01: Probability Distribution

→ Binomial Distri... = A random variable 'x' is
(BD) defined to have a binomial Distri...., its the (prob) density () (pdf) is givn by,

$$\boxed{f(x) = n C_x p^x q^{n-x}}$$

$x = 0, 1, 2, \ldots n$ $\quad p + q = 1$ $\quad 0 \le p \le 1$

here n and p → parameters of BD, whn x follows BD we can write

$X \rightarrow B(n, p)$ or

$x \sim B(n, p)$

coin tossing / throw a die

n = total trial
x = no. of success
p = (prob) of success
q = (prob) of failure

The symbol $b(x; n, p)$ also represnt the (prob) for x, success in n trials with (prob) of success p eg → 1 coin tossed 10 times, 5 get of H.

$n C_x p^x q^{n-x}$

n = 10 x = 5

p = 1/2 q = 1/2

p + q = 1

1/2 + 1/2 = 1

$(q+p)^n = q^n + n c_1 q^{n-1} p + n c_2 q^{n-2} p^2 + \ldots + p^n$

$$= \sum \frac{n!}{x!(n-x)!} p^x q^{n-x}$$

$$(q+p)^{n-1} = \sum \frac{(n-1)! \, p^{x-1} q^{n-x}}{(x-1)!(n-x)!}$$

$$(q+p)^{n-2} = \sum \frac{(n-2)! \, p^{x-2} q^{n-x}}{(x-2)!(n-x)!}$$

$= \sum_{n=0}^{n} n C_x p^x q^{n-x}$

$= \sum_{n=0}^{n} \frac{n!}{x!(n-x)!} p^x q^{n-x}$ $\quad n C_x = \frac{n!}{x!(n-x)!}$

Binomial expansion

$(a+b)^n = a^n + n c_1 a^{n-1} b + n c_2 a^{n-2} b^2 \ldots$

→ Moments of BD :-

1) Mean, − $\boxed{E(x) = np}$ (discrete)

$E(x^2) = \sum x^2 \cdot f(x)$

Pf

$E(x) = \sum x \cdot f(x)$

$= \sum_{x=0}^{n} x \cdot {}^nC_x p^x q^{n-x}$

$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x q^{n-x}$

$\boxed{{}^nC_x = \frac{n!}{x!(n-x)!}}$

$= \sum \frac{x \, n(n-1)!}{x(x-1)!(n-x)!} p^x q^{n-x}$

$n! = n(n-1)!$

$x! = x(x-1)!$

$= \sum \frac{n(n-1)!}{(x-1)!(n-x)!} p^1 \cdot p^{x-1} q^{n-x}$

p^x can be written as →

$p^x = p^1 \cdot p^{x-1}$

$\Rightarrow p^{1+x-1} = p^x$

(np out)

(we want to get np so np outside)

$= np \sum \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x}$

we know $\sum \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} p^{n-x} = (q+p)^{n-1}$

$E(x) = np \cdot (q+p)^{n-1}$

$p+q = 1$

So, $E(x) = np$

2) Variance :- $\boxed{V(x) = npq}$

Pf

$V(x) = E(x^2) - (E(x))^2$

$E(x) = np$

$E(x^2)$?

$E(x^2) = E[x(x-1) + x]$.

$= E[x(x-1)] + E(x)$. ①

x^2 can be written as →

$x^2 = x(x-1) + x$

$\therefore = x^2 - x + x = x^2$ //

$E(x) = \sum x \cdot f(x)$

$E(x(x-1)) = \sum x(x-1) f(x)$

E(x) converted to find $E(x(x-1))$

$E(x(x-1)) = \sum_{x=0}^{n} x(x-1) \cdot f(x)$

$= \sum x(x-1) \cdot f(x) + np$

$= \sum x(x-1) \cdot {}^nC_x p^x q^{n-x} + np$

${}^nC_x = \frac{n!}{x!(n-x)!}$

$= \sum x(x-1) \frac{n!}{x!(n-x)!} p^x q^{n-x} + np$

$= \sum \frac{\cancel{x(x-1)} \, n!}{\cancel{x(x-1)}(x-2)!(n-x)!} p^x q^{n-x} + np$

$p^x = p \cdot p^{x-1}$

$x! = x(x-1)(x-2)!$

$n! = n(n-1)(n-2)!$

$= \sum \frac{n(n-1)(n-2)!}{(x-2)!(n-x)!} p^2 \cdot p^{x-2} q^{n-x} + np$

$p^x = p^2 \cdot p^{x-2}$

$= n(n-1)p^2 \sum \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} q^{n-x} + np$

$= n(n-1) p^2 \cdot (q+p)^{n-2} + np$

$p + q = 1$

$q = 1 - p$

$E(x^2) = n(n-1) p^2 + np$

$V(x) = E(x^2) - (E(x))^2$

$= n(n-1)p^2 + np - (np)^2$

$= (n^2 - n)p^2 + np - n^2p^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= -np^2 + np = np(-p+1)$

$p+q=1$

$1-p = q$

$= np(1-p)$

$V(x) = npq$

$\boxed{\begin{array}{l} p+q=1 \\ q=1-p \end{array}}$

Standard Deviation, $SD = \sqrt{V(x)}$

$SD = \sqrt{npq}$

Mean, $E(x) > V(x)$ in binomial distri...

1) cal μ_3 and μ_4 (binomial distri...)

Mean, $E(x) = \mu_1' = np$

μ_1' = mean in binomial distri...

$E(x^2) = \mu_2' = n(n-1)p^2 + np$

$E(x^3) = \mu_3' = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= npq(q-p)$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$

$= 3n^2p^2q^2 + npq(1-6pq)$

$\mu_2 = V(x) = npq$

⇒ Beta and Gamma Coefficients :-

$\boxed{\gamma_1 = \sqrt{\beta_1}}$; $\boxed{\beta_1 = \frac{\mu_3^2}{\mu_2^3}}$

$= \frac{(npq(q-p))^2}{(npq)^3}$

$\beta_1 = \frac{n^2p^2q^2(q-p)^2}{n^3p^3q^3} = \frac{(q-p)^2}{npq}$

$\therefore \gamma_1 = \sqrt{\frac{(q-p)^2}{npq}}$

$\boxed{\gamma_1 = \frac{q-p}{\sqrt{npq}}}$

Skewness

∴ A binomial distri... is +vly skewed, symmetric & -vly skewed according as,

$\gamma_1 > 0$	→ +vly s	⇒ $q > p$
$\gamma_1 < 0$	→ -vly s	⇒ $q < p$
$\gamma_1 = 0$	→ symm. s	⇒ $q = p$

Kurtosis

$\boxed{\beta_2 = \frac{\mu_4}{\mu_2^2}}$ $= \frac{3n^2p^2q^2 + npq(1-6pq)}{n^2p^2q^2}$

$= \frac{3\cancel{n^2}\cancel{p^2}\cancel{q^2}}{\cancel{n^2p^2q^2}} + \frac{\cancel{npq}(1-6pq)}{n^2p^2q^2}$

$= 3 + \frac{(1-6pq)}{npq}$

$\beta_2 > 3$	→ lepto kurtic	→ $pq > \frac{1}{6}$
$\beta_2 < 3$	→ platy kurtic	→ $pq < \frac{1}{6}$
$\beta_2 = 3$	→ meso kurtic	→ $pq = \frac{1}{6}$

→ Moment generating ( ) :- (mgf).

$M_X(t)$ →

$M_x^{(t)} = E(e^{tx})$

$= \sum e^{tx} \cdot f(x)$.

$= \sum e^{tx} \cdot {}^nC_x p^x q^{n-x}$.

$e^{tx} \cdot p^x = (pe^t)^x$

$= \sum {}^nC_x (pe^t)^x q^{n-x}$.

$\boxed{M_x^{(t)} = (q + pe^t)^n}$

$E(x) = \sum x \cdot f(x)$.

$f(x) = {}^nC_x p^x q^{n-x}$

$E(x^2) = \sum x^2 \cdot f(x)$

$V(x) = E(x^2) - (E(x))^2$

$\sum {}^nC_x p^x q^{n-x} = (p+q)^n$.

${}^nC_x p^x (e^t)^x q^{n-x}$

$(q + p(e^t))^n = (q+pe^t)^n$

To get mean, μ_1' or $E(x)$.

$\frac{d}{dt}\left[M_x^{(t)}\right]_{t=0}$

$\mu_1' = E(x) = \left[\frac{d}{dt} M_x^{(t)}\right]$.

$= \frac{d}{dt}(q + pe^t)^n$

$= n(q+pe^t)^{n-1} \cdot \frac{d}{dt}(q+pe^t)$

$= n(q+pe^t)^{n-1} \cdot (0 + pe^t)$

$= n(q+pe^t)^{n-1} \cdot pe^t$.

at $t=0$,

$= n(q + pe^0)^{n-1} \cdot pe^0$. $\quad e^0 = 1$

$= n(q+p)^{n-1} \cdot p \quad$ $p+q=1$.

$\boxed{\mu_1' = E(x) = n \cdot p}$

$V(x) = E(x^2) - (E(x))^2$.

$E(x)^2 = \mu_2' = \left[\frac{d^2}{dt^2} M_x^{(t)}\right]_{t=0}$.

$= \frac{d}{dt}\left[n(q+pe^t)^{n-1} \cdot pe^t\right]$ (p.rule)

pe^t

$\frac{d}{dt}(pe^t)$

$p \cdot \frac{d}{dt} e^t = pe^t$

$= n(q+pe^t)^{n-1} \cdot pe^t + pe^t \cdot n(n-1)(q+pe^t)^{n-2} (0+pe^t)$

at $t=0$,

$= n(q+pe^0)^{n-1} \cdot pe^0 + pe^0 \cdot n(n-1)(q+pe^0)^{n-2} (pe^0)$

$(q+p=1)$

$= n(q+p)^{n-1} \cdot p + p \cdot n(n-1)(q+p)^{n-2} \cdot p$

~~$= np + p$~~

$= n \cdot 1^{n-1} \cdot p + p \cdot n(n-1) 1^{n-2} \cdot p$

$E(x^2) = np + p^2 n(n-1)$

$V(x) = E(x^2) - (E(x))^2$

$= np + p^2 n(n-1) - (np)^2$

$= np + \cancel{p^2n^2} - p^2 n - \cancel{n^2p^2}$

$= np - np^2$

$= np(1-p)$.

$\underline{V(x) = npq}$

→ Additive property of binomial distribution

(binomial distribution with n and p)

If x is $B(n_1, p)$ & y is $B(n_2, p)$ & they are independent then their sum $x+y$ also follows binomial distribution $B(n_1+n_2, p)$

(Mgf) ~~p.f~~

$X \to B(n_1, p) \Rightarrow M_x^{(t)} = (q+pe^t)^{n_1}$

$Y \to B(n_2, p) \Rightarrow M_y^{(t)} = (q+pe^t)^{n_2}$

$X+Y \to B(n_1+n_2, p) \Rightarrow M_{x+y}^{(t)} = (q+pe^t)^{n_1+n_2}$ to prove

To show $M_{x+y}^{(t)} = (q+pe^t)^{n_1+n_2}$.

$M_{x+y} = M_x^{(t)} + M_y^{(t)}$ (x & y are indpnt)

$= (q+pe^t)^{n_1} + (q+pe^t)^{n_2}$ $\quad a^n \cdot a^m = a^{m+n}$

$M_{x+y} = (q+pe^t)^{n_1+n_2}$

$X+Y \longrightarrow B(n_1+n_2, p)$

⇒ Recurrence relation for Binomial (prob) :-

binomial distri..

$$\boxed{b(x+1;n,p) = \frac{n-x}{x+1} \cdot \frac{p}{q}\, b(x;n,p).}$$

Pf/ we know, ① $b(x,n,p) = nC_x\, p^x q^{n-x}$.

② $b(x+1,n,p) = nC_{x+1}\, p^{x+1} q^{n-(x+1)}$ $\quad n-x-1$

$$\frac{②}{①}\ \frac{b(x+1,n,p)}{b(x,n,p)} = \frac{nC_{x+1}\, p^{x+1} q^{n-x-1}}{nC_x\, p^x q^{n-x}}$$

$$= \frac{\frac{\cancel{n!}}{(x+1)!\,n-(x+1)!}\ \cancel{p^x}\cdot p \cdot \cancel{q^{n-x}}\, q^{-1}}{\frac{\cancel{n!}}{x!\,(n-x)!}\ \cancel{p^x}\ \cancel{q^{n-x}}}$$

$$nC_x = \frac{n!}{x!\,(n-x)!}$$
$$p^{x+1} = p^x \cdot p^1$$
$$q^{n-x-1} = q^{n-x}\cdot q^{-1}$$
$$x!\,(n-x)! = x!\,(n-x)(n-x-1)!$$
$$(n-x) = (n-x)(n-x-1)$$

$$= \frac{1}{(x+1)!\,(n-x-1)!}\ p\, q^{-1}\ \frac{x!\,(n-x)!}{1}$$

$$= \frac{1 \cdot p\, q^{-1}\ x!\,(n-x)\cancel{(n-x-1)!}}{(x+1)\cancel{x!}\ \cancel{(n-x-1)!}}$$

$q^{-1} = \frac{1}{q}$

$$= \frac{p\, q^{-1}\,(n-x)}{(x+1)} = \frac{p\,(n-x)}{q\,(x+1)}\ \text{(split)}$$

$$= \frac{p}{q}\ \frac{n-x}{x+1}$$

$$\frac{b(x+1,n,p)}{b(x,n,p)} = \frac{p}{q}\ \frac{n-x}{x+1}$$

$$b(x+1,n,p) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\ b(x;n,p)$$

⇒ Recurrence relation for central moment :-

when $x \longrightarrow B(n,p)$

$$\boxed{\mu_{r+1} = pq\left[nr\mu_{r-1} + \frac{d\mu_r}{dp}\right]}$$

binomial ↓ discrete (Σ)

Pf/ $\mu_r = E(x-E(x))^r$ $\quad \mu = np$.

$= E(x-\mu)^r = E(x-np)^r$, $\quad f(x) = \Sigma x\, f(x)$

$= \Sigma (x-np)^r f(x)$.

$= \Sigma (x-np)^r\, nC_x\, p^x q^{n-x}$.

$$\frac{d\mu_r}{dp} = \frac{d}{dp}\left[\Sigma (x-np)^r\, nC_x\, p^x q^{n-x}\right] \quad q = (1-p)$$

$$= \frac{d}{dp}\left[\Sigma\, nC_x\, \underset{③}{p^x}\ \underset{②}{(1-p)^{n-x}}\ \underset{①}{(x-np)^r}\right]$$

$$= \sum_{x=0}^{n}\Big[nC_x\, p^x q^{n-x}\, r(x-np)^{r-1}\cdot -n + nC_x\, q^{n-x}(x-np)^r\, x\, p^{x-1} + nC_x p^x (x-np)^r (n-x)(1-p)^{n-x-1}\cdot -1\Big]$$

$$= \sum_{x=0}^{n}\Big[(-nr)\, nC_x p^x q^{n-x}(x-np)^{r-1} + x\, nC_x\, p^{x-1} q^{n-x}(x-np)^r + (-1)\, nC_x\, p^x q^{n-x-1}(n-x)(x-np)^r\Big]$$

$$= \sum_{x=0}^{n} (-nr) \cdot {}^nC_x p^x q^{n-x} (x-np)^{r-1} + \sum_{x=0}^{n} x \, {}^nC_x p^{x-1} q^{n-x}(x-np)^r + \sum_{x=0}^{n} (-1) \, {}^nC_x p^x q^{n-x-1} (n-x)(x-np)^r$$

(gives)

$$= -nr \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x}{p} + \frac{-1}{q}(n-x) \right]$$

constant out; common dist

$$= -nr \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x}{p} + \frac{-n+x}{q} \right]$$

$$\boxed{\mu_{r-1} = \sum_{x=0}^{n} {}^nC_x p^x q^{n-x} (x-np)^{r-1}}$$

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{qx + p(-n+x)}{pq} \right]$$

(μ_r not b/c we want to get μ_{r+1})

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{qx - np + px}{pq} \right]$$

($q = 1-p$)

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{(1-p)x - np + px}{pq} \right]$$

$$= \cdots \left[\frac{x - xp - np + px}{pq} \right]$$

$$= -nr \, \mu_{r-1} + \sum {}^nC_x p^x q^{n-x} (x-np)^r \left[\frac{x-np}{pq} \right]$$

$$= -nr \, \mu_{r-1} + \underline{\sum {}^nC_x p^x q^{n-x} (x-np)^{r+1}} \, \frac{1}{pq}.$$

$$\frac{d\mu_r}{dp} = -nr \, \mu_{r-1} + \mu_{r+1} \, \frac{1}{pq}.$$

$$\frac{1}{pq} \mu_{r+1} = \frac{d\mu_r}{dp} + nr \, \mu_{r-1}$$

$$\mu_{r+1} = pq \left[\frac{d\mu}{dp} + nr \, \mu_{r-1} \right]$$

→ **Fitting of Binomial distribution :-**

(q → fitting B.d / expected freq)

By fitting B.d means to determine expected / theoretical binomial freq against the gvn observed freq is givn by,

$$\boxed{\begin{aligned} f(x) &= N \cdot b(x; n, p) \\ &= N \cdot {}^nC_x p^x q^{n-x}. \end{aligned}}$$

$x = 0, 1, 2 \cdots n$

N = Total freq $p + q = 1$.

observed freq   expected freq

→ **Bernoulli distri..:-**

If $x = 0$ → failure → q
If $x = 1$ → success → p

$p(x=0) = q$
$p(x=1) = p$

mean → $\boxed{E(x) = p}$ → Bernoli distri.. $E(x) = \sum_{x=0,1} x \cdot f(x) = \sum_{x=0,1} 0 \times q + 1 \times p = p$

$E(x^2) = p$

$V(x) = E(x^2) - (E(x))^2 = p - p^2 = p(1-p)$

$\boxed{V(x) = pq}$

let 'x' be discreate R.variable taking only 2 values 0 & 1, if the pmf of X is gvn by,

$$\boxed{P(X=x) = p(x) = p^x q^{1-x}}, \quad x = 0, 1; \quad 0 < p < 1; \quad p + q = 1$$

Then x is said to follow Bernoli distri.., it is also called point binomial distri.. ($B(1,p)$)

B → Binomial D
q, 1, p.

