

Project Checkpoint: CSE 6730 Project 1

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1 Problem Statement

The efficient evacuation of physical structures is an important problem with a long history of multi-disciplinary study [15]. For this study, we focus specifically on the problem of efficiently evacuating the area around Bobby Dodd Stadium in Atlanta, GA. Bobby Dodd Stadium, the home of the NCAA Division-I Georgia Tech Yellow Jackets football team, has a seating capacity of 55,000 individuals.



Figure 1: Bobby Stadium Stadium seating chart.

Our primary objective in this study is to minimize the pedestrian evacuation time of the stadium and its surrounding area as attendees leave the area following a home football game. We do this by taking a parametric approach, in which we explore the effects of road closures, strategically placed

guidance symbols (i.e., signs), and the “takeover” of certain intersections by law enforcement in promoting an optimal result.

For purposes of this study, we define the system under investigation (SUI) as a rectangular polygon (Fig. 2) surrounding the stadium. To be clear, the SUI is defined as the area inside the polygon but outside the stadium. As pedestrians exit the stadium, they enter the SUI, and as they cross the boundary of the polygon, they exit it.

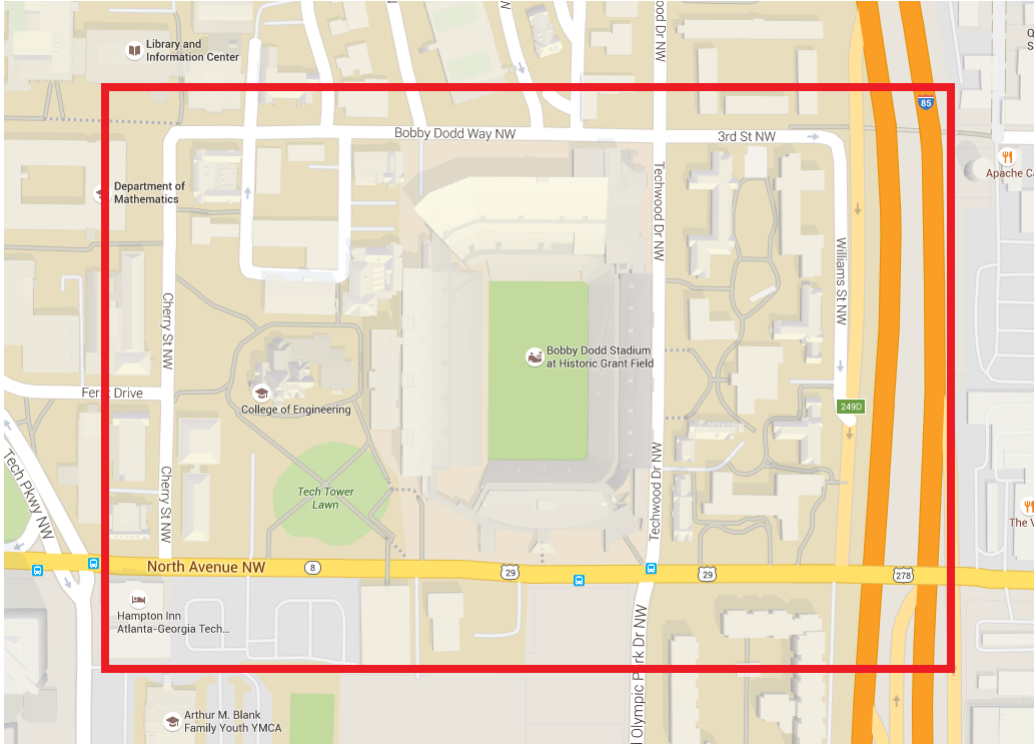


Figure 2: The system under investigation.

The stadium has 10 “gates,” which serve as ingress (pre-game) and egress (post-game) sites. These serve as the entry points to our simulation. As pedestrians enter the SUI, they are assigned a destination and then proceed to their destination by way of pedestrian walking paths (i.e., sidewalks).

2 Related Work

A variety of techniques have been used to model pedestrian movement. Cellular automata, lattice gas, social force, fluid-dynamic, agent-based, game-theoretic, and animal experimentation-based approaches have been used [15]. As noted by Zheng, Zhong, and Liu [15], models often encounter or attempt to model several common phenomena: clogging, side-stepping, lane formation, and herding behavior, among them.

