Moderation Analysis with Multicategorical Predictors

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UCLA Teaching Talk 2017

Setup Instructions:
1. Sit next to someone (anyone)
2. Boot up SPSS or SAS if you brought a computer
3. Go to github.com/akmontoya/UCLATeaching and download the files there

Overview

Advanced Regression lecture or Mediation & Moderation Course Lecture

- Motivating Example: *datafile: christmas*

- Coding multicategorical variables in regression
    - Indicator (dummy) coding
    - Sequential coding
    - Helmert coding
    - *Debate*

- Review of moderation
    - *Partners activity*

- Moderation with Multicategorical X
    - Defining models of interest
    - Estimation
    - Interpretation
    - Inference

Next Time: Probing moderation models with multicategorical predictors

Motivating Example

Schmitt, M. T., Davies, K., Hung, M., & Wright, S. C. (2010). Identity moderates the effects of Christmas displays on mood, self-esteem, and inclusion. *Journal of Experimental Social Psychology*, *46*, 1017 – 1022.

NOT REAL DATA, and only loosely based on original study

X: Religion (Christian, Buddhist, Sikh, No Religion)
W: Hours of Christmas music heard this week (voluntary or involuntary)
Y: Feelings of Inclusion

Interested in how people of different religions might feel more or less included during the winter holiday season, given the prevalence of Christmas music played on the radio or in stores.

Participants reported their religion (Christian, Buddhist, Sikh, or No Religion), how many hours of Christmas music they had heard in the last week, and how included they felt on a scale of 0 – 10.

Coding Multicategorical Variables

In a regression analysis all predictors need to be **quantitative** or **dichotomous**. So how do we include a multicategorical variable in regression analysis?

If the multicategorical variable has *k* categories, it can be represented by *k*-1 dichotomous or continuous variables.

Methods for coding multicategorical variables:

- Indicator / Dummy Coding
- Sequential Coding
- Helmert Coding

Example of Multicategorical Variables in Psychology:

- Ethnicity (e.g., Caucasian, Asian, Hispanic, African, etc.)
- Religion (e.g., Christian, Buddhist, Sikh, etc.)
- Gender Identity (e.g., female, male, non-binary, genderqueer, etc.)
- Conditions in Study
- Marital Status (married, single, widowed, divorced)

Indicator Coding

Select one group to be the "reference group," this group will be compared to all other groups. This group gets a score of zero on variables in the new coded set.

Each other group will then receive a 1 for one of the recoded variables and a zero on all others.

Example: Religion (Christian = 1, Buddhist = 2, Sikh = 3, No Religion = 4)

religion	D_1	D_2	D_3
1 (C)			
2 (B)			
3 (S)			
4 (N)			

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

Indicator Coding: Predicted Means

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

religion	D_1	D_2	D_3
1 (C)	1	0	0
2 (B)	0	1	0
3 (S)	0	0	1
4 (N)	0	0	0

Christian: $\widehat{Y_1} = b_0 + b_1(1) + b_2(0) + b_3(0) = b_0 + b_1$

Buddhist: $\widehat{Y_2} = b_0 + b_1(0) + b_2(1) + b_3(0) = b_0 + b_2$

Sikh: $\widehat{Y_3} = b_0 + b_1(0) + b_2(0) + b_3(1) = b_0 + b_3$

No Religion: $\widehat{Y_4} = b_0 + b_1(0) + b_2(0) + b_3(0) = b_0$

Indicator Coding

religion	D_1	D_2	D_3
1	1	0	0
2	0	1	0
3	0	0	1
4	0	0	0

SPSS Code:

```
compute d1 = (religion = 1).
compute d2 = (religion = 2).
compute d3 = (religion = 3).
Regression /dep = included /method = enter d1 d2 d3.
```

SAS Code:

```
data christmas; set christmas;
d1 = (religion = 1);
d2 = (religion = 2);
d3 = (religion = 2); run;
Proc reg data = christmas; model included = d1 d2 d3; run;
```

Indicator Coding: Interpreting Regression Results

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	4.433	.329		13.486	.000
	d1	2.867	.520	.500	5.515	.000
	d2	-3.717	.615	-.538	-6.043	.000
	d3	-1.908	.615	-.276	-3.103	.004

a. Dependent Variable: Included

The average feelings of inclusion for **non-religious individuals** was 4.433.

Christian individuals felt 2.867 units <u>more</u> included than non-religious individuals.

Buddhist individuals felt 3.717 units <u>less</u> included than non-religious individuals.

Sikh individuals felt 1.908 units <u>less</u> included than non-religious individuals.

Sequential Coding

Used for ordered variables, good for examining change in predicted *Y* with one step increase.

First group gets all zeros. Next group gets 1 on first variable and zeros on all other variables. Second group gets 1's on first 2 variable and zeros on all others…

Example: Religion (Christian = 1, Buddhist = 2, Sikh = 3, No Religion = 4)

religion	S_1	S_2	S_3
1 (C)			
2 (B)			
3 (S)			
4 (N)			

Sequential Coding: Predicted Means

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

religion	S_1	S_2	S_3
1 (C)	0	0	0
2 (B)	1	0	0
3 (S)	1	1	0
4 (N)	1	1	1

Christian: $\hat{Y}_1 = b_0 + b_1(0) + b_2(0) + b_3(0) = b_0$

Buddhist: $\hat{Y}_2 = b_0 + b_1(1) + b_2(0) + b_3(0) = b_0 + b_1$

Sikh: $\hat{Y}_3 = b_0 + b_1(1) + b_2(1) + b_3(0) = b_0 + b_1 + b_2$

No Religion: $\hat{Y}_4 = b_0 + b_1(1) + b_2(1) + b_3(1) = b_0 + b_1 + b_2 + b_3$

Sequential Coding in SPSS or SAS

religion	S_1	S_2	S_3
1 (C)	0	0	0
2 (B)	1	0	0
3 (S)	1	1	0
4 (N)	1	1	1

SPSS Code:

```
compute s1 = (religion ~= 1).
compute s2 = (religion > 2).
compute s3 = (religion = 4).
Regression /dep = included /method = enter s1 s2 s3.
```

SAS Code:

```
data christmas; set christmas;
s1 = (religion ^= 1);
s2 = (religion > 2);
s3 = (religion = 4); run;
proc reg data = christmas; model included = s1 s2 s3; run;
```

Sequential Coding: Interpreting Regression Results

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	7.300	.403		18.132	.000
	s1	-6.583	.657	-1.149	-10.013	.000
	s2	1.808	.735	.352	2.460	.019
	s3	1.908	.615	.368	3.103	.004

a. Dependent Variable: Included

The average feelings of inclusion for **Christian individuals** was 7.30.

Buddhist individuals felt 6.58 units <u>less</u> included than Christian individuals.

Sikh individuals felt 1.81 units <u>more</u> included than Buddhist individuals.

Non-religious individual felt 1.908 units <u>more</u> included than Sikh individuals.

Helmert Coding

Used for ordered variables. Regression coefficients quantify the difference between one group and the average of all subsequent groups.

How to code variables depends on the number of groups.

Example: Religion (Christian = 1, Buddhist = 2, Sikh = 3, No Religion = 4)

TABLE 10.7. Helmert Coding of g Categories, $g \geq 5$

Ordinal position (low to high)	D_1	D_2	D_3	$\cdots$	D_{g-1}
1	$-(g-1)/g$	0	0	$\cdots$	0
2	$1/g$	$-(g-2)/(g-1)$	0	$\cdots$	0
3	$1/g$	$1/(g-1)$	$-(g-3)/(g-2)$	$\cdots$	0
$\vdots$					
$g-1$	$1/g$	$1/(g-1)$	$1/(g-2)$	$\cdots$	$-1/2$
g	$1/g$	$1/(g-1)$	$1/(g-2)$	$\cdots$	$1/2$

Hayes & Darlington, 2017 *Regression Analysis and Linear Models*

Helmert Coding: Predicted Means

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

religion	H_1	H_2	H_3
1	-3/4	0	0
4	1/4	-2/3	0
2	1/4	1/3	-1/2
3	1/4	1/3	1/2

Christian: $\hat{Y}_1 = b_0 + b_1\left(-\frac{3}{4}\right)$

No Religion: $\hat{Y}_4 = b_0 + b_1\left(\frac{1}{4}\right) + b_2\left(-\frac{2}{3}\right)$

Buddhist: $\hat{Y}_3 = b_0 + b_1\left(\frac{1}{4}\right) + b_2\left(\frac{1}{3}\right) + b_3\left(-\frac{1}{2}\right)$

Sikh: $\hat{Y}_2 = b_0 + b_1\left(\frac{1}{4}\right) + b_2\left(\frac{1}{3}\right) + b_3\left(\frac{1}{2}\right)$

Homework: (1) Show the above equations are true based on the regression equation and Helmert coding pattern. (2) Using the Christmas dataset, show that the predicted means for each group are equal regardless of coding system used.

