02 : Large Sample test.

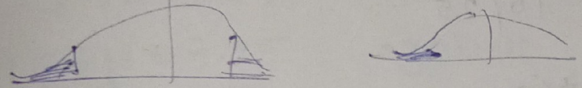
Best critical region of z-test =

To test $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$
: $\theta > \theta_0$
: $\theta \neq \theta_0$

For the significance level α the Bst c.regions are — $w = z < -z_{\alpha}$ (1-tail), $w = z > z_{\alpha}$ & $w = |z| \geq z_{\alpha/2}$ (2-tail)

eg → when $\alpha = 0.05$, the best c.regions are —

$w = z < -1.645$, $w = z > 1.645$, $w = |z| \geq 1.96$



$|z| \geq z_{\alpha/2}$ $\quad$ $z < -z_{\alpha}$

when $\alpha = 0.01$ level of significance, $w = z < -2.326$,
$w = z > 2.326$, $w = |z| \geq 2.58$

when $\alpha = 0.02$, L of S, $w = z < -2.055$,
$w = z > 2.055$, $w = |z| \geq 2.326$

→ Testing mean of a population =

By testing mean of (P), we are actually testing the significance difference b/w (P) mean & sample mean. the null hypo - H_0 :
Suppose we want to tst the alternatives H_1 : $\mu = \mu_0$ against 1 of the $H_1 : \mu \neq \mu_0$ or $H_1 : \mu > \mu_0$, $\mu < \mu_0$ on the basis of r. sample size of n from a normal (P) with non variance σ^2. the bst c.r are For the significance level α, $w = |z| \geq z_{\alpha/2}$, $w = z > z_{\alpha}$, $w = z < -z_{\alpha}$,

Tst statistic, $\boxed{z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$

using the sample data calculate the value of z. ~~if the~~ we will accept H_0 if the cal. value is < than table value.

Note

* If the (P) is gvn to be normal the tst procedure is valid even for small samples provided σ is known.
* when σ unknown & n is large, in the statistic we hve to replace σ by its estimate s

Q
1) A sample of 25 items where taken from a (P) with st-deviation 10 & sample mean is found to be 65. Can it be regarded as a sample from a normal (P) with $\mu = 60$

A) ~~$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{65 - 60}{10/25} = 5.$~~

$H_0 : \mu = 60$ against $H_1 : \mu \neq 60$

let $\alpha = 0.05$ (a-നൽകിയിട്ടില്ല) . BST CR : $|z| \geq z_{\alpha/2}$

$|z| \geq 1.96$

here tst statistic,

$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{65 - 60}{10/\sqrt{25}} = \frac{5}{10/5} = \frac{25}{10} = 2.5$

∴ $|z| = 2.5 \geq 1.96$ rejected

(i.e) Sample cannot be regarded as drawn from normal (P) with $\mu = 60$.

2) A new stereo needle was introduced enter the mrkt claiming that it has an avg life of 200 hrs with a SD of 21 hrs. This claim come under critical critsm. ~~from~~ A customer grp tested 49 needles & found that avg life of 191 hrs is the claim of the manufactural justified.

A) $\mu_0 = 200$, $\sigma = 21$, $n = 49$, $\bar{x} = 191$

$H_0 : \mu = 200$ against $H_1 : \mu_0 < 200$

let $\alpha = 0.05$

BCR is $\omega = z < -1.645$

test statistic is, $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{191 - 200}{21/\sqrt{49}}$

$= \frac{191 - 200}{21/7} = \frac{-9}{3} = -3$

$z < -1.645$

$-3 < -1.645$ ✓

~~accept~~

3) A sample of 900 members is found to hve the mean of 3.4 cm & SD = 2.61. Could it be reasonably regarded as a sample from a large population whose mean is 3.25 cm. use 2 ~~tailed~~ tailed test & $\alpha = 0.01$.

A) $n = 900$ $\bar{x} = 3.4$ $s = 2.61$ $\alpha = 0.01$

$\mu_0 = 3.25$

$H_0 : \mu = 3.25$ against $H_1 : \mu = \mu_0$.

BCR , $\omega = |z| \geq 2.58$

test statistic , $z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3.4 - 3.25}{2.61/\sqrt{900}}$

$= \frac{3.4 - 3.25}{2.61/30} = \frac{0.15}{0.087} = 1.724$

$|z| = 1.724 < 2.58$

Accept

=> Testing the equality of 2 (P) means =

By testing the equality of 2 (P) means we are actually testing the significant diffrnce b/w 2 sample means. Suppose we want to test the null hypo $H_0 : \mu_1 - \mu_2 = 0$ ($\mu_1 = \mu_2$) against 1 of the alternatives $H_1 = \mu_1 - \mu_2 < 0$, $\mu_1 - \mu_2 > 0$ or $\mu_1 - \mu_2 \neq 0$. Based on independent r.s of size n_1 & n_2 from 2 (P) having the means μ_1 & μ_2 & the non-variances σ_1^2 & σ_2^2, the BCR are for the significance level α

$\omega = z < -z_\alpha$, $\omega = z > z_\alpha$ & $\omega = |z| > z_{\alpha/2}$.

The test statistic is , ~~$z = \bar{x}_1 - \bar{x}_2$~~

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

cal the value of z using the sample info & if it lies in the CR reject Ho. otherwise accept it.

Q
1) Suppose that 64 senior girls from clg A & 81 senior girls from clg B had mean of 68.2 & 67.3. If the SD for statures of all senior girls is 2.43 is the difference b/w the 2 grp significant.

A) $n_1 = 64$, $n_2 = 81$, $\bar{x}_1 = 68.2$, $\bar{x}_2 = 67.3$, $\sigma_1 = \sigma_2 = 2.43$.

