

Explicating the role of scale-dependent processes in landscape transition & stability

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Abstract

While self-organization is often invoked in landscape change, particularly heterogeneous landscapes exhibiting multiple states, the role of scale and interpretation of vegetation heterogeneity and distributions, in particular the application of power laws to vegetation cluster distributions, remains vague. We explicitly test the role of feedback scale and feedback strength in driving self-organization of heterogeneous landscapes using a kernel-based model and a robust, nonparametric test for power-law probability, as well as performance under both continuous and event-based stress. We find three distinct stable states: one without power-law structure, one with uniform power law structure across the entire landscape, and third stable state with heterogeneous power law structure exhibited in disconnected clusters, supporting the hypothesis that such heterogeneous landscapes represent a true alternative state. Landscape resilience under consistent stress and recovery after a landscape-wide mortality event is also dependent on scale of feedback processes. Landscape-scale connectivity is broken by stress at both large scales. Less extensive feedbacks are less affected, allowing them to serve as nuclei for landscape-wide recovery. Better understanding of different morphologies can help constrain the still-unclear processes leading to the development of heterogeneity in landscapes, and insights from the breaking of feedback contiguity provide insights into landscape response to disturbance and potential for recovery.

18 1 Introduction

Landscapes can be described as linked systems of biological, climatic, geochemical and geomorphic processes that have preferred equilibria, i.e. landscapes tend to exist in relatively discrete states characterized by a dominant surface cover (*Holling*, 1973; *Noy-Meir*, 1975; *Westoby et al.*, 1989). On coarse spatial scales this means that over some spatio-temporal gradient—such as temperature, nutrient supply, or moisture—there are associated transitions in landscape systems. This dynamic is most strongly associated with the boundaries of woody ecosystems, particularly the expansion of desert shrubs (“desertification”) but also the tundra-taiga boundary, alpine treelines, tropical forest-grassland boundaries, and mangrove expansion.

27 On a *landscape scale* these can systems be considered bistable, with landscape states shift past a
28 threshold or, if there are intra-landscape negative feedback process, hysteresis that prevents such a
29 shift. The definition of these internal feedback effects is poorly understood, as is its relation with what
30 can be termed *local-scale* bistability, such as the size distribution and topology of vegetation patches and
31 gaps independent of topographical influences (Lejeune *et al.*, 1999; Rietkerk *et al.*, 2004). This dynamic

32 is most strongly associated with the boundaries of woody ecosystems, particularly the expansion of
33 desert shrubs (“desertification”) but also the tundra-taiga boundary, alpine treelines, tropical forest-
34 grassland boundaries, and mangrove expansion. While hysteresis effects can be explicitly invoked to
35 explain local instability (e.g. Yizhaq *et al.* 2007 in the case of mixed vegetated and bare dunefields), the
36 exact process basis for such facilitative feedbacks is not entirely clear (Stewart *et al.*, 2014). Potential
37 facilitative feedbacks include some combination root-zone hydraulic feedbacks, under-canopy effects
38 such as the capture of longwave radiation (reducing freezing risk at night), shading (reducing heat
39 stress in the day) and nutrient capture (D’Odorico *et al.*, 2013a,b; Runyan *et al.*, 2012; Wang *et al.*, 2012).
40 Additionally, the question of these local-scale processes may add up to an overall landscape shift—
41 or whether landscape- and local-scale shifts should be considered separate, allometric processes—is
42 generally elided.

43 To connect local heterogeneity with overall landscape change, power-law relationships ($\Pr(A \geq$
44 $a) \propto a^{-\alpha}$, where a is some minimum patch or cluster size, A some larger cluster size, and α the slope of
45 the relationship in log-log space) are often invoked, since this pattern of scale invariance is typical of a
46 classical phase transition. In cases of self-organized criticality such power-law relationships can occur
47 in response to disturbance without a system-wide catastrophic shift in state; scaling behavior arises
48 from the relationship between elements in the system, in ecological cases spatially connected feedbacks
49 (Bak *et al.*, 1987; Solé and Manrubia, 1995). While earlier studies therefore suggested power law rela-
50 tionships among patch sizes may be indicative of landscape state change (Rietkerk *et al.*, 2004), Pascual
51 and Guichard (2005) suggested the term “robust criticality” to describe systems that exhibit power-law
52 scaling in a very broad parameter space without evidence of threshold behavior. Robust criticality has
53 the potential to explain potentially very long-lived heterogeneous landscapes, with Scanlon *et al.* (2007)
54 and Moreno-de las Heras *et al.* (2011) finding evidence of robustly critical scaling in savannas and mulga
55 shrublands, respectively. The persistence of such landscapes over long terms raises the possibility that
56 such states are not transitional, but either metastable or a while third states in themselves (Okin *et al.*,
57 2009; Pascual and Guichard, 2005; Sankaran *et al.*, 2004). The relationship between scale invariance
58 and remains empirically ambiguous, with differing studies showing differing results, including areas of
59 local-scale heterogeneity and evident of transition without without scale invariance Kéfi *et al.* (2007);
60 Maestre and Escudero (2009); Moreno-de las Heras *et al.* (2011). The applicability of such power-law
61 relationships has come into questions in studies of social, transport, computational, and metabolic
62 networks, so a similar investigation of power law applicability in ecogeography is in order.

63 To untangle the scaling relationships between local- and landscape-level changes we explicitly test
64 how different scales (σ) of a simple model of spatial interaction operate at different feedback strengths
65 (ϕ) under different levels of aspatial establishment-mortality ratios (M) and external variability (β);
66 this provides a proxy for increasing interannual change of a landscape, as resources such as rain or limits
67 such as cold become more variable either under some climatic change or across some spatial variant
68 where a resource such as water or temperature becomes more variable (D'Odorico *et al.*, 2013a). For
69 weak feedback processes, we would expect changes to show a greater signature of the either initial
70 state or aspatial influences, with weak feedbacks operating at wide spatial scales acting to homogenize
71 landscapes, albeit with few signs of structure expressed by power law distributions. In contrast, we
72 would expect self-organizing dynamics from stronger feedbacks, with either local, structured hetero-
73 geneity at low feedback extents and strong threshold landscape-wide behavior for strong feedbacks
74 acting on large spatial scales (figure 1). For strong feedbacks at narrow spatial scales, we expect local
75 scale bistability, while at weak feedbacks at extensive scale large-scale heterogeneity is expected.

