As can be seen, holding age constant, boys with relatively more exposure to trauma expressed higher symptoms of PTSD, $a_1 = 0.590$, 95% CI $= 0.019$ to 1.162, $p = .043$. Furthermore, holding constant age and trauma exposure, the effect of PTSD symptoms on depression depends on loneliness, $b_3 = 0.130$, 95% CI $= 0.033$ to 0.228, $p = .009$. By the reasoning that evidence of moderation of one of the paths

TABLE 2
Unstandardized OLS Regression Coefficients With Confidence Intervals (Standard Errors in Parentheses) Estimating PTSD
Symptoms and Depression. PTSD Symptoms and Loneliness are Mean Centered

		PTSD Symptoms (M)			Depression (Y)	
		Coeff.	95% CI		Coeff.	95% CI
Trauma Exposure (X)	$a_1 \rightarrow$	0.590* (0.288)	0.019, 1.162	$c' \rightarrow$	−0.246 (0.154)	−0.551, 0.059
PTSD Symptoms (M)				$b_1 \rightarrow$	0.050 (0.050)	−0.049, 0.148
Loneliness (W)				$b_2 \rightarrow$	2.816*** (0.624)	1.580, 4.053
$M \times W$				$b_3 \rightarrow$	0.130* (0.049)	0.033, 0.228
Age (U_1)	$a_2 \rightarrow$	−4.523*** (1.020)	−6.543, −2.502	$b_4 \rightarrow$	1.065[+] (0.580)	−0.084, 2.214
Constant	$i_M \rightarrow$	47.829*** (11.793)	24.459, 71.199	$i_Y \rightarrow$	−0.025 (6.615)	−13.139, 13.088
		$R^2 = 0.166$			$R^2 = 0.209$	
		$F(2, 110) = 10.956, p < .001$			$F(5, 107) = 5.539, p < .001$	

[+]$p < .10$, *$p < .05$, **$p < .01$, ***$p < .001$.

in a mediation model is sufficient to claim moderation of mediation, this analysis as typically interpreted supports the conclusion that the indirect effect of trauma on depression through PTSD symptoms depends on loneliness.

But evidence of interaction between PTSD and loneliness does not necessarily establish whether the indirect effect depends on loneliness, as the relationship between the indirect effect of trauma and loneliness is not estimated with b_3. The indirect effect in this model is the product of the effect of X on M from Equation (16) and the conditional effect of M on Y from Equation (17):

$$\omega = a_1(b_1 + b_3 W) = a_1 b_1 + a_1 b_3 W = 0.030 + 0.077 W$$

which is a linear of function of W with intercept $a_1 b_1 = 0.030$ and slope $a_1 b_3 = 0.077$. This function is depicted visually in Figure 6. This slope of this line is the weight in the function linking the indirect effect to the moderator—the index of moderated mediation. It is positive, meaning that the indirect

effect of traumatic exposure on depression through PTSD is an increasing function of loneliness.

A bootstrap confidence interval for the index of moderated mediation that does not include zero provides more direct and definitive evidence of moderation of the indirect effect of trauma by loneliness than does a test of moderation of one of its paths. The PROCESS and Mplus code in the appendix constructs this bootstrap confidence interval. In this case, a 95% bootstrap confidence interval based on 10,000 bootstrap samples includes zero (−0.008 to 0.182). Although the bulk of the interval is above zero, we cannot definitively say (i.e., with 95% confidence) that the indirect effect depends on loneliness because the confidence interval for the index of moderated mediation includes zero.

An Illustration in a Serial Multiple Mediator Model

This last illustration shows how to apply this test to a serial multiple mediator model that includes moderation of one or more of the paths. A serial multiple mediator model has more than one indirect effect from X to Y through at least one mediator, depending on the number of mediators and which paths are freely estimated versus fixed to zero (for a discussion, see Hayes, 2013). The data come from a telephone survey of attitudes of the U.S. public regarding national security, terrorism, and stereotypes about minority groups (Nisbet, Ortiz, Miller, & Smith, 2011). The survey was administered between April and June 2011 and included various questions about endorsement of negative stereotypes about Muslim Americans (such as "fanatical," "violent"), perceptions as to how much this group represents a threat to national security, and public willingness to restrict their civil liberties (by, for example, requiring Muslims in the US to have special ID cards). Variables are scaled in the data such that higher values reflect more negative stereotypes, greater perceived threat, and higher willingness to restrict Muslim American civil liberties.