$H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$

let $\alpha = 0.05$

BCR ; $w = |z| \geq 1.96$.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{68.2 - 67.3}{\sqrt{\frac{2.43^2}{64} + \frac{2.43^2}{81}}} \quad \cancel{\frac{0.9}{0.30375 + 0.27}} \quad \cancel{= 1.558}$$

$$= 2.21$$

$|z| = 2.21 \geq 1.96$.

reject

2) A r.s of 1000 workers from a factory A shows that the mean ~~weight~~ wages where ₹ 47 per week with a SD of ₹ 23. A r.s of 1500 workers from ~~to~~ factory B gives a mean wage of ₹ 49 per week with SD ₹ 30. Is there a significance difference b/w their mean level wages?

A) $n_1 = 1000$ $\quad \bar{x}_1 = 47$ $\quad s_1 = 23$

$n_2 = 1500$ $\quad \bar{x}_2 = 49$ $\quad s_2 = 30$

$H_0: \mu_1 - \mu_2 = 0$ against $H_1: \mu_1 - \mu_2 \neq 0$.

$\alpha = 0.05$, BCR, $w = |z| \geq 1.96$

test statistic, $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{47 - 49}{\sqrt{\frac{23^2}{1000} + \frac{30^2}{1500}}}$$

$$= \frac{-2}{\sqrt{0.529 + 0.6}} = -1.88$$

$|z| = 1.88 < 1.96$.

accept

→ Testing of proportion of success of (P) =

1) In a survey of 70 business firms, it was found that 45 are planning to expand their capacities next yr. Does the sample info contradict the hypo- that 70% the firms are planning to expand the next yr?

A) $P_0 = 70\% = 0.70$ $\quad q_0 = 1 - P_0 = 0.3$

$$p' = \frac{x}{n} = \frac{\text{no. of sample}}{\text{sample size}} = \frac{45}{70} = 0.643$$

$H_0: p = 0.7$ against $H_1: p < 0.7$

let $\alpha = 0.05$ BCR $z < -1.645$.

test statistic,

$$z = \frac{p' - P_0}{\sqrt{\frac{P_0 q_0}{n}}} = \frac{0.643 - 0.7}{\sqrt{\frac{0.7 \times 0.3}{70}}} = \frac{-0.057}{0.05}$$

$$= -1.04$$

$z = -1.04 > -1.645$

accept

→ Testing the difference of 2 (P) proportions =

$$z = \frac{P_1' - P_2'}{\sqrt{p^* q^* \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \quad p^* = \frac{n_1 P_1' + n_2 P_2'}{n_1 + n_2}, \quad P_1' = \frac{x_1}{n_1}$$

$q^* = 1 - p^*$ $\quad P_2' = \frac{x_2}{n_2}$

$H_0: P_1 - P_2 = 0$ or $P_1 = P_2$

1) In a sample of 600 ~~sample~~ men from city A, 450 are found to be smokers. Out of 900 from city B, 450 are smokers. Do the data indicate that the cities are significantly different w.r.t privilence of smoking?

A) here, $p_1' = \frac{x_1}{n_1} = \frac{450}{600} = 0.75$

$p_2' = \frac{x_2}{n_2} = \frac{450}{900} = 0.50$

$H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2$

let $\alpha = 0.01$ (since no α is given, choose 0.01)

BCR $w = |z| \geq 2.58$

test statistics,

$$p^* = \frac{n_1 p_1' + n_2 p_2'}{n_1 + n_2} = \frac{600 \times 0.75 + 900 \times 0.5}{600 + 900}$$

$$= \frac{450 + 450}{1500} = \frac{900}{1500}$$

$$= \underline{0.6}$$

$$q^* = 1 - p^* = 1 - 0.6 = \underline{0.4}$$

$$\frac{1}{600 \times 3} \times 3 + \frac{1}{900 \times 2} \times 2 \qquad \frac{6}{1800}$$

$$z = \frac{p_1' - p_2'}{\sqrt{p^* q^* \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.75 - 0.5}{\sqrt{0.6 \times 0.4 \left(\frac{1}{600} + \frac{1}{900}\right)}}$$

$\cancel{0.25}$ $\cancel{\frac{0.25}{}}$ $\cancel{4.16}$ $\cancel{600}$

$$= \underline{9.6}$$

$9.6 > 2.58$

Reject

2) After an increase in excise duty on tea 800 persons out of sample 1000 persons where found to be tea drinkers. After an Ise in duty 800 people where tea drinkers 1200 persons. Test whether there is significant dse in the consumption of tea after the Ise in duty.

A) $p_1' = \frac{800}{1000} = 0.8$

$p_2' = \frac{800}{1200} = 0.667$

$H_0 : P_1 - P_2 = 0$, against $H_1 : P_1 - P_2 > 0$.

$\alpha = 0.05$, BCR $w = z > 1.645$

test stati,

$\cancel{z = 0.8 - 0.67}$

$$p^* = \frac{1000 \times 0.8 + 1200 \times 0.67}{1000 + 1200} = \frac{800 + 804}{2200}$$

$$= \underline{0.7327}$$

$$q^* = 1 - p^* = \underline{0.273}$$

$$z = \frac{0.8 - 0.67}{\sqrt{0.1984\left(\frac{1}{1000} + \frac{1}{1200}\right)}} = \frac{0.13}{\sqrt{0.1984(0.001 + 0.00083)}}$$

$$= \underline{6.8}$$

$\cancel{1.306 > 1.645}$

$6.8 > 1.645$

reject

8.3333 / 101000

0.00

1/3 / 1.33

1/10 / 10×1000

1.833 / 1000 0.18 0.019

1.8 / 1000

~~0.00183~~

0.0011