76 Early models of landscape heterogeneity, such as (Katori *et al.*, 1998)'s Ising-based model of tropical
77 rainforest canopy gaps caused by trees felled by wind, relied on direct adjacency of cells. Hydrologically-
78 based models, initially geared towards the study of banded vegetation on hillslopes (Lefever and Leje-
79 une, 1997), used Gaussian kernel-based models to model longer-range hydrological processes; such
80 models include not only short-range facilitation within vegetation patches but long-range competition
81 between them. While this method does effectively simulate landscape patterns, it has been criticized for
82 having a weak basis in process (Stewart *et al.*, 2014). Additionally, it is less clear whether facilitation-
83 competition models effectively scale up to landscape-level changes; the feedbacks commonly invoked
84 in large-scale landscape change are largely facilitative. D'Odorico *et al.* (2006)'s model is based on a
85 facilitation-competition kernel based on the subtraction of two Gaussians, but results in a stable land-
86 scapes based on local feedbacks and the strength of spatial feedback, not its spatial scale. Rietkerk *et al.*
87 (2004) makes an argument for the combined effects of feedback strength and feedback *scale* based on a
88 changing the size and values of a convolution matrix, but this is more an illustrative example for different
89 patch or gap sizes and topologies than a continuous model of landscape change, not an actual model of
90 landscape dynamics in response to outside forcings.

91 **2 Model description**

92 State is defined by a binary variable $s = 0, 1$ on the two-dimensional state grid S (here 1000×1000
93 cells with periodic boundary conditions; cover of $s = 1$ is designated c . For simplicity we describe 0
94 as bare and 1 as vegetated, but they could just as easily be reversed or describe two different kinds of
95 vegetation (e.g. grass and shrub). The maintenance or change of state at each gridcell as a function of
96 time $s(t)$ is determined by two processes: aspatial stochastic state changes and state changes governed
97 by spatially-dependent interactions among cells. Other potential factors, modeling such qualities as
98 maturity or biomass, are ignored. While this does not produce a model with a close *biological* analogue
99 to the processes outlined in section 1, it does provide guideposts for future identification of spatially-
100 dependent facilitative interactions, which also tend to exhibit a pattern of rapid initial growth with long
101 persistence modeled here, especially in such cases noted above as shrub encroachment (*Götmark et al.*,
102 2016).

103 We initialize each individual cell $S_{(x,t=0)} = 1$ based on the starting proportion c_0 , randomly dis-
104 tributed across based on ten random seeds to ensure no one starting condition biases results. The
105 probability that some site x on S at timestep t will be of either state is governed by:

$$\Pr(S_{x=1, t}) = \Pr(E_x) - M \Pr(E_x) + V_{x, t-1}(U_{x, t-1}, S_{x, t-1}) \quad (1)$$

106 The first and second terms in the equation represent aspatial, randomly-distributed establishment and
107 mortality, with the ratio between establishment and mortality given by M . $\Pr(E_x)$ is given by

$$\Pr(E_x) = C_E 10^{\log_{10} \beta - 0.5 \log_{10} M} \quad (2)$$

108 where β is a parameter controlling the amount of *aspatial* (or external) variability in the landscape
109 from timestep to timestep (analogous to [inverse] temperature in an Ising model, or in a geo-ecological
110 context landscape-wide climate variability) and C_E is a scaling constant to translate from $\Pr(E_x)$ of
111 any individual cell to fractions of the entire landscape area. Mortality parameter M is used in calcu-
112 lating the establishment rate as well, since an environment with high mortality of for vegetated cells
113 will likewise not have favorable conditions for expansion to new sites, even if there is high potential
114 for “churn” in the landscape. This approach allows for us to test constant ratios of random establish-
115 ment to random mortality even as β increases. We test three cases: an steady-state case where $M = 1$
116 ($\Pr(E_x)/M \Pr(E_x) = 1$), a rising establishment condition where $M = 0.4$ ($\Pr(E_x)/M \Pr(E_x) = 2.5$) and
117 a declining establishment condition where $M = 2$ ($\Pr(E_x)/M \Pr(E_x) = 0.5$).

118 The third term in *eq. 1*, probability of *spatially-dependent* establishment and mortality (our internal
119 dynamics), is defined by the spatial interaction grid U , a two-dimensional Gaussian blur of the
120 initial state grid controlled by the standard deviation $\sigma \in [1, 4]$ at intervals of 0.5, controlling the
121 degree of spatial interaction. We only use one Gaussian to model spatial interaction, unlike other models
122 such as *D'Odorico et al.* (2006) that use the difference of two Gaussian functions to model facilitation-
123 competition over space models with discrete areas of facilitation-competition *Rietkerk et al.* 2004. Since
124 the blur is extended over both states we are focusing on “vegetated” cells, we only need consider cases
125 where $c_0 \leq 0.5$, since $c_0 \geq 0.5$ would produce the symmetric results with “gaps.”

126 The importance of local facilitation in determining the state of each cell relative to its state in the
127 previous timestep $t - 1$ is controlled by a feedback parameter ϕ (figure 2). A new grid of potential values
128 at sites, V , is thus calculated as

$$V_{x, t-1} = \phi U_{x, t-1} + (1 - \phi) S_{x, t-1} \quad (3)$$

129 Values on sites $V_{x, t-1}$ form a continuous range (0, 1) representing the probability for an individual
130 site to be occupied by a woody plant in timestep, and drawing from these probabilities to arrive at
131 values of 0 and 1 produces $S_{x, t}$. To ensure greater model stability from timestep to timestep (reflecting
132 conditions in nature, where dramatic fluctuations in woody plant populations from year-to-year are
133 rare), after the initial calculation of U values of less than 0.02 are set to 0 and more than 0.98 set to 1 to
134 guarantee they remain bare or vegetated.