About halfway through data collection, U.S. President Barack Obama announced the death in Pakistan of Osama bin
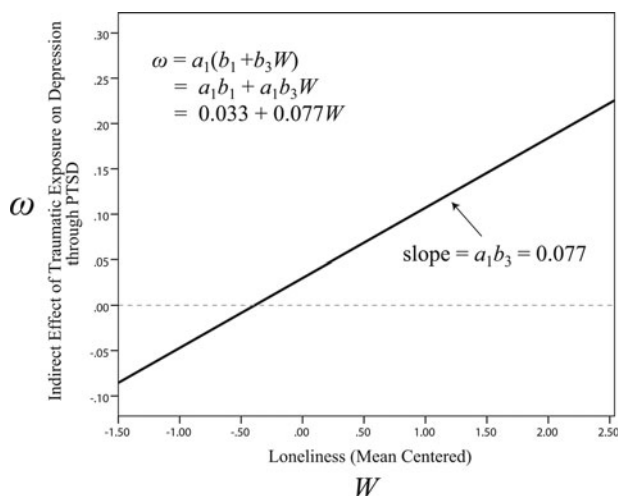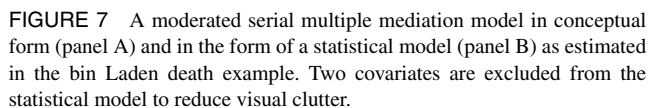

FIGURE 6   A visual representation of the linear function relating loneliness to the indirect effect of trauma exposure on depression through PTSD symptoms.

Laden, the mastermind behind the terrorist attack on the US in 2001. Much media coverage followed this announcement and its effects on national security, as well as background stories about Islam, terrorism, and so forth. As participants in the survey were in effect randomly assigned to when they were called, these data allow for an experimental test of the effect of a major national security event and its media coverage on public beliefs about Muslim Americans and national policy.

The model estimated in this example is depicted in conceptual form in Figure 7 panel A, which represents a process in which Osama bin Laden's death and following media coverage (X; 0 = interviewed before death, 1 = interviewed after death) purportedly affects public endorsement of negative stereotypes about Muslims (M_1) which in turn affects beliefs about the threat of Muslims to national security (M_2), which in turn influences willingness to restrict the civil liberties of Muslims living in the US (Y).

The model includes all possible indirect effects between X and Y and also allows the effects of X on M_1, M_2, and Y to be linearly moderated by participant age (W). Not included in the diagram are two covariates: sex of the respondent (U_1) and political ideology (U_2, scaled 1 to 7 with higher reflecting greater conservatism).

A   Conceptual Model

B   Statistical Model

FIGURE 7   A moderated serial multiple mediation model in conceptual form (panel A) and in the form of a statistical model (panel B) as estimated in the bin Laden death example. Two covariates are excluded from the statistical model to reduce visual clutter.

Figure 7, panel B depicts the model in path diagram form, which translates into three linear equations:

$$M_1 = i_{M_1} + a_{11}X + a_{21}W + a_{31}XW + a_{41}U_1 + a_{51}U_2 + e_{M_1} \tag{18}$$

$$M_2 = i_{M_2} + a_{12}X + a_{22}W + a_{32}XW + dM_1 + a_{42}U_1 + a_{52}U_2 + e_{M_2} \tag{19}$$

$$Y = i_Y + c_1'X + c_2'W + c_3'XW + b_1M_1 + b_2M_2 + b_3U_1 + b_4U_2 + e_Y \tag{20}$$

The estimated regression coefficients are displayed in Table 3. The signs of the coefficients for the product terms (a_{31}, a_{32}, c_3') reflect a trend for the time of interview (before or after Osama bin Laden's death) to have a larger positive effect on endorsement of stereotypes, perceived threat, and civil liberties restrictions among the relatively younger participants. However, only bin Laden's death on stereotype endorsement is moderated by age by a formal inferential test of moderation ($a_{31} = -0.083$, 95% CI $= -0.159$ to -0.008, $p = 0.031$). Furthermore, stereotype endorsement is positively related to perceived threat ($d = 0.700$, 95% CI $= 0.625$ to 0.774, $p < .001$) and support for restriction of Muslim American civil liberties ($b_1 = 0.105$, 95% CI $= 0.014$ to 0.195, $p = .024$), and perceived threat of Muslims is positively related to support for such restrictions ($b_2 = 0.547$, 95% CI $= 0.471$ to 0.623, $p < .001$).

But it is the product of paths and not the paths themselves that define the indirect effects, of which there are three in this model. The three indirect effects are calculated as the products of paths linking X to Y through at least one mediator. The specific indirect effect through only stereotype endorsement (M_1) is the product of the function linking time of interview to stereotype endorsement ($a_{11} + a_{31}W$) and the effect of stereotype endorsement on restriction of Muslim American civil liberties (b_1):

$$\omega_{M_1} = (a_{11} + a_{31}W)b_1 = a_{11}b_1 + a_{31}b_1W = 0.014 - 0.009W \tag{21}$$

which is a linear function of age with intercept $a_{11}b_1 = 0.014$ and slope $a_{31}b_1 = -0.009$. The index of moderation for this specific indirect effect is $a_{31}b_1 = -0.009$.