* Mean :- $E(x) = \sum_{x=0,1} x \, p(x)$

$= 0 \cdot q + 1 \times p$

$\boxed{E(x) = p}$

$\boxed{\begin{array}{l} \text{if } x=0 \\ p(x=0) = q \\ \text{if } x=1 \\ p(x=1) = p. \end{array}}$

* Variance :-

$E(x^2) = \sum x^2 p(x) = 0^2 \cdot q + 1^2 \cdot p$

$= 0 + p$

$E(x^2) = p$

$V(x) = E(x^2) - (E(x))^2$

$= p - p^2 = p(1-p)$

$\boxed{V(x) = pq}$

→ Moment generating function :- (Bernoli .D)

$M_x(t) = E(e^{tx})$

$= \sum_{x=0,1} e^{tx} \cdot p(x)$

$= e^0 \, p(x=0) + e^t \cdot p(x=1)$

$= 1 \cdot q + e^t \cdot p$

$\boxed{M_x(t) = (q + pe^t)}$

Q/ 1) Mean & variance of B.D are 2.5 & 1.875 respectively, obtain the binomial probability distri...

A) Mean, $E(x) = 2.5$

$V(x) = 1.875$

$\left.\begin{array}{l} E(x) = np = 2.5 \\ V(x) = npq = 1.875 \end{array}\right\}$ B.D

B.D = ? $= {}^nC_x \, p^x q^{n-x}$?.

$np = 2.5$

$npq = 1.875$

$2.5 q = 1.875$

$q = \frac{1.875}{2.5} = 0.75$

$p + q = 1$
$p = 1 - q$

$p = 1 - q = 1 - 0.75 = 0.25$

$np = 2.5$

$n \times 0.25 = 2.5$

$n = \frac{2.5}{0.25} = 10$

(x -q- [illegible])

∴ BD is

$f(x) = {}^nC_x \, p^x q^{n-x}$.

$= {}^{10}C_x (0.25)^x \cdot (0.75)^{10-x}$, $x = 0, 1, \dots 10$

↓
$n=10$.

2) If mean & variance of B.D are 4 & 2. find (prob).. of,

a) Exactly 2 success

b) < 2 success

c) > 6 success.

d) atleast 2 success

A) $E(x) = np = 4$

$V(x) = npq = 2$

$npq = 4 \cdot q = 2$

$q = 2/4 = \underline{0.5}$

$p = 1 - q = 1 - 0.5 = \underline{0.5}$

$np = 4$

$n \times 0.5 = 4$

$n = 4/0.5 = \underline{8}$

$f(x) = n C_x p^x q^{n-x}$.

$= 8 C_x (0.5)^x (0.5)^{8-x}$. $x = 0, 1, \ldots 8$.

a) x = no. of success

$x = 2$

$p(x=2)$ $8 C_2 (0.5)^2 (0.5)^{8-2}$

$\Rightarrow 8 C_2 \cdot 0.25 \cdot 0.015625$

$28 \cdot 3.90625 = \underline{0.1092}$

b) $p(x<2) \Rightarrow$

$p(x=0) + p(x=1)$

$= 8 C_0 (0.5)^0 (0.5)^8 +$

$8 C_1 (0.5)^1 (0.5)^7$

$= 1 \cdot 3.90625 +$

0.03125

$3.90 \times 10^{-3} + 0.03125$

$0.00390 + 0.03125$

$= \underline{0.03515}$

-ve +ve

eg $3.04 \times 10^{-2} = 0.0304$

c) >6. $x = 7, 8$

$p(x>6) = p(x=7) + p(x=8)$

$= 8 C_7 (0.5)^7 \cdot (0.5)^1 +$

$8 C_8 (0.5)^8 (0.5)^0$

$= 0.03125 + 0.00390$

$= \underline{0.03515}$

d) $x = 0, 1, 2, 3 \ldots 8$. atleast 2 → $2 \geq$.

$p(x \geq 2) =$ for easy purpose $(x \geq 2) \rightarrow 1 - p(x<2)$

$\boxed{p(x \geq 2) = 1 - p(x<2)}$

$= 1 - [p(x=0) + p(x=1)]$

$= 1 - [0.03515] = \underline{0.96485}$

(q - b)

3) Gn the mgf of the binomial variable $M_x(t) = \left(\frac{1}{3}\right)^5 (2 + e^t)^5$, obtain mean & variance.

A) Mean, $E(x) = np$

$V(x) = npq$

$M_x(t) = (q + pe^t)^n$.

$M_{x}(t) = \left(\frac{1}{3}\right)^5 (2+e^t)^5$ ഇതിനെ $(q+pe^t)^n$ എന്ന formൽ ആക്കുക.

$M_x(t) \Rightarrow \left[\frac{1}{3}(2+e^t)\right]^5 \Rightarrow \left(\frac{2}{3} + \frac{e^t}{3}\right)^5 \Rightarrow \left(\frac{2}{3} + \frac{1}{3}e^t\right)^5 = (q+pe^t)^n$

$\therefore$ we get $q = \frac{2}{3}$, $p = \frac{1}{3}$ $n = 5$

Mean, $E(x) = np$

$= 5 \times \frac{1}{3} = 5/3$

$V(x) = npq = \frac{5}{3} \times \frac{2}{3} = \frac{10}{9}$

4) Comment on the statement the mean of binomial distribution is 3 & $V(x)$ is 4 ?

A) $E(x) = 3$ $\quad V(x) = 4$

$np = 3$ $\quad npq = 4$

$\therefore 3q = 4$

$q = \frac{4}{3}$

$\therefore p = 1 - q$

$= 1 - \frac{4}{3}$

$= \frac{3-4}{3} = \frac{-1}{3}$ (prob) not be -ve

$\therefore$ It is not possible. $\quad 0 < p < 1$, $0 < q < 1$

mean always be > variance

$\boxed{E(x) > V(x)}$

here only bcz $E(x) < V(x)$, $3 < 4$

5) In 256 sets of 12 tosses of a coin, in how many cases I may expect 8 heads & 4T.

A) $\frac{\text{cases}}{\text{possi.}} \times$ total case

possibility

$N = 256$ (Total freq)

$n = 12$ $\quad (8H + 4T = 12)$ → expected freq

let success = Head

no. of success, $x = 8$.

$p = 1/2$

$q = 1/2$

$P(x) = nC_x \, p^x \, q^{n-x}$

$= 12C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^4$

$= 0.1208$

$\therefore$ no. of cases $= 0.1208 \times 256$

$= 30.9 \rightarrow 31$

$30.9 \rightarrow > 5 \Rightarrow 31$

$30.2 \rightarrow < 5 \Rightarrow 30$.

6) In litters of 4 mice the no. of litters which contained 0, 1, 2, 3, 4 females where noted, the figures are below.

No. of female mice :	0	1	2	3	4	Total
No. of litters :	8	32	34	24	5	103

If chance of obtaining female in a single trial assumed constant. Estimate this constant of unknown (prob). Find also Expected freq ?
(Always do in this method)

A)

x	f	fx
0	8	0
1	32	32
2	34	68
3	24	72
4	5	20
	103	192

$$\text{mean} = \frac{\sum xf}{\sum f} = \frac{192}{103} = 1.864$$

binomial dist → $E(x) = 1.864 = np$.

$\boxed{f(x) = n\,C_x\, p^x\, q^{n-x}}$ $x = 0, 1, \dots n$

$x = 0, 1, 2, 3, 4, \dots n$ $n = 4$

$np = 1.864$

$4p = 1.864$

$$p = \frac{1.864}{4} = 0.466$$

$q = 1 - p = 0.534$

Ex. freq,

$f(x) = n\,C_x\, p^x\, q^{n-x}$.

$= 4\,C_x (0.466)^x (0.534)^{4-x}$

∴ $\boxed{\begin{aligned}&\text{exp. freq}\\ &f(x) = N \cdot b(x; n, p)\\ &= N \cdot nC_x\, p^x\, q^{n-x}\end{aligned}}$

∴ $f(x=0) = 103 \cdot 4C_0 (0.466)^0 (0.534)^4$

$= 8.375 \Rightarrow 8.3 \rightarrow 5 < 8.3 \Rightarrow 8$

$f(x=1) = 103 \cdot 4C_1 (0.466)^1 (0.534)^3$

$= 29.23 \rightarrow 29$

$f(x=2) = 103 \cdot 4C_2 (0.466)^2 (0.534)^2$

~~0.2851856~~

~~0.061923336~~

$= 38.26 \Rightarrow 38$

$f(x=3) = 103 \cdot 4C_3 (0.466)^3 (0.534)^1$

$= 22.2 \Rightarrow 22$

$f(x=4) = 103 \cdot 4C_4 (0.466)^4 (0.534)^0$

$= 4.85 \rightarrow 5$

~~obtaining female in a single trail~~

~~$\rightarrow f(x=1) = 29$~~

2) A manfacture knows that 5% of his product is ~~deffective~~ deffective, he guarentees that not more than 10 out of 100 items in a box will be differtive. What is the (prob) that a box will fail to meet the guarenteed quality?

A) ([illegible] → success)
([illegible] %. [illegible] p.)

$p = 5\% = \frac{5}{100} = \underline{0.05}$

$q = 1 - p = 1 - 0.05 = \underline{\underline{0.95}}$.

10 out of 100 ⇒ max 10 = x.

$x = 0, 1, 2, \ldots, 10$.

$\therefore n = 100$.

success = defective.

10-[illegible] defective [illegible], so $x > 10$.

$P(x > 10) = ?$ $\left(>10 \Rightarrow P x = 11, x = 12 \ldots \text{ not possible.} \right)$

$= 1 - P(x \leq 10)$

$= 1 - \left[P(x=0) + P(x=1) + \cdots P(x=10) \right]$

$= 1 - \left[\sum_{x=0}^{10} nC_x (0.05)^x (0.95)^{100-x} \right]$

10 [illegible] stop here.

3) It has been observed from past exp that the (prob) of an electrical compound being defective is 5%

a) If sample of 25 items is taken what is the (prob) of

1) exactly 1 being defective

2) ≥ 1 being defective.

b) If 100 sample of size are taken in how many samples can be expect to hue no defective?

A) $p = \frac{5}{100} = 0.05$ $x = 0, 1, \ldots, 25$

a) $q = 1 - p = 1 - 0.05 = 0.95$

1) $P(x=1) = nC_x (0.05)^1 \cdot (0.95)^{25-1}$

$= 25C_1 (0.05)^1 (0.95)^{24}$

$= \underline{\underline{1.25 \times (0.95)^{24}}}$

2) $P(x > 1) = 1 - P(x \leq 1)$

$= 1 - \left[P(x=0) + P(x=1) \right]$

$= 1 - \left[25C_0 (0.05)^0 (0.95)^{25} + 1.25 \times (0.95)^{24} \right]$

$= 1 - \left[(0.95)^{25} + 1.25 (0.95)^{24} \right]$

$= 1 - \left[(0.95)^{24} (0.95 + 1.25) \right]$

$= \underline{\underline{1 - \left[(0.95)^{24} \, 2.20 \right]}}$

b) $N = 100$

expected no = $N \times P(x=0)$

no defective → $x = 0$.

$= 100 \cdot P(x=0)$

$= 100 \cdot 25C_0 (0.05)^0 (0.95)^{25}$

$= 100 \cdot (0.95)^{25}$

⇒ Poisson Distribution :-

binomial: $X \to B(n, p)$
(P.D) $X \to P(\lambda)$

A discrete R.V x is defined to have poisson distri... if the (prob) density () of X is gvn by,

$$\boxed{f(x) = \frac{e^{-\lambda}\lambda^{x}}{x!}} \qquad x = 0, 1, 2, \ldots \qquad \lambda > 0$$

where $\lambda \to$ paramtr of P.D.

In this case, we can write

$X \underset{(follows)}{\longrightarrow} P(\lambda)$

eg → The no. of total traffic accidents per week in a gvn ~~stage~~ state,
The no. of telephone calls per hr coming into the switch board,
The no. of defects per unit of some material.

⇒ Moments of P.D :-

1) Mean :-

$$\boxed{E(x) = \lambda}$$

$$e^{\lambda} = \sum \frac{\lambda^{x-1}}{(x-1)!}$$

Pf

$$E(x) = \sum x \, f(x).$$

$$= \sum x \cdot \frac{e^{-\lambda}\lambda^{x}}{x!} \qquad \lambda^{x} = \lambda \cdot \lambda^{x-1}.$$

$$= e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{x\,\lambda^{x-1}}{x!} \qquad x! = x(x-1)$$

$$= e^{-\lambda}\lambda \sum_{x=0}^{\infty} \frac{x\,\lambda^{x-1}}{x(x-1)!}$$

$$= e^{-\lambda}\lambda \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}}_{e^{\lambda}}$$

$$= e^{-\lambda}\lambda \cdot e^{\lambda}$$

$$= \lambda \cdot e^{-\lambda+\lambda} = \lambda \cdot e^{0} = \lambda$$

$E(x) = \lambda.$

2) Varience :-

$$\boxed{V(x) = \lambda} \qquad \boxed{\begin{array}{l} S.D = \sqrt{V(x)} \\ S.D = \sqrt{\lambda} \end{array}}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 \cdot f(x) \qquad x^2 = x(x-1) + x$$

$$= \sum_{x=0}^{\infty}\left[x(x-1) + x\right] \cdot f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot f(x) + \sum_{x=0}^{\infty} x \cdot f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^{x}}{x!} + E(x).$$

$$= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^{x}}{x!} + \lambda$$

$$= e^{-\lambda}\lambda^{2} \sum_{x=0}^{\infty} \frac{x(x-1)\,\lambda^{x-2}}{x!} + \lambda.$$

$$= e^{-\lambda}\lambda^{2} \sum_{x=0}^{\infty} \frac{x(x-1)\,\lambda^{x-2}}{x(x-1)(x-2)!} + \lambda.$$

$$= e^{-\lambda}\lambda^{2} \underbrace{\sum_{x=0}^{\infty} \frac{\lambda^{x-2}}{(x-2)!}}_{} + \lambda$$

$$= e^{-\lambda}\lambda^{2} \cdot e^{\lambda} + \lambda.$$

$$= \lambda^{2} \cdot e^{-\lambda+\lambda} + \lambda = \lambda^{2} e^{0} + \lambda.$$

$$E(x^2) = \lambda^2 + \lambda$$

$$V(x) = E(x^2) - (E(x))^2$$

(Study $\mu_1', \mu_2', \mu_3', \mu_4'$)

$= \lambda^2 + \lambda - \lambda^2$

$V(x) = \lambda$

3) Calculation of μ_3 & μ_4 :-

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$. $\mu_1 \rightarrow$ central

here $\mu_1' = E(x) = \lambda$

$\mu_2' = E(x^2) = \lambda^2 + \lambda$,

$\mu_3' = E(x^3) = ?$

$E(x^3) = \sum x^3 \cdot f(x)$

$x^3 = x(x-1)(x-2) + 3x^2 - 2x$

$= \sum_{x=0}^{\infty} [x(x-1)(x-2) + 3x^2 - 2x] f(x)$

$= \sum_{x=0}^{\infty} x(x-1)(x-2) f(x) + \sum_{x=0}^{\infty} 3x^2 f(x) - \sum_{x=0}^{\infty} 2x f(x)$

$= \sum_{x=0}^{\infty} x(x-1)(x-2) \frac{e^{-\lambda}\lambda^x}{x!} + 3\sum_{x=0}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} - 2\sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!}$

$= e^{-\lambda}\lambda^3 \sum_{x=0}^{\infty} \frac{\cancel{x(x-1)(x-2)}\lambda^{x-3}}{\cancel{x(x-1)(x-2)}(x-3)!} + 3[\lambda^2 + \lambda] - 2\lambda$

$= e^{-\lambda}\lambda^3 \sum_{x=0}^{\infty} \frac{\lambda^{x-3}}{(x-3)!} + 3\lambda^2 + \underline{3\lambda - 2\lambda}$

$= e^{-\lambda}\lambda^3 \cdot e^{\lambda} + 3(\lambda^2 + \lambda) - 2\lambda$ $\quad \boxed{e^{-\lambda+\lambda} = e^0 = 1}$

$= \lambda^3 e^{-\lambda+\lambda} + 3\lambda^2 + 3\lambda - 2\lambda$

$= \lambda^3 + 3\lambda^2 + \lambda$ //

$\mu_3' = \lambda^3 + 3\lambda^2 + \lambda$

$\therefore \mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$

$= \lambda^3 + 3\lambda^2 + \lambda - 3(\lambda^2 + \lambda)\lambda + 2\lambda^3$

$= \cancel{\lambda^3} + \cancel{3\lambda^2} + \lambda - 3\cancel{\lambda^3} - \cancel{3\lambda^2} + \cancel{2\lambda^3}$

$\mu_3 = \lambda$

Similarly we can find μ_4.