Helmert Coding in SPSS or SAS

religion	H_1	H_2	H_3
1	-3/4	0	0
4	1/4	-2/3	0
2	1/4	1/3	-1/2
3	1/4	1/3	1/2

SPSS Code:

```
compute h1 = -3/4*(religion = 1) + 1/4*(religion > 1).
compute h2 = -2/3*(religion = 4) + 1/3*((religion = 2)|(religion = 3)).
compute h3 = -1/2*(religion = 2) + 1/2*(religion = 3).
Regression /dep = included /method = enter h1 h2 h3.
```

SAS Code:

```
data christmas; set christmas;
h1 = -3/4*(religion = 1) + 1/4*(religion > 1);
h2 = -2/3*(religion = 4) + 1/3*((religion = 2)|(religion = 3));
h3 = -1/2*(religion = 2) + 1/2*(religion = 3); run;
proc reg data = christmas; model included = h1 h2 h3; run;
```

Helmert Coding: Interpreting Regression Results

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + e_i$$

Coefficients[a]

Model		Unstandardized Coefficients B	Std. Error
1	(Constant)	3.744	.225
	h1	-4.742	.484
	h2	-2.813	.493
	h3	1.808	.735

a. Dependent Variable: Included

religion	H_1	H_2	H_3
1 (C)	-3/4	0	0
4 (N)	1/4	-2/3	0
2 (B)	1/4	1/3	-1/2
3 (S)	1/4	1/3	1/2

b_0 doesn't typically map onto a nice interpretation.

Average inclusion among **Sikhs, Buddhists, and Non-religious** was 4.74 lower than **Christians**.

Average inclusion among **Sikhs and Buddhists** was 2.81 lower than **Non-religious individuals**.

Average inclusion among **Sikhs** was 1.81 higher than **Buddhists**.

A Debate on Indicator Coding

"I often say in class that "dummy codes are named after the people who use them." Dummy codes, like a ventriloquist dummy, do not speak for themselves and they are almost always more correlated with each other than other codes. We know that correlated predictors are not great in regression, so when we have the opportunity to create codes we should avoid creating correlated predictors when we don't have to."

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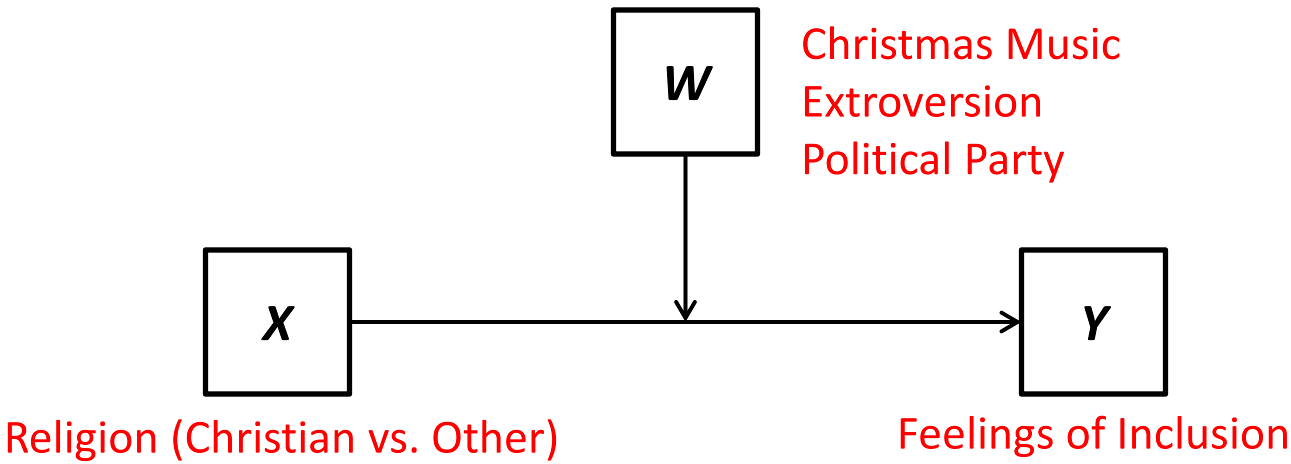
Groups of 3 or 4

Pick a Question to discuss:

1. Can you think of instances when dummy coding would give you the <u>most relevant</u> information to your hypotheses? What about sequential coding? What about Helmert coding?
2. What issues do correlated predictors bring to regression analysis? Would we <u>never</u> want to include correlated predictors?
3. If each coding method gives the same predicted means for groups, what <u>difference</u> does it make which method we use?

Moderation



Moderation is the idea that the relationship between a focal predictor (*X*) and an outcome (*Y*) may depend on some other variable (*W*).

This can be described as a *contingent relationship* or an *interaction*.

The idea being that the magnitude or the sign (or both) of the relationship between *X* and *Y* depends on *W*.

A quick example: Possible moderators!

Modeling Contingent Relationships

Predict that the relationship between religion and feelings of inclusion depends on hours of Christmas music. Thus the relationship between religion and inclusion is a *function* of gender

$$Y_i = b_0 + f(W_i)X_i + b_2 W_i$$

One popular model for $f(W_i)$ is a linear model:

$$f(M_i) = b_1 + b_3 W_i = \theta_{X \to Y|W}$$

This way we can rewrite the model:

$$Y_i = b_0 + \theta_{X \to Y|W} X_i + b_2 W_i$$

$$Y_i = b_0 + (b_1 + b_3 W_i)X_i + b_2 W_i$$

$$Y_i = b_0 + b_1 X_i + b_2 W_i + b_3 W_i X_i$$

This is a regression model which can be estimated, where the significance of b_3 reflects whether the relationship between *X* and *Y* is linearly dependent on *W.*

Test of Coefficient vs. Model Comparison

$$Y_i = b_0 + b_1X_i + b_2W_i + b_3W_iX_i$$

Typically when we think about inference for an interaction we think about a test on b_3. But we can also think about comparing two models:

Non-Contingent Model

Model 1: $Y_i = b_0^* + b_1^*X_i + b_2^*W_i + \epsilon_i^*$ where $\epsilon_i^* \sim N(0, \sigma^{*2})$

Contingent Model

Model 2: $Y_i = b_0 + b_1X_i + +b_2W_i + b_3W_iX_i + \epsilon_i$ where $\epsilon_i \sim N(0, \sigma^2)$

Test for change in R^2 from Model 1 to Model 2 is <u>equivalent</u> to test on b_3 from Model 2.

If $b_3 = 0$ in Model 2, then Model 2 = Model 1.

Test of Coefficient vs. Model Comparison

```
compute d1xhours = d1*hours.
regression /statistics = default change/dep = included
       /method = enter d1 hours /method = enter d1xhours.
```

```
data christmas; set christmas;
d1xhours = d1*hours; run;
proc reg data = christmas; model included = d1 hours; model
included = d1 hours d1xhours; test d1xhours = 0; run;
```

Test of Coefficient vs. Model Comparison

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.779[a]	.606	.583	1.66645	.606	26.158	2	34	.000
2	.790[b]	.624	.590	1.65212	.018	1.592	1	33	.216

a. Predictors: (Constant), Hours, d1

b. Predictors: (Constant), Hours, d1, d1xhours

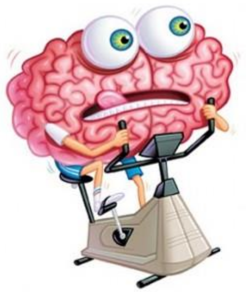
Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	1.910	.558		3.423	.002
	d1	3.837	.625	.670	6.140	.000
	Hours	.222	.080	.304	2.790	.009
2	(Constant)	2.175	.592		3.676	.001
	d1	2.074	1.529	.362	1.356	.184
	Hours	.176	.087	.241	2.020	.052
	d1xhours	.260	.206	.351	1.262	.216