The specific indirect through only perceived threat (M_2) is calculated similarly as the product of the function linking time of interview to perceived threat ($a_{12} + a_{32}W$) and the effect of perceived threat on restriction of civil liberties (b_2):

$$\omega_{M_2} = (a_{12} + a_{32}W)b_2 = a_{12}b_2 + a_{32}b_2W = 0.021 - 0.034W \tag{22}$$

This too is a linear function of age, with intercept $a_{12}b_2 = 0.021$ and slope $a_{32}b_2 = -0.034$. The index of moderation for this specific indirect effect is $a_{32}b_2 = -0.034$.

TABLE 3
Unstandardized OLS Regression Coefficients With Confidence Intervals (Standard Errors in Parentheses) Estimating
Stereotype Endorsement, Perceived Threat of Muslims, and Restriction of Muslim American Civil Liberties. Pre/Post Bin Laden
Death and Age (in 10s of Years) are Mean Centered

		Stereotype Endorsement (M_1)			Perceived Threat (M_2)			Civil Liberties Restrictions (Y)	
		Coeff.	95% CI		Coeff.	95% CI		Coeff.	95% CI
Pre or Post (X) Death	$a_{11} \rightarrow$	0.136* (0.064)	0.011, 0.261	$a_{12} \rightarrow$	0.038 (0.062)	−0.083, 0.160	$c'_1 \rightarrow$	−0.031 (0.061)	−0.151, 0.089
Stereotype (M_1) Endorsement				$d \rightarrow$	0.700*** (0.038)	0.625, 0.774	$b_1 \rightarrow$	0.105* (0.046)	0.014, 0.195
Perceived (M_2) Threat							$b_2 \rightarrow$	0.547*** (0.039)	0.471, 0.623
Age (W)	$a_{21} \rightarrow$	0.049* (0.019)	0.011, 0.087	$a_{22} \rightarrow$	0.044* (0.019)	0.008, 0.081	$c'_2 \rightarrow$	−0.011 (0.019)	−0.047, 0.026
$X \times W$	$a_{31} \rightarrow$	−0.083* (0.039)	−0.159, −0.008	$a_{32} \rightarrow$	−0.062 (0.038)	−0.136, 0.012	$c'_3 \rightarrow$	−0.030 (0.037)	−0.103, 0.043
Sex (U_1)	$a_{41} \rightarrow$	0.039 (0.063)	−0.086, 0.163	$a_{42} \rightarrow$	0.128* (0.061)	0.008, 0.248	$b_3 \rightarrow$	−0.100+ (0.061)	−0.219, 0.019
Ideology (U_2)	$a_{51} \rightarrow$	0.130*** (0.014)	0.102, 0.158	$a_{52} \rightarrow$	0.091*** (0.015)	0.062, 0.120	$b_4 \rightarrow$	0.055*** (0.015)	0.026, 0.084
Constant	$i_{M_1} \rightarrow$	2.201*** (0.089)	2.026, 2.376	$i_{M_2} \rightarrow$	−0.012 (0.120)	−0.248, 0.224	$i_Y \rightarrow$	0.658*** (0.118)	0.426, 0.890
		$R^2 = 0.133$ $F(5, 655) = 20.045, p < .001$			$R^2 = 0.460$ $F(6, 654) = 92.765, p < .001$			$R^2 = 0.453$ $F(7, 653) = 77.289, p < .001$	

$+ p < .10, * p < .05, ** p < .01, *** p < .001.$

The remaining specific indirect effect operates through M_1 and M_2 in serial and is quantified as the product of the function linking time of interview to stereotype endorsement, the effect of stereotype endorsement on perceived threat (d), and the effect of perceived threat on restriction of civil liberties:

$$\omega_{M_1 M_2} = (a_{11} + a_{31}W)db_2 = a_{11}db_2 + a_{31}db_2 W$$
$$= 0.052 - 0.032W, \qquad (23)$$

which is a linear function with intercept $a_{11}db_2 = 0.052$ and slope $a_{31}db_2 = -0.032$. The index of moderation for this specific indirect effect is $a_{31}db_2 = -0.032$.

The functions linking age to each of the specific indirect effects [Equations (21), (22), and (23)] are represented visually in Figure 8, with the indices of moderated mediation corresponding to the slopes of the lines. A bootstrap confidence interval for each slope provides an inference as to whether the specific indirect effect is moderated by age. The code in the appendix produces these bootstrap confidence intervals based on 10,000 resamples. For the serial indirect effect through both stereotype endorsement and perceived threat, the index of moderated mediation is negative with 95% confidence (−0.003 to −0.062), so we can fairly definitively say that this indirect effect depends on age. But the same cannot be said for the other two indirect effects.

A confidence interval for the index of moderated mediation for the specific indirect effect through stereotype endorsement alone is almost entirely negative but just barely crosses zero (−0.024 to 0.0004), meaning that no moderation of this specific indirect by age cannot be definitively ruled out. Similarly, the confidence interval for the index of moderated mediation through perceived threat alone includes zero and some positive values (−0.062 to 0.004), and thus no moderation of this indirect effect by age is plausible.