135 Model runs are performed for 1000 timesteps, with results presenting at 500 timesteps (depending
136 on the species of cover analogized, each timestep could represent anything from a growing season to a
137 few decades). The 500 timesteps are sufficient for a spin-up for the majority of each landscape to arrive
138 at final state while allowing for the display of some transitional areas; 1000 timestep runs were made
139 to check the persistence of intermediate states. When testing for hysteresis after a mortality event, M
140 is increased to a 8 for the time interval $t = [300, 399]$, with the assumption β remains constant over
141 the entire model run, i.e. an environment with more background variability will also experience a more
142 destructive mortality event.

143 Clusters are identified using a Hoshen-Kopelman algorithm using von Neumann neighborhoods and
144 all grids have toroidal boundary conditions. For determining the presence of power law distributions,
145 our approximate proxy for critical landscape structure, we only take cases with fifty or greater power
146 different cluster sizes and, using the powerlaw module developed by *Alstott et al.* (2014), performed

147 maximum likelihood estimates for every potential value of minimum cluster size a (eq. 1), choosing the
148 values for a and α that minimize the Kolmogorov-Smirnov statistics D , corresponding to the distance
149 between the data and fit. We use the in nonparametric test for long-tailed distributions in *Clauset et al.*
150 2009; *Virkar and Clauset* 2014; *Vuong* 1989), where a p -value is calculated by bootstrapping the distri-
151 bution, generating synthetic datasets to compare to the original model to arrive at a set of bootstrapped
152 D_b -values. By comparing the ratios to test the likelihood that we have a scale-free distribution, with one
153 being more likely. To give a precision ϵ of one decimal digit we generate twenty-five synthetic datasets
154 in accordance with the rule-of-thumb that the number of datasets should equal $(\epsilon/4)^{-2}$. Any values
155 where $p \leq 0.1$ are rejected, though such low values would only represent weak support for a power
156 law so p -values should be interpreted as rough levels of support rather than a definitive test.

157 3 Model Results

158 Results displaying cover and power law probability—our proxy for structure—are depicted in figure 3.
159 Rows 3a and 3b describe cases where the starting cover $c_0 = 0.35$ and $\beta = 1$ and $\beta = 5$, respectively.
160 3c describes where $c_0 = 0.5$ and $\beta = 1$ (increasing β at $c_0 = 0.5$ has little effect on cover and similar
161 effects on pattern as in 3b). Columns correspond to mortality ratio M , so when individual charts from
162 figure 3 are referred to we add this to the figure number (e.g. figure 3b-0.4). Figure 4 displays the spatial
163 distribution of vegetated cells at $t = 500$ for the case of $M = 1$ (even establishment and mortality), with
164 lettering following the same (c_0, β) parameters as in figure 3, with areas on the (σ, ϕ) denoted by i. (low
165 state, minimal (σ, ϕ)), ii. (intermediate state, minimal σ , maximal ϕ), iii. (transition between states,
166 medium (σ, ϕ)), and iv. (“high” state, maximal (σ, ϕ)). This convention holds through figures 5 and 6,
167 discussing response of the system after disturbance. Similarly, figure 5 shows (c_{50}, σ, ϕ) results after a
168 mortality event (M increased to 8) occurs in the time interval $t = [300, 399]$ and figure 6 corresponding
169 cover maps. Note that these are mapped in parameter spaces, not space or time, with σ and ϕ staying
170 constant for each run. We mainly report results for $M \in [0.4, 2]$ since for the former there is universal
171 growth without structure and the latter universal mortality without structure. $c_0 = 0.2$ largely follows
172 the same pattern as $c_0 = 0.35$ albeit at lower coverage, and $c_0 > 0.5$ is symmetrical but with vegetated
173 and gap areas reversed. Additionally there is a special case at where cover is the same across the entire
174 parameter space, allowing for a control case for different morphologies of cluster and gap distribution.

175 Across all cases in figure 3 we can identify either a low state or an advanced transition to one.
176 At low noise the smaller initial starting populations are overwhelmed by the initial higher levels of

177 cover. Even in this high-establishment case in 3a-0.4, small, localized feedbacks are *still* sufficient to
178 drive overall land cover down, as larger initial local gaps prevent new establishment and reinforce new
179 mortality, resulting shrinking vegetated space even when overall establishment rates are high. In figure
180 4a-i (under steady-state establishment and mortality $M = 1$) we see the effect of these dynamics: apart
181 from the small amount of snow from interannual noise, cover is in a few where vegetation only remains
182 in clusters of sufficient size to allow resilience, leading to a cluster size distribution weighted towards
183 larger clusters with well-delineated boundaries. In the special case of $c_0 = 0.5, \beta = 1$ weak feedbacks
184 lead to a “quench” sorting with clearly-delineated boundaries, albeit in a labyrinthine pattern (figure
185 4c-i). In some cases the low state extends along the σ axis as low cover is enforced by widespread
186 feedbacks, such as when $M > 1$ and in some cases where $M = 1$ (figure 3a-1.0). As β increases with
187 aspatial noise overwhelms weak local feedbacks and initial distribution of vegetation leading to neat-
188 total overwhelming of initial conditions (figure 3b-0.4). Depending on external variability, then, the low
189 state then exhibits convergence in (lack of) structure but divergence in cover.

190 The total area covered in the high state at high- σ , high- ϕ is determined by the outside parameter
191 M and the Gaussian kernel: $M = 1$ converges on 0.5, while $M = 0.4$ converges on 0.68 and $M = 2$ on
192 0.32, approximating a normal distribution around each cell. So the high state is always associated with
193 heterogeneous cover, displaying local structure universally across the landscape when the mortality
194 ratio $M \leq 1$ with little signal from initial conditions (U_{t-1} disperses and V_{t-1} diminishes the infor-
195 mation transferred from S_{t-1}). This local structure is found in both vegetated and gap spaces, leading
196 to conditions under high β as in figure 3b-i where the high state’s cover is less than the low state’s.
197 Additionally, under high β where $M \leq 1$, the areas along the σ axis take the high rather than the low
198 state. Where $M \geq 1$ power law likelihood is *reduced* by random mortality breaking networks of spatial
199 organization even as extensive feedbacks allow overall cover to be maintained. Thus when crossing
200 $M = 1$ in the high state low state we see continued convergence on cover but divergence in structure.

