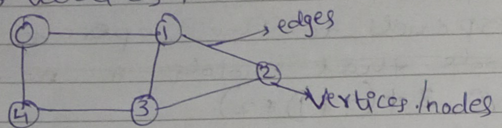
G = (V, E) → use parentheses to indicate unordered pair
G = <V, E> → angled brackets to indicate ordered pair.

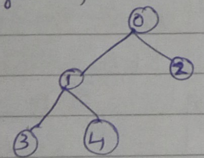
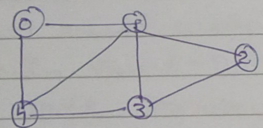
06 = Graphs

* Graph is a non-linear d.s consisting of node & edges.
* Nodes → vertices.
* eg →

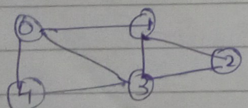


* Tree vs graph =

Every tree is a graph, but every graph is not a tree.

Tree	Graph
* Source to destination is single path	* multiple path
	
tree	Graph
ഇവിടെ 0-ൽ നിന്ന് 3-ലേക്ക് പോകണമെങ്കിൽ 0-1-3 only possible.	Here 0-ൽ നിന്ന് 3-ലേക്ക് പോകണമെങ്കിൽ multiple path ഉണ്ട്. ഉദാ:→ 0-1-3, 0-4-3 ...
* root node will be there.	* All nodes are equal.

→ Mathematical repre~ of graphs =



G = (V, E)

G = graph
V = set of vertices
E = set of edges

$V(G) = \{0, 1, 2, 3, 4\}$.
$E(G) = \{(0,1), (0,4), (0,3), (1,3), (1,2), (2,3)\}$.

→ Application of graph =
* Google maps etc
  → To find shortest path.
* Facebook → To represent user connections.

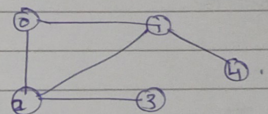
⇒ Types of graphs =

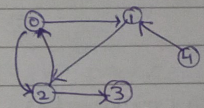
* Based on (dir) of edges → (2 types)

a) Undirected (g) ⇒ A graph in which all the edges are bi-dir~

b) Directed (g) ⇒ A graph in which all the edges are uni dir~

eg →

(1) 

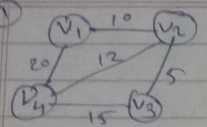
(2) 

Undirected (g)	directed (g)
ഇവിടെ നമുക്ക് 0-1 & 1-0 രണ്ടു വശത്തേക്കും പോകാം, bcz specified (dir) ഒന്നും പറഞ്ഞിട്ടില്ല.	(dir) specify ചെയ്തിട്ടുണ്ട്.

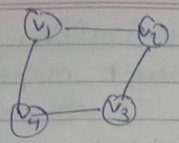
* Based on weight of edges → (2 types)

a) Weighted (g) = A (g) in which each edge is assigned with some weight/cost/value.

b) Unweighted (g) = A (g) where there is no value/weight associated with edges.



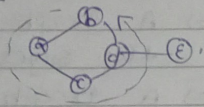
weighted (g)



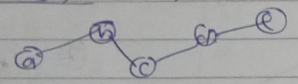
unweighted (g)

* Based on cycles in (g) — (2 types)

a) cyclic (g) → atleast 1 cycle will be there



b) acyclic (g) → no cycle will be there.

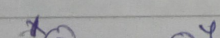


=> graph terminologies =

1) Adjacent nodes / neighbour nodes =

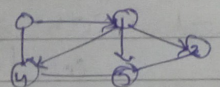
Node x is adjacent node y if there is an edge from node x to node y.

eg→



o is adjacent to 1 & vice versa.
bcz it is bi-dirn (g).

②.



o is adjacent to 1 but 1 is adjacent from o bcz it is (dirg) (g).

2) path = sequences of vertices in which each pair of successive nodes is connected by an edge.

eg →

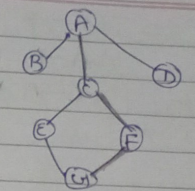


fig: 1.1

A − G =>
A − C − F − G or
A − C − E − G

Length of a path = no. of edges in that path.
here length of A − G = 3 (3 edges).
" of A − F = 2

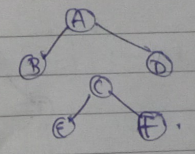
Length of path >= 1

2 types of path = ① Simple path = vertices are distinct.
eg → A C F G → in fig 1.1

② closed path = 1st & last node of that path is same.
eg → C F G E C.