PROBING MODERATED MEDIATION AND COMPARING CONDITIONAL INDIRECT EFFECTS

With evidence of moderation of an indirect effect, follow up tests can be used to "probe" moderated mediation if desired, just as is typically done in ordinary moderation analysis with an analysis of simple slopes (see, e.g., Aiken & West, 1991; Hayes, 2013). This could involve either estimating the conditional indirect effect at various values of the moderator followed by an inferential test, or a comparison of two conditional indirect effects to see if they differ from each other.

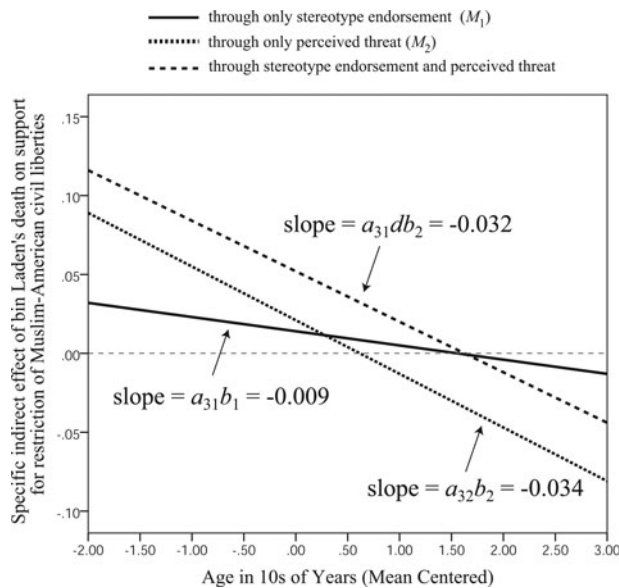Fairchild and MacKinnon (2009) discuss the comparison of indirect effects when the moderator is dichotomous by

FIGURE 8 A visual representation of the linear functions relating age to the indirect effects of bin Laden's death on support for willingness to restrict Muslim American civil liberties.

estimating the indirect effect in each group separately and conducting a hypothesis test about the difference. With a dichotomous moderator, this is a test of moderated mediation itself rather than a means of probing it. If the moderator is not dichotomous, use of their approach would require artificial categorization of the moderator prior to separate analyses of each artificially constructed group, a procedure hard to justify (nor one they recommend). Edwards and Lambert (2007) and Wang and Preacher (in press) describe the comparison of conditional indirect effects with a continuous moderator that requires the investigator to choose two values of the moderator at which to condition the estimation and inference. As with an analysis of simple slopes, it typically isn't apparent which values of the moderator are meaningful, and the choice of moderator values is often made arbitrarily by relying on conventional operationalizations of low, moderate, or high on the moderator (e.g., the mean as well as a standard deviation below and above the mean).

But such a comparison of specific indirect effects is not necessary if one has established that the indirect effect is moderated based on an inference about the index of moderated mediation. Just as in ordinary moderation analysis, where interaction implies that any two simple slopes are different from each other (see Aiken & West, 1991, p. 21), evidence that the moderator is linearly related to the indirect effect implies that *any* two conditional indirect effects defined by different values of the moderator are statistically different. Conversely, if one cannot conclude that the indirect effect is moderated, this implies that no two conditional indirect effects can be deemed different from each other. This is neither obvious nor intuitive. I illustrate the logic and mathematics of this claim using the first stage moderated mediation

model with a single mediator, but it generalizes to all of the models described thus far.

In the first stage moderated mediation model, the conditional indirect effect of X on Y through M when $W = w_1$ is $(a_1 + a_3 w_1)b_1$ and when $W = w_2$ it is $(a_1 + a_3 w_2)b_1$ [from Equation (15)]. Their difference is

$$\Delta \omega \mid (w_2, w_1) = (a_1 + a_3 w_2)b_1 - (a_1 + a_3 w_1)b_1$$
$$= a_3 b_1 (w_2 - w_1),$$

which is the index of moderated mediation multiplied by the difference between the moderator values. But because $w_2 - w_1$ is a constant in all bootstrap samples, whether the bootstrap confidence interval includes zero is determined only by the bootstrap distribution of $a_3 b_1$. It makes no difference which two values of w_1 and w_2 are used (so long as $w_2 \neq w_1$). If the bootstrap confidence interval for the index of moderated mediation does not include zero, then neither does the interval for the index multiplied by $(w_2 - w_1)$. Conversely, if the bootstrap confidence interval for the index of moderated mediation includes zero, so too does the interval for the index multiplied by $(w_2 - w_1)$.

Applying this reasoning to the first illustration, the claim based on the index of moderated mediation that anxiety moderates the indirect effect of strenuous activity on pain through pain catastrophizing leads to the additional claim that any two indirect effects conditioned on different values of anxiety are statistically different from each other. Thus, there is no need to statistically compare indirect effects conditioned on different values of anxiety.