201 In the parameter spaces between low and high state (intermediate σ , intermediate ϕ) power law
202 organization exists in the handful of cells without complete spin-up at $t = 500$, indicating power law
203 likelihood can be associated with transitions between states, even if neither the initial (random) and
204 final states show little to no evidence of power law structure (figure a,b,c-2.0). following the expectation
205 from a phase transition in a landscape, where the shift from one phase to another exhibits power-law
206 scaling. In fig. 4a-iii, for instance, vegetation levels dropping towards a low states generates coarse-
207 scale heterogeneity across the landscape as superclusters with higher initial density persist longer. The

208 margins and interiors of these superclusters are themselves heterogeneous as the vegetation clusters
209 shrink in accordance with a power law distribution. Since there is no trend towards higher or lower land
210 cover in figure 4c-iii there are similar areas of both uniform vegetation and uniform gap surrounded
211 by power-law-structured vegetation and gap areas. These transitionary areas also have the highest
212 coefficient of variation in total cover from seed to seed albeit still low, with $CoV \approx 0.01$ at $\beta = 1$.
213 Increasing β reduces the importance of distributions by increasing spatially unbiased inputs, resulting
214 in more spatially uniform, locally heterogeneous new establishment, again pushing towards a global
215 shift in state where initial spatial distribution has little influence.

216 The intermediate state at low σ , high ϕ (ii. in figures 4,6) consistently exhibits stable power law
217 structure regardless of overall cover in cases where $M \geq 1$. Rather than spreading fine-grained hetero-
218 geneity, at low β this area exhibits uniform structured heterogeneity across the landscape but
219 without landscape-wide connectivity at low starting populations, and even in with higher landscape
220 populations there is a lesser degree of global connectivity and an increase in near-adjacent clusters
221 (compare the relatively reticulated patterns in figure 4c,i ii.). High ϕ increases the importance of spatial
222 interactions even as low σ keeps their extent tighter, resulting in noisier boundaries between vegetated
223 areas than in the low state as random establishment is only retained by these strong-but-local spatial
224 interactions At $M = 1$ and low β this area is indistinguishable by landscape coverage or power law
225 likelihood and is only apparent from spatial organization, visible both in the case of (figure 4a-ii., 4c-ii.).
226 This results in long-term persistence of isolated-but-structured clusters even under long time spans,
227 evidence this state truly is an alternate one, albeit only arising under stressed conditions ($M \geq 1$).
228 Even at high β , where there is a convergence in cover but still some difference in how each landscape
229 approaches maximum cover, there remains some difference in organization of cell networks, with a
230 median cluster size lower in under inextensive feedbacks case 4b-ii than under extensive ones case
231 4b-iv (25974 vs. 33081 cells, respectively).

232 After a mortality event is introduced, only $M \leq 1$ exhibits any recovery past a universal low state.
233 The low state remains at low cover, though at higher β there is some measure of spatially-facilitated
234 recovery, leading to a higher p -value. At low values of β the high state is most resilient in cover, even
235 as power law likelihood disappears as in the high mortality cases in figures 5,6. As β increases, though,
236 strong and extensive feedbacks maintain gaps created in the mortality event and leave the area with low
237 cover. The most resilient in *structure* remains in the intermediate state, where close ranges of strong
238 facilitation lead to some residual survival and preferential nearby establishment in cover, which is also

evidenced by the distribution of cluster sizes visible in figure 6. This effect is most pronounced in bases of higher β (figure 5b), where the intermediate state has the highest levels of cover relative to the rest of the parameter space. The appearance final states here are not dissimilar to those in under regular, non-hysteresis conditions (figure 6 a,c-i,ii), with those in the $\beta = 5$ well on their way to recovery; given that $M = 0.4$ in these examples such recovery is like to be towards a low-organization, but high cover, state as in figure 3a,b-i. While there remains greater cover than other cells of higher σ , in $\beta = 5$ cover is at the high- σ , high- ϕ corner is further attenuated, as connectivity between clusters is lost both at the scale of fine networks but also between large areas of the landscape (figure 6b-iv). This may be compared to the unstable states described in figure 4a-iii, where consistent intermediate-scale feedbacks lead to slow change towards the high or low state.

Based on this we can revise our conceptual model from figure 1 and add how it responds under disturbance (figure 7). Initial conditions and outside conditions are dominant in the low- σ , low- ϕ corner, though it is worth emphasizing that both of these are still mediated through spatial feedbacks, only the transition to a final state exhibits spatial self-organization that manifests as a power law structure, consistent with a more classical phase transition. Transitions in cover at higher (σ, ϕ) evolve not from discrete clusters growing but rather a mosaic of high and gap states that either spread or, in the cases of fig. 4a-iii, 4b-iii, eventually settle into one of the states; scale of process, not strength seems more important here (thus the revision in the high- σ , low- ϕ corner in figure 7b). Cover in this high state is relatively stable under a wide variety of parameter spaces in the absence of a ϕ , and the more heterogeneously-clustered intermediate cover exhibits stable organization even under high M , with or without a mortality event. While there is a relatively clear split in parameter space between high, low, and intermediate state, within each case there is little evidence of threshold behavior within the run of each case, consistent with some recent observations of landscape change supporting more gradual than catastrophic shifts over time (*Bhattachan et al., 2014*). The resilience of power-law structure under a wide parameter space, especially in the intermediate state, is consistent with *Pascual and Guichard (2005)*'s robust criticality.