These phenomena manifest themselves in different ways and at different magnitudes depending on the system being modeled. In one of the early explorations of 2D cellular automata for simulating traffic flow in the related domain of vehicular traffic simulation, Biham and Middleton [1] note the presence of a sharp *jamming transition* in which all cars in the simulation transition from moving at maximal speed to being stuck. Similar effects were noted in simulations of bi-directional pedestrian movement, with Weifeng, Lizhong, and Weicheng [13] finding that as total pedestrian density increases, a critical value is reached at which the system transitions into a jammed state where only a few pedestrians are able to proceed.

In a slightly different take on the problem, Okazaki and Matsushita [12] modeled pedestrian evacuation using Coulomb’s Law, with actors magnetically moving toward their goals and away from obstacles that would lead to collisions. In another application of equations of natural phenomena, Helbing [7] derived fluid dynamic equations for explaining the movement of pedestrian crowds, observing phenomena such as the development of lanes, jamming, and crossing. Helbing [9] further notes an explicit “faster-is-slower” effect, in which attempting to move faster results in a lower average speed of leaving at high pedestrian density in evacuation scenarios.

In a more recent study, Helbing et al. [8] use a “social force” model, which states that pedestrians operate in some sense automatically when reacting to obstacles and other pedestrians, applying strategies that have been learned to be most effective over time. Several suggestions are made to alleviate the three most common problems in pedestrian crowds: counterflows, bottlenecks, and intersecting flows. The presence of strategically placed obstacles are found to actually reduce these negative phenomena, leading to improved flow.

Building on previous cellular automata-based models, Burstedde et al. [3] introduce a *floor field*, a secondary grid of cells which underlies the main grid and acts as a substitute for pedestrian intelligence. These fields can be either

static or dynamic, and are capable of promoting the avoidance of jams, as well as simulating attractive effects, in which pedestrians are more likely to follow in the paths of pedestrians ahead of them.

Taking a more algorithmic approach, Fang et al. [6] have designed a modified ant colony optimization (ACO) algorithm for optimizing pedestrian evacuation, seeking to minimize evacuation time, evacuation distance, and congestion. Kemloh Wagoum, Seyfried, and Holl [10] utilize an observation principle approach in modeling pedestrian evacuation, in which pedestrians first observe their environment and then make a final decision on strategy based on obtained data.

There are several takeaways from the literature described above that have potential applicability to the model at hand. First, on the topic of modeling pedestrian walkways, several papers focused on either a single passageway, enclosed areas with obstacles (walls, pillars, etc.), or evacuation from a room with a single doorway. However, modeling walkways as a undirected graph [6] seems feasible given the problem at hand, where there are multiple pathways one can take to get to a single destination. In papers where cellular automata was utilized, the median size of each cell on the walkway was 0.4m^2 [2, 3, 13].

Second, on the topic of pedestrian modeling, multiple walking speed techniques were used, ranging from a constant speed of 1 m/s [13] or 1.3 m/s [3], to using a distribution resembling a step function [2] or a Normal distribution [11]. In either distribution, the median walking speed was also approximately 1.3 m/s. For reaction to stimuli, 0.3 seconds was utilized [3], which was one time step in that particular simulation. Incorporation of opportunistic side-stepping was done [2], in addition to using least-cost algorithms for establishing the path a pedestrian would like to take [6]. In the context of cellular automata, analyzing a given pedestrian’s cell’s neighbors in either a 4-connected [13] or 8-connected fashion [3] to decide in which direction the pedestrian would like to walk for the next time step was utilized. This, coupled with a dynamic floor field [3] to factor in long-distance forces, could be utilized.

Finally, simulation execution practices were also gleaned. Care must be taken to do parallel-based updating to avoid sequential updating from interfering with underlying model execution [2]. The practice of using a deterministic model running with randomized initial conditions has precedent [1]. Finally, to achieve statistical significance, 20 runs per configuration has been used [2].

3 Conceptual Model

For our simulation, we utilize a 2-dimensional cellular automata (CA)-based approach. We construct a discrete time-stepped model, in which the simulation clock advances by a fixed interval every time step. We utilize a stochastic (probabilistic) approach, in which the output of each individual simulation run is a random variable. Thus, multiple simulation runs will be performed and statistical analysis will be used to provide confidence levels in our results.

3.1 Inputs

The primary input to our simulation model is a stream of football game attendees (i.e., pedestrians) exiting Bobby Dodd Stadium following a home football game. For purposes of modeling this input stream, we assume pedestrian interarrival time at the stadium gate boundary is a homogeneous stochastic process that follows a Poisson distribution. If the average rate of people exiting the stadium is known, the Poisson distribution will give a probability distribution of when the next person will exit within a set interval of time, independent of the last person who exited the stadium. As pedestrians arrive at the gate boundary, they enter the SUI.

