Chapter = 02

Control Chart for Variables

⇒ **Mean Chart / $\bar{x}$ -chart =**

→ Procedure for the construction →

* combute the mean of each sample say $\bar{x}_1, \bar{x}_2, \bar{x}_3 \ldots \bar{x}_k$, where k denotes the no. of samples.
* Compute $\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k}$
* choose the central line as $\bar{\bar{x}}$.
* Fix UCL (uppr control limit) & LCL using any of the appropriate formula,

a) $\boxed{UCL_{\bar{x}} = \mu' + A\sigma'}$ $\boxed{LCL_{\bar{x}} = \mu' - A\sigma'}$ $\boxed{A = 3/\sqrt{n}}$

μ ও σ [illegible]

This is used when the standards μ & σ are known.

μ' & σ' → specified values of μ & σ &

$A = \frac{3}{\sqrt{n}}$, which is tabulated for different values of n from 2 to 25

b) $\boxed{UCL_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R}}$ $\boxed{LCL_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}}$ $\boxed{CL = \bar{\bar{x}}}$

μ ও σ [illegible]

This is used when standards are not gvn.

A_2 is obtained from table 4 different values of n from 2 to 25.

c) $\boxed{UCL_{\bar{x}} = \bar{\bar{x}} + A_1\bar{s}}$ $\boxed{LCL_{\bar{x}} = \bar{\bar{x}} - A_1\bar{s}}$

μ ও σ [illegible]

This is also used when the standards are not gvn.

A_1 is tabulated for diffnt values of n from 2 to 25

* After fixing LCL, CL (central limit) & UCL the samples means are plotted on chart.

⇒ **Range Chart / R-chart =**

→ procedure for the construction →

* calculate the range for the each sample
* compute $\bar{R} = \frac{R_1 + R_2 + \cdots R_k}{k}$

where k → sample no.

* set the CL as follows –

a) when standards are gvn say, $SD = \sigma'$

$\boxed{CL = d_2\sigma', \quad UCL_R = D_2\sigma', \quad LCL_R = D_1\sigma'}$

b) standards are not gvn,

$\boxed{CL = \bar{R}, \quad UCL_R = D_4\bar{R}, \quad LCL_R = D_3\bar{R}}$

c) 3σ CL for R-chart are,

$$\boxed{\begin{aligned} CL &= \bar{R} \\ UCL_R &= \bar{R} + 3\sigma_R \\ LCL_R &= \bar{R} - 3\sigma_R \end{aligned}}$$

The values d_2, D_1, D_2, D_3 & D_4 are obtained from the table.

⇒ Interpretation of $\bar{x}$ & R-chart =

* A process is said to be under statistical control if both $\bar{x}$ & R chart show control. (i.e) if all the sample points fall with in the control limits, we say that the process is in control.
* If 1/more of the points in $\bar{x}$-chart / in R-chart / in both fall outside CL we say that the process is out of control.
* $\bar{X}$-chart shows the undesirable variation b/w samples while R-chart reveals any undesirable variations with in the samples.

Q
1) The follow are the values of mean $\bar{x}$ & range are for 20 subgroups of 5 readings each taken from an inspection.

$\bar{x}$	1.85	1.81	1.75	1.76	1.83	1.76	1.71	1.80	~~1.77~~
R	.28	.14	.23	.35	.26	.25	.21	.08	~~.19~~
$\bar{x}$	1.82	1.68	1.69	1.81	1.78	1.57	1.72	1.74	~~1.79~~
R	.36	.10	.23	.29	.22	.05	.23	.23	~~.39~~

~~1.77~~ ~~1.79~~
~~.19~~ ~~.39~~

$\bar{x}$ —	1.77	1.79
R —	.19	.39
$\bar{x}$ —	1.55	1.57
R —	.32	.47

Draw the $\bar{x}$ & R chart with the warning & actual limits & explain?