4 Implications for process scale & inferring landscape change

Even weak and local stable process external climatic variability. When the overall trend in the landscape is more hostile to vegetation, this still results the sort of heterogeneous landscapes as in figure 4a-i. This non-power law distribution of cluster sizes is in accord with *Maestre and Escudero (2009)*'s

observation that semiarid landscapes in Spain do not exhibit scale-free relationships but are skewed towards larger cluster sizes which serve as facilitative sites for maintenance of vegetation. Under higher variability in death and establishment even the low-feedback area can exhibit some evidence of power law cluster size distributions while in transition, as in figure 4b-i, as clusters grow and are chained together, increasing the connectivity of feedback processes. It may be expected that cluster size distributions in landscapes with very localized, weaker feedbacks may approach and retreat from power law probabilities across growing cycles, with a large cluster-skewed frequency distribution of clusters remaining an attractor as larger clusters serve as loci of resilience.

The “uniform heterogeneity” in is largely a consequence of how the high-state can be considered a system with well-mixed disturbance. If we substitute “establishment” and “mortality” with “recovery” and “disturbance” we see a model fairly similar to the classic Ising-based models of forest disturbance pioneered by *Katori et al.* (1998) at $M \leq 1$, with the breaking of structure at $M > 1$ consistent with the lack of a recovery phase in our landscape. A comparison can be made to widespread feedbacks, such as local recycling of moisture. The slow transition to one state or another in each discrete (σ, ϕ) parameter space is consistent with the dynamics of landscapes sustained by large-scale feedbacks *without disturbance* (*Ahstrom et al.*, 2017). It is often under cases of *disturbance*—either natural as by fire or anthropogenic as with grazing—that abrupt boundaries appear between grass states and forest states still sustained in a well-mixed “high state” organized evaporation-precipitation feedbacks (*Oliveras and Malhi*, 2016). This is comparable to the relatively mosaic hard-edged mosaics in the cases of mortality or unstable spatial transitions, where larger-scale productivity is locally broken when interacting with another feedback-supported state; even though the unstable case in figure 4a-iii was not maintained over the long term, a constant disconnection and reconnection of feedback connectivity may have the effect of a constant instability, or true landscape-scale bistability.

The intermediate state in figures 4a-ii and 4c-ii is, like the high state, uniform, but the individual clusters lack the “uniform heterogeneity,” especially at low β instead consisting of solid core clusters as in the low state but with noisy, power-law distributed clusters adjacent and near-adjacent without landscape-wide connectivity. Ecohydrological feedbacks are considered the most likely candidates for such robust, relatively short-range (meters) feedbacks. In arid regions overlapping root systems a combination of root-facilitated soil water uptake, canopy shading of soil, canopy prevention of rainsplash soil compaction, as well as recycling of nutrients via leaf litter (*Caylor et al.*, 2006; *D'Odorico et al.*, 2005, 2007; *Scanlon et al.*, 2007; *Wainwright et al.*, 2009). The appearance of a state consisting of isolated

300 clusters stressed is also in accord with the such patterns occurring under conditions of water stress
301 (figure 3, column 2.0), such as in with the decreasing area under facilitation roughly to the thresholds
302 for facilitation seen in more process-based models (*Caylor et al.*, 2006; *D'Odorico et al.*, 2005).

303 After a mortality event and likewise can serve as connected networks for expansion back to a high
304 state, as the similarity between strong recovery in the low σ parameter spaces across ϕ , both in the
305 high- β and post-mortality case also demonstrate how such short-range feedbacks can chain into larger
306 ones still exhibiting power law structure without long-range facilitation, consistent with the dense
307 savannah landscapes under increasing moisture described by *Scanlon et al.* (2007). Recovery after a
308 mortality event is likewise strong in this area of the parameter space both due to intrinsic parameters
309 for facilitation and greater survival of facilitation networks (best observed in figures 5b, 6b-ii), but
310 it is worth noting this depends on the the uniformity of our landscape's underlying conditions, with
311 spatial facilitation and regrowth happening in much the same way as before the disturbance. In an
312 actual mortality event the full hydro-ecogeomorphic system is may be disrupted, leading to changes
313 in facts such as soil structure or nutrient supply. As such, after a mortality event resilient clusters may
314 the distribution found in in figures 4a-i,6a-i (corresponding to cases where $M > 1$ after a disturbance,
315 when no recovery occurred).

316 An alternative mechanism—and one more applicable beyond arid environments—is the abilities
317 of canopies to capture longwave radiation emitted at night. One of the clearest examples of a purely
318 facilitative feedback is the role of radiative feedback. *He et al.* (2010) and *D'Odorico et al.* (2010)'s
319 studies of microclimates in the northern Chihuahuan Desert compared grassland temperatures with
320 those dominated by the shrub *Larrea tridentata*. While the mechanism for warming is under-canopy,
321 the effect diffuses to a landscape scale of of ~ 100 m. On average the shrub-dominated landscapes were
322 2° relative to nearby grasslands, with differences of 8° on the coldest nights, reducing winter freeze-
323 induced mortality across the landscape. Similar facilitation processes—both in preventing mortality
324 winter and enhancing summer growth—have been observed in boreal forests (*MacDonald et al.*, 2008).
325 Empirical studies and models of alpine treelines have shown a similar dynamic, both in terms of pre-
326 venting mortality and facilitating growth, with chains of weakly-connected clusters advancing north-
327 wards, again likely sustained by such radiative feedbacks (*Resler and Fonstad*, 2009; *Zeng and Malanson*,
328 2006) While the feedback processes in this model are isotropic, new establishment beyond northern or
329 altitudinal margins of boreal and alpine landscapes produce similar, high power law likelihood patterns
330 of establishment resembling the robustly critical cases in the intermediate state. Such feedbacks are

likely to occur on the scale of meters to tens of meters.

The above feedbacks highly specific, both in different scales of feedback (e.g. the radiative facilitation under mangroves, another cold-limited woody plant species currently expanding its range, are probably restricted to the immediate area around under-canopy spaces, pers. comm K. Cavanaugh and R. Bardou, 2018), so radiative feedback may be associated with the intermediate state or a transition to a high state. Additionally, the presence of a feedback process is dependent not just on plant species but the facilitative effects of the surrounding environment, e.g. for ecohydrological facilitation soil properties need to comport with infiltration and root system properties, for boreal and alpine facilitation the effectiveness of the above litter-microbial feedbacks to prime soil for future expansion. The capability for recovery these factors remaining constant, which is not necessarily the case in a large-scale mortality event. Additionally, there needs to be some base for recovery to continue, either from small surviving clusters or new establishment creating new clusters for recovery.