$\therefore \mu_4 = 3\lambda^2 + \lambda$. (proof $\rightarrow$ last pg)

$\rightarrow$ Poisson distri... as a limiting form of B.D :-

Poisson distri... is obtained as an approximation to the B.D under the condi...

a) n is very large $(n \rightarrow \infty)$

b) p is very small $(p \rightarrow 0)$

c) $np = \lambda$, a finite quantity

Pf/ $\quad f(x) = {}^nC_x p^x q^{n-x}$ $\qquad {}^nC_x = \frac{n!}{x!(n-x)!}$

$q^{n-x} = (1-p)^{n-x} = \frac{(1-p)^n}{(1-p)^x}$

$= \frac{n!}{x!(n-x)!} p^x q^{n-x}$

$= \frac{n(n-1)(n-2)\dots(n-x-1)\cancel{(n-x)!}}{x!\cancel{(n-x)!}} p^x (1-p)^{n-x}$

n outside

$= \frac{n(n-1)(n-2)\dots(n-x-1)(1-p)^n}{x!(1-p)^x}$

$$= \frac{n^x \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) p^x (1-p)^n}{x!\,(1-p)^x}$$

$$f(x) = \frac{(np)^x \left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right](1-p)^n}{x!\,(1-p)^x} \quad \text{--- (1)}$$

apply 3 condi~

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right) = 1$$

$np = \lambda \quad \therefore \quad p = \frac{\lambda}{n}. \qquad (np)^x = \lambda^x$

$$\lim_{n\to\infty} (1-p)^x = \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^x = 1$$

$$\lim_{n\to\infty} (1-p)^n = \boxed{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n = e^{-\lambda}} \text{ (def)}$$

(1) becomes

$$f(x) = \frac{\lambda^x \times 1 \times e^{-\lambda}}{x!}$$

$$\boxed{f(x) = \frac{e^{-\lambda}\cdot\lambda^x}{x!}} \longrightarrow \text{poisson distri~}$$

=> Fitting of poisson distri... :- (P.D)

(multiply N with f(x))

By fitting (P.D), we mean to calculate the expected poisson freq against the gvn observed freq.

If f(x) denotes the poisson freq ( ) given by,

$$f(x) = N \cdot p(X = x)$$

$$\boxed{f(x) = N \cdot \frac{e^{-\lambda}\cdot\lambda^x}{x!}} \qquad x = 0, 1, \ldots, \quad \lambda > 0$$

1) A random variable X follows a P.D with mean = 1. cal (prob) that a) $X = 0$ (b) $X = 1$ (c) $X \geq 2$

A)

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$E(X) = \lambda = 1$

a) $p(X=0) = \frac{e^{-1}\cdot 1^0}{0!} = e^{-1} = 0.3678$ $\quad \boxed{0! = 1}$

b) $p(X=1) = \frac{e^{-1}\cdot 1^1}{1} = e^{-1} = 0.3678$

$e + \frac{-1}{} = 0.367$

c) $p(X \geq 2) = p(X=2) + p(X=3) \ldots$ (difficult)

$\therefore = 1 - [p(X=0) + p(X=1)]$

$= 1 - [0.3678 + 0.3678]$

$= 0.2644$

1) If x & y are Poisson variates such that $P(x=1) = P(x=2)$ &
$P(y=2) = P(y=3)$
find variance of $x - 2y$?

A) $V(x-2y) =$ always + in $V(x)$

$V(ax \pm b) = a^2 V(x)$

$V(x) + V(2y)$

$V(2y) = 2^2 V(y) = 4V(y)$

$= V(x) + 4V(y)$ — (1)

$V(x)$, $V(y)$?. $x \to P(\lambda_1)$, $y \to P(\lambda_2)$

$$P(x) = \frac{e^{-\lambda_1} \cdot \lambda_1^x}{x!}$$

$$P(y) = \frac{e^{-\lambda_2} \cdot \lambda_2^y}{y!}$$

→ $P(x=1) = P(x=2)$ (given)

$$\frac{\cancel{e^{-\lambda_1}} \cdot \cancel{\lambda_1}}{1!} = \frac{\cancel{e^{-\lambda_1}} \cdot \cancel{\lambda_1} \lambda_1}{2!}$$

$2! \to 1 \times 2 = 2$

$$1 = \frac{\lambda_1}{2}$$

$\lambda_1 = 2$

→ $P(y=2) = P(y=3)$ (given)

$$\frac{\cancel{e^{-\lambda_2}} \cancel{\lambda_2^2}}{2!} = \frac{\cancel{e^{-\lambda_2}} \cancel{\lambda_2^2} \lambda_2}{3!}$$

$$= \frac{1}{2} = \frac{\lambda_2}{6}$$

$6 = \lambda_2 2$
$\lambda_2 = 6/2 = 3$

$\lambda_2 = \frac{6}{2} = 3$

$V(x) = 2$
$V(y) = 3$

$V(x-2y) = V(x) + 4V(y)$ — (1)
$= 2 + 4 \times 3$
$= 2 + 12 = 14$

2) A P.D has double mode at $x=2$ & $x=3$. what prob that x will have 1 / the other value of the 2 values ?

A) Since P.D has double mode at $x=2$ & $x=3$ clearly $\lambda = 3$

equivalent 2 mode 2, 3, take highest no. as $\lambda \to \lambda = 3$

$$P(x=2 \text{ or } 3) = P(x=2) + P(x=3)$$

$$= \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!}$$

$$= \frac{e^{-3} 9}{2} + \frac{e^{-3} 27}{6}$$

$= 0.04979 \times 4.5 +$
0.04979×4.5

$= 0.448$

$$P(x = 2 \text{ or } 3) = P(x=2) + P(x=3)$$

$$= \frac{e^{-3}\, 3^2}{2!} + \frac{e^{-3}\, 3^3}{3!}$$

$$= e^{-3}\frac{9}{2} + e^{-3}\frac{27}{6}$$

$$= 0.04979 \times 4.5 + 0.04979 \times 4.5$$

$$= 0.448$$

⇒ **Mode of P.D :-**

Mode is the value of the random variable having max (prob)

If x is the mode, then $f(x)$ will be max. This gives 2 conditions

$f(x)$ always ↑.

1) $f(x) \geq f(x+1)$

2) $f(x) \geq f(x-1)$

Pf/

1) $f(x) \geq f(x+1)$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{\cancel{e^{-\lambda}}\lambda^x}{x!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{x+1}}{(x+1)!}$$

$\lambda^{x+1} = \lambda^x \lambda^1$

$x!$ & $(x+1)!$ → big. $\therefore (x+1)\,x!$

$$\frac{\lambda^x}{x!} \geq \frac{\cancel{\lambda^x}\lambda}{(x+1)!}$$

$$\frac{\cancel{\lambda^x}}{\cancel{x!}} \geq \frac{\lambda}{(x+1)\cancel{x!}}$$

$$1 \geq \frac{\lambda}{(x+1)} \quad \text{or}$$

$$(x+1) \geq \lambda \quad \text{or}$$

$$\lambda \leq (x+1) \quad x \geq \lambda - 1 \text{ — (1)}$$

2) $f(x) \geq f(x-1)$

$$\frac{e^{-\lambda}\lambda^x}{x!} \geq \frac{\cancel{e^{-\lambda}}\lambda^{x-1}}{(x-1)!}$$

$\lambda^{x-1} = \lambda^x \cdot \lambda^{-1} = \frac{\lambda^x}{\lambda}$

$$\frac{\lambda^x}{x!} \geq \frac{\lambda^x}{\lambda(x-1)!}$$

$x!$ & $x-1$ → big. $\therefore x(x-1)$

$$\cancel{1 \geq \frac{\lambda}{(x-1)}}$$

$$\cancel{(x-1) \geq \lambda}$$

$$\cancel{\lambda \leq (x-1)}$$

$$\frac{1}{x\cancel{(x-1)!}} \geq \frac{1}{\lambda\cancel{(x-1)!}}$$

$$\frac{1}{x} \geq \frac{1}{\lambda}$$

$\lambda \geq x.$

$$x \leq \lambda \text{ — (2)}$$

x → mode
λ → integr

from (1) & (2),

$$\lambda - 1 \leq x \leq \lambda$$

~~$x = r$~~

~~$r \leq \lambda \leq r+1$~~

* If x is a integr. ⇒ 2 mode
* if x is a float ⇒ 1 mode. eg → 4.5 mode is at 4

* If λ is an integr 'r' we get 2 modes r & $r-1$.
* If λ is not an integr, the intgr part of λ will be mode.

⇒ Addive property :- If x follows P.D λ_1 & y follows P.D λ_2,

If $x \to P(\lambda_1)$; $y \to P(\lambda_2)$ & if x & y are independent then $x+y \to P(\lambda_1+\lambda_2)$

$$\boxed{M_x^{(t)} = e^{\lambda(e^t-1)} \quad \text{(moment)}}$$

$x \to P(\lambda_1)$

$$M_x^{(t)} = e^{\lambda_1(e^t-1)}$$

$y \to P(\lambda_2)$

$$M_y^{(t)} = e^{\lambda_2(e^t-1)}$$

$x+y \rightarrow P(\lambda_1+\lambda_2) \longrightarrow$ To prove

$$M_{x+y}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}$$

Since x & y are independent,

$$\boxed{M_{x+y}(t) = M_x(t) \cdot M_y(t)}$$

$$= e^{\lambda_1(e^t-1)} \cdot e^{\lambda_2(e^t-1)}$$

$$= e^{\lambda_1(e^t-1)+\lambda_2(e^t-1)}$$

$[a^m \cdot a^n = a^{m+n}]$

(common) $= e^{(e^t-1)(\lambda_1+\lambda_2)}$

$$\therefore \boxed{M_{x+y}(t) = e^{(\lambda_1+\lambda_2)(e^t-1)}}$$

$\Rightarrow$ Moment generative ( ) :-

$$M_x(t) = e^{\lambda(e^t-1)}$$

pf

$$M_x(t) = E(e^{tx}) \qquad E(x) = \sum x \cdot f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx}\lambda^x}{x!} \qquad \boxed{\sum_{x=0}^{\infty} \frac{\lambda^x}{x!} = e^{\lambda}}$$

$e^{tx}\lambda^x = (\lambda e^t)^x$

$$= e^{-\lambda} \cdot e^{\lambda e^t}$$

$$= e^{-\lambda+\lambda e^t} \Rightarrow e^{\lambda e^t - \lambda}$$

$$\Rightarrow e^{\lambda(e^t-1)}$$

$\Rightarrow$ Recurrence relation for central moments :-

when $X \rightarrow P(\lambda)$

$$\boxed{\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]}$$

pf

$$\mu_r = E(x - E(x))^r \qquad E(x) = \sum x \cdot fx.$$

$$= E(x-\lambda)^r$$

$$= \sum_{x=0}^{\infty} (x-\lambda)^r f(x).$$

$$\mu_r = \sum_{x=0}^{\infty} (x-\lambda)^r \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$\frac{d\mu_r}{d\lambda} = ?$ (at a time u v $\frac{d}{d\lambda}$)

$$\frac{d}{d\lambda}\left[\sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$$= \sum_{x=0}^{\infty} \frac{1}{x!}\left[\frac{d}{d\lambda}\underset{①}{(x-\lambda)^r}\ \underset{②}{e^{-\lambda}} \cdot \underset{③}{\lambda^x}\right]$$

$$= \sum_{x=0}^{\infty} \frac{1}{x!}\Big[r(x-\lambda)^{r-1} \cdot -1 \cdot e^{-\lambda} \cdot \lambda^x + (x-\lambda)^r \cdot e^{-\lambda} \cdot -1 + \lambda^x + (x-\lambda)^r e^{-\lambda}\, x\lambda^{x-1}\Big]$$

$$= -r\sum_{x=0}^{\infty} (x-\lambda)^{r-1}\frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda} \cdot -1 \cdot \lambda^x}{x!} + \sum_{x=0}^{\infty} x(x-\lambda)^r \frac{e^{-\lambda}\lambda^{x-1}}{x!}$$

common $\quad \lambda^{x-1} = \lambda^x \cdot \lambda)$

$$= -r\, \mu_{r-1} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left[-1+\frac{x}{\lambda}\right]$$

$$= -r\, \mu_{r-1} + \sum_{x=0}^{\infty} (x-\lambda)^r \frac{e^{-\lambda}\lambda^x}{x!}\left[\frac{x-\lambda}{\lambda}\right].$$

$$= -r\mu_{r-1} + \frac{1}{\lambda}\underbrace{\sum_{x=0}^{\infty}(x-\lambda)^{r+1}\frac{e^{-\lambda}\lambda^x}{x!}}_{\mu_{r+1}}$$

$$\text{i.e. } \frac{d\mu_r}{d\lambda} = -r\mu_{r-1} + \frac{1}{\lambda}\mu_{r+1}$$

$$\frac{1}{\lambda}\mu_{r+1} = \frac{d\mu_r}{d\lambda} + r\mu_{r-1}$$

$$\mu_{r+1} = \lambda\left[r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right]$$

Q
1) fit a P.D to following data. →

No. of accident (x)	No. of men (f)	fx
0	95	0
1	75	75
2	44	88
3	18	54
4	2	8
5	1	5
	235	230

A)
$\lambda \to$ Mean of P.D.

$$\lambda = \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{230}{235} = 0.978$$

$$\lambda = 0.978 \approx 0.98$$

$$\boxed{f(x) = N \cdot \frac{e^{-\lambda}\lambda^x}{x!}}$$

$$= 235 \cdot \frac{e^{-0.98}\cdot(0.98)^x}{x!}$$

$$f(x=0) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^0}{0!} \quad 0! \to 1.$$

$$= 88.1$$

$$f(x=1) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^1}{1!}$$

$$= 86.37$$

$$f(x=2) = 235 \cdot \frac{e^{-0.98}\cdot(0.98)^2}{2!}$$

fact (5)
$5! \to 5 + \text{shift} + x^{-1}$

$$= 225.694 \times 2.664$$

$$= 42.$$

$f(x=3) = 13.81$

$f(x=4) = 3.38$

$f(x=5) = 0.66$

2) The record of birth over the last 100 yrs maintained by the muncipal council of a town showed that 200 children were born blind during that period. On the assumption that no. of children borned blind in a yr follows P.D. Estimate no. of yrs. in which there were -

1) no blind births (4) more than 2 b. birth.
2) 1 " "
3) 2 " "

$(e^{-2} = 0.13534)$

A) (Using Prob × N.)

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

avg $= \lambda =$ mean $= \frac{200}{100} = 2$

1) $f(x=0) = \frac{e^{-2}2^0}{0} = e^{-2} = 0.13534$

$\therefore$ no. of yrs $= 100 \times 0.13534$
$= 13.534 = 14$

2) $f(x=1) = \frac{e^{-2}2^1}{1!} = 0.27$

yrs $= 100 \times 0.27 = 27.06 = 27$

3) $f(x=2) = \frac{e^{-2}2^2}{2!} = 0.27$

yr $= 100 \times 0.27 = 27.06$
$= 27$

4) $f(x>2) = 1 - [f(x=0) + f(x=1) + f(x=2)]$

$= \cancel{1 - [14 + 27 + 27]}$

(no yrs)

$= 1 - [0.13534 + 0.27006 + 0.27006]$

$= 32$ yrs.

3) If x has PD with parameter $\lambda > 0$ & $\mu_k' = E(x^k)$ then S.T

$$\mu_{k+1}' = \lambda\left[\mu_k' + \frac{d\mu_k'}{d\lambda}\right].$$

A) $\mu_k' = E(x^k)$

$= \sum x^k \cdot f(x)$

$\mu_k' = \sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!}$

$$\frac{d\mu_k'}{d\lambda} = \frac{d}{d\lambda}\left[\sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!}\right]$$

$= \sum_{x=0}^{\infty} \frac{1}{x!}\left[x^k \cdot e^{-\lambda}\lambda^x\right]$ $\quad \frac{d}{dx}(x-1) = 1-0 = 1$

$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\cdot\lambda^x\right]$ (uv rule).

$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\cdot x\lambda^{x-1}\times 1 + (-e^{-\lambda}\cdot\lambda^x)\right]$

$e^{-\lambda}\cdot x\lambda^{x-1} + (e^{-\lambda}\lambda^x)$

$= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\lambda^x\left[\frac{x}{\lambda}\right] - 1\right]$

$\cancel{= \sum_{x=0}^{\infty} \frac{x^k}{x!}\left[e^{-\lambda}\lambda^x\left[\frac{x-\lambda}{\lambda}\right]\right]}$

$= \sum_{x=0}^{\infty} x^k \cdot \frac{e^{-\lambda}\lambda^x}{x!}\frac{(x-\lambda)}{\lambda}$

$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot \left[-1 + \frac{x}{\lambda}\right]$

$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot -1 + \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \cdot \frac{x}{\lambda}$

$\cancel{= \frac{1}{\lambda}\sum x^k}$

$= \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x \frac{x}{\lambda} - \sum \frac{x^k}{x!} e^{-\lambda}\lambda^x - 1$

$$\frac{d\mu_k'}{d\lambda} = \frac{1}{\lambda}\underbrace{\sum x^{k+1}\frac{e^{-\lambda}\lambda^x}{x!}} - \underbrace{\sum x^k \frac{e^{-\lambda}\lambda^x}{x!}}_{\mu_k'}$$

$$\frac{d\mu_k'}{d\lambda} = \frac{1}{\lambda}\mu_{k+1}' - \mu_k'$$

$$\frac{1}{\lambda}\mu_{k+1} = \mu_k' \cdot \frac{d\mu_k'}{d\lambda}$$

$$\mu_{k+1}' = r\left[\mu_k' + \frac{d\mu_k}{d\lambda}\right]$$

Q) If X & Y are independent poisson variate S.T conditional distri- of X given x+y is binomial?

A) ~~$P(X/X+Y) = f(x)$~~ ?

