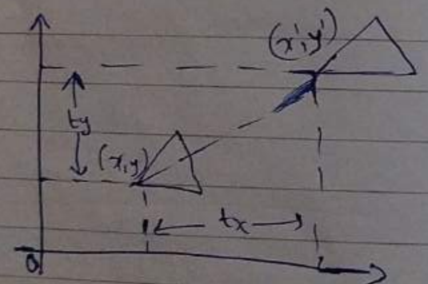
Module-3

2D Transformation

* changing position shape, size / orientation of an obj → Transformaⁿ.
* Transformⁿ means changing some graphics into something else by applying rules.
* we can hve various types of trans~ like translation, scaling up/down, rotation, shearing, reflection, etc.
* when a trans~ takes place on a 2D plain then it is → 2D trans~ (2D plain → (x,y)).

I Translation :



* It is trans~ that is used to re-position the obj along the straight line path from 1 coord loc to another.
* It is a ~~reject~~ rigid body trans~, so we need to translate the whole obj.

* So we translate 2D points by adding translation distance 't_x' & 't_y' to the original coord. position (x,y) to move ~~to~~ at new position (x',y') as—

$$\boxed{\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}}$$

* Translation distance pair (t_x, t_y) → a translaⁿ vector / shift vector.
* we can represnt it into single matrix eq. in column vector as

$$\boxed{P' = P + T}$$

$P' \to$ new position (x',y')
$P \to$ old position (x,y)

Matrix represnⁿ of colum vector :

$$\underset{P'}{\begin{bmatrix} x' \\ y' \end{bmatrix}} = \underset{P}{\begin{bmatrix} x \\ y \end{bmatrix}} + \underset{T}{\begin{bmatrix} T_x \\ T_y \end{bmatrix}}$$

M. r~ of row vector :

$$P' = P + T$$
$$\Rightarrow [x' \; y'] = [x \; y] + [T_x \; T_y]$$

eg → Translate the Δ A $(10,10)$, B $(15,15)$, C $(20,10)$. 2 unit in x dir~ & 1 unit in y dir

$P' = P + T$ $\quad t_x = 2$ (x → move 2 units) $\quad t_y = 1$ (y → " ")

$A = A' = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 12 \\ 11 \end{bmatrix}$.

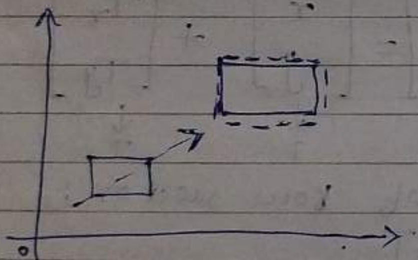
$B = B' = \begin{bmatrix} 15 \\ 15 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 17 \\ 16 \end{bmatrix}$

$C = C' = \begin{bmatrix} 20 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 22 \\ 11 \end{bmatrix}$

Final coord after translan —

$A'(12,11)$, $B'(17,16)$, $C'(22,11)$

II Scaling:



* It is a transf that used to alter the size of an obj.
* This opr is carried out by multipling coord value (x, y) ~~which~~ with scaling factor (S_x, S_y) respectively. So the eq for scaling

$$\boxed{\begin{aligned} x' &= x * S_x \\ y' &= y * S_y \end{aligned}}$$

This eq can be represented in column ~~vector~~ vector matrix eq —

$$\boxed{P' = P * S}$$

Column vector:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \underbrace{\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}}_{\text{Scaling matrix}}$$

* Any +ve value can be assigned to (S_x, S_y).
* values <1 reduce the size while values >1 enlarge the size of the obj. & obj remains unchanged when values of both factor is 1.

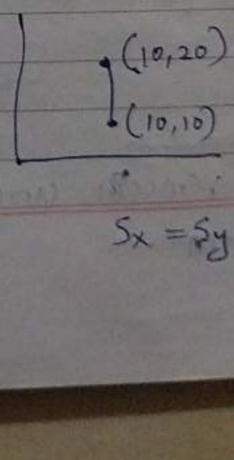
$(S_x, S_y) < 1$ → reduce obj size

$(S_x, S_y) > 1$ → enlarge "

$(S_x, S_y) = 1$ → remain unchanged.

* Same values of S_x & S_y will produce uniform scaling & different values S_x & S_y will produce differntial scaling.

eg →



$S_x = 2$

$S_y = 2$.