In addition, we select each pedestrian’s target destination and walking speed at random. For selecting a target destination, we sample from an empirical distribution. For determining a walking speed for each pedestrian, we sample from a distribution previously referenced in the literature by Blue and Adler [2] (5% fast walkers (4 cells/time step), 90% standard (3 cells/time step), 5% slow (2 cells/time step)).

Pedestrians in our model can be considered a consumer entity class *Pedestrian* with attributes which will be set and updated during the course of our simulation. Table 1 provides an overview of the attributes of the *Pedestrian* class.

New instances of the *Pedestrian* class are created and initialized as pedestrians exit the stadium.

3.2 Output

For each simulation run, when all pedestrians were evacuated from the SUI we determined the simulation to be complete. As this is a stochastic simulation, the output results must be interpreted as samples of a random process.

At the conclusion of a series of simulation runs for a given set of parameters, we compute an average of the total egress duration across all runs from that series. More formally, the average egress time E_{avg} for n simulation runs can be represented as:

$$E_{avg} = \frac{1}{n} \sum_{i=1}^n d_i \quad (1)$$

where d_i is the total egress duration of the i th simulation run. This output can be considered a derived scalar output variable (DSOV). We then define the optimal parameter strategy to be the set of parameters P that give the the minimum value of E_{avg} .

3.3 Content

3.3.1 Approach

Pedestrians exit the stadium from one of four potential exits. To build the simulation space, we overlay a 2-dimensional cellular automata grid on a map of the SUI. We model this space as a weighted graph, a type of graph

Attribute	Description
Current	<i>Node</i> object corresponding to the current location of the pedestrian.
Destination	<i>Node</i> object (see Table 2) corresponding to the final destination of the pedestrian.
TargetNext	The desired next node to move to in the pedestrian's shortest path to his destination, as computed by Dijkstra's algorithm.
Speed	Walking speed, formulated in grid cells traversed per time step.
EgressComplete	A boolean value. This value is false when the pedestrian has not yet egressed, and true when egress is complete.

Table 1: Attributes of the *Pedestrian* entity class.

in which each graph edge is assigned a weight corresponding to the spatial distance between the two nodes [14]. Note that in this section, the terms “cell” and “node” are used interchangeably.

As pedestrians exit the stadium (and thus enter the SUI), they are probabilistically assigned one of the graph’s destination nodes. When a pedestrian reaches a destination node, that constitutes them exiting the SUI. Destination nodes are chosen based on their spatial meaning when overlaid on a map of the SUI, and constitute one of three possible classes: on-campus housing (dormitories), parking lots, or the North Avenue Metropolitan Atlanta Rapid Transit Authority (MARTA) station.

Each node in the graph is a member of a *Node* class, which has attributes as shown in Table 2.

As part of the simulation initialization, the node and edge information is populated. For each node in the graph, we pre-compute the least-cost path to every possible destination node N_d . In general, we can compute the shortest path between any given node N_i and destination node N_d using Dijkstra’s well-known algorithm [4]. In our simulation, we use a publicly available Python implementation of Dijkstra’s algorithm [5]. A hash map *Paths* attribute of each node stores the next node in the least-cost path to each destination.

At each time step, a pedestrian occupying a given node N_i uses the *Paths* attribute to identify the next desired cell to walk towards given their destination node N_d . This drives the pedestrian’s instinct to move toward their destination, based strictly on the shortest distance walk from their current position to N_d .

This concept of pre-computing and saving shortest-path data is known as creating a static floor field (that is, a floor field that does not change as the simulation progresses). However, a given pedestrian must decide where to move in light of other pedestrians and other modeled factors. Such factors do change with time, and requires the use of a dynamic floor field [3].

The chosen implementation of the dynamic floor field is to be determined. However, the static floor field will return the next node desired based on the shortest path; due to the 8-connected nature of the nodes this corresponds to a specific direction of travel spatially (for example, North or North East). Thus, it is reasonable to also consider directions adjacent to the “shortest path” direction when deciding a pedestrian’s next move (for example, a “shortest path” direction of East would yield adjacent directions of North East and South East).

It will likely occur that more than one pedestrian will decide to pursue the same node in a given time step. In such cases, a probabilistic approach is taken to resolve this conflict (since only one pedestrian can occupy a cell at a time), depending on their respective walking speeds. If the walking speeds are the same, each pedestrian will get an equal chance of getting the desired node. However, if there is a disparity, pedestrian(s) featuring higher walking speeds will have a higher chance of getting the desired node, and pedestrian(s) featuring lower walking speeds will have a lower chance of getting the desired node.