5 Conclusion

We establish a model that reproduces heterogeneous landscapes based on a combination of local feedbacks and global tendency towards change, many of which exhibited power-law scaling laws in their distribution of cluster size. This was done without some of the common assumptions made in studies of landscape heterogeneity, such as the presence of separate facilitative and competitive processes or zones of interaction. Furthermore, in addition to generating clearly distinct landscape states at high and low levels of cover we also, by using the nonparametric power law test from (*Clauset et al.*, 2009) we found thresholds across our extent, scale (σ, ϕ) parameter space. Furthermore, this model reproduced different scales of and structures of heterogeneity in agreement with different environmental types: both unstructured and structured (not exhibiting and exhibiting power laws) observations in semi-arid to arid environments and structured, but high-cover examples in forested woody landscapes.

This has implications for long-standing assumptions on the applicability of power law relations in landscapes. Heterogeneity is not evidence in itself of incipient change, although it is always associated with shifts in landscape (see the emergence of power laws in figure 3b-0.4, 4b-i., 5b-0.4, 6b-i.), in accordance with classical models of state change. These final states can still exhibit stable heterogeneity, resulting in a “quenched” state typically with hard boundaries between states depending on the starting population and a rank-order distribution weighted towards larger (relative to overall cluster area) clusters; such states . Power-law structure can be maintained, though, either in the presence of strong

361 long-range feedbacks, maintaining power-law structure over long areas as external variability is mixed
362 into the structure, making for a uniformly self-organized structure.

363 The presence of long-lived spatial heterogeneity in stressed (equal or higher ratios of mortality to
364 establishment) imply the presence of short-range, sustaining feedbacks. The observations of such dis-
365 tributions of woody plants in arid and semi-arid and arid landscapes over long periods of time suggests
366 dynamics akin to *Pascual and Guichard* (2005)'s concept of robust criticality. Yet, given the presence of
367 similar dynamics during transition, even under weak feedback strength, there remains some ambiguity.
368 As such, power law relationships are not enough to discern the state or direction of a landscape—they
369 are not necessarily associated with stable heterogeneity they can represent change or stability, and
370 long-term resilience of cover can still occur in their absence. While evidence of spatial self-organization
371 can provide insights into the processes shaping landscapes and their *potential* dynamics, they must be
372 put in the context of other studies of landscape history and geo-ecological process.

373 Additionally, this model started with random initial distributions of vegetated cells and, while spatial
374 feedbacks influenced the establishment or mortality of interannual noise to the landscape, the noise
375 inputs themselves were likely aspatial. While such processes can occur on the landscape scale, such as
376 landscape-scale interannual variability in precipitation or temperature, but while event-based changes
377 are important for understanding landscape structure and resilience at centennial or shorter timescales
378 longer-term changes require, in the terms of this study, a shift in M , and given increasing global favor-
379 ability towards woody plants a shift from $M = 1$ to $M < 1$. With constant mortality ratios stressed,
380 intermediate-state environments will either spread into either high-cover, low-structure state or a high
381 state with well-mixed power law structure without catastrophic shift in state over time. Furthermore,
382 while spread in this model would be even across the landscape, in real landscapes they would be asso-
383 ciated with landscape features facilitating longer within-landscape connectivity (e.g. preferential cor-
384 ridors for water, nutrient, or propagule transport) or expansion along a gradient, such as expansion of
385 the "fuzzy" edge of a treeline northward with increasing temperatures.

386 Similarly, disruption often takes a spatially heterogeneous form beyond the uniformly-forced mor-
387 tality in this model. Such disruption, when well-mixed across a landscape, simply breaks structure and
388 reduces cover, with the lack of any explicit recovery mechanism (typical in such models; *Katori et al.*
389 1998; *Pascual and Guichard* 2005) in the face of high external mortality breaking structure. Strong
390 breaks in structure during mortality events lead to the emergence of hard-edged borders between gap
391 spaces (figures 6b-i.), much as in the high state-gap mosaics that temporarily occur in cases without ex-

392 ternal disturbance (figure 4a-iii.,; in 4c-iii. there are also hard edges between uniformly vegetated areas
393 due to equal starting ratios of vegetation to gap). Depending on their location on the parameter space
394 such shifts could lead to an abrupt shift. However, given the overall similarity between each surface
395 this zone is likely small unless coupled with some change in interannual variability either promoting or
396 preventing new establishment.

397 While large-term external variability across the M parameter may be a consequence of change in time, *within* each (σ, ϕ) , such as year-to-year changes in connectivity of feedback processes due to such factors as shifting levels of ecological competition or changing disturbance-recovery patterns over time, 400 provide potential explanation for landscape heterogeneity between small clusters and large-scale organized stability. Without even forcing or a single discrete mortality event, though, the scale of interaction—401 and thus level of connectivity—can change dynamically over time, leading to shifting mosaics at the borders of landscape types, such as those observed on the borders between forest and savanna landscapes 404 (*Oliveras and Malhi, 2016*). Such localized disturbance, between the level of close-scale patterns in vegetation clusters and landscape-scale structure, provide a fruitful avenue of study both in modeling how 405 such disturbance stabilizes mosaics and responses from either short- or long-range self-organization.
406

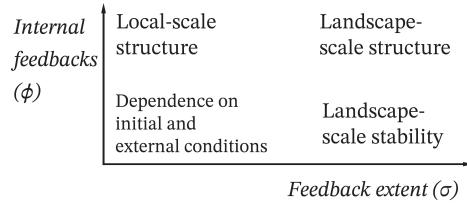


Figure 1: We expect the following dynamics after running the model at varying feedback extents (σ) and strengths (ϕ). At low values of both initial conditions are expected to persist, or changes due to external forcings will be purely dependent on them. In the opposite high- σ , high- ϕ quadrant we expect landscapes to exhibit widespread self-organized structure (with *structure* defined as exhibiting a power-law size distribution.) as internal feedbacks are dominant. At high- σ , low- ϕ we expect landscape-wide stability without organization as they are stabilized by long-range feedbacks without strong enough short-range facilitation to lead to structure, and short-range structure without landscape-level effects in the opposite low- σ , high- ϕ corner.

