D7 : Axiomatic Definition of Probability.

→ **Sample Space** :- It is the set of all (S.S) possible outcomes of a random expt. Denoted by S or Ω.

* Every indecomposable outcome of a random expt → sample point / elementry outcome.

eg → Sample space obtained in the throw of a single die is the finite sample space. (ie)

$$S = \{1, 2, 3, 4, 5, 6\}.$$

$S = \{1, 2, 3\}$ → S-point

eg → Sample space obtained in connection with random expt (ie), tossing a coin again & again until a head appears is a countably infinite sample space. (ie)

$$S = \{H, TH, TTH, TTTH \ldots\}$$

→ **Event** :- It is a subset of a S.S.
In other words of all the possible outcome of an expt, some outcome in S.S satisfy a specified discription. → Event.

$S = \{1, 2, 3, 4\}$
$\{1, 3\}$ → Event
$S = \{2, 4\}$

→ **Field of an event** :- (F) → field.

$S = \{1, 2\}$
subset = $\{1\}, \{2\}, \{1, 2\}$
ϕ null set

ϕ compliment → S
S → ϕ.

Let S be S.S of a random expt then the collection of sets 'F' is → Field / Algebra, if it satisfies the following condit~ →

1) F is non empty.
2) the elements of F are the subset of S.
3) If $A \in F$, then $A' \in F$ (compliment)
4) If $A \in F$ and $B \in F$ the $A \cup B \in F$ (union (combination))

compliment → S-ൽ ഉള്ളതിൽ ...

eg) $S = \{1, 2, 3, 4\}$

$F = \{\phi, S, \{1\}, \{2,3,4\}\}$. ϕ, S → Sure.

→ σ - field / σ Algebra of events :-

Let S be a non empty set & F be a collection of subset of S then F → <u>σ field / σ algebra</u>, if follow~ condit~ →

1) F is non empty
2) the elements of F are the subset of S
3) If $A \in F$ then $A' \in F$.
4) The union of any countable collection of F is an elements of F.

(i.e) If $A_i \in F$, $i = 1, 2, \dots n$ then $\bigcup_{i=1}^{\infty} A_i \in F$.

(Ai F ...)

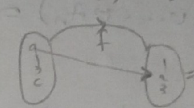
* σ algebra also → <u>boral field</u>.

eg → $B = \{\phi, S\}$.

$B = \{\phi, A, A', S\}$.

→ () and <u>measure</u> :-

() → maping from domain to range. (starting point; where it go)



a, b, c (set) 1, 2, 3.. (real no)

$\mu(A_1 \cup A_2 \dots \cup A_n) = \mu(A_1) \cup \mu(A_2) \dots \cup \mu(A_n)$

$\mu\left(\bigcup_{i=1}^{n} A_i\right) = \bigcup_{i=1}^{n} \mu A_i$

$\mu\left(\bigcup_{i=1}^{\alpha} A_i\right) = \bigcup_{i=1}^{\alpha} \mu A_i$

* when the elements of the domain are sets & the elements of the range are real no. the () is said to be a set (), denoted by $P(A)$ / $\mu(A)$.

* In a set () $A_1, A_2, \dots A_n$ are disjoint at the domain, & if $\mu(A_1 \cup A_2 \cup \dots \cup A_n) = \mu(A_1) \cup \mu(A_2) \cup \dots \cup \mu(A_n)$ then the set () → <u>additive</u>

* If a set 'S' is partion into a countable no. of disjoin set $A_1, A_2, \dots$ & a set () defined on satisfies the property

$\mu(A_1 \cup A_2 \cup \dots) = \mu(A_1) + \mu(A_2) + \dots\dots$

Then the set ( ) → Countably additive

→ Measure :- A set ( ) which is non-negative & totally additive → a measure

* A measure will be called probability measure if $\mu(A_1 \cup A_2 \cup \dots \cup A_n) = \mu(A_1) + \mu(A_2) \dots + \mu(A_n) = 1$

where $\bigcup_{i=1}^{n} A_i = S$, $\underset{\text{disjoint (common and)}}{A_i \cap A_j} = \phi$, $i = j$

→ Axiomatic Definition :-

(Prob) for the occurance of 'A' if P(A) satisfies the follow 3 axioms –

* Axiom 1 (Non negativity).