a. Dependent Variable: Included

$$t^2 = F$$
$$1.262^2 = 1.592$$

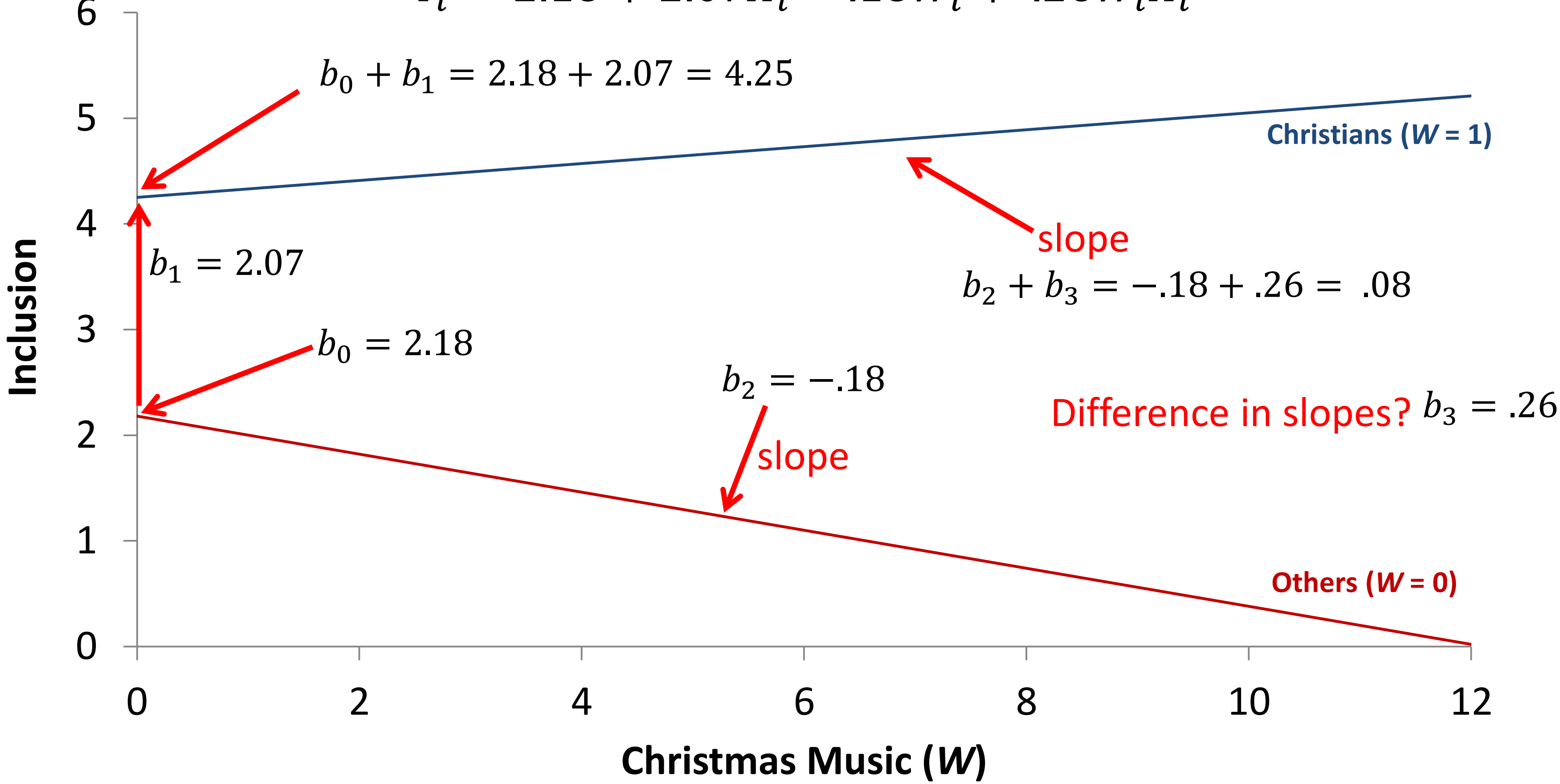
Practice Interpreting Coefficients



What if instead we felt that the relationship between religion and inclusion depends on Christmas Music?

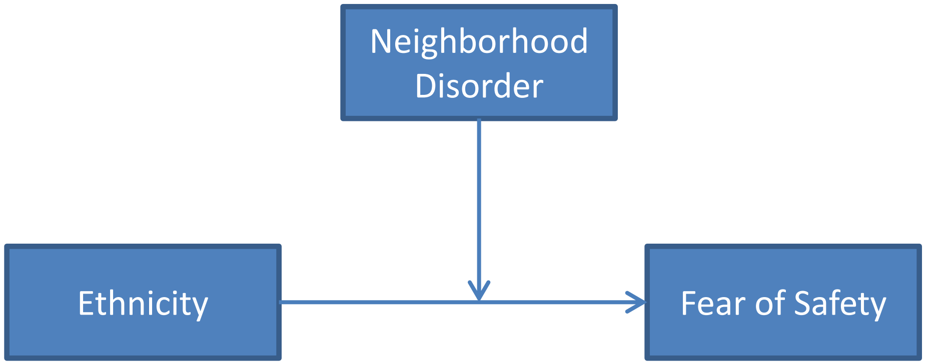
$$Y_i = b_0 + b_1 X_i + b_2 W_i + \; b_3 W_i X_i$$

$$Y_i = 2.18 + 2.07 X_i - .18 W_i + \; .26 W_i X_i$$



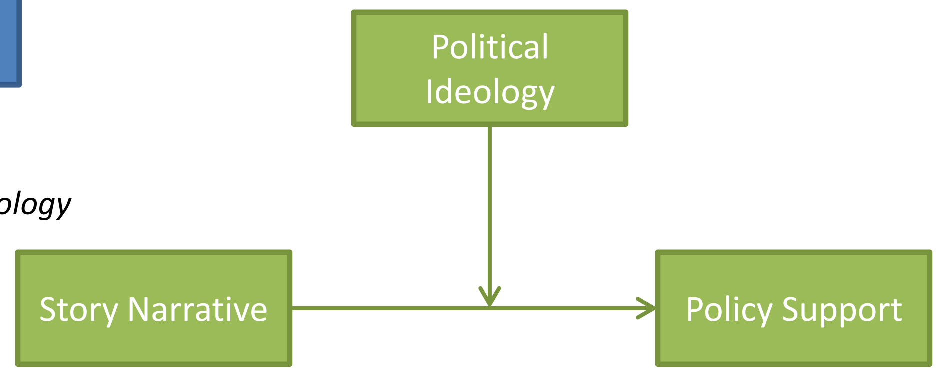
What if *X* is Multicategorical?

If X is a multicategorical variable (e.g. race, experimental condition (when there are more than two), the process is similar.