$S_x = S_y$ → uniform scaling

$$\left.\begin{aligned} x' &= x \cdot S_x \\ y' &= y \cdot S_y \end{aligned}\right\} — ①$$

$S_x = 2$

$S_y = 2$

1st point — (10,10)
in ①

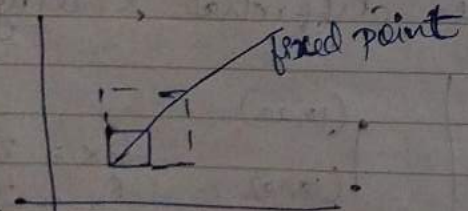
$x' = 10 * 2 = 20$
$y' = 10 * 2 = 20$
(20, 20)

2nd point — (10,20)
in ①

$x' = 10 * 2 = 20$
$y' = 20 \times 2 = 40$
(20, 40)

* Scaling according with a fixed point —

* we can control the position of obj after scaling by keeping 1 position fixed → fix point (x_f, y_f)

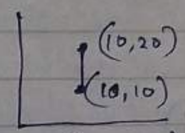


* Fixpoint (x_f, y_f) will remain unchanged after the scaling trans.

* eq for scaling the fixed point position —

$$\boxed{\begin{aligned} x' &= x_f + (x - x_f) * S_x \\ y' &= y_f + (y - y_f) * S_y \end{aligned}}$$

* eq →



$x_f = 10$, $y_f = 10$
$S_x = 2$, $S_y = 2$

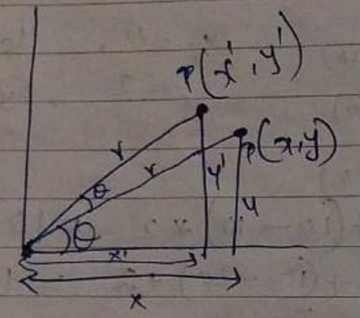
1st point — (10,10)
$x' = 10 + (10 - 10) * 2 = 10$
$y' = 10 + (10 - 10) * 2 = 10$
(10,10)

2nd point — (10,20)
$x' = 10 + (10 - 10) * 2 = 10$
$y' = 10 + (20 - 10) * 2 = 10 + 10 * 2$
$= 10 + 20 = 30$
(10,30).

III Rotation :

* It is a trans. that used to reposition the obj along the ○ path in the (x,y) plain.
* To generate a rotation we specify a rotation angle 'θ' & position of the rotation point (pivot point) (x_r, y_r) about which the obj is to be rotated.

* the value of rotation angle defines anticlock wise rotation & -ve value of R. angle defines clock wise dir.
* we 1st find the eq. when pivot point is at coord origin (0,0)



θ → rotate angle

φ → angle

In order to find the value of x', y'

$\cos\theta = \frac{\text{adj side}}{\text{hypo}}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(\theta+\phi) = x'/r$

$r(\cos(\theta+\phi)) = x'$

$\cos\theta = x/r$

$x = r(\cos\theta) = r\cos\theta$

$x' = r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$$\boxed{x' = x\cos\phi - y\sin\phi}$$

$\sin(\theta+\phi) = y'/r$

$y' = r(\sin(\theta+\phi))$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$\sin(\theta+\phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

$\sin\theta = y/r$, $y = r\sin\theta$

$$\boxed{y' = y\cos\phi + x\sin\phi}$$

we can write it in the form column vector matrix eq. as—

$x' = x\cos\phi - y\sin\phi$

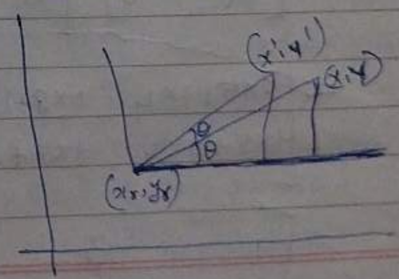
$y' = y\cos\phi + x\sin\phi$.

$p' = p \cdot r(\phi)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} * \begin{bmatrix} \cos\phi & -\sin\phi \\ \underset{(x\text{ term})}{\sin\phi} & \underset{(y\text{ term})}{\cos\phi} \end{bmatrix}$$

$\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \longrightarrow$ rotation matrix.

* Rotation about Arbitrary point :



Transⁿ eq for rotation of a point about pivot point (x_r, y_r):

$$x' = x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta$$
$$y' = y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta.$$

This eq are differing frm rotation about origin & it's matrix represⁿ is also diffrnt

eg → locate the new position of the triangle A[(5,4), B(8,3), C(8,8)], after it's rotation by 90° clockwise about the origin.