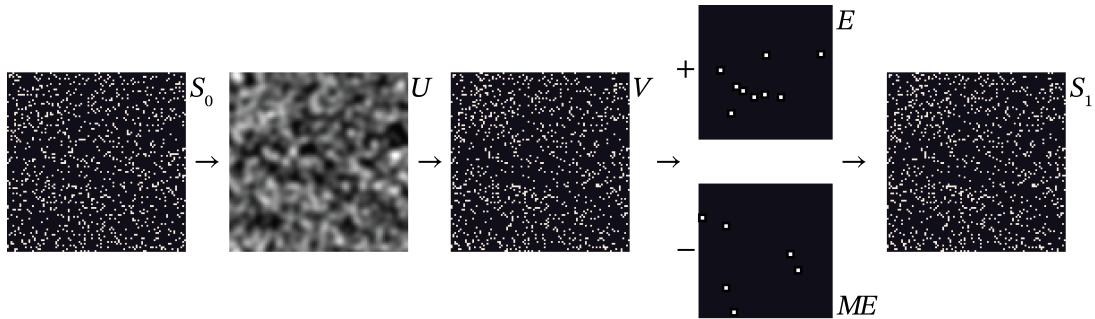


Figure 2: Diagram of model progression over each timestep: moving from S_0 to U we Gaussian blur the initial state grid at a variance σ . The probability that any given site x on V will be occupied is given by $\phi U_x + (1 - \phi)S_x$, resolving into values of 0 or 1. Grids randomly generated for establishment E and mortality ME (with cells magnified here for demonstration) are then added and subtracted, yielding S_1 . Landscapes depicted are 1% as large as landscapes used in model.

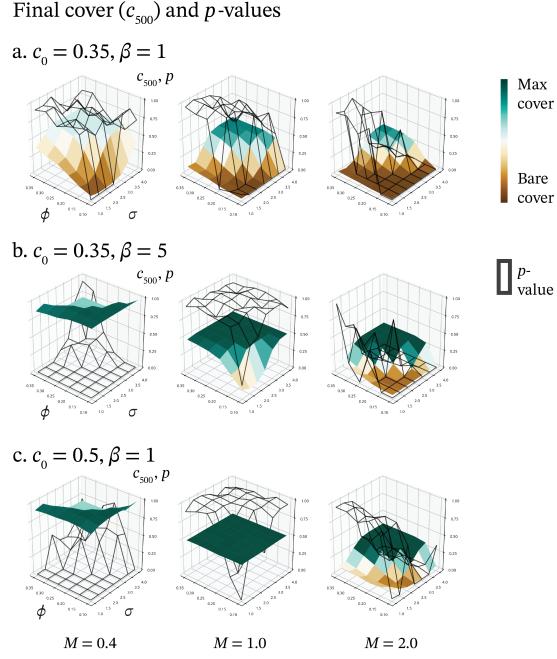


Figure 3: Land cover c_{500} and power law probability p for each (σ, ϕ) space, with each (σ, ϕ) surface arranged according to mortality/establishment ratio M and rows based on initial starting population c_0 and outside variability β ; row numbering is consistent across figures (e.g. row a. always describes $c_0 = 0.35, \beta = 1$ in figure 4 as well); $c_0 = 0.5, \beta = 5$ is excluded since it has similar cover dynamics as $c_0 = 0.5, \beta = 5$ and the effects of increasing β on cluster morphology are similar to $c_0 = 0.35, \beta = 5$ (figure 5). Since cover in the “high” state is regulated by the overall mortality ratio, the colorbars indicating state are scaled by mortality ratio, not absolute levels of cover. The wireframe surface represents power law probability p . Note the special case at $M = 1, c_0 = 0.5, \beta = 1$ where cover is uniform across the (σ, ϕ) surface but power law probability varies.