To prove $\rightarrow P(X/x+Y) = nC_x \, p^x q^{n-x}$?.

$$P(X/Y) = \frac{P(X,Y)}{P(Y)}$$

$$P(X=x/X+Y=n) = \frac{P(X=x, Y=n-x)}{P(X+Y=n)}$$

$$= \frac{P(X=x) + P(Y=n-x)}{P(X+Y=n)} \quad \text{—(1)}$$

$X \rightarrow P(\lambda_1) \quad \rightarrow P(x) = \dfrac{e^{-\lambda_1}\lambda_1^x}{x!}$

$Y \rightarrow P(\lambda_2)$

$(y=n-x) \quad \rightarrow P(y) = \dfrac{e^{-\lambda_2}\lambda_2^{n-x}}{(n-x)!}$

$X+Y \rightarrow P(\lambda_1+\lambda_2)$

$\rightarrow P(X+Y) = \dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}$

(a) $(x+y=n)$

(1) becomes,

$$= \frac{\dfrac{e^{-\lambda_1}\lambda_1^x}{x!} \cdot \dfrac{e^{-\lambda_2}\lambda_2^{n-x}}{(n-x)!}}{\dfrac{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}{n!}}$$

$$~~= \frac{e^{-\lambda_1}\lambda_1^x \cdot e^{-\lambda_2}\lambda_2^{n-x}\, n!}{x!(n-x)!\, e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}~~$$

$$~~= \frac{n!}{x!(n-x)!}\,\lambda_1^x\lambda_2^{n-x}~~$$

$$= \frac{e^{-\lambda_1} \cdot e^{-\lambda_2} \cdot \lambda_1^x \lambda_2^{n-x}}{x!(n-x)!} \cdot \frac{n!}{e^{-(\lambda_1+\lambda_2)}(\lambda_1+\lambda_2)^n}$$

$$= \frac{\cancel{e^{-(\lambda_1+\lambda_2)}}\,\lambda_1^x\,\lambda_2^{n-x}\,n!}{x!(n-x)!\,\cancel{e^{-(\lambda_1+\lambda_2)}}(\lambda_1+\lambda_2)^n}$$

$$= \frac{n!}{x!(n-x)!}\,\frac{\lambda_1^x\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^n} \qquad \text{let } n = x+n-x.$$

$$= \frac{n!}{x!(n-x)!}\,\frac{\lambda_1^x\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^{x+n-x}}$$

$$= \frac{n!}{x!(n-x)!}\,\frac{\lambda_1^x\,\lambda_2^{n-x}}{(\lambda_1+\lambda_2)^x\,(\lambda_1+\lambda_2)^{n-x}}$$

$$= nC_x\left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)^x\left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right)^{n-x}.$$

p^x & q^{n-x}

$$= nC_x\, p^x\, q^{n-x}.$$

where, $p = \dfrac{\lambda_1}{\lambda_1+\lambda_2}$

$q = \dfrac{\lambda_2}{\lambda_1+\lambda_2}$.

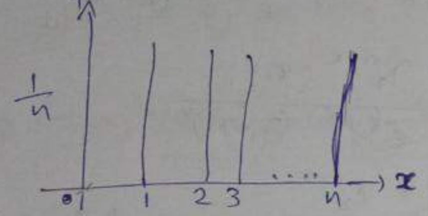
⇒ Uniform Distribution :-

This is the simplest type of discrete (prob) distri..., a discrete r.v 'x' is said to hve a uniform distri... if the (prob) density ( ) is gvn by,

$$\boxed{f(x) = \frac{1}{n}} \qquad x = 1, 2, \ldots, n.$$

$$= 0 \quad \text{elsewhere.}$$

graph



→ Mean :-

$$E(x) = \sum_{x=0}^{n} x\, f(x)$$

$$= \sum_{x=0}^{n} x \cdot \frac{1}{n}$$

$$= \frac{1}{n}\left[1+2+3+\cdots+n\right]$$

$$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)}{2}\right] = \frac{n+1}{2}$$

$$\boxed{E(x) = \frac{n+1}{2}}$$

→ Varience :-

$$V(x) = E(x^2) - (E(x))^2.$$

$$E(x^2) = \sum_{x=0}^{n} x^2 \cdot f(x)$$

$$= \sum_{x=0}^{n} x^2 \cdot \frac{1}{n}$$

$$= \frac{1}{n}\left[1^2+2^2+\cdots+n^2\right]$$

$$= \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$E(x^2) = \frac{(n+1)(2n+1)}{6}$$

$$V(x) = \frac{(n+1)(2n+1)}{6_{2\times 3}} - \left(\frac{n+1}{2}\right)^2 \quad \text{(common)}$$

$$= \frac{n+1}{2}\left[\frac{2n+1}{3} \times \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n+3}{6}\right] \qquad +\cdot - = -$$

$$= \frac{n+1}{2}\left[\frac{n-1}{6}\right]$$

$$\boxed{V(x) = \frac{n^2-1}{12}}$$

→ Moment generating ( ) :-

$$M_x(t) = E(e^{tx})$$

$$= \sum_{x=0}^{n} e^{tx} \cdot f x \quad = \sum e^{tx} \cdot \frac{1}{n}$$

$$= \sum_{x=0}^{n} \frac{1}{n}\left[e^{t} + e^{2t} + e^{3t} \cdots + e^{nt}\right]$$

$$= \sum_{x=0}^{n} \frac{1}{n} e^t \underbrace{\left[1 + e^t + e^{2t} + \dots e^{nt}\right]}_{\text{GP}} \quad \text{--- (1)}$$

$$\text{GP} = \frac{a(r^n - 1)}{r-1}$$

$$\text{sum} = \frac{a(r^n-1)}{r-1} \qquad a = 1, \quad r = e^t \rightarrow \frac{e^{2t}}{e^t} = e^t$$

$$= \frac{1\,(e^t)^n - 1}{e^t - 1}$$

(1) →

$$= \frac{1}{n} e^t \left[\frac{e^{tn} - 1}{e^t - 1}\right]$$

$$\boxed{M_x(t) = \frac{e^t (e^{tn} - 1)}{n(e^t - 1)}}$$

→ Geometric Distribution :-

A r.v 'x' is defined to have (Geo) / pascal, distribution if pdf of x is given by,

$$\boxed{f(x) = q^x \cdot p} \qquad x = 0, 1, 2 \dots$$

$$q + p = 1$$

0, elsewhere.

(x-এ 0, 1, 2 ... [illegible] $q^x \cdot p$ [illegible] 0)

→ Mean :-

$$E(x) = \sum x \cdot f(x)$$

$$= \sum_{x=0}^{\infty} x \cdot q^x \cdot p \quad \text{--- (a)}$$

$$= p \sum_{x=0}^{\infty} x \cdot q^x$$

$$= p\left[0 + q + 2q^2 + 3q^3 + \dots\right]$$

$$= p \cdot q \left[1 + 2q + 3q^2 + \dots\right]$$

$$1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}$$

$$1 + 3x + 6x^2 + \dots = \frac{1}{(1-x)^3}$$

$$1 + 4x + \dots = \frac{1}{(1-x)^4}$$

$$= p \cdot q \cdot \frac{1}{(1-q)^2}$$

$1 - q = p$

$$= \cancel{p} \cdot q \cdot \frac{1}{p^{\cancel{2}}}$$

$$\boxed{E(x) = \frac{q}{p}}$$

→ Varience :-

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 \cdot f(x) = \sum x^2 \cdot q^x \cdot p$$

$$\left[x^2 = x(x-1) + x\right]$$

$$= p \sum_{x=0}^{\infty} x^2 \cdot q^x$$

$$= p \sum_{x=0}^{\infty} \left[x(x-1) + x\right] \cdot q^x$$

$$= p \sum_{x=0}^{\infty} x(x-1)\, q^x + \sum_{x=0}^{\infty} x\, q^x$$

$$= p \sum_{x=0}^{\infty} x(x-1)\, q^x + \sum_{x=0}^{\infty} x\, q^x p \quad \text{--- (a)}$$

$$= p\left[2 \times 1\, q^2 + 3 \cdot 2\, q^3 + 4 \cdot 3\, q^4 + \dots\right] + \frac{q}{p}$$

$$= p\left[2q^2 + 6q^3 + 12q^4 + \dots\right] + \frac{q}{p}$$

$$= 2q^2 p\left[1 + 3q + 6q^2 + \dots\right] + \frac{q}{p}$$

$$= 2q^2 p \cdot \frac{1}{(1-q)^3} + \frac{q}{p}$$

$$= 2q^2 \cancel{p} \cdot \frac{1}{p^{\cancel{3}\,2}} + \frac{q}{p}$$

$$E(x^2) = \frac{2q^2}{p^2} + \frac{q}{p}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{2q^2}{p^2} + \frac{qp}{p \times p} - \frac{q^2}{p^2}$$

$$= \frac{2q^2 + qp - q^2}{p^2} = \frac{q^2 + qp}{p^2}$$

$$= \frac{q(q+p)}{p^2}$$

$$\boxed{V(x) = \frac{q}{p^2}}$$

→ Moment generating ( ) :-

$$M_x(t) = E(e^{tx})$$

$$= \sum e^{tx} \cdot f(x) = \sum e^{tx} \cdot q^x \cdot p$$

$$= p \sum e^{tx} \cdot q^x$$

$$= p \sum_{x=0}^{\infty} (qe^t)^x$$

$$= p \left[1 + (qe^t)^1 + (qe^t)^2 + \dots \right].$$

$$1 + x + x^2 + \dots = \frac{1}{(1-x)}$$

$$= p \frac{1}{(1-qe^t)}$$

$$\boxed{M_x(t) = \frac{p}{1-qe^t}}$$

⇒ Lack of memory property :-

If 'x' has geometric density with paramtr 'p', then,

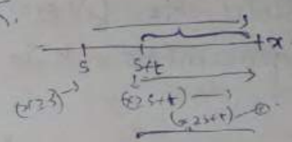
$$\boxed{P[x \geq s+t \,/\, x \geq s] = P[x \geq t]}$$

for $s, t = 0, 1, 2 \dots$

Pf/

$$P[\underset{(A)}{x \geq s+t} \,/\, \underset{(B)}{x \geq s}] = \frac{P(x \geq s+t,\ x \geq s)}{P(x \geq s)}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



$$= \frac{P(x \geq s+t)}{P(x \geq s)}$$

$$= \frac{\sum_{x=s+t}^{\infty} q^x \cdot p}{\sum_{x=s}^{\infty} q^x \cdot p}$$

$$= \frac{\cancel{p} \sum_{x=s+t}^{\infty} q^x}{\cancel{p} \sum_{x=s}^{\infty} q^x} = \frac{\sum_{x=s+t}^{\infty} q^x}{\sum_{x=s}^{\infty} q^x}$$

$$= \frac{q^{s+t} + q^{s+t+1} + q^{s+t+2} + \dots}{q^s + q^{s+1} + q^{s+2} + \dots}$$

$$= \frac{q^{s+t} \left[\cancel{1 + q + q^2 + \dots} \right]}{q^s \left[\cancel{1 + q + q^2 + \dots} \right]}$$

$$= \frac{q^{s+t}}{q^s} = \frac{\cancel{q^s} \cdot q^t}{\cancel{q^s}} = q^t = P(X \geq t)$$

⇒ Negative binomial distribution :-

Let 'x' be the discrete R.V assuming the values 0, 1, 2 ....., if the pmf of x is given by,

~~$p(x) = {}^{x+k-1}C_{k-1}$~~

$$\boxed{p(x) = {}^{x+k-1}C_{k-1}\, p^k q^x}$$ $x = 0, 1, 2 \ldots$, $p+q = 1$

then x is said to follow a -ve b.distri... with paramters k & p.

Note,

* -ve b.distri... can also be written as

$$\boxed{p(x) = {}^{-k}C_x\, p^k (-q)^x}, \quad x = 0, 1, 2 \ldots$$

* In the def... of -ve b.distri...., if we take $k = 1$, p(x) becomes

$$p(x) = {}^{x+1-1}C_{1-1}\, p^1 q^x$$

$$= {}^{x}C_0\, p\, q^x$$

$$p(x) = p\, q^x \qquad \boxed{p(x) = q^x p} \quad x = 0, 1, 2 \ldots$$

This is the (prob) () of Geometric distri...

→ properties :-

1) Mean = $\boxed{E(x) = \frac{kq}{p}}$

$$E(x^2) = \frac{k(k+1)q^2}{p^2} + \frac{kq}{p}$$

2) Variance = $\boxed{V(x) = \frac{kq}{p^2}}$

3) -ve b.distri... tends to poission distri... under certain condi...

4) Geometric distri... is a particular case of -ve b.distri...

5) -ve b.distri... ⟶ binomial waiting time distri...

Discrete

① Binomial ② poission ③ Geometric ④ Bernoulli

⑤ uniform ⑥ -ve binomial

⇒ Continuous (prob) distri... :-

1) Normal distri... :-

A R.V 'x' is defined to be normally distributed, if its PDF is given by,

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}} \quad -\infty < x < \infty$$

μ = mean

σ = S.D

μ & σ satisfy $-\infty < \mu < \infty$ & $\sigma > 0$

when x follows N. distri... we write its symbolically as,

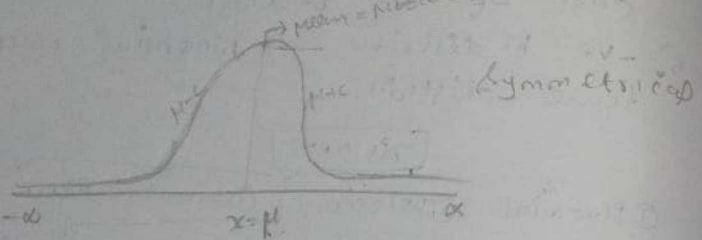
$$X \to N(\mu, \sigma) \quad \text{or}$$
$$X \to N(\mu, \sigma^2)$$

→ Properties :-

* Graph of N. distri... is gvn by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

is a bell-shaped smooth symmetrical curve → normal curve as shown below.



* Some of imp pro) are. →

* Normal curve symmetrical about $x = \mu$, (i.e) $f(\mu + c) = f(\mu - c)$ for any c.
* Mean, Mode & Median are identical.
* Normal curve $f(x)$ has a max at $x = \mu$ & max value of the ordinates is $\frac{1}{\sigma\sqrt{2\pi}}$

point $= \mu$, value $= \frac{1}{\sigma\sqrt{2\pi}}$

* Normal curve extends from $-\infty$ to $+\infty$.
* curve touches the x axis only at $\pm\infty$. (i.e) X-axis is an asymptote to the curve.
* for a N. distri... $\boxed{\beta_1 = 0, \beta_2 = 3}$
* $\boxed{QD = \frac{2}{3}.SD, \quad MD = \frac{4}{5}.SD}$

evn → 2r
odd → 2r+1

* All odd order central moments are vanish, (i.e) $\boxed{\mu_{2r+1} = 0}$, $r = 0, 1, 2 \ldots$
* The evn order central moments are gvn by, $\mu_{2r} = 1 : 3 : 5 : \ldots (2r-1)\sigma^{2r}$.
* The points of inflection of curve are $x = \mu \pm \sigma$
* The lower & upper quartiles are equidistance from median.

$Q_1 \quad Q_D \quad Q_3$

2) ~~Rectangular distribution~~ :-

unif... contin...

2) Rectangular distri... :- (uniform distri...) (continous)

* A very simple distri... for a contin... R.V is the uniform distri...
* It is perticularly useful in theoretical statistics, bcz it is convenient to deal with mathematically.

def →

A ~~that~~ contin... R.V 'x' is said to hve rectangular distri..., if its pdf givn by,

$$\boxed{f(x) = \frac{1}{b-a}}, \quad a \leq x \leq b$$
$$= 0, \quad \text{elsewhere.}$$

Rmrk →

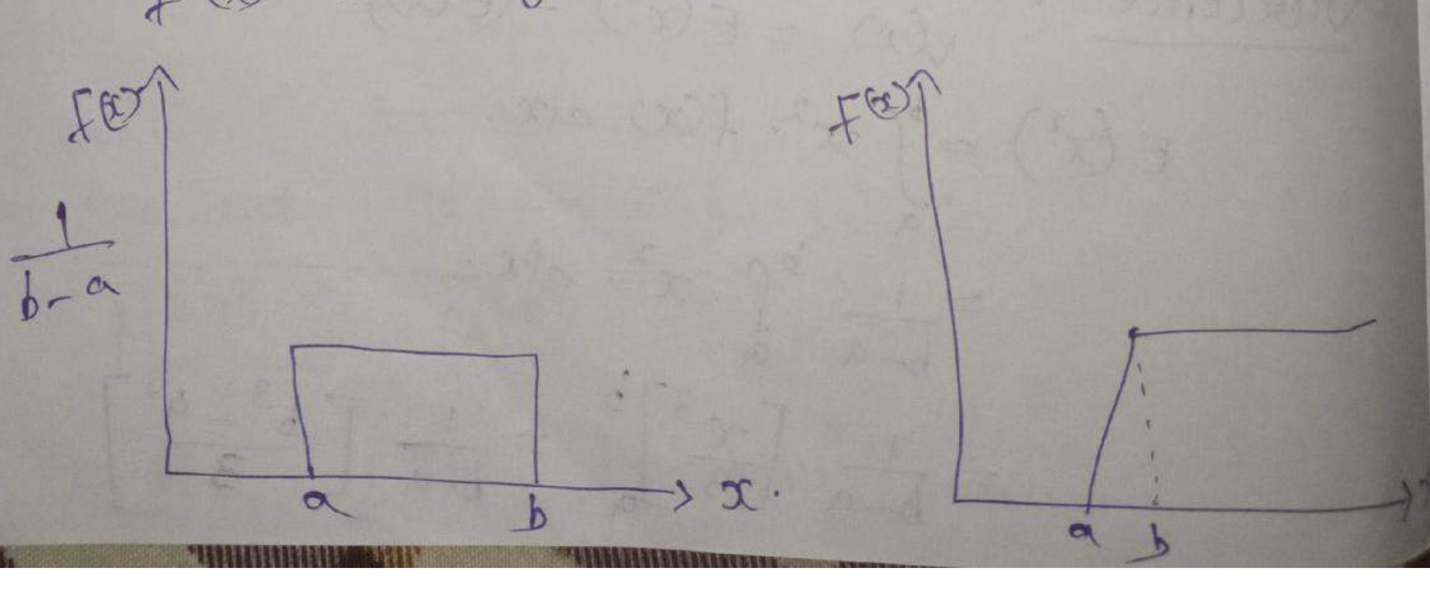
* a and b be (a<b) are 2 paramtr of uniform distri... on a, b.
* The distri... → rectangular distri... since the curve, $y = f(x)$, discribes a ▭ over the x-axis and b/w the ordinates at $x = a$ & $x = b$.
* The distri... () $F(x) = \begin{cases} 0, & \text{if } -\infty < x < a \\ \frac{x-a}{b-a}, & a < x < b \\ 1, & b < x < \infty. \end{cases}$