A) mean gvn,

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 \cdots}{k} = \frac{\Sigma \bar{x}}{k} \rightarrow (\Sigma \bar{x} \text{ add})$$

(mean)
$$= \frac{1.85 + 1.81 + 1.75 + \cdots + 1.79 + 1.82 + \cdots 1.74}{20}$$

$$= \frac{34.76}{20} = \underline{\underline{1.738}} \rightarrow 1.74.$$

(range)
$$\bar{R} = \frac{\bar{R}_1 + \bar{R}_2 + \cdots}{k} = \frac{\Sigma \bar{R}}{k}.$$

$$= \frac{.28 + .14 + \cdots + .08 + .36 + .10 + \cdots .23}{20}$$

$$= \frac{4.88}{20} = \underline{\underline{0.244}}$$

(for subgroups of 5) → 20.

from CL for the sample of size 5,
[μ & σ ... , so (b)] ∴ we gt value
of $A_2 = 0.577$ (log table)
$D_3 = 0$. & $D_4 = 2.115$

CL for $\bar{x}$-chart,

$$UCL = \bar{\bar{x}} + A_2\bar{R}$$
$$= 1.738 + 0.577 \times 0.244 = \underline{\underline{1.8787}}$$

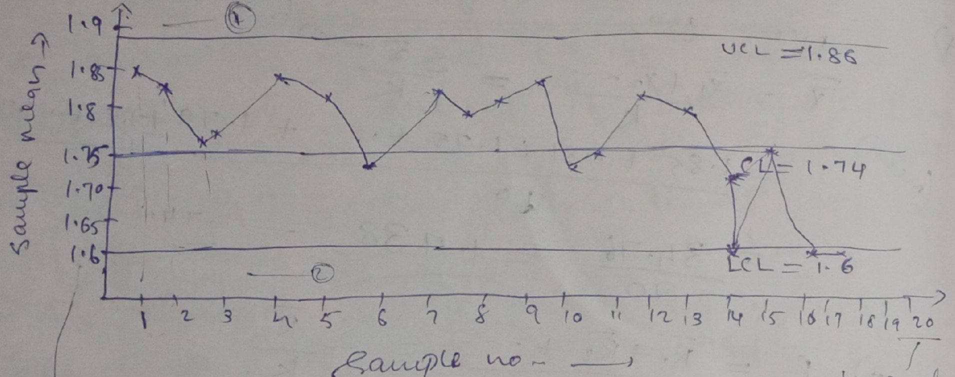
$$LCL = \overline{\overline{x}} - A_2 \overline{R}$$
$$= 1.738 - 0.577 * 0.244 = \underline{\underline{1.5972}}$$

CL for R-chart,

$$UCL = D_4 \overline{R} = 2.115 \times 0.244 = \underline{\underline{0.51606}}$$

$$LCL = D_3 \overline{R} = 0 \times 0.244 = \underline{\underline{0.}}$$

Control chart for mean $\overline{x}$,



K is value 20, 86 1-20

$\overline{\overline{x}} = 1.738 \rightarrow 1.74$

$LCL = 1.59 \rightarrow 1.6$

$UCL = 1.87 \rightarrow 1.88$

① & ② lines are are no then only under control

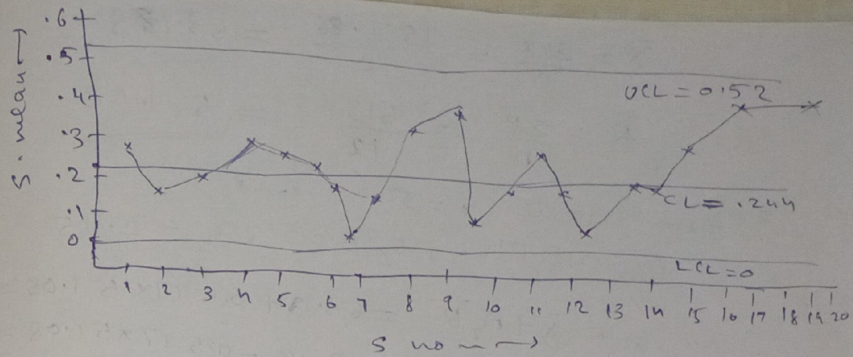
from the table given we observe that all mean values except 3 values inside the C.L. The 3 values which lie outside the LCL suggest that the process is out of control.