$0 \le P(A) \le 1$ 0 → 1

* Axiom 2 (Norming)

$P(S) = 1$ Total prob = 1.

* Axiom 3 (Countably additivity)

If $A_1, A_2 \dots A_n$ are finite / infinite sequence of elements in B such that,

$A_i \cap A_j = \phi$, $i \ne j$

$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$.

→ Theorum 1 :

The prob of a impossible event is 0 (ie) $P(\phi) = 0$

Pf let ϕ be the impossible event

$S \cup \phi = S$ ($P(A \cup B) = P(A) + P(B)$ only if $A \cap B = \phi$.)

(prob) on 2 sides →

$P(S) + P(\phi) = P(S)$ (since $S \cap \phi = \phi$)

$P(S) = 1$ (axiom 2)

$1 + P(\phi) = 1$

$P(\phi) = 1 - 1$

$P(\phi) = 0$

→ Theorum 2 :- (prob) is infinitely additive

(i.e) $P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$

where $A_i \cap A_j = \phi$, $i = j$

Pf consider an infinite seq. of events $A_1, A_2 \dots A_n, \phi, \phi \dots$ which are pairwisely disjoined since A_i's are disjoint.

$P(A_1 \cup A_2 \cup \dots \cup A_n \cup \phi \cup \phi \dots) =$

(as axiom 3) $P(A_1) + P(A_2) \dots P(A_n) + P(\phi) + P(\phi) \dots$

$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) \dots P(A_n)$

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$.

here + → Σ

$A_1 \cup A_2 \cup \dots \cup A_n$ → +

→ Theorum (Monotonicity) :-

If $A \underset{\text{subset}}{\subset} B$ then $P(A) \le P(B)$

Pf $B = A \cup (A' \underset{\text{intersection}}{\cap} B)$

$P(B) = P(A) + P(A' \cap B)$

$P(B) = P(A) + (\text{+ve quantity})$

$P(B) \geq P(A)$ (or)

$P(A) \leq P(B)$

→ **Addition Therum :-**

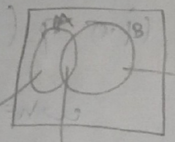
If A and B are any 2 events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Pf

$A \cup B = A \cup (A' \cap B)$

$P(A \cup B) = P(A) + P(A' \cap B)$ — ①

(that's why we taken B)



$B = A \cap B \cup A' \cap B$

$P(B) = P(A \cap B) + P(A' \cap B)$

$P(A' \cap B) = P(B) - P(A \cap B)$ — ②

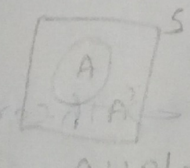
Sub ② in ①

$P(A \cup B) = P(A) + P(A' \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

→ **Complimentation (therum) :-**

$P(A') = 1 - P(A)$.



$A \cup A' = S$

$P(A \cup A') = P(S)$

$P(A) + P(A') = 1$ $P(S) = 1$.

$P(A') = 1 - P(A)$

Note

1) If $A \cap B = \phi$, $P(A \cup B) = P(A) + P(B)$

2) $P(A \cup B) = 1 - P(A \cup B)'$

$P(A \cup B) = 1 - P(A' \cup B')$

→ **Theorem :-** For any 2 events A and B s.t. $\underbrace{P(A \cap B) \leq P(A)}_{①} \leq \underbrace{P(A \cup B)}_{②} \leq \underbrace{P(A) + P(B)}_{③}$.

① $A \cap B \subset (A)$

$P(A \cap B) \leq P(A)$ $A \subset B \Rightarrow P(A) \leq P(B)$.

② $A \subset (A \cup B)$

$P(A) \leq P(A \cup B)$

③ ~~$A \cup B$~~ $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$P(A \cup B) \leq P(A) + P(B)$

Q

1) Given $P(A) = 0.3$, $P(B) = 0.78$ and $P(A \cap B) = 0.16$.