③ cycle = starting & ending nodes are same, all other nodes are distinct
eg → C F G E C.

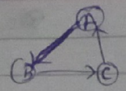
3) connected (g) = if there is a path from any node to any other node.
eg → fig: 1.1



B − C ✓
A − G ✓
A − C X

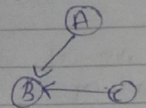
2 types →

1) Strongly c. (g) =



A — C ✓
A — B ✓
C — A ✓

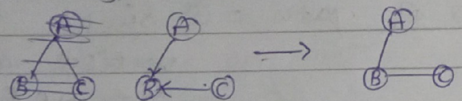
3) **weakly C.(G)** =



A — C ✗.
C — A ✗.

* not a strongly C.(G).
* But if we are converting this to undi. (G) all nodes are connected.

eg →



A — C = ABC ✓
C — A = CBA ✓

that's why it is **weakly C.(G)**

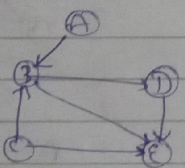
4) **Degree** = No. of edges connected to a node..

2 types →

a) Indegree of a node = no. of edges coming to that node.

b) Outdegree of a node = no. of edges going outside from that node

eg →

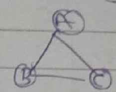


Indegree (D) = 1
Outdegree (D) = 1
Indegree (E) = 3
Outdegree (E) = 0

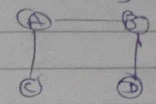
5) **Complete graph** =

It is a simple undi. (G) in which every pair of distinct vertices is connected by a unique edge.

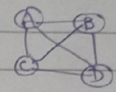
eg → 1)



✓



✗



✓

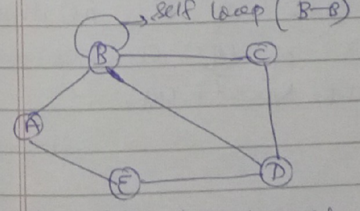
⇒ **Representation of graph** =

1) **Adjacency matrix** =

* It is a sequential repre.
* used to represent which nodes are adjacent to each other. (i.e) there any edge connecting nodes to a graph.
* In this repre. we have to construct a n×n matrix A.
* If there is any weighted graph then instead of 1s and 0s, we can store weights of the edge.

eg →

1) **Undirected (G)** →



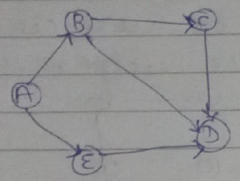
⇒

	A	B	C	D	E
A	0	1	0	0	1
B	1	1	1	1	0
C	0	1	0	1	0
D	0	1	1	0	1
E	1	0	0	1	0

* Consider the rows.
→ A ⇒ connected to B & E
so 1, others 0

Adjacency Matrix → Directed (g)
→ Undirected (g)
→ weighted (g)

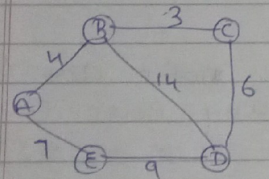
② Di graph (g) →



	A	B	C	D	E
A	0	1	0	0	1
B	0	0	1	1	0
C	0	0	0	1	0
D	0	0	0	0	0
E	0	0	0	1	0

⇒ Consider the outgoing nodes. take it as 1, others all 0.

③ Undirected weighted (g) →



	A	B	C	D	E
A	0	4	0	0	7
B	4	0	3	14	0
C	0	3	0	6	0
D	0	14	6	0	9
E	7	0	0	9	0

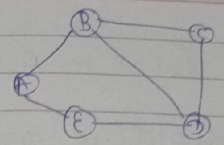
consider the weight instead of 1 & 0.

② Adjacency list =

* It is a linked repre~
* In this repre~, for each vertex in (g), we maintain the list of its neighbours. It means, every vertex of (g) contains list of its adjacent vertices.

eg →

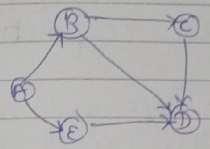
① Undirected (g) →



A is connected to B & E.