A similar derivation in the second stage moderated mediation model leads to the claim in the second illustration that one cannot definitively claim that any two indirect effects of traumatic exposure on depression through PTSD symptoms conditioned on different values of loneliness are statistically different from each other. This is because whether the bootstrap confidence interval for the difference between conditional indirect effects in this model contains zero is determined by whether the bootstrap confidence interval for the index of moderated mediation contains zero. A comparable logic applies to the models with multiple mediators, whether operating in parallel or in serial (as in the bin Laden effect example).

But this test need not replace the procedure described by Preacher et al. (2007) for probing moderated mediation. They describe a direct analogue of a simple slopes analysis that involves estimating the conditional indirect effect of X on Y through M for various values of moderator W and then testing whether the indirect effect is different from zero at those moderator values. Applying their procedure to the pain catastrophizing example, plugging values of anxiety into Equation (15) corresponding to a standard deviation below the mean, the mean, and a standard deviation above the mean ($W = -6.889, 0.000,$ and 6.889) results in conditional indirect effects of strenuous exercise on pain experience

through pain catastrophizing equal to 0.365, −2.097, and −4.561, respectively. A bootstrap confidence interval for each of these conditional indirect effects excludes zero only when anxiety is relatively high, 95% confidence interval for $\omega \mid (W = 6.889) = -9.412$ to -0.430. That is, only among the more anxious can we say fairly definitively that there is a *negative* indirect effect of strenuous exercise on pain experience through pain catastrophizing.

Knowledge that the indirect effect is different from zero for certain values of the moderator but not others can be useful supplementary information when describing the conditional nature of a mechanism, and thus Preacher et al. (2007)'s procedure can be a useful means of probing the moderation of mediation. In principle, one could engage in such conditional estimation and inference even absent evidence of moderation of the indirect effect, but moderation cannot be inferred from a pattern of significant and nonsignificant conditional indirect effects. Difference in significance does not imply significantly different (Gelman & Stern, 2006). Thus, it is important to choose one's words carefully so as not to imply moderation of the indirect effect based on the pattern observed when a formal test of moderation of the indirect effect has not been conducted or when such a test reveals no definitive evidence of moderation.

Extensions, Cautions, and Future Research

The index of moderated mediation described and illustrated in this article quantifies the relationship between a proposed moderator and the indirect effect of X on Y through M. It applies to any mediation model in which the indirect effect is estimated as linearly moderated, including many of the models described by Edwards and Lambert (2007), Preacher et al. (2007), and others. An interval estimate of the index of moderated mediation provides an inferential test as to whether the indirect effect depends linearly on the moderator. This test is easy to conduct using SEM software such as Mplus, and it is implemented in the PROCESS macro for SPSS and SAS. The mathematics of this method apply whether the moderator is dichotomous or continuous. Though not illustrated in the prior examples, when the moderator is dichotomous, this test of moderated mediation serves as a test of difference between the conditional indirect effects in the two groups coded by the moderator variable.

Echoing a point first made by Fairchild and MacKinnon (2009), the outcome of an inference about moderation of a specific path in a mediation model does not automatically lead to a corresponding inference about moderation of the corresponding indirect effect. However, the outcome of an inference about moderation of a single path in a mediation model may be diagnostic of the likely outcome of the test described here. The relationship between the indirect effect and the moderator includes the regression coefficient for a product term in the model, but this coefficient does not quantify the relationship between the indirect effect and the modera-

tor. In two of my illustrations, it was shown that even without evidence of moderation of one of the paths, the indirect effect may still be moderated and, by the same token, just because a path is moderated, that does not mean the indirect effect is. Thus, one should be cautious testing moderation of an indirect effect by relying only on a test of moderation of one of the constituent paths.

In addition, I illustrated that this test of moderated mediation can also be interpreted as an inferential test of the difference between any two conditional indirect effects that can be constructed from the model coefficients and different values of the moderator. Thus, there is no need to probe the moderation of an indirect effect by comparing conditional indirect effects derived from the model to each other. If the indirect effect is moderated, then a corollary inference is that any two conditional indirect effects from the model are different. Conversely, if the indirect effect is not moderated, then no two conditional indirect effects can be said to be different from each other.

Relaxation of the often-used requirement for claiming moderated mediation that a path in the model must be moderated according to a formal statistical test is likely to be of comfort to most researchers and not particularly controversial, seeing as tests of linear interaction in regression analysis are notoriously low in power. Less comforting, and more controversial, will be the occasion when a hypothesis test or confidence interval supports a claim that the $X \rightarrow M$ or $M \rightarrow Y$ path is moderated but a confidence interval for the index of moderated mediation includes zero. Which outcome to trust will depend on one's modeling and analysis philosophy. I have argued that an inference about the index of moderated mediation, as a direct estimator of the linear relationship between moderator and indirect effect, is most pertinent and should be given more weight in the judgment when a conflict arises. This is consistent with the current movement in inference in mediation analysis away from tests of the individual paths in the model toward tests of the indirect effect as quantified with the product of paths (see, e.g., Hayes, 2009; Hayes & Preacher, 2013; MacKinnon, Lockwood, & Williams, 2004; Zhao, Lynch, & Chen, 2010). But not everyone will agree with this perspective.