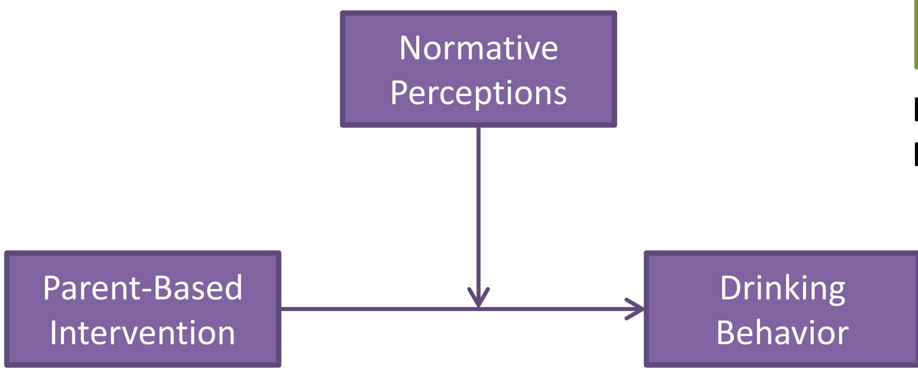
White, Black, Latino

Barajas-Gonzales & Brooks-Gunn, 2014 , *Journal of Family Psychology*



High, Moderate, or Low
Personal Responsibility

Niederdeppe et al. 2014, *Health Communications*



Pre-college, Precollege + Booster,
After Starting College

Cleveland et al., 2013, *Alcoholism: Clinical and Experimental Research*

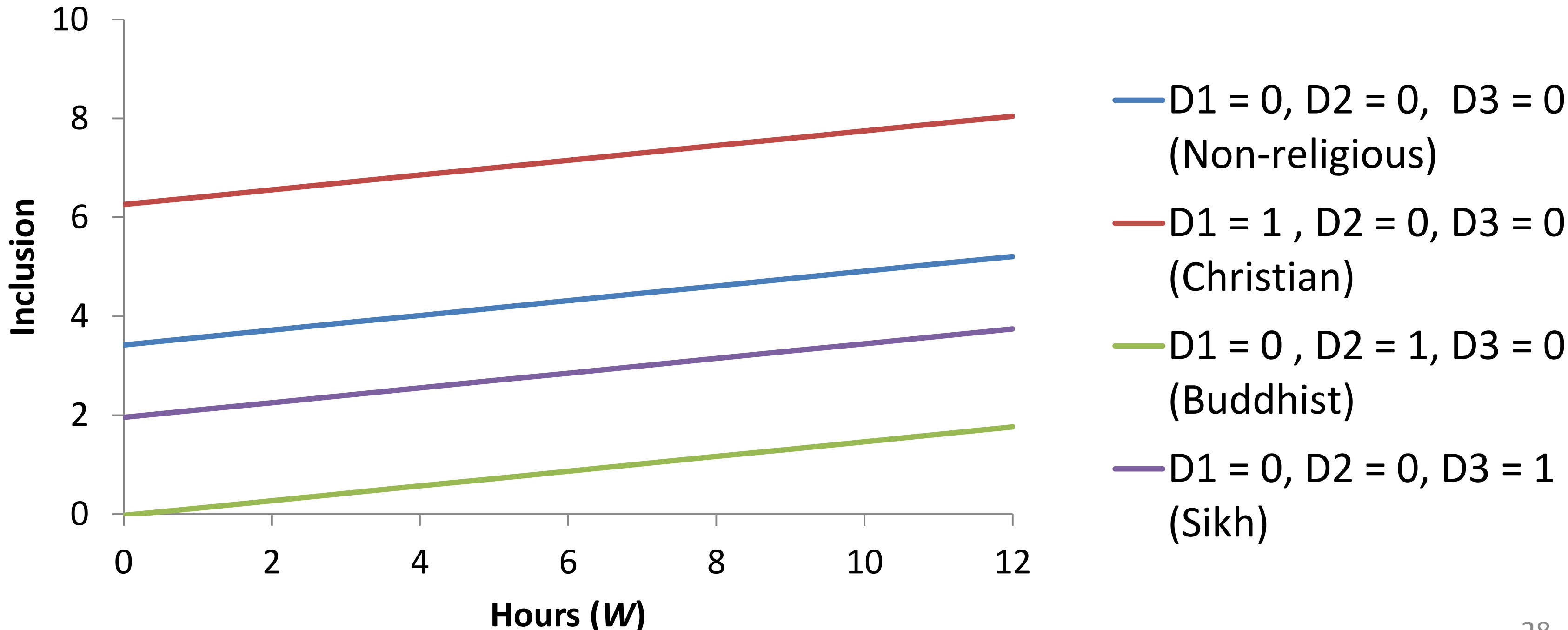
Setting up the Model

Begin by coding *X* into *k*-1 new variables. I'll use dummy coded variables for now.

Non-Contingent Model

$$Y_i = b_0^* + b_1^* D_{1i} + b_2^* D_{2i} + b_3^* D_{3i} + b_4^* W_i + \epsilon_i^* \text{where } \epsilon_i^* \sim N(0, \sigma^{*2})$$

This model fixes the slopes of the 4 lines to be equal!



Setting up the Model: Varying Slopes

Allow the relationships between each dummy code to depend on W.

$$Y_i = b_0 + \theta_{D_1 \to Y|W} D_{1i} + \theta_{D_2 \to Y|W} D_{2i} + \theta_{D_3 \to Y|W} D_{3i} + b_4 W_i + \epsilon_i \text{where } \epsilon_i \sim N(0, \sigma^2)$$

$$\theta_{D_1 \to Y|W} = b_1 + b_5 W_i \qquad \theta_{D_2 \to Y|W} = b_2 + b_6 W_i \qquad \theta_{D_3 \to Y|W} = b_3 + b_7 W_i$$

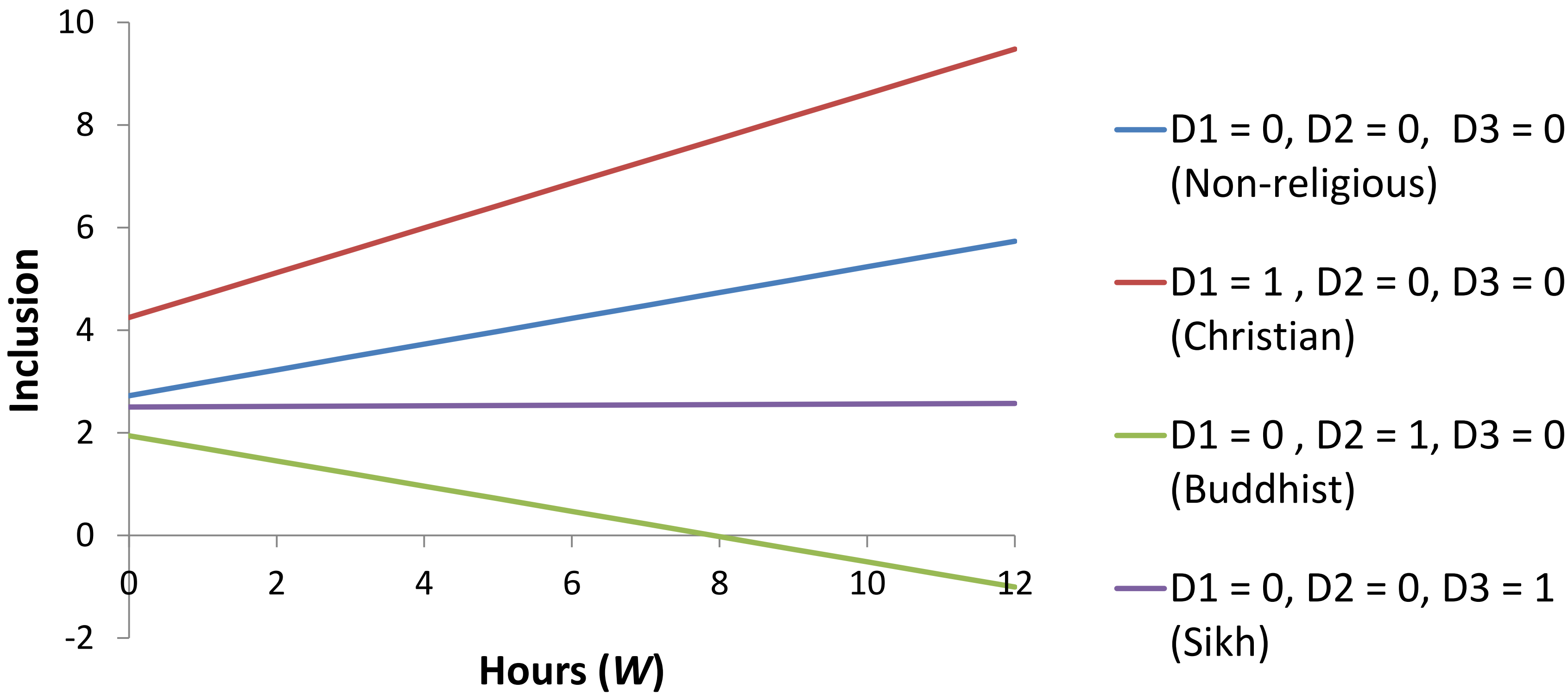
Contingent Model

$$Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} + (b_3 + b_7 W_i) D_{3i} + b_4 W_i + \epsilon_i$$

Setting up the Model: Varying Slopes

Contingent Model

$$Y_i = b_0 + (b_1 + b_5 W_i)D_{1i} + (b_2 + b_6 W_i)D_{2i} + (b_3 + b_7 W_i)D_{3i} + b_4 W_i + \epsilon_i$$



Estimating Contingent Model

```
compute d2xhours = d2*hours.
compute d3xhours = d3*hours.
regression /dep = included /method = enter d1 d2 d3 hours
d1xhours d2xhours d3xhours.
```

```
data christmas; set christmas;
d2xhours = d2*hours; d3xhours = d3*hours; run;
Proc reg data = christmas; model included = d1 d2 d3 hours
d1xhours d2xhours d3xhours; run;
```

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	2.725	.533		5.110	.000
	d1	1.524	.941	.266	1.619	.116
	d2	-.780	.847	-.113	-.921	.365
	d3	-.222	.754	-.032	-.295	.770
	Hours	.251	.070	.344	3.566	.001
	d1xhours	.185	.125	.249	1.481	.149
	d2xhours	-.497	.129	-.449	-3.839	.001
	d3xhours	-.245	.122	-.198	-2.009	.054

a. Dependent Variable: Included

Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} + (b_3 + b_7 W_i) D_{3i} + b_4 W_i$$