Attribute	Description
NodeID	Unique identifier for the node.
XCoordinate	X-coordinate of the node on the 2-dimensional grid.
YCoordinate	Y-coordinate of the node on the 2-dimensional grid. (Note: for purposes of this simulation, the y-axis is inverted.)
Paths	A dictionary (hash map) relating each possible destination to the NodeID of the next node in the shortest path as determined by Dijkstra's algorithm.
Neighbors	Array of <i>Node</i> objects corresponding to nodes that are one edge away from the node.
Available	Boolean value indicating whether the node is available for a pedestrian (true) or not available (false).
NodeType	Enumeration of the type of node, corresponding to its spatial significance. The NodeType can be one of four possibilities: sidewalk, road, SUI entrance, or SUI exit.

Table 2: Attributes of the *Node* class.

3.3.2 Parameters

To optimize the egress of everyone exiting the stadium in our SUI, there are several parameters that can be altered. The first parameter is closing certain streets or intersections meant for vehicular traffic and making them for use by pedestrians only (i.e., roads become sidewalks). The second is having traffic police take over certain intersections to control the pedestrian flow (i.e., overrule traffic lights). The third is placing signs along the sidewalk to direct pedestrians to walk a certain direction to their destination. The optimal time for everyone to exit the stadium and the SUI could be a combination of the three parameters. For example, directing pedestrian traffic to an intersection that is closed to all vehicles or controlled by a police officer is a combination that is likely to produce better results than only directing pedestrians to a regular crosswalk.

To simulate these parameters, in addition to a *Node* class, there is also an *Intersection* class, which has attributes as shown in Table 3.

Attribute	Description
IntId	Integer value. A unique identifier for the intersection.
Nodes	Array of <i>Node</i> objects corresponding to nodes that are part of this intersection.
IsOpen	Boolean value indicating whether the intersection is open for pedestrian access (true) or not open (false).

Table 3: Attributes of the *Intersection* class.

To model intersections where vehicles are allowed to pass, the nodes in the intersections are assigned to a pre-defined intersection. The intersection is therefore made up of a group of nodes. To simulate the crosswalk opening and closing, the nodes will be either forced closed or available for pedestrians to occupy them. If the road is closed and vehicles are passing, then no nodes in the intersection will be available for pedestrians. The duration of intersection closure can be determined stochastically or by the number of pedestrians waiting for the crosswalk to open.

One parameter in the simulation that can be modified is determining

which intersections will be completely closed to vehicles and always available for pedestrian use. To model this case, all nodes in that intersection will behave in the same way as the nodes that make up the sidewalk. There will be no need to create an intersection object for that intersection. On the other hand, if there is an intersection that is closed off for pedestrians and only available for vehicular traffic, then the initial creation of nodes will not include that intersection - there will be no nodes at that intersection as no pedestrian can ever occupy those nodes.

If there are pedestrians that need to cross an intersection, but the intersection is closed, then the pedestrians will not be able to advance to their next node. This will cause a list of pedestrians to stay in their current node until the crosswalk opens up (i.e., queue). Once that occurs, then the pedestrians can continue to their next nodes.

3.3.3 Random Numbers

To model the egress of the stadium, we take a stochastic (probabilistic) approach. In the context of a computer simulation, this necessitates the use of a pseudorandom number generator.

In this initial project checkpoint, we utilize a pre-existing random number generator. However, for our final simulation model, we intend to use the simple linear congruential approach to create our own generator. This generator is given a *seed* value, and then produces a uniformly distributed pseudorandom number. This value is then used to sample from a given probability distribution.

3.4 Assumptions and Simplifications

We make a number of assumptions and simplifications in constructing our simulation model.

Firstly, our simulation is simplified by focusing only on *pedestrian* traffic, avoiding the inclusion of vehicular traffic which could be significant in a football game evacuation scenario.

We also assume the stadium to be at or near full seating capacity of 55,000 individuals. While the mean empirical value of football game attendance at Bobby Dodd Stadium is somewhat lower than 55,000, the choice of 55,000 allows us to “stress test” our model by simulating the upper bound.

We assume that pedestrians stay on sidewalks and obey traffic lights, as the inclusion of other types of pedestrian behaviors would likely add complexity to the model without yielding much additional value. We also assume that main roads such as North Avenue remain open to vehicular traffic, avoiding a more unrealistic simulation that would be possible if all roads could be closed.

In the realm of our “updating rule” which specifies the next cell for a given pedestrian to travel to in the simulation, we assume that pedestrians are following a shortest-path approach. While logical, this may be an oversimplification, as pedestrians (particularly from the home team) may follow more indirect paths to their destinations following a victory.

