• Introduction to probability

01 : Classical Definition of probability.

→ **Random Experiment :-**

There are some exp. called deterministic exp., whose outcomes can be predicted.
If the exact outcomes of the trials of the exp. is unpredictable → Random exp.

eg → Tossing a coin, Rolling a die, lifetime of a machine, wheather condin.. of a certain region, etc.

→ **Trial & Event :-**

Trial is an attempt to produce an outcome of a random exp.

eg → If we toss a coin / throw a die, we are performing ~~triess~~ trials.

An outcomes in an exp. → Event / cases.

eg → Getting a head / tale in tossing a coin is a event.
usually events are denoted by A, B, C ....

Random

cannot predict result.

Determinist

predict (r).

Trial

[illegible]

Event

[illegible] result

→ **Equally likely events :-**

Events / cases are said to be equally likely when we have no reason to expect 1 rather than other.

eg → In tossing an unbiased coin, the 2 events head & tail are equally likely.

→ **Exhaustive events :-** The set of all possible outcomes in a trial constitute the set of exhaustive cases.

eg → In the case of tossing a coin there are 2 (exh) cases — H or T

In throwing a die, there are 6 (exh) cases

→ **Mutually exclusive event :-**

M. E. e / disjoint → If the happening of any 1 of them precludes / excludes, the happening of all the others in a trial. (i.e) If no 2 / more of them can be happened simultaneously in the same trial.

eg → The events of turning a H or T in tossing a coin are M. ex.

→ **Favourable cases :-**

The cases which entail the occurrence of an event → F. to the Event

eg → while tossing a die, the occurance of 2 / 4 / 6 are the favourable events which entail ~~entale~~ the occurance of a even no.

(P) definition ⇒

→ **classical definition / Mathematical / apriori defi :-**

If a trial results in 'n' mutually exclusive, equally likely & exhaustive cases & 'm' of them are favourable (m < n) to the happening of an event 'A', then the probability of 'A', denoted as, P(A) defined as —

$$\boxed{P(A) = \frac{m}{n}} = \frac{\text{no of favourable cases}}{\text{Total no of cases}}$$

$\therefore\ 0 \le P(A) \le 1$

→ **Frequency definition of (P) :-**

$$\boxed{P(A) = \lim_{n \to \infty} \frac{m}{n}}$$

Let the trial being repeated over a large no. of times under essentialy homogeneous condiⁿ, the limit of the ratio of the no. of times, an event a happens (A) to the total no. of trials (n), as the no. of trials tends to ∝

→ (P) of the event A.

Note :-

* If 'A' is impossible event, when $P(A)=0$
* If 'A' is sure event, when $P(A) = 1$
* If 'A' is random event, $0 < P(A) < 1$.

→ Limitations of classical defiⁿ :-

* In c. defiⁿ of (P) only equally likely cases are taken into consideration.
If the events cannot be considered equally likely c. defiⁿ ~~fails~~ fail to give a gd account of the concept of (P).

* When total no. of possible outcomes 'n' becomes infinite / countabelly infinite this defiⁿ fails to give a measure of (P).

* If we are deviating from the games of chances like tossing a coin, throwing a die, etc. this defiⁿ cannot be applied.

* Anther limitation is that which does not contribute much to the growth of (P) theory.

Q/

1) 3 coins are tossed wht is it (P) of getting

coin → 2^n.
here 3 coin $= 2^3 = 8$

Die
6^n

a) all head
b) exactly 1 H
c) exactly 2 H.
d) atleast 1 H
e) at least 2 H.
f) atmost 1 head
g) No H.

A)

$S = \{HHH, HHT, HTT, TTT, THT, THH, HTH, TTH\}$.

2^n
$2^3 = 8$

a) $P(\text{all head}) = \frac{1}{8}$ (HHH)

b) $P(\text{exactly } 1H) =$ (only 1 H needed) (HTT, THT, TTH)
$= \frac{3}{8}$

c) $P(\text{exactly } 2H) =$ HHT, THH, HTH
$= \frac{3}{8}$.

d) $P(\text{atleast } 1\,H) = 7/8$ (except TTT other all)

e) $P(\text{atleast } 2\,H) = \frac{4}{8}$ (HHH, HHT, THH, HTH)

f) $P(\text{atmost } 2\,H) = 7/8$ (except HHH) max = 2H. $3H$ ✗

g) $P(\text{atmost } 1\,H) = \frac{4}{8}$

h) $P(\text{no head}) = \frac{1}{8}$ (TTT)

2) A box contains 8 red, 3 white, 9 blue balls. If 3 balls are drawn at random determined (P) that

a) all 3 are blue.

b) 2 are red & 1 is white

c) atleast 1 is white

d) 1 of each colour is drawn.