Since $F(x)$ is not contin... at $x = a$ & $x = b$, it is not differentiable at this point. thus,

$$\boxed{F'(x) = f(x) = \frac{1}{b-a} \neq 0}$$

exist everywhere except at the point $x=a$ & $x=b$. & consequenly we get pdf $f(x)$

* The graph of uniform pdf $f(x)$ & the corresponding distri.... () $F(x)$ are gvn below,



* for a ▭-lar / uniform variate 'x' in $(-a, a)$ pdf is gvn by,

$$f(x) = \begin{cases} \frac{1}{2a} & , \quad -a < x < a \\ 0 & , \quad \text{elsewhere} \end{cases}$$

~~moments~~

* Mean :- $E(x)$

$$= \int_a^b x \cdot f(x)\, dx.$$

$$= \int_a^b x \cdot \frac{1}{b-a}\, dx.$$

$$= \frac{1}{b-a}\int_a^b x \cdot dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$= \frac{1}{b-a} \times \frac{b^2-a^2}{2} \qquad \boxed{b^2-a^2 = (b+a)(b-a)}$$

$$= \frac{1}{(\cancel{b-a})} \times \frac{(b+a)(\cancel{b-a})}{2}$$

$$\boxed{\text{mean} = \frac{b+a}{2}.}$$

* Varience :- $V(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_a^b x^2 \cdot f(x)\, dx.$$

$$= \frac{1}{b-a}\int_a^b x^2 \cdot dx.$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a}\left[\frac{b^3-a^3}{3}\right]$$

$$= \frac{b^3-a^3}{3(b-a)} \qquad b^3-a^3 = (b-a)\cdot(b^2+ab+a^2)$$

$$= \frac{(\cancel{b-a})(b^2-ab+a^2)}{3(\cancel{b-a})}$$

$$E(x^2) = \frac{b^2-ab+a^2}{3}$$

$$\therefore \quad V(x) = \frac{b^2-ab-a^2}{3} - \left[\frac{b+a}{2}\right]^2$$

$$= \frac{b^2-ab-a^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{b^2-ab-a^2}{3} - \frac{b^2+2ab+a^2}{4}$$

$$= \frac{4(b^2-ab-a^2) - 3(b^2+2ab+a^2)}{12}$$

$$= \frac{4b^2-4ab-4a^2-3b^2-6ab-3a^2}{12}$$

$$= \frac{b^2+2ab-a^2}{12} \qquad (b-a)^2 = b^2+2ab-a^2$$

$$= \frac{(b-a)^2}{12}$$

⇒ Moment generating ( ) :-

$$M_x(t) = E(e^{tx})$$

$$= \int_a^b e^{tx} \cdot f(x)\, dx.$$

☆ $\Gamma_{n+1} = n! = n(n-1)! = n\Gamma_n$

$\Gamma_{n+1} = n\Gamma_n$ —— ⓐ

Putting $m=1$ & $p=\frac{1}{2}$

$\cancel{f(x) = \frac{m^p}{\Gamma p} \cdot e^{-mx} \cdot x^{p-1}}$

—① $\int_0^\infty e^{-mx} \cdot x^{p-1} \, dx = \frac{\Gamma p}{m^p}$

$m=1$, $p=1/2$ $\Rightarrow \int_0^\infty e^{-x} x^{p-1} \, dx = \Gamma_{1/2} = \sqrt{\pi}$

⟹ moments :-

(i) Mean →

$E(x) = \int x \cdot f(x) \, dx$

$= \int_0^\infty x \frac{m^p}{\Gamma p} e^{-mx} \cdot x^{p-1} \, dx$

$= \frac{m^p}{\Gamma p} \int_0^\infty x e^{-mx} \cdot x^{p-1} \, dx$

$x \cdot x^{p-1} \Rightarrow x^{(p+1)-1}$

$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-mx} x^{(p+1)-1} \, dx$

$\int e^{-mx} x^{p-1} dx = \frac{\Gamma p}{m^p}$

$\int_0^\infty e^{-mx} x^{(p+1)-1} dx = \frac{\Gamma_{p+1}}{m^{p+1}}$

$= \frac{m^p}{\Gamma p} \cdot \frac{\Gamma_{p+1}}{m^{p+1}}$

—ⓐ like, $\Gamma_{n+1} = n\Gamma_n$

$\therefore \Gamma_{p+1} = p\Gamma p$

ⓑ

$= \frac{m^p}{\Gamma p} \cdot \frac{p\Gamma p}{m^{p+1}} = \frac{\cancel{m^p}}{\cancel{\Gamma p}} \cdot \frac{p\cancel{\Gamma p}}{\cancel{m^p} \cdot m}$

$= \frac{p}{m}$ //

$= \int_a^b e^{tx} \cdot \frac{1}{b-a} \, dx.$

$= \frac{1}{b-a} \left[\frac{e^{tx}}{t} \right]_a^b = \frac{1}{b-a} \left[\frac{e^{tb} - e^{at}}{t} \right]$

$\boxed{M_x^{(t)} = \frac{e^{bt} - e^{at}}{t(b-a)}}$, $t \neq 0$

⟹ Gamma distri... :-

A contin... R.V 'x' is said to hve a gamma distri... , if its pdf is gvn by,

$\boxed{f(x) = \frac{m^p}{\Gamma p} \cdot e^{-mx} \cdot x^{p-1}}$, $x > 0$.

gamma p

$= 0$, elsewhere.

where $m > 0$, $p > 0$ ⟶ paramtr of G.distri.

Γp ⟶ Gamma p.

Note
Being a pdf we know that $\int f(x) \, dx = 1$.

$= \int_0^\infty \frac{m^p}{\Gamma p} \cdot e^{-mx} \cdot x^{p-1} \, dx = 1$

$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-mx} \cdot x^{p-1} \, dx = 1$

☆ ① $\boxed{\Rightarrow \int_0^\infty e^{-mx} \cdot x^{p-1} \, dx = \frac{\Gamma p}{m^p}}$

If $p = n$, a +ve integr $\Gamma n = (n-1)!$

$\boxed{\text{if } p = n, \text{ then } \Gamma n = (n-1)!}$

using $\int$-tion by parts we can S.T

$$= \int_0^\infty e^{tx} \cdot \frac{m^p}{\Gamma p} \cdot e^{-mx} \cdot x^{p-1}\, dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty \underline{e^{tx}} \cdot \underline{e^{-mx}} \cdot x^{p-1}\, dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-(m-t)x} \cdot x^{p-1}\, dx$$

$$= \frac{m^p}{\cancel{\Gamma p}} \cdot \frac{\cancel{\Gamma p}}{(m-t)^p}$$

$$= \frac{m^p}{(m-t)^p} = \left(\frac{m}{m-t}\right)^p$$

$$= \left(\frac{\cancel{m}}{\cancel{m}(1-\frac{t}{m})}\right)^p$$

$$= \left(\frac{1}{1-\frac{t}{m}}\right)^p = \frac{1}{(1-\frac{t}{m})^p}$$

$$= \left(1-\frac{t}{m}\right)^{-p}$$

$$e^{tx} \cdot e^{-mx} = e^{tx-mx}$$
$$= e^{-mx+tx}$$
$$= e^{-x(m-t)}$$
$$= e^{-(m-t)x}$$

$$\int_0^\infty e^{-mx} x^{p-1}\, dx = \frac{\Gamma p}{m^p}$$

$$\therefore \int_0^\infty e^{-(m-t)x} x^{p-1}\, dx = \frac{\Gamma p}{(m-t)^p}$$

$\Rightarrow$ Exponential distri.... :-

If a contin... r.v x has a pdf

$$\boxed{f(x) = \theta e^{-\theta x}} \quad x > 0,\ \theta > 0$$

* Mean :- $\boxed{E(x) = \frac{1}{\theta}}$

$$E(x) = \int_0^\infty x \cdot f(x)\, dx = \int_0^\infty x \cdot \theta e^{-\theta x} \cdot dx$$

$$= \int_0^\infty x^{2-1} \cdot \theta e^{-\theta x} = \theta \int_0^\infty \underbrace{x^{2-1} \cdot e^{-\theta x}}\, dx$$

$x' = x^{2-1}$

(2) Varience $\rightarrow$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^\infty x^2 \cdot \frac{m^p}{\Gamma p} \cdot e^{-mx} \cdot x^{p-1}\, dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty \underline{x^2} \cdot e^{-mx} \cdot \underline{x^{p-1}}\, dx$$

$$= \frac{m^p}{\Gamma p} \int_0^\infty e^{-mx} \cdot x^{(p+2)-1}\, dx$$

$$\Rightarrow \frac{m^p}{\Gamma p} \times \frac{\Gamma p+2}{m^{p+2}}$$

$$\Rightarrow \frac{m^p}{\Gamma p} \cdot \frac{(p+1)\, p\, \Gamma p}{m^{p+2}}$$

$$\Rightarrow \frac{\cancel{m^p}}{\cancel{\Gamma p}} \cdot \frac{(p+1)\, p\, \cancel{\Gamma p}}{\cancel{m^p} \cdot m^2}$$

$$E(x^2) \Rightarrow \frac{(p+1)\, p}{m^2}$$

$x^2 \cdot x^{p-1}$ $(p+2)-1$
$2+p-1 \Rightarrow x$

(5)

$$\int_0^\infty e^{-mx} x^{(p+1)-1}\, dx = \frac{\Gamma p+1}{m^{p+1}}$$

$$\therefore \int_0^\infty e^{-mx} \cdot x^{(p+2)-1}\, dx = \frac{\Gamma p+2}{m^{p+2}}$$

$$\Gamma n+1 = n\Gamma n$$

$$\therefore \Gamma p+2 = (p+1)\, \Gamma p+1$$
$$= (p+1) \cdot p\, \Gamma p$$

$$V(x) = \frac{(p+1)\, p}{m^2} - \left(\frac{p}{m}\right)^2$$

$$= \frac{(p+1)\, p}{m^2} - \frac{p^2}{m^2} = \frac{\cancel{p^2} + p - \cancel{p^2}}{m^2}$$

$$\boxed{V(x) = \frac{p}{m^2}}$$

(3) Moment generating ( ) :-

$$M_x(t) = E(e^{tx})$$
$$= \int_a^b e^{tx} \cdot f(x)\, dx$$

* **Mgf** :- $\boxed{M_x(t) = \left[1 - \frac{t}{\theta}\right]^{-1}}$

$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} \cdot f(x)\, dx.$

$= \int_0^\infty e^{tx} \cdot \theta e^{-\theta x}\, dx$

$= \theta \int_0^\infty e^{tx} \cdot e^{-\theta x}\, dx = \theta \int_0^\infty e^{tx - \theta x}\, dx.$

$= \theta \int_0^\infty e^{-(\theta - t)x}\, dx.$

$= \theta \cdot \left[\frac{e^{-(\theta - t)x}}{-(\theta - t)} \right]_0^\infty \quad (x \to \infty, 0)$

$e^{\infty} = \infty$
$e^{-\infty} = 0$

$= \cancel{\theta \left[\frac{e^{-\infty}}{-\theta - t} - 0 \right]} = \cancel{\theta}$

$= \theta \left[\frac{e^{-\infty}}{-\theta - t} - \frac{e^{0}}{-\theta - t} \right] = \theta \left[\frac{e^{-\infty} - e^{0}}{-(\theta - t)} \right]$

$\theta \left[\frac{0 - 1}{-(\theta - t)} \right] = \theta \left[\frac{-1}{-(\theta - t)} \right] = \theta \cdot \frac{1}{\theta - t} = \frac{\theta}{\theta - t}$ (split not possible)

$= \left[\frac{\theta - t}{\theta} \right]^{-1} = \left[\frac{\theta}{\theta} - \frac{t}{\theta} \right]^{-1}$

$M_x(t) = \left[1 - \frac{t}{\theta} \right]^{-1}$

* Exp. distri. sometimes is the -ve Expo. distri.

$\boxed{\int_0^\infty e^{-mx} \cdot x^{p-1}\, dx = \frac{\Gamma p}{m^p}}$

$m = \theta$
$p = 2$

$\therefore \Rightarrow \theta \cdot \frac{\Gamma 2}{\theta^2} = \frac{\Gamma 2}{\theta}$ $\quad \Gamma(n+1) = n(n-1)!$

$\boxed{\Gamma n = (n-1)\Gamma(n-1)}$

$\therefore \Rightarrow \Gamma 2 = (2-1)\Gamma(2-1) = 1\Gamma 1 = 1$

$\therefore \Rightarrow \frac{1}{\theta}$ $\quad \boxed{\text{mean} = \frac{1}{\theta}}$

* **Variance** :- $\boxed{V(x) = \frac{1}{\theta^2}}$

$V(x) = E(x^2) - (E(x))^2$

$E(x^2) = \int_0^\infty x^2 \cdot f(x)\, dx = \int_0^\infty x^2 \cdot \theta e^{-\theta x} \cdot dx.$

$\boxed{x^2 = x^{3-1}}$

$= \theta \int_0^\infty x^{3-1} \cdot e^{-\theta x} \cdot dx$ $\quad \int_0^\infty e^{-mx} \cdot x^{p-1}\, dx = \frac{\Gamma p}{m^p}$

$= \theta \cdot \frac{\Gamma 3}{\theta^{3}} = \frac{\Gamma 3}{\theta^2}$

$\Gamma n = (n-1)\Gamma(n-1)$
$\Gamma 3 = (3-1)\Gamma(3-1)$
$= 2\Gamma 2 = 2$

$E(x^2) \Rightarrow \frac{2}{\theta^2}$

$V(x) = E(x^2) - (E(x))^2$

$= \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{2-1}{\theta^2} = \frac{1}{\theta^2}$

1) <u>Mean</u> :-

$$E(x) = \int_0^{1} x \cdot f(x)\, dx = \int_0^{\infty} x \cdot \frac{1}{\beta(m,n)} x^{m-1} (1-x)^{n-1} dx$$

$$= \frac{1}{\beta(m,n)} \int_0^{1} x^1 \cdot x^{m-1} (1-x)^{n-1} dx.$$

$$= \frac{1}{\beta(m,n)} \int_0^{1} x^{(m+1)-1} (1-x)^{n-1} dx.$$

$\cancel{\frac{\Gamma p}{m^p} = \Gamma m+1}$

$$= \frac{\beta(m+1,n)}{\beta(m,n)}$$ 1

$$\boxed{\beta(m+r,n) = \frac{\Gamma m+r \; \Gamma n}{\Gamma m+r+n}}$$ 2

$$= \frac{\dfrac{\Gamma m+1 \; \Gamma n}{\Gamma m+1+n}}{\dfrac{\Gamma m \; \Gamma n}{\Gamma m+n}}$$

$\Gamma n+1 = n\Gamma n$

$\Gamma m+1 = m\Gamma m$

$\Gamma m+1+n = m+n \Gamma m+n$

$$= \frac{\Gamma m+1 \; \cancel{\Gamma n}}{\Gamma m+1+n} \cdot \frac{\cancel{\Gamma m \; \Gamma n}}{\cancel{\Gamma m \; \Gamma n}} \frac{\Gamma m+n}{\Gamma m \; \Gamma n}$$

$$= \frac{m \cancel{\Gamma m} \; \cancel{\Gamma n}}{m+n \; \cancel{\Gamma m+n}} \frac{\cancel{\Gamma m+n}}{\cancel{\Gamma m} \; \cancel{\Gamma n}} = \frac{m}{m+n}$$

$$\boxed{E(x) = \frac{m}{m+n}}.$$

Q) If x is a r.v with continuous distribution, F the $F(x)$ has a uniform distribution on $[0,1]$?

A) Let $f(x)$ be the pdf of x.

Let $y = F(x)$.

$$\therefore f(x) = \frac{dF(x)}{dx} = \frac{dy}{dx}$$

Let $g(y)$ be the pdf of y.

$$g(y) = f(x) \cdot \left|\frac{dx}{dy}\right|$$

$$= f(x) \cdot \frac{1}{\frac{dy}{dx}} = \frac{f(x)}{f(x)} = 1$$

$0 < y < 1$.

$\therefore y \to$ a uniform distribution in $[0,1]$.