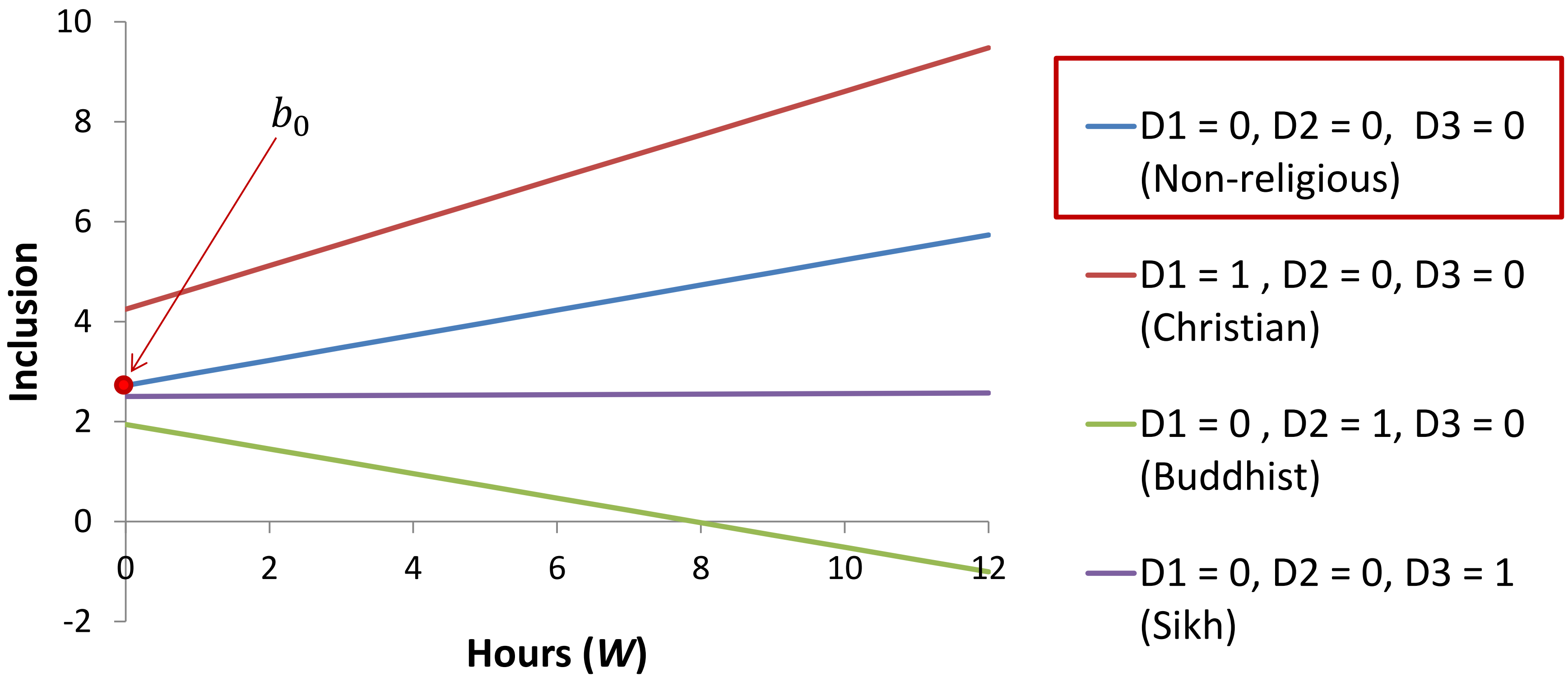
b_0 : Predicted *Y* when D_1, D_2, D_3, and *W* are all zero

$$E(Y | D_1 = D_2 = D_3 = W = 0)$$
$$= b_0 + (b_1 + b_5 0)0 + (b_2 + b_6 0)0 + (b_3 + b_7 0)0 + b_4 0 = b_0$$

Interpreting Coefficients

Contingent Model

$$Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} + (b_3 + b_7 W_i) D_{3i} + b_4 W_i + \epsilon_i$$



Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5 W_i)D_{1i} + (b_2 + b_6 W_i)D_{2i} + (b_3 + b_7 W_i)D_{3i} + b_4 W_i$$

b_0 : Predicted *Y* when D_1, D_2, D_3, and *W* are all zero

b_1 : Increase in *Y* with a one unit increase in D_1 when D_2, D_3, and *W* are all zero

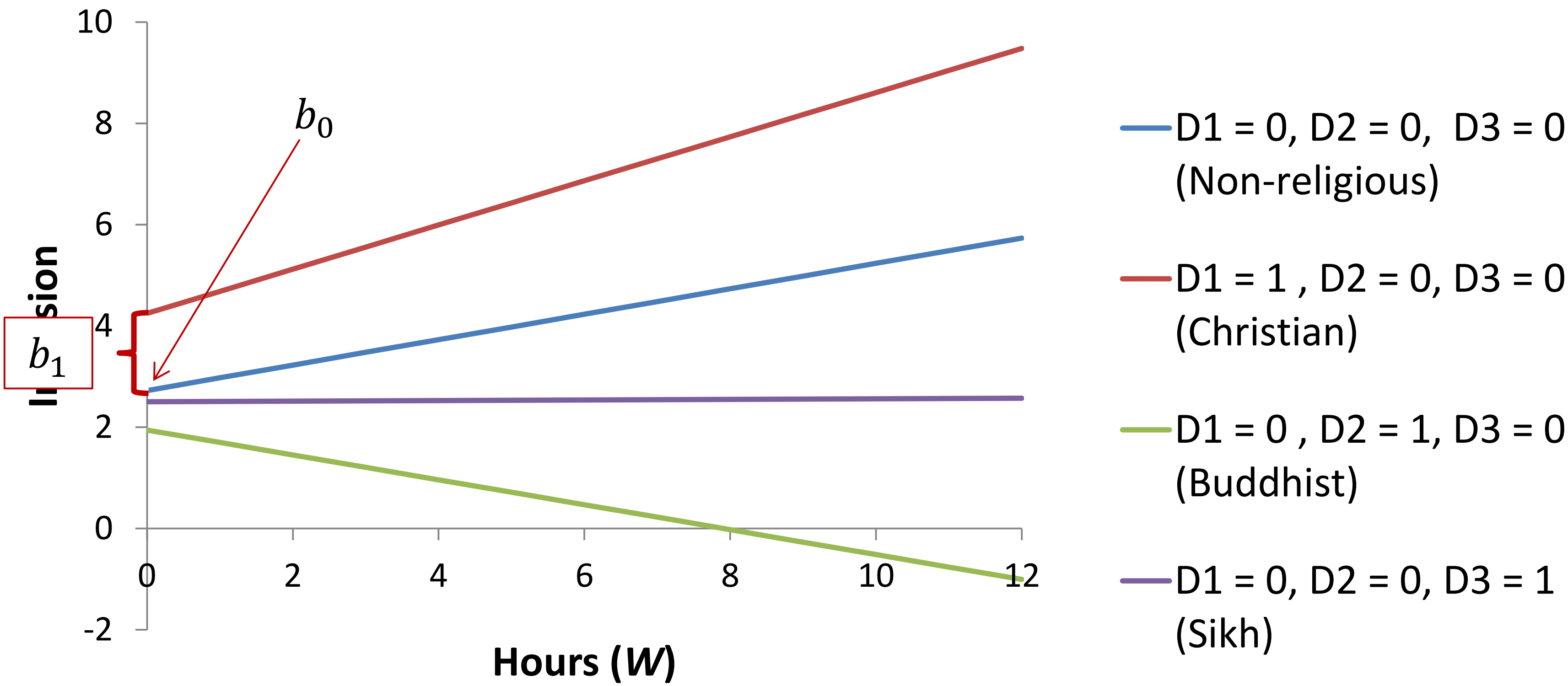
$$E(Y|D_2 = D_3 = W_i = 0)$$
$$= b_0 + (b_1 + b_5 0)D_1 + (b_2 + b_6 0)0 + (b_3 + b_7 0)0 + b_4 0 = b_0 + \underbrace{b_1}D_1$$

Difference between D_1 = 0 and D_1 = 1 (non-religious and Christian) when *W* = 0

Interpreting Coefficients

$$E(Y|D_2 = D_3 = W_i = 0)$$
$$= b_0 + (b_1 + b_5 0)D_1 + (b_2 + b_6 0)0 + (b_3 + b_7 0)0 + b_4 0 = b_0 + b_1 D_1$$

Difference between D_1 = 0 and D_1 = 1 (non-religious and Christian) when W = 0



Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5 W_i)D_{1i} + (b_2 + b_6 W_i)D_{2i} + (b_3 + b_7 W_i)D_{3i} + b_4 W_i$$

b_0 : Predicted Y when D_1, D_2, D_3, and W are all zero

b_1 : Increase in Y with a one unit increase in D_1 when D_2, D_3, and W are all zero

b_2 : Increase in Y with a one unit increase in D_2 when D_1, D_3, and W are all zero