A	→ B → E X
B	→ A → C → D X
C	→ B → D X
D	→ B → C → E X
E	→ A → D X

② Directed (g) →



A	→ B → E X
B	→ C → D X
C	→ D X
D	X
E	→ D X

⇒ Graph traversal =

* It is a technique used for searching a vertex in (g).
* Also used to decide the order of vertices in the search process.
* 2 types →
  a) DFS (Depth First search) → (stack)
  b) BFS (Breadth First search) → (Queue)

a) DFS =

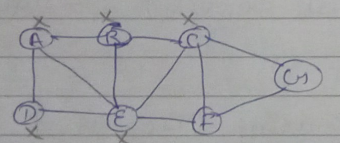
* It produces spanning tree as final result. (for) a graph without loops.
* We use stack ds with max size of total no of vertices in (g) to implement DFS traversal of a (g).

stack → LIFO
queue → FIFO

* (A) →

1) Define a stack of size total no. of vertices in (G).
2) Select any vertex as starting point for traversal, visit that vertex, i.e push it on to the stack.
3) visit any 1 of the adjacent vertex of the vertex which is at top of the stack, which is not visited, i.e push it on to the stack.
4) Repeat stp 3 until there are no new vertex to be visit from the vertex on top of the stack.
5) when there is no new vertex to be visit then use backtracking i.e pop 1 vertex from stack.
6) Repeat stp 3,4,5 until stack becomes empty.
7) when stack becomes empty, then produce final spanning tree by removing unused edges from the graph.

eg →
(1)

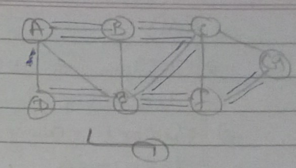


* select any starting node → A.
* adjacent node of A → B, D, E → B.
* then push to stack.
* B → C
* C → E
* E → D
* D → A X so backtracking (B) i.e pop top element.

D
E
C
B
A

stack.

stack empty ആകും വരെ element add ചെയ്ത് pop ചെയ്ത് last stack empty ആവുമ്പോ → we visited all nodes



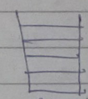
→

E ← Top.
C
B
A

→ visit F.

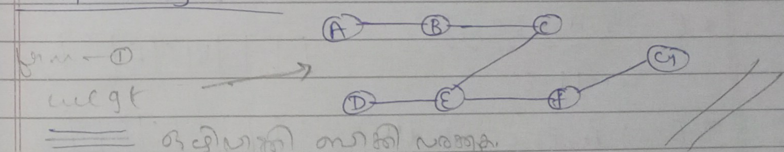
F	pushes →	G ← Top.
E		F
C		E
B		C
A		B
		A

* G → No visited node so (B) i.e pop.
→ G X
* Check F " " so (B) i.e pop.
→ F X
× until A. so (B) i.e pop
* stack become empty.



stack

Spanning tree



A	B	C	E	D	F	G

visited

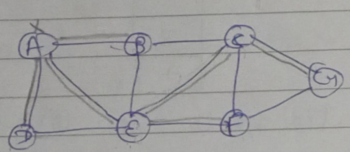
stack.

b) BFS =
* BFS produces a spanning tree as a final result (i.e) graph without loops.
* we use queue d.s with max size of total no. of vertices in graph to implement BFS of a (G).

* (A1) -

1) Define a queue of size total no. of vertices in G.
2) Select any vertex as starting point of traversal. visit that vertex & ins it onto the Q.
3) visit all adjacent vertices of vertex which is at point of Q which is not visited & ins them into the Q.

eg → ①



→ visited edges only

visit all vertex of A

* Select any vertex → A. [A | | | | |] queue

* adj vertex of A → D, E, B ins them & dlt A from Q. [D | E | B | |]

* select front vertex → D
& adj vertices of D → A & E (A visited) & dlt D. [| | E | B | |]

* E adj A, D, B, C, F & dlt E. [. | . | B | C | F |]

* B → A, E, C & dlt B [| | | | C | F |]

* C → B, E, G, F & dlt C [. | . | . | . | F | G]

* F → G & delt F [| | | | | G]

* G → no vertex. So dlt G [| | | | | |]

queue.

Spanning tree of BFS →