$\Rightarrow$ <u>Beta Distribution</u> :- [1st kind]

If a r.v 'x' has pdf given by

$$\boxed{f(x) = \frac{1}{\beta(m,n)} x^{m-1} (1-x)^{n}} \quad 0 < x < 1$$

$m > 0$ & $n > 0$ then x is said to have beta distribution,

$$\boxed{\beta(m,n) = \int_0^1 x^{m-1} (1-x)^{n-1} \cdot dx} \to \beta()$$

$\boxed{\beta(m,n) \text{ or } \beta_1(m,n)}$

1

2) Variance :-

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^1 x^2 \cdot \frac{1}{\beta(m,n)} x^{m-1}(1-x)^{n-1}\, dx$$

$$= \frac{1}{\beta(m,n)} \int_0^1 x^2 \cdot x^{m-1}(1-x)^{n-1}\, dx.$$

$$= \frac{1}{\beta(m,n)} \underbrace{\int_0^1 x^{m+2-1}(1-x)^{n-1}\, dx}_{\beta(\;)}.$$

— ①

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\, dx.$$

$$\therefore \beta(m+2,n) = \int_0^1 x^{m+2-1}(1-x)^{n-1}\, dx.$$

$$\beta(m,n) = \frac{\Gamma m\, \Gamma n}{\Gamma m+n} \quad \text{— ⓑ}$$

$$\text{ⓐ } \beta(m+2,n) = \frac{\Gamma m+2\, \Gamma n}{\Gamma m+2+n}$$

$$\Rightarrow \frac{1}{\beta(m,n)} \cdot \beta(m+2,n)$$

$$\Rightarrow \frac{\beta(m+2,n)}{\beta(m,n)} = \frac{\dfrac{\Gamma m+2\, \Gamma n}{\Gamma m+2+n}}{\dfrac{\Gamma m\, \Gamma n}{\Gamma m+n}} \quad \begin{matrix}\text{ⓐ}\\ \text{ⓑ}\end{matrix}$$

$$\Gamma m+2 = m+1\, \Gamma m+1$$
$$= (m+1)\, m\, \Gamma m$$

$$= \frac{\Gamma m+2\, \Gamma n}{\Gamma m+2+n} \cdot \frac{\Gamma m+n}{\Gamma m\, \Gamma n}$$

$$= \frac{(m+1)\, m\, \Gamma m\, \Gamma n\, \Gamma m+n}{\Gamma m+2+n\, \Gamma m\, \Gamma n}.$$

$$= \frac{(m+1)\, m\, \Gamma m+n}{\Gamma m+2+n} = \frac{(m+1)\, m\, \Gamma m+n}{(m+n+1)\, \Gamma m+n+1}$$

$$E(x^2) = \frac{(m+1)\, m\, \Gamma m+n}{(m+n+1)(m+n)\, \Gamma m+n} = \frac{(m+1)\, m}{(m+n+1)(m+n)}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{(m+1)\, m}{(m+n+1)(m+n)} - \left(\frac{m}{m+n}\right)^2$$

$$= \frac{m}{m+n}\left[\frac{(m+1)}{m+n+1} - \frac{m}{m+n}\right]$$

$$= \frac{m}{m+n}\left[\frac{(m+1)(m+n) - m(m+n+1)}{(m+n+1)(m+n)}\right]$$

$$= \frac{m}{m+n}\left[\frac{m^2 + mn + m + n - m^2 - mn - m}{(m+n+1)\; m+n}\right]$$

$$= \frac{m}{m+n}\left[\frac{n}{(m+n+1)\; m+n}\right]$$

$$V(x) = \frac{mn}{(m+n+1)(m+n)^2}$$

⇒ Beta Distribution of 2nd kind :-

Let 'x' be a continous r.v assuming values from 0 to ∞, if the pdf of x is gvn by;

$$\boxed{f(x) = \frac{1}{\beta(m,n)} \frac{x^{m-1}}{(1+x)^{m+n}}} \quad , m,n>0 \, , \quad 0<x<1$$

then x is said to follow a Beta distri... of 2nd kind & is denoted by $\beta_2(m,n)$,

Note →

* β distri... of 2nd kind can be transformed to β distri.. of 1st kind by the transformation,

$$\boxed{1+x = \frac{1}{y}} \quad (i.e)$$

if $x \to \beta_2(m,n)$ then y is defined above follow $\beta_1(m,n)$

* As above we can show that,

$$\boxed{\begin{aligned} Mean = E(x) &= \frac{m}{n-1} \\ Varience &= \frac{m(m+n-1)}{(n-1)^2(n-2)} \end{aligned}}$$

~~2) Rectangular distribution :~~

→ Moments of normal distri.... =.

1) Mean :-

$$\boxed{E(x) = \mu}$$

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

$\left(x = (x-\mu) + \mu \right)$

$$= \int_{-\infty}^{\infty} \left[(x-\mu)+\mu\right] \cdot \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

$$= \int_{-\infty}^{\infty} (x-\mu) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \mu \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx + \mu \underbrace{\int_{-\infty}^{\infty} f(x)\, dx}_{1}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} (x-\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx}_{①} + \mu .$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \times 0 + \mu .$$

$$\boxed{E(x) = \mu}$$

$$① \int_{-\infty}^{\infty} (x-\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} \rightarrow \text{odd } ().$$

$$\Rightarrow 0$$

$f(-x) = -f(x)$. by the (pro) $\int_{-\infty}^{\infty}$ odd $() = 0$

⇒ <u>varience</u> :-

$$\boxed{V(x) = \sigma^2}$$

$$V(x) = E(x - E(x))^2$$

$$V(x) = E(x-\mu)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu) \cdot f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-\mu) \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

$$-\left(\frac{x-\mu}{\sigma}\right)^2 \frac{1}{2} = -\frac{1}{2} z^2 \quad ©$$

by <u>Rule method</u>

$$z = \frac{x-\mu}{\sigma} \Rightarrow x-\mu = \frac{1}{\sigma}$$

$$\therefore\ x-\mu = z\sigma \quad ⓐ$$

diff. on both side,

$$d(x-\mu) = d(z\sigma)$$

$$dx - 0 = dz\,\sigma. \Rightarrow dx = dz\,\sigma \quad ⓑ$$

$$\therefore = \int_{-\infty}^{\infty} (\sigma z)^2 \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}\, \sigma\, dz$$

$$(\sigma z)^2 = \sigma^2 z^2$$

$$= \int_{-\infty}^{\infty} (\sigma z)^2 \frac{1}{\cancel{\sigma}\sqrt{2\pi}}\, e^{-\frac{z^2}{2}}\, \cancel{\sigma}\, dz .$$

$$\cancel{= \frac{\sigma z^2}{\sqrt{2\pi}} \cdot e^{-\frac{z^2}{2}} dz}$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma^2 z^2\, e^{-z^2/2}\, \cancel{\sigma}\, dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2\, e^{-z^2/2}\, dz \quad ⓕ$$

ⓓ both side are =.

$$\int_{-\infty}^{\infty} \Rightarrow 2\int_{0}^{\infty}$$

Rule method

$$\text{let } u = \frac{z^2}{2} . \quad ⓔ$$

dz?. $\therefore z^2 = 2u \Rightarrow z = \sqrt{2u}$

$$d(z^2) = d(2u)$$

$$\cancel{2} z\, dz = \cancel{2}\, du$$

$$z\, dz = du$$

$$dz = \frac{du}{z} \Rightarrow \frac{du}{\sqrt{2u}} \quad ⓕ$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2\, e^{-z^2/2}\, dz$$

becomes

$$\Rightarrow \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} 2u\, e^{-u} \frac{du}{\sqrt{2u}}$$

$$= \frac{2\sigma^2}{\sqrt{2}\cdot\sqrt{\pi}} \int_{0}^{\infty} 2u\, e^{-u} \frac{du}{\sqrt{2}\cdot\sqrt{u}}$$

$$\sqrt{2} \times \sqrt{2} = 2$$

$$= \frac{2\sigma^2 \times \cancel{2}}{\cancel{2}\sqrt{\pi}} \int_0^\infty \cancel{u}\, e^{-u} \frac{du}{u^{1/2}}$$

$\frac{u}{u^{1/2}} \Rightarrow u^{1-1/2} \Rightarrow u^{1/2}$

$\frac{x^a}{x^b} = x^{a-b}$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty u^{1/2} e^{-u}\, du$$

$u^{1/2} \Rightarrow u^{\frac{3}{2}-1}$

$\Rightarrow u^{\frac{3-2}{2}}$

$\Rightarrow u^{1/2}$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty u^{\frac{3}{2}-1} e^{-u}\, du.$$

$$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-u} u^{\frac{3}{2}-1}\, du$$

$\int_0^\infty e^{-mx} \cdot x^{p-1} = \frac{\Gamma p}{m^p}$, $m = 1$, $p = 3/2$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{\Gamma 3/2}{1^{3/2}}$$

$\Gamma n = (n-1)!$

$\Gamma 3/2 = \frac{3}{2} - 1 = \frac{1}{2}$

$\Gamma n+1 = n \Gamma n.$

$= \left(\frac{3}{2}-1\right) \Gamma \frac{3}{2}-1$

$= \frac{1}{2} \Gamma 1/2$

$$= \frac{\cancel{2}\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{\cancel{2}} \Gamma 1/2$$

$$= \frac{\sigma^2}{\cancel{\sqrt{\pi}}} \cdot \cancel{\sqrt{\pi}}$$

$\boxed{\Gamma 1/2 = \sqrt{\pi}}$

$\boxed{V(x) = \sigma^2}$

$SD(x) = \sqrt{V(x)}$

$= \sqrt{\sigma^2}$

$\boxed{SD(x) = \sigma}$

$\Rightarrow$ odd order moments about mean μ

$\boxed{\mu_{2r+1} = 0}$ for $r = 1, 2, 3 \ldots$

$2r \to$ even
$2r+1 \to$ odd
$2r-1 \to$ odd.

$$\mu_{2r+1} = E(x-\mu)^{2r+1}$$

$$= \int_{-\infty}^\infty (x-\mu)^{2r+1} \cdot f(x)\, dx.$$

$$= \int_{-\infty}^\infty (x-\mu)^{2r+1} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \cdot dx.$$

Euler method

Put $z = \frac{x-\mu}{\sigma}$.

$\sigma z = x - \mu$

$d(\sigma z) = d(x-\mu)$

$\sigma dz = dx$

$$\Rightarrow \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^\infty (\sigma z)^{2r+1} e^{-1/2 z^2} \sigma dz$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^\infty \sigma^{2r+1} \cdot z^{2r+1} \cdot e^{-1/2 z^2} \cancel{\sigma}\, dz.$$

$$= \frac{\sigma^{2r+1}}{\sqrt{2\pi}} \int_{-\infty}^\infty z^{2r+1} \cdot e^{-1/2 z^2}\, dz$$

$z^{2r+1} \to$ odd f^n

$(z^r \to$ even $f^n)$

$$= \frac{\sigma^{2r+1}}{\sqrt{2\pi}} \times 0$$

$[f(-x) = -f(x)$ by the prop

$\int_{-\infty}^\infty$ odd $f^n = 0]$

$\boxed{\mu_{r+1} = 0}$

→ Evn order central moment :-

when $x \to N(\mu, \sigma)$

$$\boxed{\mu_{2r} = 1\cdot 3\cdot 5 \cdots (2r-1)\,\sigma^{2r}.}$$

$$\mu_{2r} = E(x-\mu)^{2r}$$

$$= \int_{-\infty}^{\infty} (x-\mu)^{2r} \cdot \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^{2r} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^{2r} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$r=1$

$\underline{\mu_2} = 2r-1 = \sigma^2$
$r=2$

$\underline{\mu_4} = 2r-1$
$= 2\times 2-1 = 3$
$\Rightarrow 1\cdot 3\,\sigma^2$
$= 3\sigma^2$

put, $z = \frac{x-\mu}{\sigma}$

$x-\mu = \sigma z$

$dx = \sigma\, dz$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^{2r} \cdot e^{-\frac{1}{2}z^2}\, \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma^{2r} \cdot z^{2r} \cdot e^{-z^2/2}\, dz$$

$$= \frac{\sigma^{2r}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^{2r} \cdot e^{-z^2/2}\, dz.$$

Evn ()

$$\int_{-\infty}^{\infty} \Rightarrow 2\int_{0}^{\infty}$$

$$= \frac{2\sigma^{2r}}{\sqrt{2\pi}} \int_0^{\infty} z^{2r} \cdot e^{-z^2/2}\, dz.$$

put $u = \frac{z^2}{2}$

$z^2 = 2u \Rightarrow z = \sqrt{2u}$

$\cancel{2}z\, dz = \cancel{2}du$

$z\,dz = du$

$dz = \frac{du}{z}$

$z = \sqrt{2u}$. $= \frac{du}{\sqrt{2u}}$.

$$= \frac{2\sigma^{2r}}{\sqrt{2\pi}} \int_0^{\infty} (2u)^r \cdot e^{-u} \cdot \frac{du}{\sqrt{2u}}$$

$$= \frac{2\sigma^{2r}}{\sqrt{2}\cdot\sqrt{\pi}} \int_0^{\infty} 2^r \cdot u^r \cdot e^{-u} \cdot \frac{du}{\sqrt{2}\cdot\sqrt{u}}.$$

$$= \frac{\cancel{2}\,\sigma^{2r}}{\cancel{\sqrt{2}\sqrt{2}}\sqrt{\pi}}\, 2^r \int_0^{\infty} \frac{u^r \cdot e^{-u} \cdot du}{u^{1/2}}$$

$\frac{u^r}{u^{1/2}} = u^{r-1/2}$

$$= \frac{\sigma^{2r}\, 2^r}{\sqrt{\pi}} \int_0^{\infty} u^{r-1/2} \cdot e^{-u}\, du$$

x^{p-1} gamma form

$u^{r-1/2} \Rightarrow u^{r+\frac{1}{2}-1}$

$$= \frac{\sigma^{2r}\, 2^r}{\sqrt{\pi}} \int_0^{\infty} u^{(r+\frac{1}{2})-1} \cdot e^{-u}\, du.$$

$$\left(\int_0^{\infty} e^{-mx} \cdot x^{p-1} = \frac{\Gamma p}{m^p}\right)$$

$p = r+1/2$

$m = 1$

$$= \frac{\sigma^{2r}\, 2^r}{\sqrt{\pi}} \cdot \frac{\Gamma(r+1/2)}{1^{r+1/2}}$$

$\Gamma n = (n-1)\,\Gamma(n-1)$

$\Gamma(r+\frac{1}{2}) = (r+\frac{1}{2}-1)\,\Gamma(r+\frac{1}{2}-1)$

$= (r-1/2)\,\Gamma(r-\frac{1}{2})$

$$= \frac{2^r\sigma^{2r}}{\sqrt{\pi}} \cdot (r-\tfrac{1}{2})(r-\tfrac{1}{2}-1)\,\Gamma(r-\tfrac{1}{2}-1)$$

$$= \frac{2^r\sigma^{2r}}{\sqrt{\pi}} \cdot (r-\tfrac{1}{2})(r-\tfrac{3}{2})\,\Gamma(r-3/2)$$

$$= \frac{2^r\sigma^{2r}}{\sqrt{\pi}} (r-\tfrac{1}{2})(r-3/2) \cdots \tfrac{1}{2}\Gamma(1/2)$$

will end,

$$= \frac{2^r\sigma^{2r}}{\cancel{\sqrt{\pi}}} \left(\frac{2r-1}{2}\right)\left(\frac{2r-3}{2}\right) \cdots \frac{1}{2}\cancel{\sqrt{\pi}}$$

$$= \frac{\cancel{2^r}\, \sigma^{2r} (2r-1)(2r-3)\cdots 1}{\cancel{2^r}}$$

$$= 1 \cdot 3 \cdot 5 \cdots (2r-3)(2r-1)\sigma^{2r}$$

→ Recurrance relation for even order central moments :-

(even) $\mu_{2r} = 1 \cdot 3 \cdot 5 \cdots (2r-1)\sigma^{2r}$. $\quad \mu_{2r} \quad \mu_{2r+1} \quad \mu_{2r+2}$

(even) (odd) (even)

(even) $\mu_{2r+2} = 1 \cdot 3 \cdot 5 \cdots (2r-1)(2r+1)\sigma^{2r+2}$

$$\frac{\mu_{2r+2}}{\mu_{2r}} = \frac{1 \cdot \cancel{3} \cdot \cancel{5} \cdots \cancel{(2r-1)}(2r+1)\sigma^{2r+2}}{1 \cdot \cancel{3} \cdot \cancel{5} \cdots \cancel{(2r-1)}\sigma^{2r}}$$

$$= \frac{2r+1 \cdot \cancel{\sigma^{2r}} \cdot \sigma^2}{\cancel{\sigma^{2r}}} \qquad \left[\sigma^{2r+2} = \sigma^{2r} \cdot \sigma^2\right]$$

$$\frac{\mu_{2r+2}}{\mu_{2r}} = (2r+1) \cdot \sigma^2$$

$\mu_{2r} \longrightarrow$

$$\boxed{\mu_{2r+2} = (2r+1)\,\sigma^2 \cdot \mu_{2r}}$$

⇒ Moment generating ( ) :- (normal mgf)/ (raw mgf)

$$\boxed{M_x(t) = e^{\mu t + \frac{1}{2}t^2\sigma^2}}$$

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx.$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{tx} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$z = \frac{x-\mu}{\sigma}$

$z\sigma = x - \mu$

$x = z\sigma + \mu$

$dx = \sigma\, dz$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t(\sigma z + \mu)} \cdot e^{-\frac{1}{2}z^2} \cancel{\sigma}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z + t\mu} \cdot e^{-\frac{1}{2}z^2} dz \qquad \boxed{a^{m+n} = a^m \cdot a^n}$$

(split) $$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z} \cdot e^{t\mu}\, e^{-z^2/2}\, dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z} \cdot e^{-z^2/2}\, dz$$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{t\sigma z - z^2/2}\, dz$$

$\sigma t z - \frac{z^2}{2} - \frac{1}{2}t^2\sigma^2 + \frac{1}{2}t^2\sigma^2$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2 + \frac{1}{2}t^2\sigma^2}\, dz.$$

$= \frac{-1}{2}\left[-2\sigma t z + z^2 + t^2\sigma^2\right] + \frac{1}{2}t^2\sigma^2$

$$= \frac{e^{t\mu}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2}\, e^{\frac{1}{2}t^2\sigma^2}\, dz$$

$= -\frac{1}{2}\left[-2\sigma t z + z^2 + (t\sigma)^2\right] + \frac{1}{2}t^2\sigma^2$

$= \frac{-1}{2}(z - t\sigma)^2 + \frac{1}{2}t^2\sigma^2$

$$= \frac{e^{t\mu}\, e^{\frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2}\, dz$$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2}\, dz.$$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}u^2}\, du.$$

$u = z - t\sigma$
$du = dz$

$\int_{-\infty}^{\infty} \to$ ②

$$= 2\frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_0^{\infty} e^{-u^2/2}\, du.$$

Let $v = \frac{u^2}{2}$

$2v = u^2$
$u = \sqrt{2v}$
$\cancel{2}dv = \cancel{2}u\,du$
$dv = u\,du$
$du = \frac{dv}{u}$
$du = \frac{dv}{\sqrt{2v}}$

$$= 2\frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{2\pi}} \int_0^{\infty} e^{-v} \frac{dv}{\sqrt{2v}}$$

$$= \cancel{2}\frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{\cancel{2}}\sqrt{\pi}} \int_0^{\infty} e^{-v} \frac{dv}{\sqrt{\cancel{2}}\sqrt{v}}$$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \int_0^{\infty} e^{-v} \frac{dv}{v^{1/2}}$$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \int_0^{\infty} v^{-1/2} e^{-v}\, dv.$$

