A Careful Consideration of the St. Petersburg Paradox

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1 The paradox

If heads appears for the first time on turn n, the player is awarded $\$2^n$.

$$\begin{aligned} EMV &= \sum_{n=1}^{\infty} \frac{1}{2^n} 2^n \\ &= \sum_{n=1}^{\infty} 1 \\ &= \infty \end{aligned}$$

The St. Petersburg game's possible winnings are geometrically distributed with $Pr(n) = \frac{1}{2}^n$, where $Pr(n)$ is the probability of $\$2^n$.

2 Simulating for world

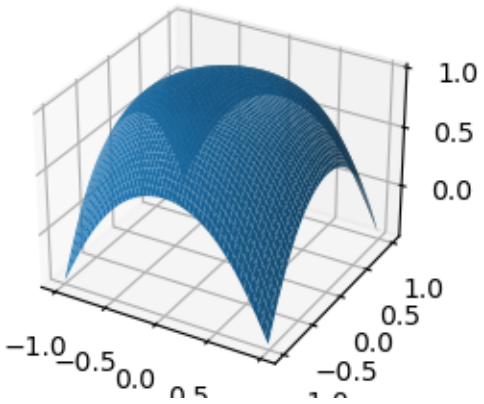

Figure 1: Distribution

Winnings ($\$2^n$)	Number of People
1	5022
2	2521
3	1255
4	609
5	314
6	142
7	68
8	34
9	15
10	11
11	5
12	3
13	1
14	0
15	0
16	0
17	0

Table 1: World

3 Lottery

Winnings ($\$2^n$)	Number of People
$150,000,000$	$1:292,291,338$

Table 2: Lottery

Hello

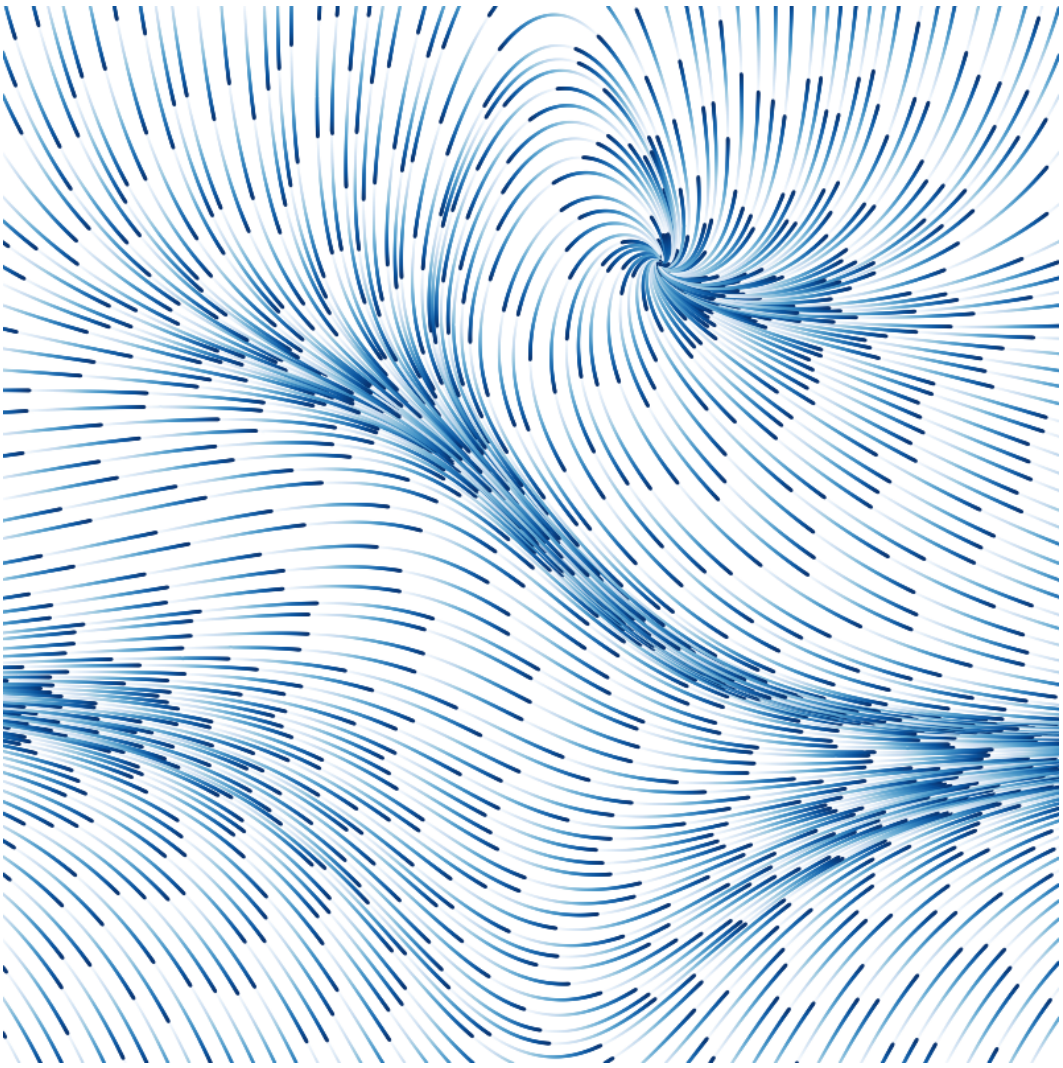

Figure 2: Distribution