Of course, the possibility of contradictory outcomes of related tests in the same analysis is not unique to this problem. For instance, different methods used for inference about indirect effects in unmoderated mediation models also can conflict, although they tend to agree more often than they disagree (see Hayes & Scharkow, 2013). Research examining the causes of contradictions between the outcome of the test described here and the outcome of other approaches is worth undertaking. For instance, in the first two examples, perhaps the differences in conclusions yielded by the traditional piecemeal approach and the test I describe is caused exclusively by the use of bootstrap methods for inference about the index of moderated mediation but ordinary inferential procedures using OLS or ML estimators of standard

errors, and symmetric confidence intervals or p-values in the piecemeal approach. Given that bootstrapping a confidence interval for the index of moderated mediation does not require the estimation and use of an analytical standard error for any of the paths in the model, it may be less susceptible to inferential problems caused by such things as heteroscedasticity (see e.g., Long & Ervin, 2000). Or perhaps bootstrapping is more (or less) susceptible to high-leverage cases or other forms of outliers than traditional inferential methods.

This test is restricted to moderation models in which the indirect effect is a linear function of the moderator. As noted earlier, in the first and second stage moderation model that allows both the $X \rightarrow M$ and $M \rightarrow Y$ paths to be moderated by a common variable, the indirect effect is a nonlinear function of W [see Equation (12)]. In principle, a hypothesis test that the weights for W and W^2 in this function are both equal to zero could be construed as a formal test of moderated mediation in this model. But this could not be done with a single bootstrap confidence interval (or any other single-parameter test), and so this test of moderated mediation cannot be directly applied to such a model. When W is continuous, moderation of mediation can be tested in this model using the procedure described by Edwards and Lambert (2007) and Wang and Preacher (in press), which requires choosing two values of W and testing whether the indirect effects conditioned on these two values are statistically different from each other. If yes, this implies that the indirect effect depends on W. But if not, this does not mean W does not moderate the indirect effect because the outcome of this test, and therefore the inference about moderated mediation, will be dependent on which two values of W are chosen.

When moderator W is dichotomous, the index of moderated mediation corresponds to the difference between the indirect effects in the two groups coded by the moderator variable. In the first and second stage model when W is dichotomous and coded 0 and 1, $\Delta\omega \mid (w_2 = 1, w_1 = 0)$ based on Equation (12) simplifies to a $a_1b_3 + a_3b_1 + a_3b_3$ (see Hayes, 2013). A bootstrap confidence interval for this sum can be used as a test of moderated mediation in this model, is fairly easy to generate in Mplus, and is produced automatically by the PROCESS macro for SPSS and SAS.

All derivations, examples, and discussion thus far have been based on models of observed variables. But extension to latent variable models is straightforward. To do so, latent variables and their corresponding indicators replace observed variables in the derivations, and the mathematics for the computation of the index of moderated mediation generalizes without modification. The usual complexities associated with latent variable interactions apply whenever a product term in one of the equations involves a latent variable (for a discussion of latent variable interactions see, for example, Marsh, Wen, Hau, & Nagengast, 2013). An example of a fully latent second stage moderated mediation model, including Mplus code for estimation, can be found in Hayes and Preacher (2013).

In the examples presented here, the data were from cross-sectional studies or a natural experiment with condition confounded with time. Serious reservations have been expressed about the ability of mediation analysis when conducted using cross-sectional data and designs to properly recover the parameters of a process that evolves over time, as a mediation process by definition does (see, e.g., Cole & Maxwell, 2003; Maxwell & Cole, 2007). Maxwell, Cole, and Mitchell (2011) go so far as to characterize cross-sectional mediation analysis as "almost certainly futile" (p. 836). Autoregressive models with at least M and Y measured over time and that include variables at a prior time as statistical controls have advantages over cross-sectional designs (Cole & Maxwell, 2003; Selig & Preacher, 2009). But extending this method to autoregressive models is straightforward, and no modifications to the mathematics are required to accommodate lagged values as controls. Implementation requires simply adding these lags to the equations for M and Y in (for example) Mplus code or using the PROCESS macro. An alternative is to allow for the possibility that the paths in the model may be contingent on prior values. That is, rather than (or in addition to) controlling for information available in prior lags, use lag variables as moderators (Selig, Preacher, & Little, 2012). The method described here could be used to establish evidence as to whether the indirect effect is contingent on prior measurements of X, M, or Y, depending on how the model is specified.