$-\frac{1}{2} = \frac{1}{2} - 1$

$$= \frac{e^{t\mu + 1/2\, t^2\sigma^2}}{\sqrt{\pi}} \int_0^{\infty} v^{\frac{1}{2}-1} e^{-v}\, dv.$$

$$\left[\int e^{-mx} x^{p-1}\, dx = \frac{\Gamma p}{m^p}\right]$$

$p = \frac{1}{2}$ $m = 1$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\sqrt{\pi}} \frac{\Gamma 1/2}{1}$$

$$\boxed{\Gamma 1/2 = \sqrt{\pi}}$$

$$= \frac{e^{t\mu + \frac{1}{2}t^2\sigma^2}}{\cancel{\sqrt{\pi}}} \cdot \cancel{\sqrt{\pi}}$$

$$M_x(t) = e^{\mu t + 1/2\, t^2\sigma^2}$$

⇒ Central mgf :-

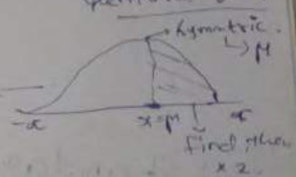
$$M_x(t) = E\left(e^{t(x-\mu)}\right)$$
$$= E\left(e^{tx - t\mu}\right)$$
$$= E\left(e^{tx} \cdot e^{-t\mu}\right)$$
$$= e^{-t\mu}\ E\left(e^{tx}\right)$$
$$= e^{-t\mu} \cdot e^{\mu t + 1/2\, t^2\sigma^2}$$
$$= e^{-t\mu + \mu t + 1/2\, t^2\sigma^2}$$
$$= e$$
$$\boxed{M_x(t) = e^{\frac{1}{2}t^2\sigma^2}}$$

⇒ Mean deviation of normal distri...:-

$E(x) = \mu$
$V(x) = \sigma^2$
$SD = \sigma$

$$\boxed{MD = E\,|x - E(x)|}$$
$$= E\,|x - \mu|$$
$$= \int_{-\infty}^{\infty} |x - \mu| f(x)\, dx.$$
$$= \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx.$$

Remove |...|



Curve is symmetric ←
$$= 2\int_{\mu}^{\infty} (x-\mu) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx.$$

$$= \frac{2}{\sigma\sqrt{2\pi}} \int_{\mu}^{\infty} (x-\mu)\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx.$$

Put $z = \frac{x-\mu}{\sigma}$
$\sigma z = x - \mu$
$\sigma dz = dx.$

(Limit Random) $\int \to$ when $x = \mu$ in z
$\Rightarrow z = 0$
$x = \infty$, $z = \infty$.

$$= \frac{2}{\cancel{\sigma}\sqrt{2\pi}} \int_0^{\infty} \cancel{\sigma} z\, e^{-\frac{1}{2}z^2}\, \sigma\, dz$$

$$= \frac{2\sigma}{\sqrt{2}\sqrt{\pi}} \int_0^{\infty} z\, e^{-z^2/2}\, dz.$$

put $u = \frac{z^2}{2}$

$2u = z^2$

$\cancel{\sqrt{2u} = z}$

$d(2u) = d(z^2)$

$\cancel{2}\, du = \cancel{2} z\, dz$

① $du = z\, dz$

$dz = \frac{du}{z}$

when $z=0, u=0$

when $z=\infty$, $u=\infty$

$$= \frac{2\sigma}{\sqrt{2}\sqrt{\pi}} \int_0^{\infty} e^{-u}\, du \quad ①$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \left[\frac{e^{-u}}{-1}\right]_0^{\infty}$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \left[\frac{e^{-\infty} - e^{0}}{-1}\right] = \frac{2\sigma}{\sqrt{2\pi}} \left[\frac{e^{-\infty}}{-1} - \frac{e^{0}}{-1}\right]$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \left[0 + 1\right] = \frac{2\sigma}{\sqrt{2\pi}} \cdot 1$$

$2 = \sqrt{2}\sqrt{2}.$

$$= \frac{\cancel{\sqrt{2}}\cdot\sqrt{2}\,\sigma}{\cancel{\sqrt{2}}\sqrt{\pi}} = \frac{\sqrt{2}\,\sigma}{\sqrt{\pi}}$$

$$= \sqrt{\frac{2}{\pi}}\,\sigma \doteq 0.79788\sigma \approx \frac{4}{5}\sigma$$

⇒ **Standard Normal distri.... :-**

(for doing prbm, we hue to do in S.Nrdi)

$x \rightarrow N(\mu, \sigma)$

(x-ని z-లోకి మార్చాలి)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

$z \rightarrow N(0,1)$

$$f(z) = f(x) \left|\frac{dx}{dz}\right|$$

$$z = \frac{x-\mu}{\sigma}$$

$\sigma z = x - \mu.$

$\sigma\, dz = dx.$

$$\sigma = \frac{dx}{dz}$$

$$\therefore f(x) = \frac{1}{\cancel{\sigma}\sqrt{2\pi}}\, e^{-z^2/2} \cdot \cancel{\sigma}$$

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}}$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2} \quad \text{of} \quad \sigma = \frac{dx}{dz}$$

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}}$$

⇒ Additive (pro) :- (Addition → 2 variables)

(a) If (a) $X_1 \longrightarrow N(\mu_1, \sigma_1)$

(b) $X_2 \longrightarrow N(\mu_2, \sigma_2)$ and if.

x_1, x_2 are indepdent then,

$$\boxed{x_1 + x_2 \longrightarrow N\left(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}\right)}$$

Pf/ $x_1 \longrightarrow N(\mu_1, \sigma_1)$

$M_x(t) = e^{\mu t + 1/2 t^2 \sigma^2}$

$\therefore M_{x_1}(t) = e^{\mu_1 t + 1/2 t^2 \sigma_1^2}$

$X_2 \longrightarrow N(\mu_2, \sigma_2)$

$M_{x_2}(t) = e^{\mu_2 t + 1/2 t^2 \sigma_2^2}$

x_1 & x_2 are indepent,

$$M_{x_1+x_2}(t) = M_{x_1}(t) \cdot M_{x_2}(t)$$

$$= e^{\mu_1 t + 1/2 t^2 \sigma_1^2} \cdot e^{\mu_2 t + 1/2 t^2 \sigma_2^2}$$

$$= e^{\mu_1 t + 1/2 t^2 \sigma_1^2 + \mu_2 t + 1/2 t^2 \sigma_2^2}$$

$$= e^{\mu_1 t + \mu_2 t + 1/2 t^2 \sigma_1^2 + 1/2 t^2 \sigma_2^2}$$

$$M_{x_1+x_2}(t) = e^{(\mu_1 + \mu_2)t + 1/2 t^2 (\underbrace{\sigma_1^2 + \sigma_2^2}_{V(x)})}$$

$X_1 \to N(\mu_1, \sigma_1)$

$M_x(t) = e^{\mu t + 1/2 t^2 \sigma^2}$

SD.

$$x_1 + x_2 \longrightarrow N\left(\mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2}\right)$$

(b) If x_i, $i=1,2,\dots,n$ are n independent normal variate with mean μ_i & SD σ_i respectively, then the variate $y=\sum_{i=1}^{n} x_i$ is an normal variate with mean $\sum \mu_i$ & variance $\sum \sigma_i^2$.

Pf/

$x_i \longrightarrow N(\mu_i, \sigma_i)$

$$M_{x_i}(t) = e^{\mu_i t + \frac{1}{2} t^2 \sigma_i^2}$$

Then $M_y(t) = M_{\sum_{i=1}^{n} x_i}(t)$. $\qquad y=\sum_{i=1}^{n} x_i$ — (i)

([illegible] Σ [illegible] π)

$$= \prod_{i=1}^{n} M_{x_i}(t)$$

$$= \prod_{i=1}^{n} e^{\mu_i t + \frac{1}{2} t^2 \sigma_i^2}$$

([illegible] Σ) $= \cancel{\prod} e^{\Sigma(\mu_i t + \frac{1}{2} t^2 \sigma^2)}$

$$= \cancel{\prod} e^{\Sigma \mu_i t + \frac{1}{2} t^2 \Sigma \sigma^2}$$

$$y \longrightarrow N\left(\Sigma \mu_i, \sqrt{\Sigma \sigma_i^2}\right) \quad \text{SD}$$

$$\sum_{i=1}^{n} x_i \longrightarrow N\left(\Sigma \mu_i, \sqrt{\Sigma \sigma_i^2}\right)$$

(c) If x_i $i=1,2,3 \dots n$ are n independent normal variate with mean μ_i & SD σ_i, then the variate $y=\sum_{i=1}^{n} a_i x_i$ is a normal distri… with ~~mean~~ mean $\sum_{i=1}^{n} a_i \mu_i$ & variance $\sum_{i=1}^{n} a_i^2 \sigma_i^2$ where a_i constnt.

Pf/ let $x_i \longrightarrow N(\mu_i, \sigma_i)$

$$M_{x_i}(t) = e^{\mu_i t + \frac{1}{2} t^2 \sigma_i^2}$$

$$M_y(t) = M_{\sum_{i=1}^{n} a_i x_i}(t) \qquad M_{a_i x_i}(t) = M_{x_i}(a_i t).$$

$$= \prod_{i=1}^{n} M_{a_i x_i}(t) = \prod_{i=1}^{n} M_{x_i}(a_i t)$$

$$= \prod_{i=1}^{n} e^{\mu_i a_i t + \frac{1}{2} a_i^2 t^2 \sigma_i^2}$$

$$= e^{\Sigma(\mu_i a_i t + \frac{1}{2} a_i^2 t^2 \sigma_i^2)}$$

$$= e^{\Sigma \mu_i a_i t + \frac{1}{2} t^2 \Sigma a_i^2 \sigma_i^2} \quad \text{, v60.}$$

$$y \longrightarrow N\left(\mu_i a_i, \sqrt{\Sigma a_i^2 \sigma_i^2}\right)$$

$\Rightarrow$ Moment generating ( ) standard N distri:- $\rightarrow z$

$$M_z(t) = E(e^{tz})$$

$$= \int_{-\alpha}^{\alpha} e^{tz} \cdot f(z)\, dz$$

$$= \int_{-\alpha}^{\alpha} e^{tz} \cdot \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \, ,$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} e^{tz - z^2/2}\, dz. \quad \text{take } -\tfrac{1}{2} \text{ outside.}$$

$(tz \to 2tz$ [illegible]$)$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} e^{-\frac{1}{2}(z^2 - 2tz)}\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z^2 - 2tz + t^2 - t^2)} dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z-t)^2 - t^2]} dz.$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t)^2 + t^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(z-t)^2}{2}} \cdot \underline{e^{t^2/2}} dz$$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(z-t)^2/2} dz.$$

Rule.
$u = z - t$
$du = dz.$

$$= \frac{e^{t^2/2}}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-u^2/2}}\, dz.$$

$\int \to z = -\infty, u = -\infty$
$z = \infty, u = \infty$

$\int e^{-mx} x^{p-1} dx = \frac{\sqrt{p}}{m^p}.$

$\int_{-\infty}^{\infty} e^{-u^2/2} du = \sqrt{2\pi}.$

$$= \frac{e^{t^2/2}}{\cancel{\sqrt{2\pi}}} \cdot \cancel{\sqrt{2\pi}}$$

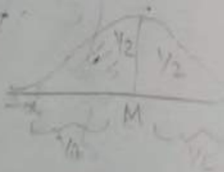
$$\underline{\underline{M_z(t) = e^{t^2/2}}}$$

⇒ Median :-

$$\boxed{\int_{-\infty}^{M} f(x)\, dx = \frac{1}{2}}$$

(or)

$$\int_{M}^{\infty} f(x)\, dx = \frac{1}{2}.$$



$$\int_{-\infty}^{M} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \frac{1}{2}$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{M} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2} dx = \frac{1}{2}$$

$z = \frac{x-\mu}{\sigma}$
$z\sigma = x - \mu$
$\sigma dz = dx$
$x = -\infty, z = -\infty$
$x = M, z = \frac{M-\mu}{\sigma}$

$$\frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\frac{M-\mu}{\sigma}} e^{-z^2/2} \cancel{\sigma} dz = \frac{1}{2}$$

$$\int_{-\infty}^{\frac{M-\mu}{\sigma}} \underbrace{\frac{1}{\sqrt{2\pi}} e^{-z^2/2}} dz = \frac{1}{2} \text{ --- (1)}$$

$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$
standard normal
so. $\int \to \int_{-\infty}^{0}$
S. normal's median $= 0$

$$\int_{-\infty}^{\frac{M-\mu}{\sigma}} f(z)\, dz = \frac{1}{2}. \text{ --- (1)}$$

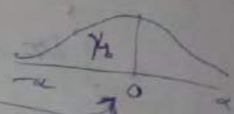
$$\int_{-\infty}^{0} f(z)\, dz = \frac{1}{2}. \text{ --- (2)}$$

comparing (1) & (2),

$\frac{M-\mu}{\sigma} = 0$ → 0 is at center so.

$M - \mu = 0$

$M = \mu.$

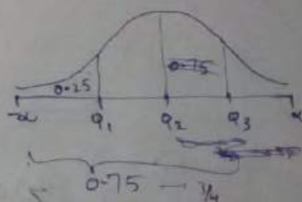


⇒ Quartile deviation :-

$$QD = \frac{Q_3 - Q_2}{2}$$

$$\int_{-\infty}^{Q_1} f(x)\, dx = 0.25 \text{ --- (1)}$$

$$\int_{-\infty}^{Q_3} f(x)\, dx = 0.75 \text{ --- (2)}$$



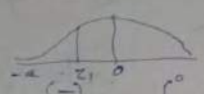
by standising (1) & (2). (into z)

$$\int_{-\infty}^{\frac{Q_1-\mu}{\sigma}} f(z)\,dz = 0.25 = \int_{-\infty}^{z_1} f(z)\,dz = 0.25 \quad \text{--- (3)}$$

$$\int_{-\infty}^{\frac{Q_3-\mu}{\sigma}} f(z)\,dz = 0.75 = \int_{-\infty}^{z_2} f(z)\,dz = 0.75 \quad \text{--- (4)}$$

$$z_1 = \frac{Q_1-\mu}{\sigma}$$

$$z_2 = \frac{Q_3-\mu}{\sigma}$$



$$\int_{-\infty}^{0} - \int_{z_1}^{0} = \int_{-\infty}^{z_1}$$

— (3) $\underline{Z_1}$

$$\int_{-\infty}^{z_1} f(z)\,dz = \int_{-\infty}^{0} f(z)\,dz - \int_{z_1}^{0} f(z)\,dz$$

$$0.25 = 0.5 - \int_{z_1}^{0} f(z)\,dz.$$

$$\int_{z_1}^{0} f(z)\,dz = 0.5 - 0.25 = 0.25$$

$$\therefore z_1 = -0.6745$$

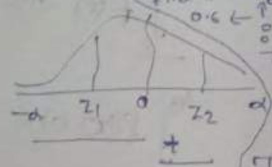
From log table.
$\int f(z)dz = 0.25$
0.25 table-ல் இல்லை. we find 2 value
0.2486 0.2518
then find differce
0.2486 − 0.25 = 0.0014 ; 0.2518 − 0.25 = 0.0018
close value to 0.25
so choose 0.2486
find the value 0.67
0.6 + 0.07 = 0.67
Then weight 0.67

$$\frac{Q_1-\mu}{\sigma} = -0.6745$$

$$Q_1 - \mu = -0.6745\sigma$$

$$Q_1 = -0.6745\sigma + \mu.$$

$\underline{Z_2}$



— (4)

$$\int_{-\infty}^{z_2} f(z)\,dz = 0.75$$

$$\int_{-\infty}^{z_2} f(z)\,dz = \int_{-\infty}^{0} f(z)\,dz + \int_{0}^{z_2} f(z)\,dz.$$

$$0.75 = 0.5 + \int_{0}^{z_2} f(z)\,dz$$

$$\int_{0}^{z_2} f(z)\,dz = 0.75 - 0.5 = 0.25$$

$$z_2 = 0.6745$$

$$\frac{Q_3-\mu}{\sigma} = 0.6745$$

$$Q_3 = 0.6745\sigma + \mu$$

$$\therefore QD = \frac{Q_3-Q_1}{2} = \frac{0.6745\sigma + \mu - (-0.6745\sigma + \mu)}{2}$$

$$= \frac{0.6745\sigma + \cancel{\mu} + 0.6745\sigma - \cancel{\mu}}{2}$$

$$QD = 0.6745\sigma$$

$$QD \approx \frac{2}{3}\sigma$$

⟹ **Mode :-**

It is that value of r.v having max (prob). (i.e) if x is the mode $f(x)$ will be max.

Mode is the soln, $f'(x)=0$ & $f''(x)<0$

$f(x) \to \max \to$ find $f'(x)=0$, $f''(x)<0 \to$ max
$f''(x)>0 \to$ min

$$\therefore f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

$$= c \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$ take log on both side.