control chart for range R,

$\overline{R} = .244$

$UCL = 0.52$

$LCL = 0.$



From the fig we find that all the values lie b/w the ULC & LCL. Hence the Range is under control.

) construct a c.chart for mean ie range for which samples,

x_1	19	36	42	51	60	18	42	42	19	36	42	51
x_2	24	54	51	74	60	20	65	45	34	50	50	74
x_3	80	69	57	75	72	27	75	68	70	69	58	75
x_4	81	77	59	78	95	42	78	72	81	81	59	78
x_5	81	84	78	132	138	60	87	90	81	84	78	60

A)

Sample no.	sample values.					mean	range
	x_1	x_2	x_3	x_4	x_5	$\overline{x}$	
1	19	24	80	81	81	57	81-19 =62
2	36	54	69	77	84	64	48
3	42	51	57	59	78	57.6	36
4	51	74	75	78	132	82	81
5	60	60	72	95	138	85	78
6	18	20	27	42	60	33.4	42
7	42	65	75	78	87	69.4	45
8	42	45	68	72	90	63.6	48
9.	19	34	70	81	81	57	62
10	36	50	69	81	84	64	48
11	42	50	58	59	78	57.6	36
12	51	74	75	78	60	67.6	27

$$\bar{\bar{x}} = \frac{\Sigma \bar{x}}{n} = \frac{757.86}{12} = 63.13$$

$$\bar{R} = \frac{\Sigma R}{n} = \frac{613}{12} = 51.08$$

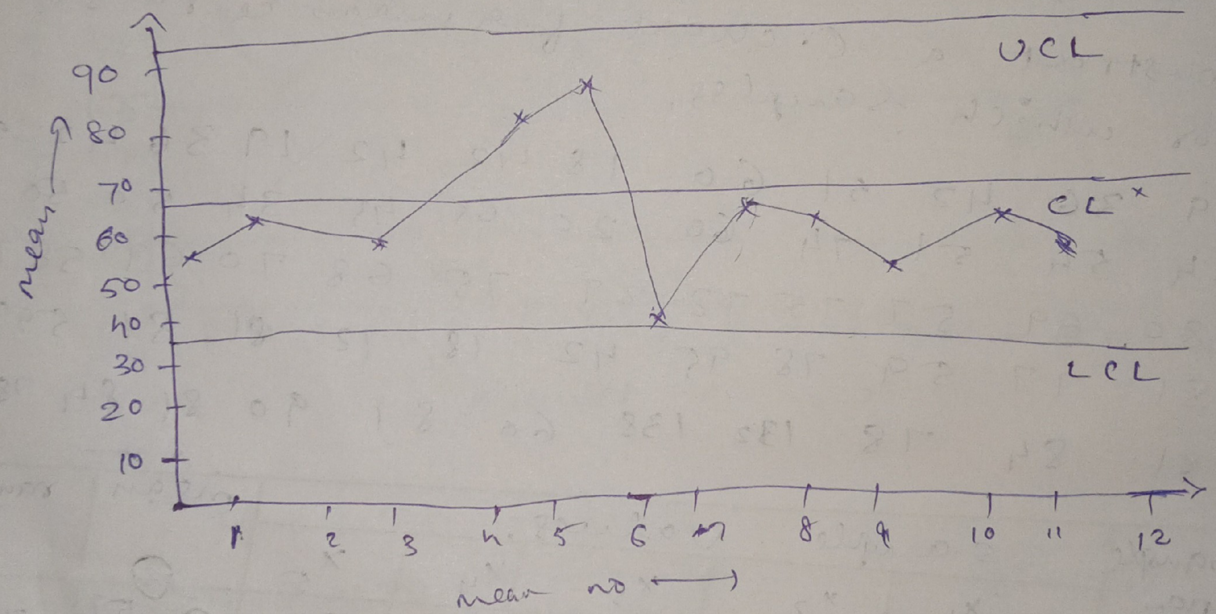
From table $n = 5$ $(x_1, x_2 \ldots x_5)$,