Find (a) $P(A' \cap B')$ (b) $P(A' \cup B')$ (c) $P(A \cap B')$

A) a) $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$ $\boxed{P(A') = 1 - P(A)}$

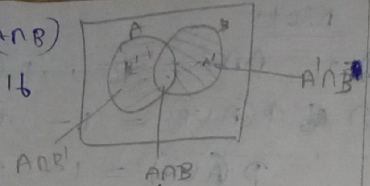
$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - [0.3 + 0.78 - 0.16] = 0.08$

b) $P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$

$= 1 - 0.16 = 0.84$

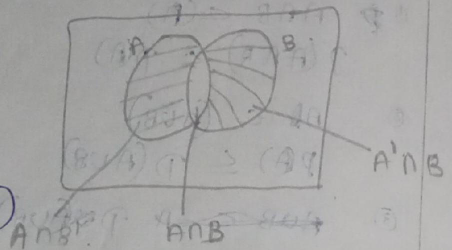
c) $P(A\cap B') = P(A) - P(A\cap B)$
$= 0.3 - 0.16$
$= 0.14$



d) $P(B'\cap A) = P(B) - P(A\cap B)$
$= 0.78 - 0.16 = 0.62$

2) for any 2 events A & B, P.T
$P[(A\cap B')\cup(B\cap A')] = P(A) + P(B) - 2P(A\cap B)$.

LHS = RHS



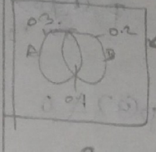
A) from graph.
$A = (A\cap B')\cup(A\cap B)$
$P(A) = P(A\cap B') + P(A\cap B)$
$\hookrightarrow P(A) = P(A) + P(B) - P(A\cap B)$
$P(A\cap B') = P(A) - P(A\cap B)$ — ①   common

$B = (A\cap B)\cup(A'\cap B)$
$P(B) = P(A\cap B) + P(A'\cap B)$ $(\because (A\cap B)\cap(A'\cap B) = \phi)$
~~$P(A'\cap B)$ = $P(A\cap B)$ $P(B)$~~
$P(A'\cap B) = P(B) - P(A\cap B)$ — ②
$P[(A\cap B')\cup(A'\cap B)] \overset{(split)}{=} P(A\cap B') + P(A'\cap B)$
$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$
$P[(A\cap B')\cup(A'\cap B)] = P(A) + P(B) - 2P(A\cap B)$

3) Given $P(A) = 0.3$, $P(B) = 0.2$ and $P(A\cap B) = 0.1$. find $P[(A\cap B')\cup(B\cap A')]$.

A) $P[(A\cap B')\cup(B\cap A')] = P(A) + P(B) - 2P(A\cap B)$
$= 0.3 + 0.2 - 2\cdot 0.1$
$= 0.3 + 0.2 - 0.2$
$= 0.3$

ok we don't know the formula



$= (A - 0.1) + (B - 0.1)$
$= (0.3 - 0.1) + (0.2 - 0.1) = 0.3$

4) Given $P(A) = 0.3$, $P(B) = 0.2$ and $P(A\cap B) = 0.1$
find a) $P(A\cup B)$ (b) $P(A\cap B')$ (c) $P(A'\cap B)$
(d) $P(A'\cap B')$

A) a) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$= 0.3 + 0.2 - 0.1 = 0.4$

b) $P(A\cap B') = P(A) - P(A\cap B)$
$= 0.3 - 0.1 = 0.2$

~~c) $P(A'\cap B)$~~ $P(A'\cap B)$ $= P(B) - P(A\cap B)$
$0.2 - 0.1 = 0.1$

$P(A\cap B') + P(A\cap B) \Rightarrow 0.2 + 0.1 = 0.3$

d) $P(A'\cap B') = P(A\cup B)'$
$= 1 - P(A\cup B)$
$= 1 - 0.4 = 0.6$

5) Given $P(A) = 0.12$ $P(B) = 0.29$ $P(A\cap B) = 0.07$
find a) $P[(A\cap B')\cup(A'\cap B)]$ (b) $P(A'\cap B')$