$\log(ab) = \log a + \log b$

$$\log f(x) = \log(c) + \log e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

log + e = cancel.

$$\therefore \log f(x) = \log(c) - \frac{(x-\mu)^2}{2\sigma^2}$$

differenciate both side.

$$\frac{1}{f(x)} \cdot f'(x) = 0 - \frac{1}{\sigma^2}(x-\mu)$$

$$\frac{f'(x)}{f(x)} = -\frac{(x-\mu)}{\sigma^2}$$

$$f'(x) = -(x-\mu)\, f(x).$$

$$f'(x) = 0 \Rightarrow -\frac{(x-\mu)}{\sigma^2} f(x) = 0. \quad \therefore f(x) \neq 0$$

$$x-\mu = 0$$

$$x = \mu$$

1) x is normal variate with mean 42 & SD 4, find probability a value taken by x is,

a) < 50 (b) > 50 (c) < 40 (d) > 40

(e) b/w 43 & 45 (f) b/w 40 & 44. (g) b/w 37 & 41

A) mean, $\mu = 42$ $\quad \sigma^2 = V(x)$

SD, $\sigma = 4$ $\quad \sigma = SD$

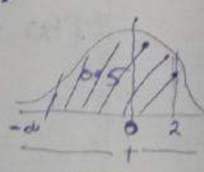
~~a) P (x<5)~~ ഇതിനെ standard normal ലേക്ക് മാറ്റുക.

$$z = \frac{x-\mu}{\sigma} = \frac{x-42}{4}$$

a) $P(x<50) = P\left(\frac{x-42}{4} < \frac{50-42}{4}\right)$

$(-\mu, \div\sigma)$

$= P(z<2)$



$= P(-\infty<z<0) + P(0<z<2)$ $\quad a \to \underset{a}{2}.\underset{b}{00}$ $\quad \to b\ 2.00$

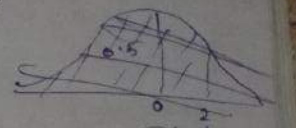
$= 0.5 + 0.4772$ $\quad$ table

$= 0.9772$

$\overset{a}{2}.\overset{b}{00}$
$a \to$ 2 places needed
$1.5 \to 1.50$

b) $P(x>50) = P\left(\frac{x-42}{4} > \frac{50-4}{4}\right)$

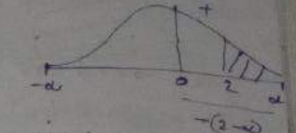
$= P(z>2)$



$= P(0<z<\infty) - P(0<z<2)$

$= 0.5 - 0.4772$

$= 0.0228$



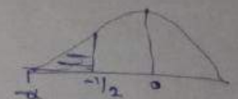
$(2-\infty) = (0-\infty)(0-2)$

c) $P(x<40) = P\left(\frac{x-42}{4} < \frac{40-42}{4}\right)$

$= P(z < -1/2)$ $\quad \to 0.5$ $\quad \to 3.00$

$= 0.5 - 0.1915.$

$= 0.3085$



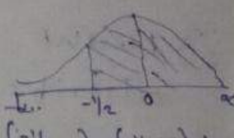
$(\alpha, -1/2) = (\alpha, 0) - (1/2, 0)$

d) $P(x>40) = P\left(\frac{x-42}{4} > \frac{40-42}{4}\right)$

$= P(z > -1/2) \to \underset{a}{0.5}\underset{b}{0}$

$= 0.5 + 0.1915$

$= 0.6915$



$(-1/2, \infty) = (-1/2, 0) + (0, \infty)$

e) $P(43 < x < 45) =$

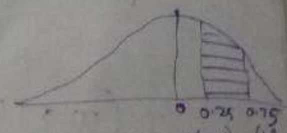
$P\left(\frac{43-42}{4} < \frac{x-42}{4} < \frac{45-42}{4}\right)$

$\frac{1}{4} = 0.25$

$\frac{3}{4} = 0.75$

$= P(0.25 < z < 0.75)$

$= P(0<z<0.75) - P(0<z<0.25)$ $\quad$ Table



$(0.25, 0.75) = (0, 0.75) - (0, 0.25)$

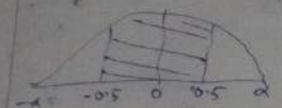
$= 0.2734 - 0.0987$ $\quad$ table $\quad 0.05$ $\quad 0.7$

$= 0.1747$

1) $P(40 < x < 44) = P\left(\frac{40-42}{4} < \frac{x-42}{4} < \frac{44-42}{4}\right)$

$= P(-0.5 < z < 0.5)$

$= P(-0.5 < z < 0) + P(0 < z < 0.5)$



$(-0.5, 0.5) = (-0.5, 0) + (0, 0.5)$

(take only the values)

$= 0.1915 + 0.1915$

$= 0.3820$

1) $P(37 < x < 41) = P\left(\frac{37-42}{4} < \frac{x-42}{4} < \frac{41-42}{4}\right)$

$= P(-1.25 < z < -0.25)$

$= P(0 < z < 1.25) - P(0 < z < 0.25)$

$(1.25, 0.25) = (0, 1.25) - (0, 0.25)$

$= 0.3944 - 0.0987$

$= 0.2957$

2) Height of students is normally distributed with mean 165 cm & SD 5 cm, find (prob) height of student is

a) more than 177 cm

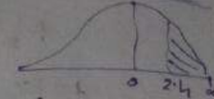
b) < 162 cm

) $\sigma = 5$, $\mu = 165$ $z = \frac{x-\mu}{\sigma} = \frac{x-165}{5}$

$P(x > 177) = P\left(\frac{x-165}{5} > \frac{177-165}{5}\right)$

$= P(z > 2.4)$

$= P(0 < z < \infty) - P(0 < z < 2.4) = 0.5 - 0.4918 = 0.0082$



$(2.4, \infty) = (0, \infty) - (0, 2.4)$

b) $P(x < 162) = P\left[\frac{x-165}{5} < \frac{162-165}{5}\right]$

$= P[z < -0.6] = P(-\infty < z < 0) - P(-0.6 < z < 0)$

$= 0.5 - 0.2258 = 0.2742$

⟹ Normal distri... as a limiting form of binomial distri... :-

Binomial distri... tends to normal distri... under the following condi...

a) n is very large $(n \to \infty)$

b) neither p nor q is very small.

Pf/ let $x \to B(n, p)$

$f(x) = {}^nC_x p^x q^{n-x}$

$E(x) = np = \mu$

$V(x) = npq$, $SD = \sqrt{npq}$

$M_x(t) = (q + pe^t)^n$

define $z = \frac{x - E(x)}{SD} = \frac{x - np}{\sqrt{npq}}$

$\mu = np$

$\sigma = \sqrt{npq}$

$z = \frac{x-\mu}{\sigma}$

$M_x(t) = E(e^{tx})$

$M_{\frac{x-\mu}{\sigma}}(t) = E\left(e^{t\left(\frac{x-\mu}{\sigma}\right)}\right)$

$= E\left(e^{\frac{tx}{\sigma} - \frac{t\mu}{\sigma}}\right)$

$= E\left(e^{tx/\sigma} \cdot e^{-t\mu/\sigma}\right)$

$= e^{-t\mu/\sigma} E\left(e^{tx/\sigma}\right)$

$M_x(t) = E(e^{tx})$

$M_x(t/\sigma) = E\left(e^{\frac{t}{\sigma}x}\right)$

$M_z(t) = e^{\frac{-t\mu}{\sigma}} \cdot \left(q + pe^{t/\sigma}\right)^n \qquad M_x(t) = (q+pe^t)^n$

take Log on both sides.

$\log M_z(t) = \log e^{-t\mu/\sigma} + \log\left(q + pe^{t/\sigma}\right)^n$ $\boxed{\log x^n = n \log x.}$

$= \frac{-t\mu}{\sigma} + n\log\left(q + pe^{t/\sigma}\right)$

$= \frac{-t\mu}{\sigma} + n\log\left(q + p\left(1 + \frac{t}{\sigma} + \frac{t^2}{2!}\right)\right)$ $\boxed{e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots \quad \therefore e^{t/\sigma} = 1 + \frac{t}{\sigma} + \frac{t^2}{2\sigma^2} + \dots}$

$= \frac{-t\mu}{\sigma} + n\log\left(q + p + \left[p + \frac{pt}{\sigma} + \frac{pt^2}{\sigma^2}\right] + \dots\right)$

$\boxed{\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots}$

$= \frac{-t\mu}{\sigma} + n\log\left(1 + p\left(\frac{t}{\sigma} + \frac{t^2}{2\sigma^2} + \dots\right)\right)$

$= \frac{-t\mu}{\sigma} + n\left[p\left(\frac{t}{\sigma} + \frac{t^2}{2\sigma^2} + \dots\right) - \frac{p^2}{2}\left(\frac{t}{\sigma} - \frac{t^2}{\sigma^2} + \dots\right)^2\right]$

$= \frac{-t\mu}{\sigma} + n\left[\frac{pt}{\sigma} + \frac{pt^2}{2\sigma^2} - \frac{p^2t^2}{2\sigma^2} + O\left(\frac{1}{n^{3/2}}\right)\right]$

order of

$= \frac{-t\mu}{\sigma} + \frac{npt}{\sigma} + \frac{npt^2}{2\sigma^2} - \frac{np^2t^2}{2\sigma^2} + O\left(\frac{1}{n^{1/2}}\right)$

$= \frac{-t\mu}{\sigma} + \frac{npt}{\sigma} + \frac{npt^2}{2\sigma^2}\left(1-p\right) + O\left(\frac{1}{n^{1/2}}\right)$

$= \frac{-t\mu}{\sigma} + \frac{npt}{\sigma} + \frac{npt^2}{2\sigma^2} \cdot q + O\left(\frac{1}{n^{1/2}}\right)$

$\mu = np$
$\sigma^2 = npq$

$= \cancel{\frac{-t\mu}{\sigma}} + \cancel{\frac{\mu t}{\sigma}} + \frac{\cancel{\sigma^2} t^2}{2\cancel{\sigma^2}} + O\left(\frac{1}{n^{1/2}}\right)$

$= \frac{t^2}{2} + O\left(\frac{1}{n^{1/2}}\right) = t^2/2$

as $n \to \infty$.

$\log M_z(t) = \frac{t^2}{2}$ (to cut log $\to$ e)

$e^{\log M_z(t)} = e^{t^2/2}$

$\boxed{M_z(t) = e^{t^2/2}}$

This is the mgf of standard normal distri-

So $Z \to N(0,1)$

(i.e) , $\frac{x - np}{\sqrt{npq}} \to N(0,1)$ as $n \to \infty$

$\therefore$ BD $\to$ ND

$\log M_z(t) = \frac{t^2}{2}$ (to cut log → e).

$e^{\log M_z(t)} = e^{t^2/2}$

$$\boxed{M_z(t) = e^{t^2/2}}$$

This is the mgf of standard normal distri-

So $Z \to N(0,1)$

(i.e), $\frac{x-np}{\sqrt{npq}} \to N(0,1)$ as $n \to \infty$

∴ BD → ND

1) If x_1 & x_2 are independent r.v with mean 3 & 4 & varience 4 & 5. $P(4 \le x_1 + x_2 \le 13)$?

A) $P(4 \le x_1 + x_2 \le 13) = ?$

$E(x_1) = 3$ $\quad V(x_1) = 4$
$E(x_2) = 4$ $\quad V(x_2) = 5$ } given.

eg
$P(40 < x < 50)$
[illegible] $-E(x)$ & $\div$ SD
∴ $Z = \frac{x-\mu}{\sigma}$

$E(x_1+x_2) = E(x_1) + E(x_2)$
$= 3 + 4 = 7$

$V(x_1+x_2) = V(x_1) + V(x_2)$
$= 4 + 5 = 9$

$V(x_1+x_2) = 9$
$SD = \sqrt{9} = 3$

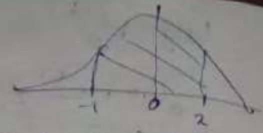
let $y = x_1 + x_2$.

$P(4 \le y \le 13) =$

$P\left(\frac{4-7}{3} \le \frac{y-7}{3} \le \frac{13-7}{3}\right)$

$Z = \frac{y-\mu}{\sigma}$
$= \frac{y-7}{3}$

$= P\left(\frac{-1}{} \le Z \le \frac{2}{}\right)$



$= P(-1<Z<0) + P(0<Z<2)$

$(-1,2) = (-1,0) + (0,2)$

1.00

2.00

$= 0.3412 + 0.4772$

$= 0.8185$

2) ~~In~~ In a normal distrin 15% items are below 35 & 10% items are above 65, find mean & SD (prob)

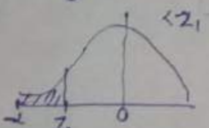
A) $P(x<35) = \frac{15}{100}$

$Z = \frac{x-\mu}{\sigma}$

$P\left(\frac{x-\mu}{\sigma} < \frac{35-\mu}{\sigma}\right) = 0.15$

$P(Z < Z_1) = 0.15$

where $Z_1 = \frac{35-\mu}{\sigma}$



$(-\infty, Z_1) = (-\infty, 0) - (Z_1, 0)$

$P(-\infty < Z < 0) - P(Z_1 < Z < 0) = 0.15$

$0.5 - P(Z_1 < Z < 0) = 0.15$

$P(Z_1 < Z < 0) = 0.5 - 0.15$

$P(Z_1 < Z < 0) = 0.35$

$Z_1 = ?$

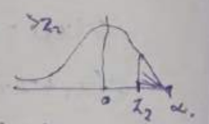
0.35 0.3485 & 0.3508

$P(x>65) = \frac{10}{100}$

$P\left(\frac{x-\mu}{\sigma} > \frac{65-\mu}{\sigma}\right) = 0.10$

$P(Z > Z_2) = 0.10$

where $Z_2 = \frac{65-\mu}{\sigma}$



$(Z_2, \infty) = (0,\infty) - (0, Z_2)$

$P(0<Z<\infty) - P(0 < Z < Z_2) = 0.10$

$0.5 - P(0 < Z < Z_2) = 0.10$

$P(0 < Z < Z_2) = 0.5 - 0.10$

$P(0 < Z < Z_2) = 0.4$

$Z_2 = ?$

$0.3485 - 0.35 = 0.015$, $0.3508 - 0.35 = 0.008$

$Z_1 = -1.04$

$Z_1 = \frac{35-\mu}{\sigma}$

$\frac{35-\mu}{\sigma} = -1.04$

$35 - \mu = -1.04\sigma$

$-\mu = -1.04\sigma - 35$

$\mu = 35 + 1.04\sigma$ (a)

(a) + (b)

$\mu = 35 + 1.04\sigma$

$-\mu = -65 + 1.28\sigma$

$0 = -30 + 2.32\sigma$

$2.32\sigma = 30$

$\sigma = \frac{30}{2.32} = 12.93$

$\mu = ?$

(a)

$\mu = 35 + 1.04\sigma$

$\Rightarrow 35 + 1.04 \times 12.93$

$\Rightarrow 35 + 13.4492$

$\mu \Rightarrow 48.4472$

$Z_2 = 1.28$

$\frac{65-\mu}{\sigma} = 1.25$

$65 - \mu = 1.25\sigma$

$-\mu = 1.25\sigma - 65$

$-\mu = -65 + 1.28\sigma$ (b)

Chapter centis

1) Avg IQ of a grp of 800 children is 98, SD is 8, assuming normality find expected no. of children hving IQ b/w 100 & 120

normal dist

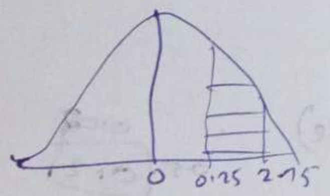
A) mean, $\mu = 98$, $N = 800$, $\sigma = 8$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-98}{8}$$

$$P(100 < x < 120) = P\left(\frac{100-98}{8} < \frac{x-98}{8} < \frac{120-98}{8}\right)$$

$$= P(0.25 < Z < 2.75)$$

$$= P(0 < Z < 2.75) - P(0 < Z < 0.25)$$



(log table) $\rightarrow$ $= 0.4970 - 0.0987 = \underline{0.3983}$

$\therefore$ expected no. of children $= N \times 0.3983$

$= 800 \times 0.3983$

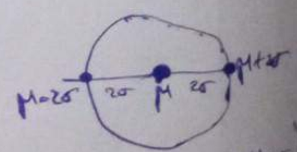
$= \underline{319}$

2) x is a normal variate with mean μ & SD σ, (a) prob that x takes a value in 2 σ neighbourhood of μ. (b) wht % of observations would you expect to lie within the 2 σ limits.

A) mean $= \mu$ S.D $= \sigma$

2σ neighbourhood

$$Z = \frac{x-\mu}{\sigma}.$$



$(\mu - 2\sigma < x < \mu + 2\sigma)$

(a) $P(\mu-2\sigma < x < \mu+2\sigma) = P\left(\frac{\cancel{\mu}-2\cancel{\sigma}-\cancel{\mu}}{\cancel{\sigma}} < \frac{x-\mu}{\sigma} < \frac{\cancel{\mu}+2\cancel{\sigma}-\cancel{\mu}}{\cancel{\sigma}}\right)$

$= P(-2 < z < 2)$

$= P(-2 < z < 0) + P(0 < z < 2) = 0.4772 + 0.4772$

$= \underline{0.9544}$

(b) $P(\mu - 2\sigma < x < \mu + 2\sigma)$ -ന്റെ % കാണുക.

% of 2σ limit $= 100 \times P(\mu - 2\sigma < x < \mu + 2\sigma)$

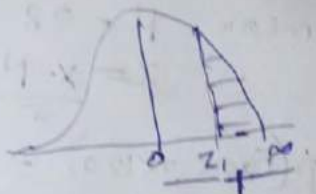
$= 100 \times 0.9544 = 95.44\ \%$

3) If x is normally distributed with mean 11 & SD 1.5, find the no. x_0 such that (a) $P(x > x_0) = 0.3$ (b) $P(x < x_0) = 0.09$

A) $\mu = 11$ $\quad \sigma = 1.5$

(prob) തന്നിട്ടുണ്ട് → find x_0 (table ൽ) $\quad z = \frac{x - \mu}{\sigma} = \frac{x - 11}{1.5}$

a) $P(x > x_0) = 0.3 \longrightarrow \downarrow \boxed{0.3}$



$P(x > x_0) = 0.3$

$P\left(\frac{x - 11}{1.5} > \frac{x_0 - 11}{1.5}\right) = 0.3$

$P\left(z > \frac{x_0 - 11}{1.5}\right) = 0.3$ $\quad$ let $z_1 = \frac{x_0 - 11}{1.5}$

$P(z > z_1) = 0.3$

$P(0 < z < \infty) - P(0 < z < z_1) = 0.3$

$0.5 - P(0 < z < z_1) = 0.3$

$P(0 < z < z_1) = 0.5 - 0.3 = \underline{0.2} \to$ (table)

$0.5 \boxed{0.2} \Rightarrow 0.52$

$z_1 = \underline{0.52}$

$\frac{x_0 - 11}{1.5} = 0.52$

$x_0 - 11 = 0.52 \times 1.5$ $\quad = $

$x_0 = 0.52 \times 1.5 + 11$