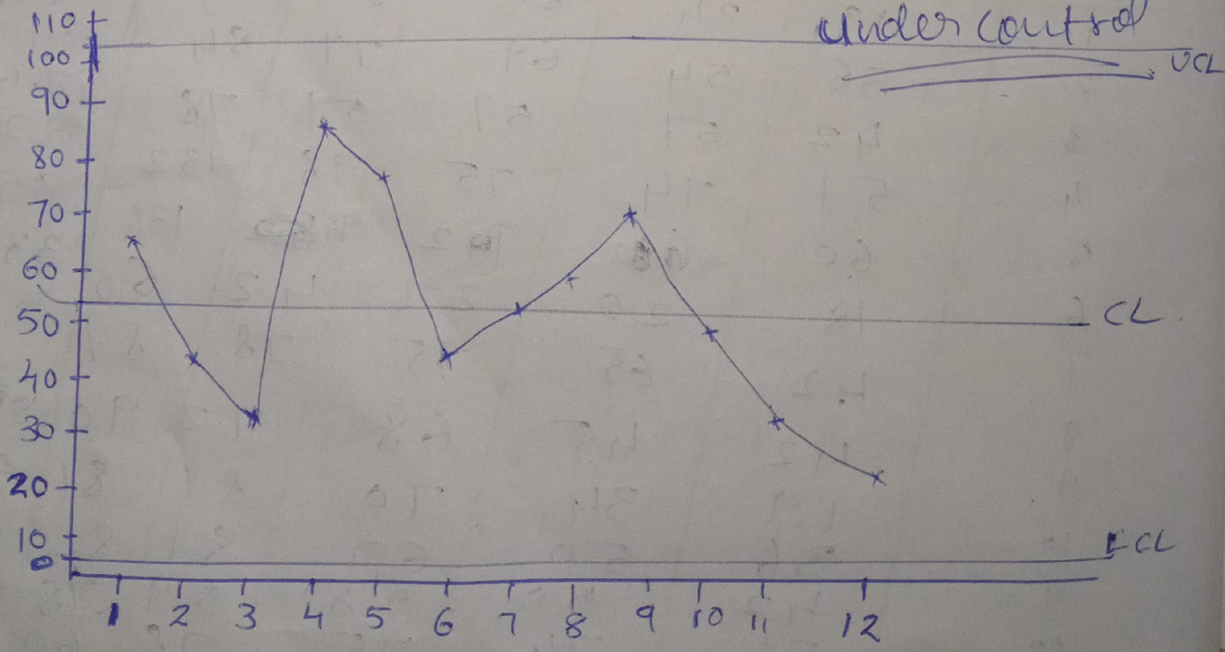
$A_2 = 0.577$, $D_3 = 0$, $D_4 = 2.115$

$$UCL_{\bar{x}} = \bar{\bar{x}} + A_2 \bar{R} = 63.13 + 0.577 \times 51.08 = 92.6$$

$$LCL_{\bar{x}} = \bar{\bar{x}} - A_2 \bar{R} = 63.13 - 0.577 \times 51.08 = 33.6$$

$$CL_{\bar{x}} = \bar{\bar{x}} = 63.13$$





c. chart for R,

$$CL = \overline{R} = 51.08$$

$$UCL_R = D_4\overline{R} = 108.03$$

$$LCL_R = D_3\overline{R} = 0$$

$\therefore$ R chart is under control.