A) a) $P(\text{all 3 are B}) = \frac{9C_3}{20C_3}$ C = combination

8 R
3 W
9 B

Total = 20 (8+3+9)

$9C_3 \to 9 + \text{shift} + \div\ 3 = 84.$

$= \frac{84}{1140} = 0.07$

b) $P(2 \text{ are R } \& \ 1 \text{ is W}) = \frac{8C_2 \times 3C_1}{20C_3}$

R = 8 W = 3

$= 3C_1$

$= \frac{28 \times 3}{1140} = \frac{84}{1140}$

$= 0.07$

c) $P(\text{atleast } 1 \text{ is } w)$

0, 1, 2, 3

none

$= 1 - P(\text{None is } w)$

constant.

at least → 1 − P (none).

8R
3W → subtract
9B

8+9=17

$= 1 - \frac{17C_3}{20C_3}$

$= 1 - \frac{680}{1140} = 0.41$

d) $P(1 \text{ of each clr is draw})$

$= \frac{8C_1 \times 3C_1 \times 9C_1}{20C_3}$

$= \frac{216}{1140} = 0.18$

((P) values b/w 0 & 1)

(Total (P) always 1)

3) From a truck load basket of eggs 1 basket is taken at random & found that 3 out of 100 egg are rotten. Obtain an estimate of (P) of getting an rotten egg of 1 egg is taken at random from that truck load.

$= \frac{3 C_1}{100 C_1} = \frac{3}{100} = 0.03$

4) wht is (P) that a leap (366) yr selected at random will contain 53 Sundays?

A) 366 Days

(52 week-[illegible] [illegible] sun [illegible])

week = ?

$= \frac{366}{7} = 52.28 \Rightarrow 52$ week.

$52 \times 7 = 364$ (Remaining 2 days.)

That 2 days (366) may ~~be~~ →

$\boxed{\text{Sun + Mon}}$	1
Mon + Tue	2
Tue + wed	3
wed + Thru	4
Thru + fri	5
fri + sat	6
$\boxed{\text{sat + Sun.}}$	7

(P) to get Sundays = 2.

∴ ~~Proba~~ P (53 Sundays) = $\frac{2}{7}$

5) wht is the (P) of getting a spade / an ace from a pack / cards?

A) ~~Pack~~ Total cards = 52. (1 pack)

Spade	Club	Diamond	Heart
King – 1	King – 1		
Queen – 1	Q – 1	"	"
Jack – 1	J – 1		
Ace – 1	Ace – 1		

2–10 cards 2–10 cards

(13) (13) (13) (13)

Total King = 4

Total Q = 4

Total J = 4

Total Ace = 4

∴ ~~P (Spade | Ace)~~

([illegible] → +)

P (Spade) + P (ace)

(1 Ace Spade-[illegible] [illegible]) P (Spade) = $\frac{13}{52}$

P (Ace) = $\frac{3}{52}$

([illegible] 1 Ace [illegible] ∴ [illegible] [illegible] – 3)

∴ P (Spade | Ace) =

$\frac{13}{52} + \frac{3}{52} = \frac{13+3}{52} = \frac{16}{52}$

6) If the letters of word 'REGULATIONS' be arranged at random. wht is the chance will be exactly 4 letters b/w R & E?

(position [illegible] [illegible])

A) REGULATIONS

P (R..... E) = ?.

Total letters in RECULATIONS = 11

(arranged → permutations).

R & E are constnt.

$^{11}P_2 = 110$ (11 + shift x + 2)

(Total 11 നിന്ന് 2 എണ്ണം constant ആണ്)

position → $\overline{1}$ $\overline{2}$ $\overline{3}$ $\overline{4}$ $\overline{5}$ $\overline{6}$ $\overline{7}$ $\overline{8}$ $\overline{9}$ $\overline{10}$ $\overline{11}$

$6\begin{cases} \text{If } R \to 1^{st} \text{ position } \& E \to 6^{th} \text{ position (4 Letters gap)} \\ R \to 2^{nd}\ P,\ E \to 7^{th}\ P \\ R \to 3^{rd}\ P,\ E \to 8^{th}\ P. \\ R \to 4^{th}\ P,\ E \to 9^{th}\ P \\ R \to 5^{th}\ P,\ E \to 10^{th}\ P \\ R \to 6^{th}\ P,\ E \to 11^{th}\ P \text{ (Total 11 P).} \end{cases}$

Then R & E will be exchanged.

$6.\begin{cases} E \to 1^{st}\ P,\ R \to 6^{th}\ P \\ \ldots\ldots \end{cases}$

R _ _ _ _ E = 6

E _ _ _ _ R = 6

∴ Total cases = 12

∴ P (getting 4 letters b/w R & E) $= \frac{12}{110}$

$= \frac{6}{55}$