Final cover maps at $M = 1$

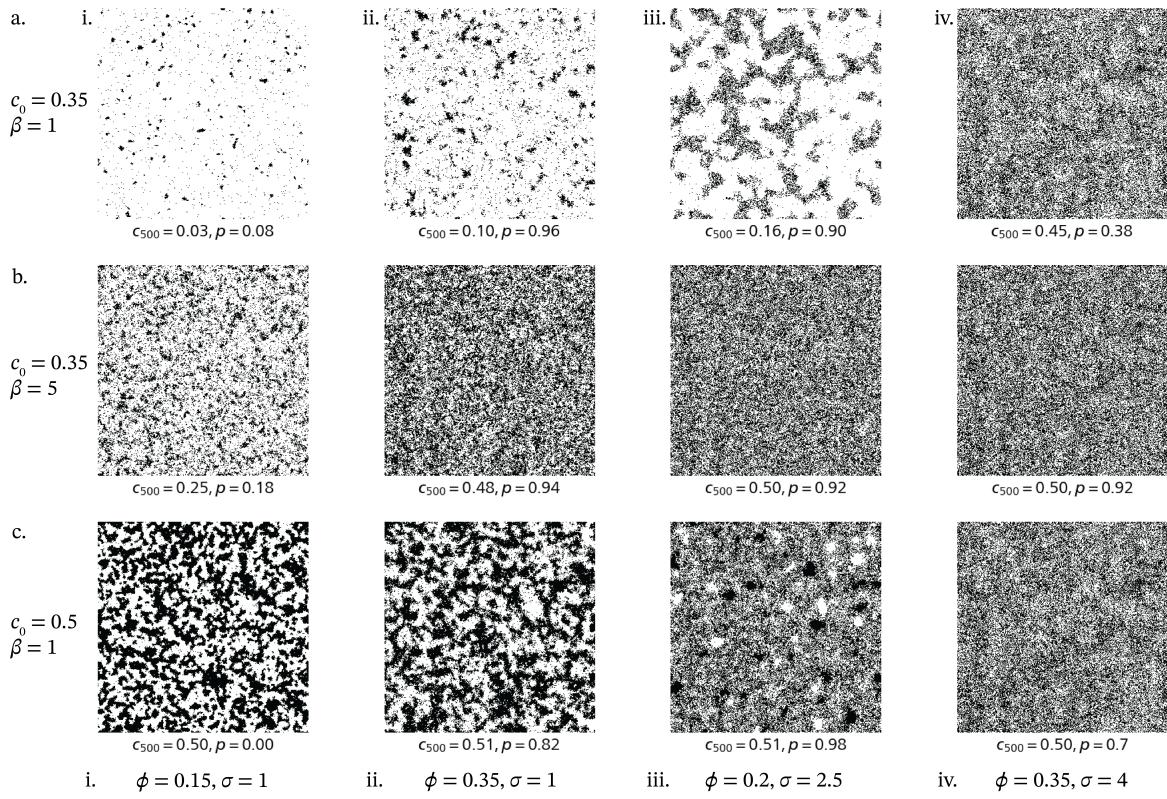


Figure 4: Examples of land cover c_{500} at $M = 1$. Column i. is the low state at minimal (σ, ϕ) . ii. is the intermediate state and minimal σ , maximal ϕ , iii. an unstable at $t = 500$ state at $(\sigma, \phi) = 2.5, 0.2$, declining to the low state in row a., increasing to the high state in row b, and in a steady state of cover row c., and column iv. is the high state at maximal (σ, ϕ) .

Post-mortality event ($M = 8$),
recovery at $M = 0.4$

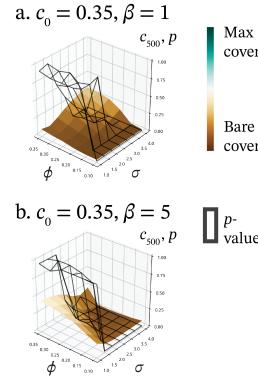


Figure 5: (σ, ϕ) surfaces for land cover c_{500} after a mortality event at $M = 8$ and a recovery condition of $M = 0.4$ for greater clarity of recovery dynamics; similar dynamics are seen at $M = 1$ and $c_0 = 0.5$ and are excluded for conciseness, as are cases when $M > 1$ as they exhibit zero recovery after the mortality event.

Final cover maps at high-growth $M = 0.4$ after disturbance event ($M_{\text{dist}} = 8$)

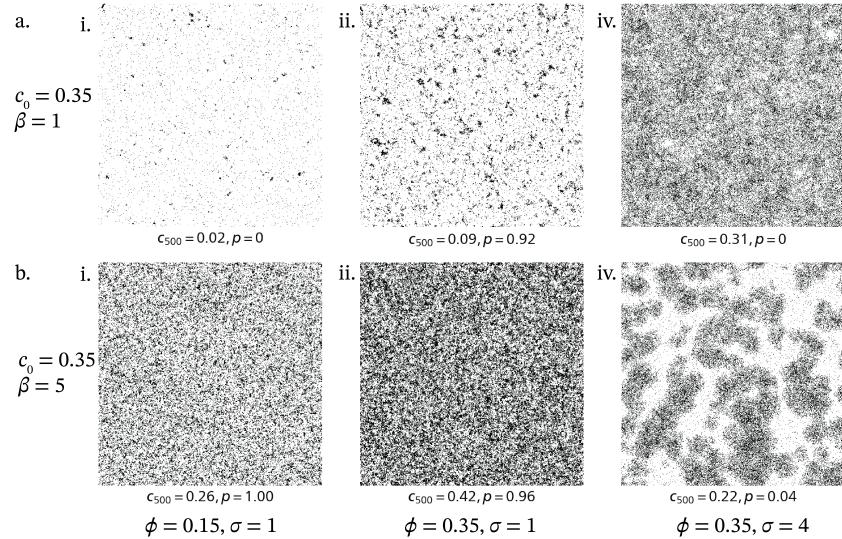


Figure 6: Examples of recovering land cover c_{500} on the (σ, ϕ) surface after a mortality event at $M = 8$ and a recovery condition of $M = 0.4$ for greater clarity of recovery dynamics. Examples from the middle of the surface $((\sigma, \phi) = 0.25, 2.0)$ are less relevant so column iii. is excluded.

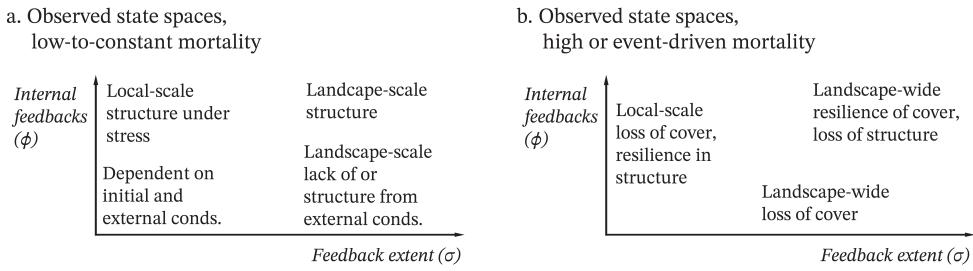


Figure 7: Revising the expected change space in figure 1, in diagram a. we find a broad similarity with expectations from figure 1 with some revisions. Only the minimal and maximal corners in (σ, ϕ) space consistently manifest. The high σ , low ϕ corner can take on the characteristics of either the minimal or maximal corner depending on starting population and outside conditions, with a greater tendency toward the unstructured state. The low- σ , high- ϕ corner only manifests a distinct set of local-scale dynamics under stress, but converging towards the unstructured state when $M < 1$ (figure 3). Under greater stress or after a mortality event the dynamics in structure and cover are often reversed, with some cover remaining at the maximal (σ, ϕ) corner without structure but stronger resilience or growth of locally self-organized cells in the higher- ϕ area, which lead the recovery of the landscape (figure 5)

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