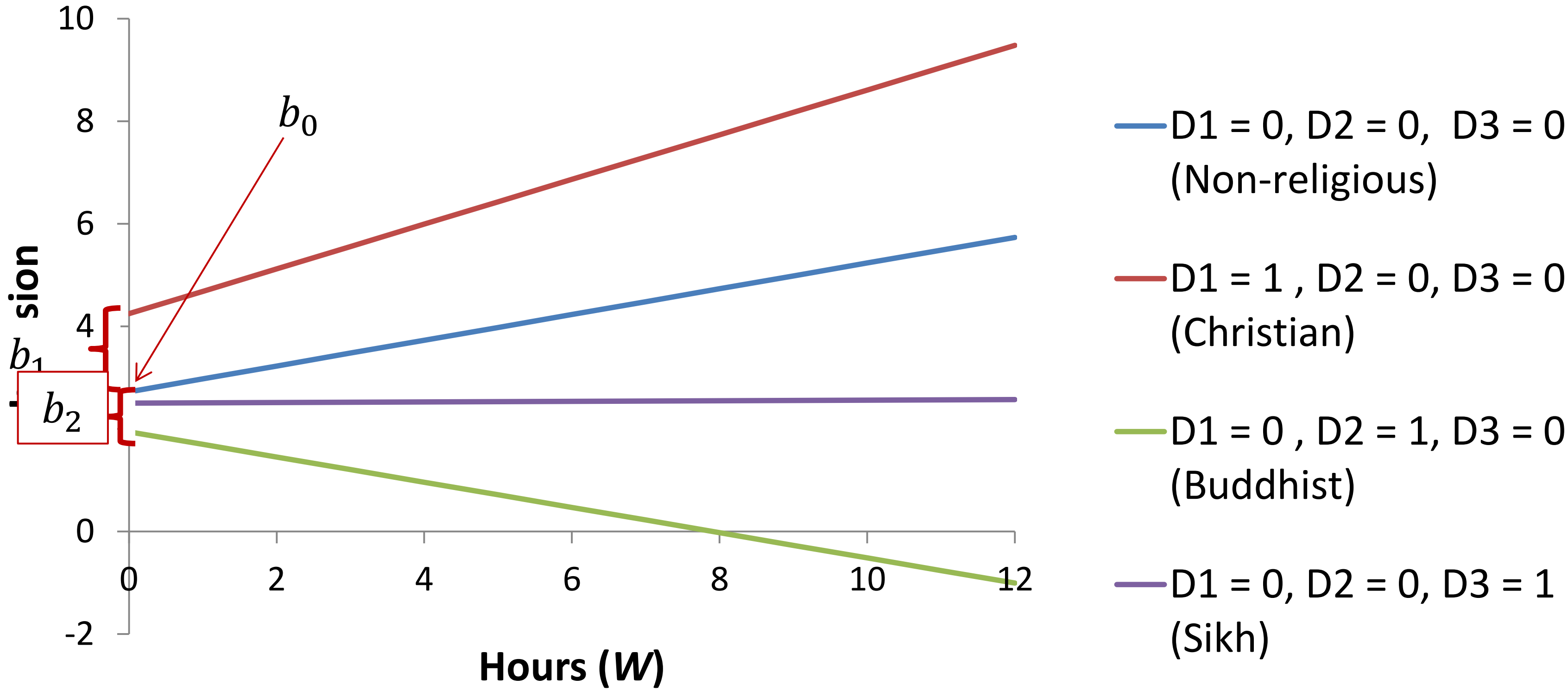
$$E(Y|D_1 = D_3 = W_i = 0) = b_0 + \underbrace{b_2}D_2$$

Difference between D_2 =0 and D_2 = 1 (non-religious and Buddhist) when W = 0

Interpreting Coefficients

$$E(Y|D_1 = D_3 = W_i = 0) = b_0 + b_2D_2$$

Difference between D_2 =0 and D_2 = 1 (non-religious and Buddhist) when W = 0



Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5W_i)D_{1i} + (b_2 + b_6W_i)D_{2i} + (b_3 + b_7W_i)D_{3i} + b_4W_i$$

b_0 : Predicted *Y* when D_1, D_2, D_3, and *W* are all zero

b_1 : Increase in *Y* with a one unit increase in D_1 when D_2, D_3, and *W* are all zero

b_2 : Increase in *Y* with a one unit increase in D_2 when D_1, D_3, and *W* are all zero

b_3 : Increase in *Y* with a one unit increase in D_3 when D_1, D_2, and *W* are all zero

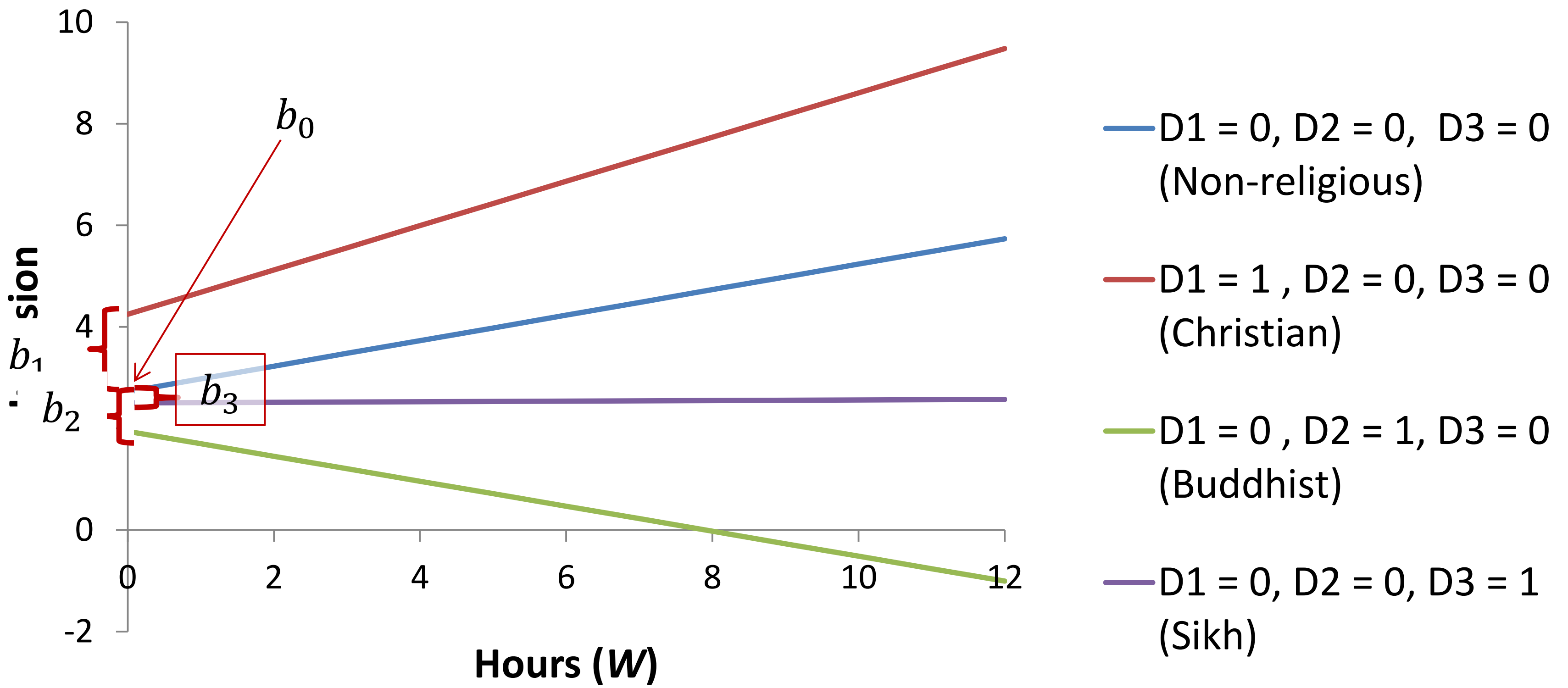
$$E(Y|D_1 = D_2 = W_i = 0) = b_0 + \underbrace{b_3}_{}D_2$$

Difference between D_3 = 0 and D_3 = 1 (non-religious and Sikh) when *W* = 0

Interpreting Coefficients

$$E(Y|D_1 = D_2 = W_i = 0) = b_0 + b_3 D_2$$

Difference between D_3 = 0 and D_3 = 1 (non-religious and Sikh) when W = 0



Interpreting Coefficients

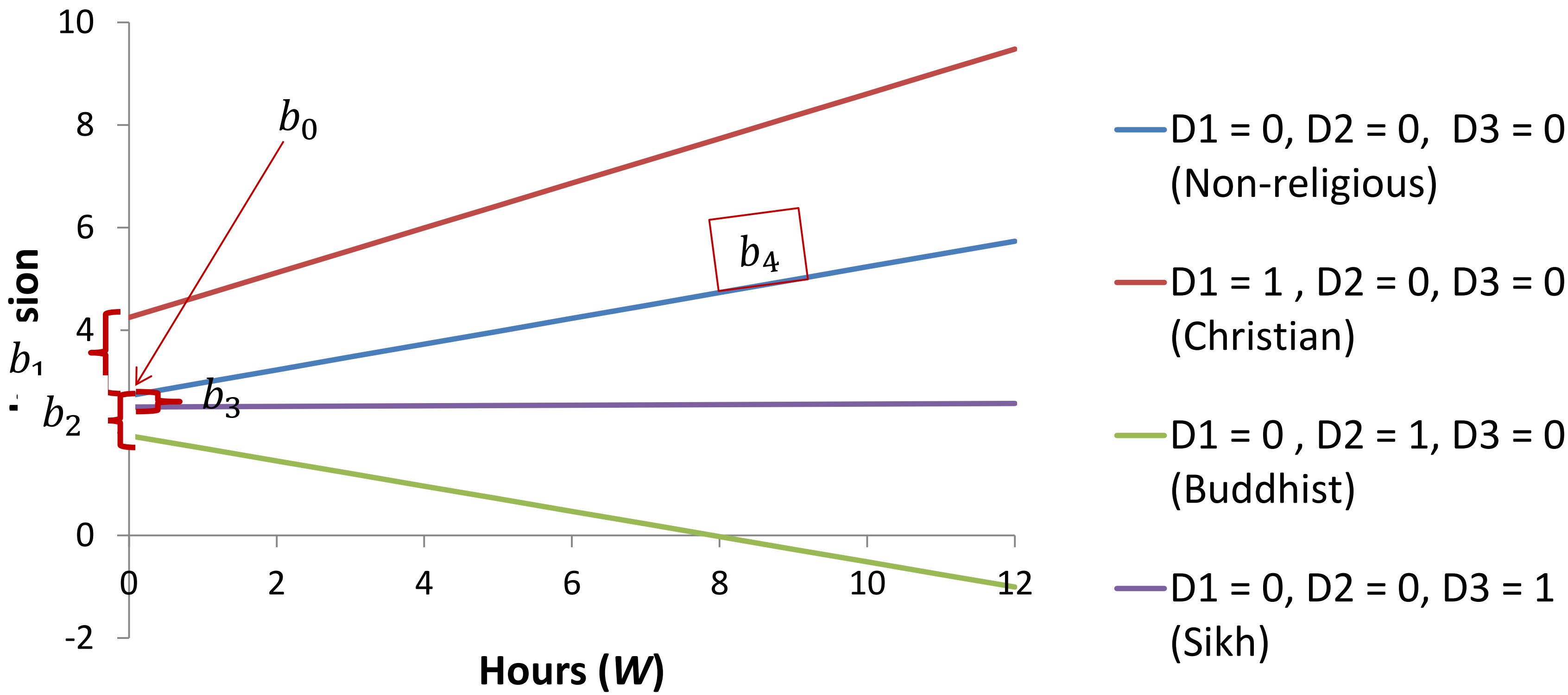
$$Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} + (b_3 + b_7 W_i) D_{3i} + b_4 W_i$$