As rotation is clockwise we will take $\theta = -90°$, so $P' = R \cdot P$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(-90) & -\sin(-90) \\ \sin(-90) & \cos(-90) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = \begin{bmatrix} \cos(-90) & -\sin(-90) \\ \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 8 & 8 \\ 4 & 3 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times5+1\times4 & 0\times8+1\times3 & 0\times8+1\times8 \\ -1\times5+0\times4 & -1\times8+0\times3 & -1\times8+0\times8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 & 8 \\ -5 & -8 & -8 \end{bmatrix}$$

final coords after rotation A(4,-5), B(3,-8), C(8,-8)

→ **Homogenous coord represⁿ of geometric Transⁿ** = metrix represⁿ of basic geometric represⁿ

[Homogneous repsⁿ = geometric repsⁿ]

1) Translation:

$$P' = P + T$$
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

2) Scaling: $P' = P * S$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

3) Rotation: $P' = P \cdot R(\theta)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

* Homogenous coord system mean expressing each coord as a homogenous coord to represent all geometric transⁿ eqs as matrix multiplication.
* Basics geometric reprsⁿ of transⁿ can be represented in general matrix form –

$$\boxed{P' = M_1 * P + M_2}$$

P' & P are coord position represented as column vectors.

$M_1 \to 2\times2$ matrix containing multiplicative terms.

$M_2 \to$ 2-element column vector containing translational terms.

* For translaⁿ 'M_1' is the <u>identity matrix</u>. For Normal rotation & Scaling 'M_2' is a 0 matrix.

* In ordr to process complex transⁿ (Sequnce of geometric transⁿ) easier we must calculate all sequnce of transⁿ in 1 step & for that reason, we reformulate above eq to eliminate the matrix addition associated with translation terms in matrix 'M_2'.

* we can combine that the by expanding 2x2 matrix represⁿ into 3x3 matrixes. It will allows us to convert all transⁿ into matrix multiⁿ, bt we need to reprsnt vertex position (x,y) with homogenous coord triple (x_h, y_h, h),

write triple as $(h\cdot x, h\cdot y, h)$

A convinent choice is to set <u>$h=1$</u>, each 2D position is represnted with homogenous coord

(Expressing coord in homogenous coords form allows us to reprsnt all geometric transⁿ eq as matrix represⁿ)

1) <u>Translation</u> :

$$p' = p * T\,(T_x\, T_y)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}$$

$h = 1$

identity matrix

2) <u>Rotation</u> :

$$p' = p * R(\theta)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} x \\ y \end{bmatrix}} * \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$h = 1$

3) <u>Scaling</u> :

$$p' = p * (S_x, S_y)$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} * \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ <u>Composite Transⁿ</u> :-

* we can set up a matrix for any sequnce of transformⁿ as a C. transⁿ matrix by calculating matrix product of individual transⁿ

* for column matrix repres^n of coord position we form composite trans^n by multipling matrices in order from R–L.

1) **Translations:**

2 successive transla^n are performed as

$$P' = T(tx_2, ty_2)\{T(tx_1 \cdot ty_1) \cdot P\}$$
$$P' = \{T(tx_2, ty_2) \cdot T(tx_1, ty_1)\} \cdot P$$
$$P' = \begin{bmatrix} 1 & 0 & tx_2 \\ 0 & 1 & ty_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & tx_1 \\ 0 & 1 & ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$P' = \begin{bmatrix} 1 & 0 & tx_2 + tx_2 \\ 0 & 1 & ty_1 + ty_1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$P' = T(tx_1 + tx_2, ty_1 + ty_2) \cdot P$$

2) **Rotations:**

2 successive rotations applied to point P produce the transformed position.

$$P' = R(\theta_2) \cdot \{R(\theta_1) \cdot P\}$$
$$= \{R(\theta_2) \cdot R(\theta_1)\} \cdot P.$$

$$P' = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_2\cos\theta_1 - \sin\theta_2\sin\theta_1 & -(\cos\theta_2\sin\theta_1 + \sin\theta_2\cos\theta_1) & 0 \\ \sin\theta_2\cos\theta_1 + \cos\theta_2\sin\theta_1 & -\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_2+\theta_1) & -\sin(\theta_2+\theta_1) & 0 \\ \sin(\theta_2+\theta_1) & \cos(\theta_2+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta_2) \cdot R(\theta_1) = R(\theta_1 + \theta_2)$$
$$\therefore P' = R(\theta_1 + \theta_2) \cdot P$$

3) **Scaling:**

Concatenating trans^n matrices for 2 successive scaling op^r produces follo^n composite scaling matrix –

$$\begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Sx_1 Sx_2 & 0 & 0 \\ 0 & Sy_1 Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S(Sx_2, Sy_2) \cdot S(Sx_1, Sy_1) = S(Sx_1 \cdot Sx_2, Sy_1 \cdot Sy_2)$$

The successive op^r are multiplicative.