Finally, bootstrap confidence intervals are not the only possible approach to inference for the index of moderated mediation, and I have provided no evidence that a bootstrap confidence interval is superior in this application relative to other methods that have been advanced for inference about the product of regression coefficients that could be used instead (such as permutation-based tests or Bayesian methods). I favor bootstrap confidence intervals here because they are frequently used in mediation analysis and are already implemented in popular software researchers are using. Furthermore, it is reasonable to generalize to this test what is already known about the superiority of bootstrap confidence intervals relative to methods that require the assumption that the sampling distribution of the index of moderated mediation is normal (as in Morgan-Lopez & MacKinnon, 2006). But research examining other methods is worth undertaking, especially when applying the method described here to more complex models with multiple mediators, latent variables, or lagged variables as moderators.

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APPENDIX

SPSS, SAS, and Mplus Implementation

This appendix contains code for SPSS, SAS, and Mplus that implements the analyses described in the examples. SPSS and SAS code assumes that the PROCESS macro has been activated. See Appendix A of Hayes (2013) for instructions.

Example 1: Strenuous Exercise and Pain Perception

Using PROCESS for SPSS, the command below estimates the model in Figure 3. The variable names in the data used in the PROCESS command are **pain** (Y: pain perception), **pcs** (M: pain catastrophizing), **anxiety** (W: anxiety), **activity** (X: strenuous activity), **sex** (U_1), **bdi** (U_2: depression), and **immerse** (U_3: immersion time in seconds).

```
process vars=pain pcs anxiety activity sex bdi immerse/y=pain/x=activity/m=pcs/w =anxiety/ model=7/boot=
    10000/percent=1/center=1.
```

The comparable command for the SAS version of PROCESS, assuming the data file is named "icy," is

```
%process (data=icy, vars=pain pcs anxiety activity sex bdi immerse, y=pain, x=activity, m=pcs,
    w=anxiety, model=7, boot=10000, percent=1, center=1);
```

This code mean centers strenuous exercise and anxiety prior to estimation and then generates the regression coefficients and standard errors reported in Table 1, along with conditional indirect effects at various values of the moderator and the index of moderated mediation (a_3b_1) with bootstrap confidence intervals using the percentile method (see excerpt below). Removing `percent=1` from the command produces a bias-corrected bootstrap confidence interval.

```
********INDEX OF MODERATED MEDIATION********
Mediator
        Index     SE(Boot)    BootLLCI    BootULCI
pcs    −.3575      .1830      −.7585      −.0249
        *********************************************
```

The Mplus code below estimates model coefficients and the index of moderated mediation. Mplus generates a standard error for the index using the delta method as well as a p-value assuming normality of the sampling distribution of a_3b_1. To generate a 95% bootstrap confidence interval for the index, remove the two exclamation points in the ANALYSIS: and OUTPUT: sections of the code. Replace (`bootstrap`) with (`bcbootstrap`) for bias-corrected rather than percentile bootstrap confidence intervals.

```
DATA:
    file is 'c:\pain.txt';
VARIABLE:
    names are sex bdi immerse anxiety activity pcs pain;
    usevariables are sex bdi immerse pcs pain actxanx anxmc actmc;
DEFINE:
    anxmc=anxiety-18.329;
    actmc=activity-0.538;
    actxanx=actmc*anxmc;
ANALYSIS:
    bootstrap=10000;
MODEL:
    pcs ON actmc (a1)
        actxanx (a3)
        anxmc sex bdi immerse;
    pain ON actmc sex bdi immerse
        pcs (b1);
MODEL CONSTRAINT:
    new (a3b1);
    a3b1=a3*b1;
OUTPUT:
    !cinterval (bootstrap);
```

Example 2: Trauma and Mental Health

The SPSS PROCESS code below estimates the model depicted in Figure 5 and produces output used to construct Table 2. The variable names in the data that are used in the PROCESS command are **depress** (*Y*: depression), **ptsd** (*M*: symptoms of posttraumatic stress disorder), **lonely** (*W*: loneliness), **trauma** (*X*: exposure to trauma), and **age** (*U*: age of child). PROCESS model 14 requires that the moderator in the second stage moderated mediation model be denoted as *V* in the syntax rather than *W*.

```
process vars=ptsd lonely trauma depress age/y=depress/m=ptsd/x=trauma/v=lonely/model=14/boot=10000/
   percent=1/center=1.
```

The comparable command for the SAS version of PROCESS is

```
%process (data=alaqsa,vars=ptsd lonely trauma depress age,y=depress,x=trauma,m=ptsd,v=lonely,model=14,
   boot=10000,percent=1,center=1);
```

The resulting output contains the regression coefficients, model summary information, conditional indirect effects at various values of the moderator, and the index of moderated mediation (a_1b_3) along with a bootstrap confidence interval for the index. See the output excerpt below.

```
********INDEX OF MODERATED MEDIATION********
Mediator
        Index    SE(Boot)    BootLLCI    BootULCI
ptsd    .0769     .0505      −.0077      .1820
        *********************************************
```

