01: Test of Significance

→ Test of hypothesis =

* Rules / procedures which enable as to decide whether to accept / reject the hypothesis / to determine whether observed samples defer significantly from expected rslt → Test of hypothesis / Test of significnts.

* 2 types —

a) Null hypo.. =

hypo.n to be tested is usually refer to as the null hypo.n, denoted by H_0.
The null hypo.n is a preposition of zero differnces.

eg → If we want to show that stdnts of clg A hve a highr avg IQ than stdnt of clg B.
Then might formulate the hypo.n that there is no diffence.

$$H_0 : \mu_A = \mu_B$$

b) Alternative hypo.. =

The statemnt that is being tested against the null hypo.. → A. hypo. & is denoted by H_1

$H_1 : \mu_A \neq \mu_B$ (or), $H_1 : \mu_A > \mu_B$ | (or) $H_1 : \mu_A < \mu_B$

→ Type I & Type II errors =

Research requires testing of hypothesis.

In this process 2 types of wrong inferences can be drawn, This are → type I & type II errors.

Rejecting a null hypo 'H_0' when it is actually true → Type I error or error of 1^{st} kind.

Accepting a null hypo 'H_0' when it is false → Type II error / error of 2^{nd} kind.

(Type I) $\alpha = P$(Type I error)

$= P$(rejecting H_0 given H_0 is true)

$= P$(Rejecting H_0 / H_0 is true)

(Type II) $\beta = P$(Type II error)

$= P$(Accepting H_0 given H_1 is true)

$= P$(Accepting H_0 / H_1 is true)

⇒ **Test Statistics =**

A appropriate ( ) of sample observation is chosen & the decision either to accept / reject the hypo. is taken based on the value of the ( ) → test statistics / test criteria.

→ **Critical Region =**

The basis of testing a hypothesis is the partition of the sample ~~size~~ space into 2 exclusive regions, namely region of acceptance & region of rejection. If the sample point falls in the region of rejection, H_0 is rejected, ~~the~~, region of rejection is → critical region (w).

Acceptance region is the set of those values of test statistics for which we are accepting the hypothesis.

Here we want to find Best critical region [BCR] guided only by the principle of minimising the (prob) of errors of type I & type II

→ **Level of Significants =**

It is defined as the (prob) of rejecting the null hypothesis 'H_0' when it is true [type I].

The significants level is also → The size of the critical region, size of the test / producer's risk

Denoted by 'α' (i.e)

$\alpha = P$(Rejecting H_0/H_0 true value)

$= P(x \in w \mid H_0)$

→ **Critical value = (cv)**

The value of test statistic which seperates the critical region & acceptance region → critical value

cv is usually → Z_α (or) t_α.

→ **power of a test =**

The (prob) of rejecting null hypothesis 'H_1' when it is actually not true → power of a test & it is given by $1-\beta$.

$$\text{Power} = P(\text{Rejecting } H_0 / H_1 \text{ is true})$$
$$= 1 - P(\text{acceptance } H_0 / H_1 \text{ is true})$$
$$= 1 - P(\text{acceptance } H_0 \; (H_0 \text{ is not true}))$$
$$= 1 - P(\text{Type II error})$$
$$= 1 - \beta$$

→ **Neymann pearson Lemma =**

If $x_1, x_2 \ldots x_n$ are n independent obs^n^ from population $f(x, \theta)$, we shall denote the likelyhood of this obs^r^ when the H_0 is true by $L(x, H_0)$ & H_1 is true by $L(x, H_1)$.

Let $H_0 = \theta = \theta_0$ & $H_1 = \theta = \theta_1$, be the to be performed $L(x, H_0) = f(x_1, \theta_0) \cdot f(x_2, \theta_0) \ldots f(x_n, \theta_0)$.

$$L(x, H_1) = f(x_1, \theta_1) \cdot f(x_2, \theta_1) \ldots f(x_n, \theta_1)$$

Then BCR is that for which $k(x, H_1) \geq L(x, H_0)$ inside it, where k is true & is to be chosen such that the size of CR is α.

→ **Most powerful Test =**

If there exist a C.Region 'C_0' with greater power than any other region giving size of type I error then C_0 is → Best C.region & the test based on it → most powerful test.

→ **P-value = (prob-value)**

A P-value is a statistical measurement used to validate a hypothesis against observed data.
P-values are used in hypothesis testing to decide whether to reject null hypo^n^

→ **Steps involved in testing a hypo^n^ =**

Step 1: State null hypo^n^ 'H_0' & alternative hypo^n^ H_1
Step 2: Choose the level of significance α.
Step 3: Determine the test statistic.
Step 4: Determine the (prob) distribution of test statistic.
Step 5: Determine the best critical region
Step 6: Calculate the value of test statistic
Step 7: **Decision**: If the calculated value of test statistic falls in the c.region reject the null hypo^n^ H_0, otherwise accept it.

Q
1) Let $X \to B(10, p)$ consider the follow test for testing $H_0: p = 1/2$ against $H_1: p = \frac{1}{4}$ "Reject H_0 if $X \leq 2$" find the significance level (α) & power of test ?