b_4 : Increase in Y with one unit increase in W when D_1, D_2, D_3 are all zero

$$E(Y | D_1 = D_2 = D_3 = 0)$$
$$= b_0 + (b_1 + b_5 W)0 + (b_2 + b_6 W)0 + (b_3 + b_7 W)0 + b_4 W = b_0 + \underbrace{b_4}_{\text{Slope of } WY \text{ line for non-religious individuals}} W$$

Interpreting Coefficients

$$E(Y|D_1 = D_2 = D_3 = 0)$$
$$= b_0 + (b_1 + b_5W)0 + (b_2 + b_6W)0 + (b_3 + b_7W)0 + b_4W = b_0 + \underbrace{b_4}_{\text{Slope of } WY \text{ line for non-religious individuals}}W$$



Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5W_i)D_{1i} + (b_2 + b_6W_i)D_{2i} + (b_3 + b_7W_i)D_{3i} + b_4W_i$$

b_4 : Increase in *Y* with one unit increase in *W* when D_1, D_2, $D_3$are all zero

b_5 : Increase in *MY* relationship with 1 unit increase in D_1 when D_2 and D_3 are zero

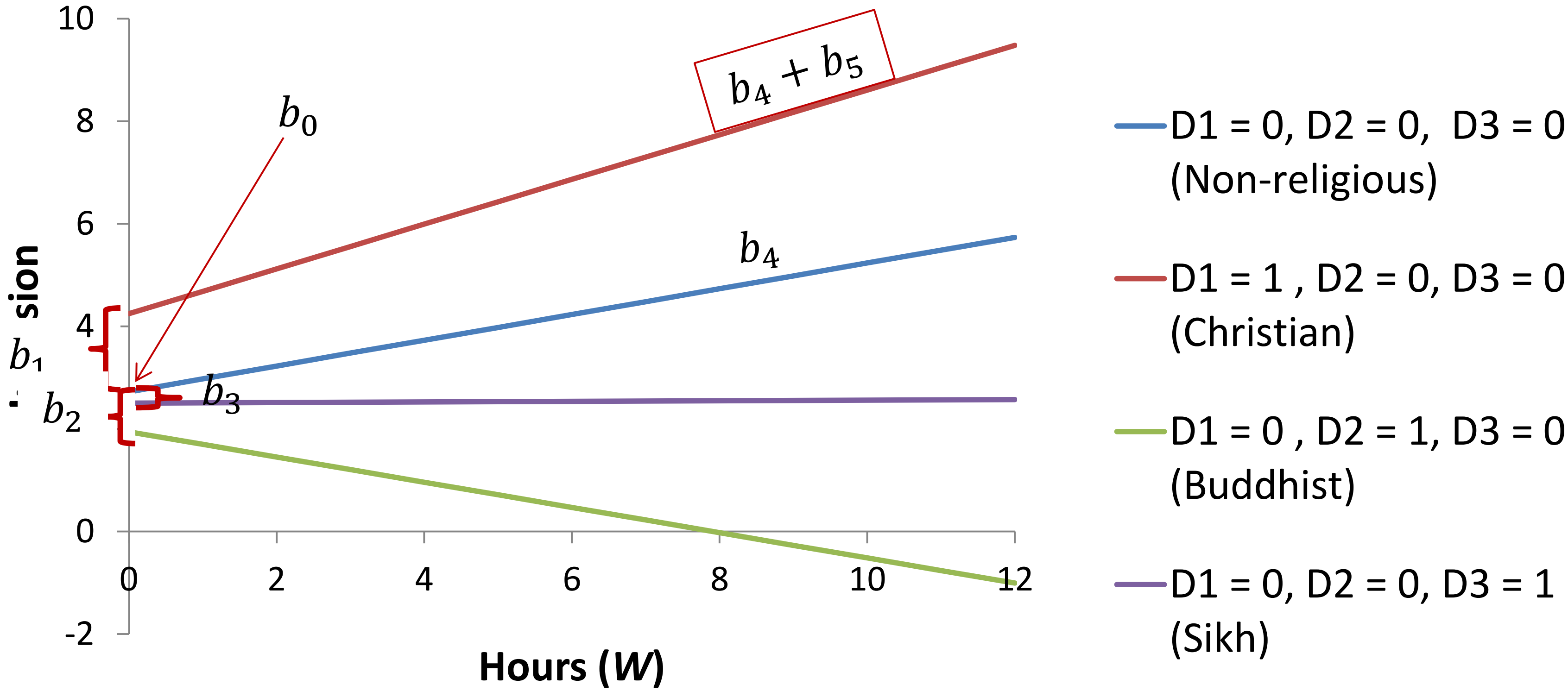
$$\begin{aligned} E(Y|D_2 = D_3 = 0) \\ &= b_0 + (b_1 + b_5W)D_1 + (b_2 + b_6W)0 + (b_3 + b_7W)0 + b_4W \\ &= b_0 + b_1D_1 + b_4W + b_5WD_1 = b_0 + b_1D_1 + (b_4 + \underbrace{b_5}D_1)W \end{aligned}$$

Difference in slope of *WY* line for Christian vs. non-religious individuals

Interpreting Coefficients

$$E(Y|D_2 = D_3 = 0)$$
$$= b_0 + b_1 D_1 + b_4 W + b_5 W D_1 = b_0 + b_1 D_1 + (b_4 + \underbrace{b_5}_{} D_1) W$$

Difference in slope of WY line for Christian vs. non-religious individuals



Interpreting Coefficients

$$Y_i = b_0 + (b_1 + b_5 W_i)D_{1i} + (b_2 + b_6 W_i)D_{2i} + (b_3 + b_7 W_i)D_{3i} + b_4 W_i$$

b_4 : Increase in *Y* with one unit increase in *W* when D_1, D_2, $D_3$are all zero

b_5 : Increase in *MY* relationship with 1 unit increase in D_1 when D_2 and D_3 are zero

b_6 : Increase in *MY* relationship with 1 unit increase in D_2 when D_1 and D_3 are zero

Difference in slope of *WY* line for Buddhist vs. non-religious individuals

b_7 : Increase in *MY* relationship with 1 unit increase in D_2 when D_1 and D_3 are zero