The following Mplus code estimates model coefficients as well as the index of moderated mediation. To generate a bootstrap confidence interval for the index of moderated mediation, remove the two exclamation points (!) in the ANALYSIS: and OUTPUT: sections of the code.

```
DATA:
   file is 'c:\alaqsa.txt';
   VARIABLE:
   names are age ptsd lonely trauma depress;
   usevariables are age trauma depress ptsdmc lonelymc ptsdxlon;
DEFINE:
   ptsdmc=ptsd-28.212;
   lonelymc=lonely-2.231;
   ptsdxlon=ptsdmc*lonelymc;
ANALYSIS:
   !bootstrap=10000;
MODEL:
   ptsdmc ON age
        trauma (a1);
   depress ON trauma ptsdmc lonelymc age
        ptsdxlon (b3);
MODEL CONSTRAINT:
   new (a1b3);
   a1b3=a1*b3;
OUTPUT:
   !cinterval (bootstrap);
```

Example 3: The Bin Laden Effect

The PROCESS macro does not have any models preprogrammed that combine moderation with serial mediation. However, it can be tricked into estimating the model in Figure 7 by using a moderator and product as covariates in an unmoderated serial mediation model (PROCESS model 6). The **save** option produces a file of bootstrap estimates of all the regression coefficients that can be used to construct confidence intervals for the indices of moderated mediation.

The SPSS code below accomplishes the analysis and produces the output summarized in Table 3. The variables used in the model are **mcivil** (Y: restriction on civil liberties), **binladen** (X: interviewed before (0) or after (1) Osama Bin Laden's death), **stereo** (M_1: negative stereotype endorsement), **rthreat** (M_2: perceived threat of Muslims), **age** (W: age), **sex** (U_1: gender), and **ideo** (U_1: political ideology). The first few lines of code mean center X and W and then construct the product of these mean centered variables.

```
compute binmc=binladen-0.410.
compute agemc=age-4.846.
compute binage=binmc*agemc.
process vars=binmc mcivil rthreat stereo agemc ideo sex binage/y=mcivil/x=binmc/m=stereo rthreat/
   model=6/boot=10000/save=1/percent=1.
```

Upon execution, PROCESS generates a data file containing the 10,000 bootstrap estimates of all 21 regression coefficients (three intercepts and 18 regression weights) named COL1, COL2, COL3, and so forth up, to COL21. The regression coefficients in the file are in the order left to right as they appear in the PROCESS output from top to bottom. In the notation of Figure 7 and Equations (18), (19), and (20), the construction of the three indices of moderation require a_{31}, a_{32}, d, b_1, and b_2, which are held in COL6, COL13, COL8, COL15, and COL16, respectively.

Clicking on this file to make it active and then running the code below produces 95% bootstrap confidence intervals for the indices of moderated mediation for the specific indirect effects through M_1 only (**omegam1**), through M_2 only (**omegam2**), and through M_1 and M_2 in serial (**omegm1m2**).

```
compute omegam1=col6*col15.
compute omegam2=col13*col16.
compute omegm1m2=col6*col8*col16.
frequencies variables=omegam1 omegam2 omegm1m2/format notable/percentiles 2.5 97.5.
```

The comparable commands in SAS are

```
data ble;set binladen;binmc=binladen-0.410;agemc=age-4.846;binage=binmc*agemc;run;
%process (data=ble,vars=binmc mcivil rthreat stereo agemc ideo sex binage,y=mcivil,x=binmc,
   m=stereo rthreat,model=6,boot=10000, save=boots,percent=1);
data boots;set boots; omegam1=col6*col15;omegam2=col13*col16; omegm1m2=col6*col8*col16;run;
proc univariate data=boots noprint;var omegam1 omegam2 omegm1m2; output out=percent pctlpts=2.5 97.5
   pctlpre=omegam1P omegam2P omegm1m2P;run;
proc print data=percent;run;
```

The Mplus code below estimates the model coefficients and constructs the indices of moderated mediation for all specific indirect effects along with standard errors for the indices based on the delta method. Removal of the exclamation points produces bootstrap confidence intervals.

```
DATA:
   file is 'c:\ble.dat';
VARIABLE:
   names are binladen rthreat stereo mcivil age ideo sex;
   usevariables are stereo rthreat mcivil ideo sex binmc agemc binage;
DEFINE:
   binmc=binladen-0.410;
   agemc=age-4.846;
   binage=binmc*agemc;
ANALYSIS:
   !bootstrap=10000;
MODEL:
   stereo ON binmc agemc sex ideo
      binage (a31);
   rthreat ON binmc agemc sex ideo
         binage (a32)
         stereo (d);
```

```
mcivil ON binmc agemc binage sex ideo
      stereo (b1)
      rthreat (b2);
MODEL CONSTRAINT:
   new (omegam1 omegam2 omegm1m2);
   omegam1=a31*b1;omegam2=a32*b2;omegm1m2=a31*d*b2;
OUTPUT:
   !cinterval(bootstrap);
```