A) $X \to B(10, p)$

Significant level, $\alpha = P(\text{rejecting } H_0 / \text{when } H_0 \text{ is T})$
$= P(\text{rejecting } H_0 \,(H_0))$
$= P(X \leq 2 \mid p = 1/2)$
$= P(X=0) + P(X=1) + P(X=2)$ when $p = 1/2$

$f(x) = {}^nC_x p^x q^{n-x}$, $n = 10$, $B(10, p)$

$q = 1 - p = 1 - 1/2 = 1/2$

$= {}^{10}C_0 p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2 p^2 q^8$
$= {}^{10}C_0 (\tfrac{1}{2})^0 (\tfrac{1}{2})^{10} + {}^{10}C_1 (\tfrac{1}{2})^1 (\tfrac{1}{2})^{9} + {}^{10}C_2 (\tfrac{1}{2})^2 (\tfrac{1}{2})^{8}$

$= (\frac{1}{2})^{0}\, {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2$

$= (\frac{1}{2})^{10}(1 + 10 + 45) = \frac{1}{2^{10}}(56) = \frac{56}{1024} = 0.0546$

power $= p(\text{rejecting } H_0 \mid H_1)$

$= P(x \leq 2 \mid p = \frac{1}{4})$

$= p(x=0) + p(x=1) + p(x=2) \mid p = \frac{1}{4})$

$q = 1-p = 1 - \frac{1}{4} = 3/4$

$= {}^{10}C_0 (\frac{1}{4})^0 (\frac{3}{4})^{10-0} + {}^{10}C_1 (\frac{1}{4})^1 (\frac{3}{4})^{10-1} + {}^{10}C_2 (\frac{1}{4})^2 (\frac{3}{4})^{10-2}$

$= 1 \times 1 \times 0.05631 + 10 \cdot \frac{1}{4} \times 0.07508 + 45 \times \frac{1}{16} \times 0.1001$

$= 1 \times 0.05631 + \frac{10}{4} \times 0.07508 + \frac{45}{16} \times 0.1001$

$= 0.05631 + 2.5 \times 0.07508 + 2.8125 \times 0.1001$

$= 0.05631 + 0.1877 + 0.28$

$= 0.52401$

(If $\alpha \to$ size of test $\to$ level of significan)

2) It is desired to test a hypo $H_0 : p = \frac{1}{2}$ $H_1 : p = 3/4$ on the basis of tossing a coin once, where p is the prob of getting a head in a single trial & agreeing to accept H_0 if a tail appears & to accept H_1 otherwise. find values of α & β.

A) $\alpha = p(\text{rejecting } H_0 \mid H_0)$

$= p(\text{Head appears} \mid p = \frac{1}{2})$

$= p \Rightarrow \frac{1}{2}$

$\beta = P(\text{accepting } H_0 \mid H_1)$

$= p(\text{tail appears} \mid p = 3/4)$

$= 1 - p$ when $p = 3/4$

$= 1 - \frac{3}{4} = \frac{1}{4}$

3) If $x \geq 1$ in critical region for testing $H_0 : \theta = 2$ against $H_1 : \theta = 1$ on the basis of a single obs^r from $f(x,\theta) = \theta e^{-\theta x}$, $x \geq 0$ obtain the probs of type I & Type II errors?

A) Given $f(x,\theta) = \theta e^{-\theta x}$, $x \geq 0$

$p(\text{type I error}) = p(\text{rejecting } H_0 \mid H_0)$

$= p(x \geq 1 \mid \theta = 2)$

$= \int_1^\infty f(x)dx = \int_1^\infty \theta e^{-\theta x} dx \,/\, \theta = 2$

$= \int_1^\infty 2 e^{-2x} dx = 2\left[\frac{e^{-2x}}{-2}\right]_1^\infty$

$= \frac{2}{-2}\left[e^{-\infty} - e^{-2}\right] = -\left[-e^{-2}\right]$

$= e^{-2}$

$p(\text{type II error}) = p(x < 1 \mid \theta = 1) \to p(\text{accepting } H_0 \mid H_1)$

$= \int_0^1 f(x)dx = \int_0^1 \theta e^{-\theta x} dx \,/\, \theta = 1$

$= \int_0^1 e^{-x} dx = -\left[e^{-x}\right]_0^1$

$= -(e^{-1} - e^{-0})$

$= -(e^{-1} - 1)$

$= -e^{-1} + 1 = 1 - e^{-1} = 1 - \frac{1}{e}$

4) find the size & power of follow test. rejecting $H_0 : \lambda = 1$ (null hypo) in favour of $H_1 : \lambda = 2$ (alternative hypo) whenever $x_1 + x_2 \geq 2$, assume that x_1 & x_2 are independent obs^r from poission distri?

A) Given $x_1 \to P(\lambda)$, $x_2 \to P(\lambda)$

$\therefore y = x_1 + x_2 \to P(2\lambda)$ (by additive prop)

Size of the test $= p(\text{rejecting } H_0 \mid H_0)$

$= p(y \geq 2 \mid \lambda = 1)$

$= 1 - P(\cancel{Y<2} / \lambda = 1)$

$= 1 - P(y=0) + \cancel{P}(y=1) / \lambda=1)$

$= 1 - \left(\frac{e^{-1}}{1} + \frac{e^{-1}}{1} \times 1\right) = 1 - (2e^{-1})$

$= 1 - \frac{2}{e}$

$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$

Power $= P(\text{rejecting } H_0 / H_1)$

$= P(Y \geq 2 / \lambda = 2) = 1 - P(y=0) + P(y=1) | \lambda = 2$

$= 1 - \left(\frac{e^{-2} \cdot 2^0}{0!} + \frac{e^{-2} \, 2^1}{1!}\right) = 1 - e^{-2}(1+2)$

$= 1 - 3e^{-2}$