Difference in slope of *WY* line for Sikh vs. non-religious individuals

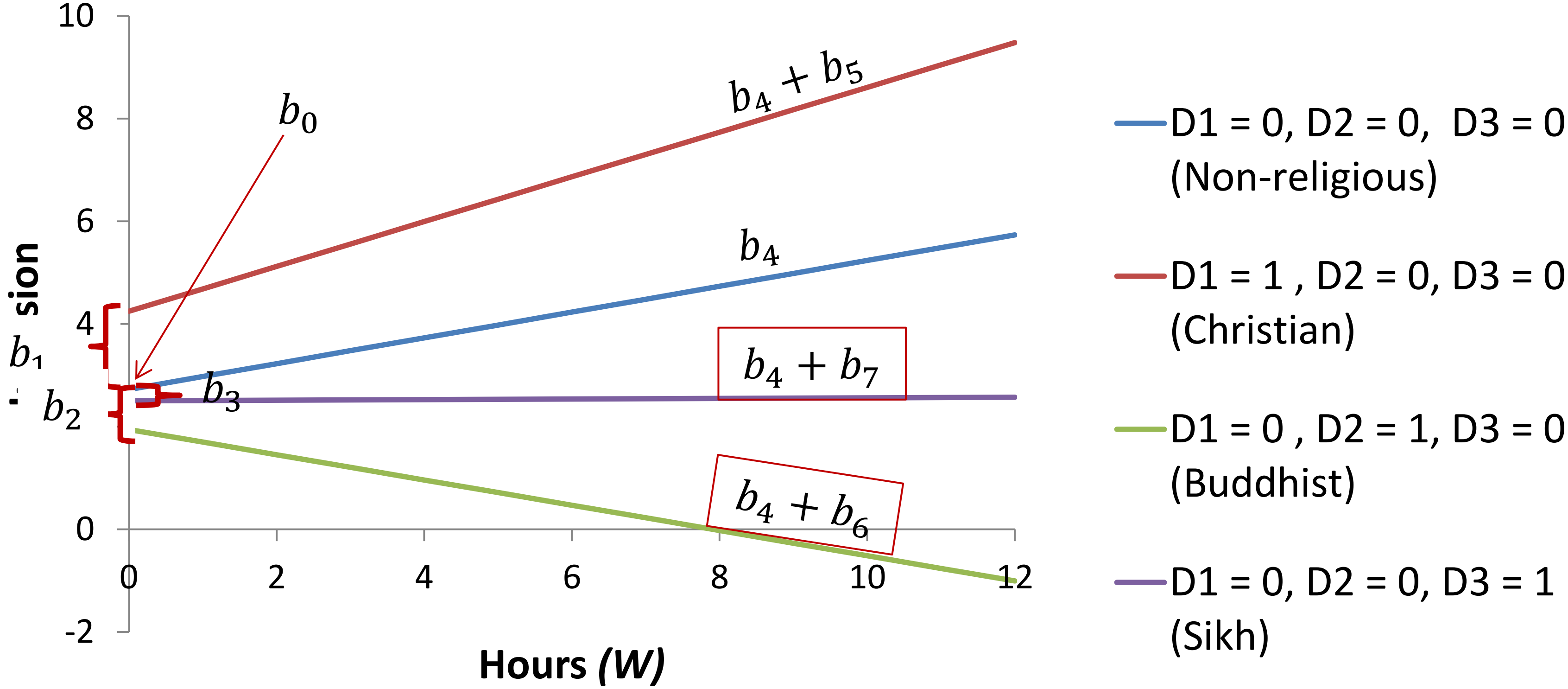
Interpreting Coefficients

b_6 : Increase in *MY* relationship with 1 unit increase in D_2 when D_1 and D_3 are zero

Difference in slope of *WY* line for Buddhist vs. non-religious individuals

b_7 : Increase in *MY* relationship with 1 unit increase in D_2 when D_1 and D_3 are zero

Difference in slope of *WY* line for Sikh vs. non-religious individuals

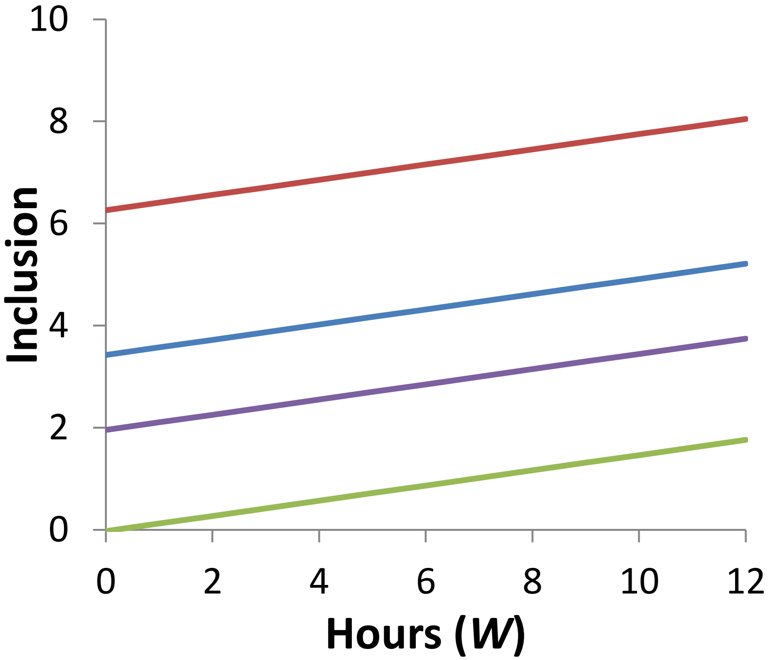


Inference on Interaction

How can we tell if there is an interaction?
No single coefficient tells us if the slopes are equal.

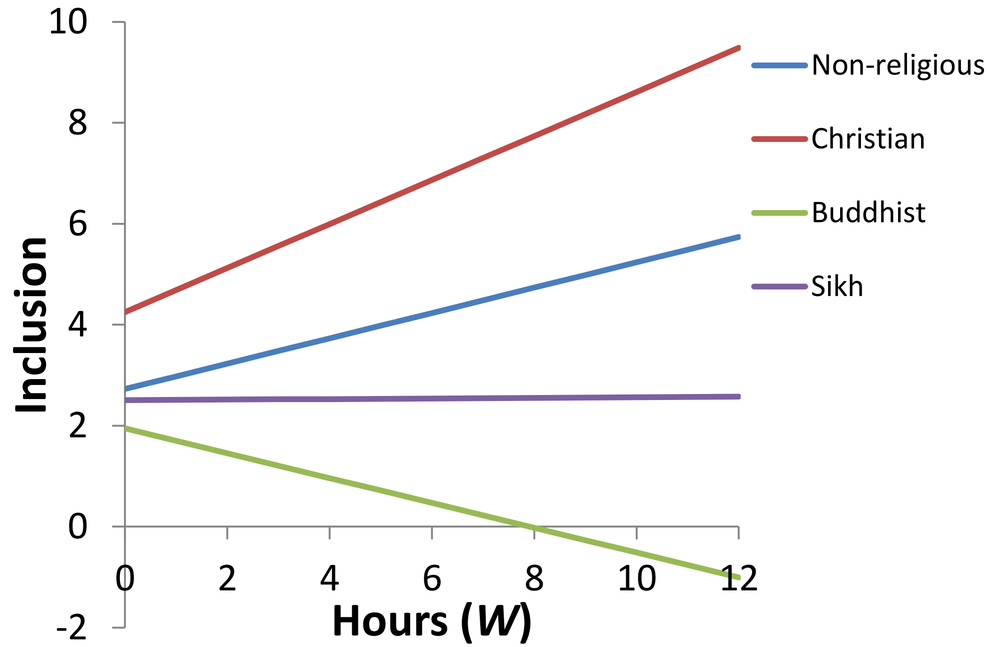
Non-Contingent Model

$$Y_i = b_0^* + b_1^* D_{1i} + b_2^* D_{2i} + b_3^* D_{3i} + b_4^* W_i + \epsilon_i^*$$



Contingent Model

$$Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} + (b_3 + b_7 W_i) D_{3i} + b_4 W_i + \epsilon_i$$



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Contingent Model

$$\begin{aligned} Y_i = b_0 + (b_1 + b_5 W_i) D_{1i} + (b_2 + b_6 W_i) D_{2i} \\ + (b_3 + b_7 W_i) D_{3i} + b_4 W_i + \epsilon_i \end{aligned}$$

If $b_5 = b_6 = b_7 = 0$ then models are equivalent

Use model comparison to select contingent or non contingent model.

Use increase in R^2 to compare models.

Inference on Interaction

```
regression /statistics = default change /dep = included
/method = enter d1 d2 d3 hours
/method = enter d1xhours d2xhours d3xhours.
```

```
Proc reg data = christmas; model included = d1 d2 d3 hours
d1xhours d2xhours d3xhours; test d1xhours = 0, d2xhours = 0,
d3xhours = 0; run;
```

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.902[a]	.813	.790	1.18196	.813	34.895	4	32	.000
2	.949[b]	.900	.876	.90859	.087	8.384	3	29	.000

a. Predictors: (Constant), Hours, d2, d1, d3

b. Predictors: (Constant), Hours, d2, d1, d3, d3xhours, d2xhours, d1xhours

Test is on 3 degrees of freedom because there are 3 new parameters.

Homework: (3) Show that test statistics F-change, df1, df2, and Sig. F Change do not depend on what type of coding scheme you use (i.e. replicate with sequential and Helmert coding)

Next Time

- Review testing an interaction with a multicategorical variable using a new dataset, and different coding scheme

- Probing moderation models with multicategorical predictors
    - Pairwise differences between groups
        - Pick-a-point technique
        - Johnson-Neyman technique

    - Omnibus tests of group differences
        - Pick-a-point technique
        - Johnson-Neyman technique

- Conducting these analyses using PROCESS