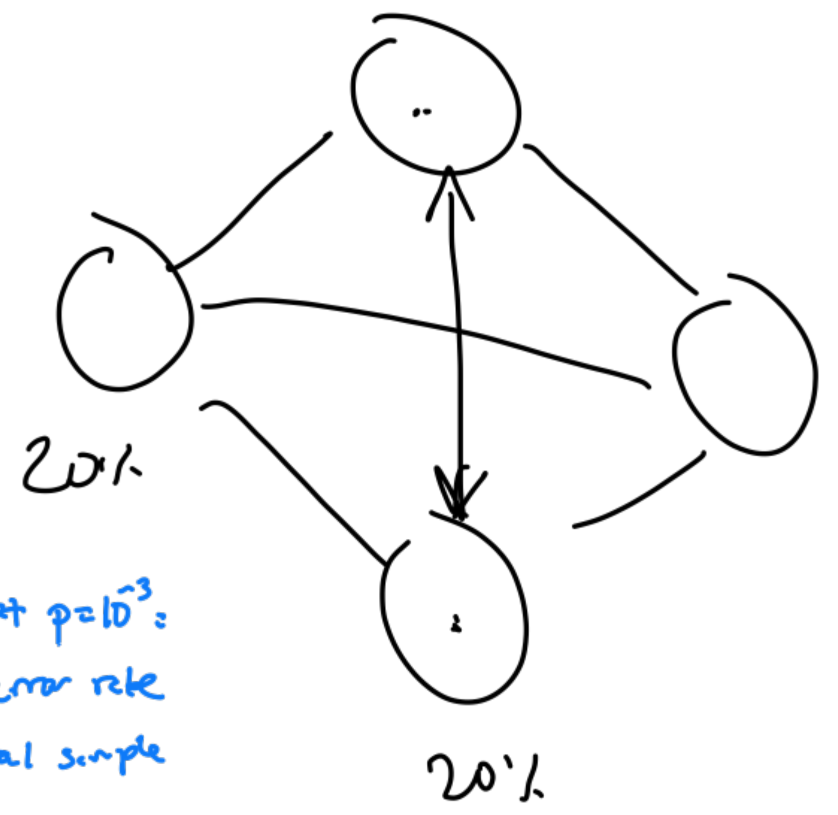

Circuit-Level Metropolis Algorithm

Initially:
MC sampling at $p = 10^{-3}$:
→ Logical error rate
→ And initial sample $\vec{E}$

① Start w/ a set of errors $\vec{E}$ which is known to cause a logical error

② Select a (gate, fault) pair at random from the circuit, (g, f)

(2a) If g is not an element of $\vec{E}$, then set:

$$\vec{E}' = \vec{E} \cup \{(g,f)\}$$

$$Pr(\vec{E}' | \vec{E}) = \frac{Pr(g)}{1 - Pr(g)} Pr_g(f)$$

Definitions

① $Pr(g)$:= probability that the gate g has some failure

② $Pr_g(f)$:= probability that, given g has failed, the failure corresponds to f.

$\vec{E} = \{(g_0, f_0), \dots, (g, h), \dots\}$
$\{(g_0, f_0), \dots, (g, h), \dots, (g, f)\}$
$\{(g_0, f_0), \dots, (g, f)\}$

(2b) If $\exists\, (g, h) \in \vec{E}$ for some h, then set:

$$\vec{E}' = \{\vec{E} \cup \{(g,f)\}\} \setminus \{(g,h)\}$$

(a) $f = h \Rightarrow$ we've effectively removed an element from $\vec{E}$

$$\Rightarrow Pr(\vec{E}' | \vec{E}) = 1$$

(b) $f \neq h$

$$\Rightarrow Pr(\vec{E}' | \vec{E}) = Pr_g(f)$$

Circuit (noiseless)
↓
NoiseModel (circuit) ← probs
↓
noisy version ckt

Through this process, each $\vec{E}'$ that gets generated appears as though it's a sample from the $\pi_{i|\mathcal{J}}$ distribution (eventually)

What we're trying to find is:

$$r = \frac{\bar{p}_{i+1}}{\bar{p}_i} = \frac{\pi_{i+1}(\mathcal{J})}{\pi_i(\mathcal{J})} = C \frac{\mathbb{E}_{i|\mathcal{J}}\left[g\left(C \frac{\pi_i}{\pi_{i+1}}\right)\right]}{\mathbb{E}_{i+1|\mathcal{J}}\left[g\left(C^{-1} \frac{\pi_{i+1}}{\pi_i}\right)\right]} = C \frac{\frac{1}{N}\sum(\,)}{\frac{1}{N}\sum(\,)}$$

$\bar{p}_1 = r \bar{p}_0$

We approximate the numerator & denominator using empirical averages:

$$\mathbb{E}_{i|\mathcal{J}}\left[g\left(C \frac{\pi_i}{\pi_{i+1}}\right)\right] \approx \mathbb{E}^{est}_{i|\mathcal{J}}\left[g\left(C \frac{\pi_i}{\pi_{i+1}}\right)\right] := \frac{1}{N} \sum_{j=1}^{N} g\left(C \frac{\pi_i(E_j)}{\pi_{i+1}(E_j)}\right)$$

How can we estimate the empirical average if this quantity is unknown?

These terms are known from the ckt probs.

This is given from the paper as $\frac{1}{1+x}$

where $E_1, \dots, E_N \in \mathcal{J}$ drawn from $\pi_{i|\mathcal{J}}$, i.e., the samples obtained from running the Metropolis algorithm. → Have to run MCMC some number of times until convergence, and then take N samples

$$g(x) = \frac{1}{1+x} = \frac{1}{1+C(\,)}$$

From these estimates, we want to find C s.t.:

$$\mathbb{E}^{est.}_{i|\mathcal{J}}\left[g\left(C \frac{\pi_i}{\pi_{i+1}}\right)\right] = \mathbb{E}^{est.}_{i+1|\mathcal{J}}\left[g\left(C^{-1} \frac{\pi_{i+1}}{\pi_i}\right)\right]$$

$$\Rightarrow \frac{\bar{p}_{i+1}}{\bar{p}_i} \approx C \frac{\mathbb{E}^{est.}_{i|\mathcal{J}}\left[g\left(C \frac{\pi_i}{\pi_{i+1}}\right)\right]}{\mathbb{E}^{est}_{i+1|\mathcal{J}}\left[g\left(C^{-1} \frac{\pi_{i+1}}{\pi_i}\right)\right]} = C \cdot 1 = C$$

DEM:

error (p) D0 D1
⋮
error (p) DN-1 DN

stim. circuit.explain

[Error 1 : → DEPOLARIZE(1/2)(p)
Error 2
⋮
Error N:]

0, error ( ) [detectors]
⋮
N error (p) [detectors]

gate$_i^0$, X
⋮
gate$_i^n$, Z