We make the assumption that all pedestrians will be traveling to one of three possible classes of destinations - local housing, MARTA, or a parking facility. While an important simplification, it is likely true in reality that pedestrians may be traveling to other destinations as well, such as social venues. It is also likely that some percentage of pedestrians may not evacuate the area for an extended time period, as they sit idle while waiting for friends or planning their destination in an ad-hoc manner.

We assume the stochastic processes in this simulation to be homogenous stochastic processes, which can be defined as stationary stochastic processes where the random variables $X = X_1, X_2, \dots, X_n$ are independent and identically distributed. While not always realistic in the context of simulation model-building, we do believe this assumption to be reasonable in the context of this simulation.

There are inherent simplifications in the choice of a cellular automata-based simulation approach. Specifically, it is assumed that each individual travels a constant distance per time step through the simulation. Nevertheless, we attempt to mitigate this simplification by drawing the speed for each individual from a probability distribution, which does introduce some variation, albeit only on an individual-to-individual basis.

4 Description of Simulation Software

4.1 Architecture

In generating our simulation, we take an object-oriented design approach. We use the Python programming language for our implementation.

Distinct types of entities in our simulation are represented as *classes*. For interacting with these classes, we define a number of methods that function as their public interfaces. It should be noted that while we may define explicit “getter” and “setter” methods in our conceptual model, the Python programming language provides these interfaces on declared object attributes without the need to provide explicit method definitions in code.

4.2 Interfaces

4.2.1 User Interface

To run our simulation, first ensure Python 2.7 is installed on the testing machine. Python’s matplotlib module will also need to be installed for visualization purposes. If visualization is not needed, it can be turned off by setting the *Visualization* option to False in the driver program, *driver.py*.

If Python and the matplotlib module are installed, you can simply enter the *code* subdirectory and run the following at a Unix command prompt:

```
$ python driver.py
```

The output will initially state that a preprocessing step is being performed to prepare the simulation. Then, the actual simulation will automatically begin. Pedestrians will be created and will begin moving toward their destinations. As they do, the number of “active pedestrians” remaining in the SUI will be displayed at every time step.

To use our simulation in a less rote manner, we have provided a simple interface by which a user can easily use our software. First, the user creates a *Grid* object, which is initialized from a maximum number of rows, maximum number of columns, node file, edge file, and type map giving translations between integer node types (such as SUI entrance, SUI exit, sidewalk, and road). An optional file in *pickle* format can contain previously produced shortest path information.

The node file should be in a CSV format, of the form:

```
cell_x, cell_y, pixel_x, pixel_y, meters_x, meters_y, node_type
```

where *cell_x* and *cell_y* correspond to *x* and *y* coordinates in a 2D-grid space. *pixel_x*, *pixel_y*, *meters_x*, and *meters_y* translate the *x* and *y* coordinates into pixel space and physical space, respectively. The *node_type*

is an integer value that specifies whether the node is a sidewalk (1), road (2), SUI entrance (3), or SUI exit (4) node.

The edge file should also be in a CSV format, of the form:

node_a_id	node_b_id	edge_weight
-----------	-----------	-------------

where the node ids refer to zero-based line indices in the node file. In other words, if the node file contained two rows, such as:

1	4	4	16	4	16	2
2	6	8	24	8	24	2

A corresponding edge file might look as follows:

0	1	7.000
---	---	-------

where 0 refers to the node in the first line of the nodes file, and 1 refers to the node in the second line.

The type map should be a dictionary, with string keys that are human-readable descriptions of node types, and integer values that correspond to in the node file.

Once the grid is created, it is passed to the *Simulation* constructor, along with a parameter dictionary with options such as the number of pedestrians that should be simulated, and whether the visualization engine will be enabled.

A typical driver program might look as follows:

```
# driver.py
from grid import Grid
from simulation import Simulation

# Specify the max number of rows and columns in the grid.
max_rows = 66
max_cols = 139

# Create a type map mapping human-readable node types to integer
# ids.
type_map = { "sidewalk": 1, "crosswalk": 2, "entrance": 3, "exit":
4 }

# Create a grid object that contains the underlying nodes and path
```

```

information.
grid = Grid(max_rows, max_cols, "vertices.csv", "edges.csv",
            type_map)

# Set up a simulation object.
simulation = Simulation(grid, {"num_pedestrians": 500,
                              "visualization": True})

# Run the simulation.
simulation.run()

```

4.2.2 Pedestrian Class Interfaces

For simulating pedestrian movement, we define a *Pedestrian* class as shown in Table 4. Pedestrians are initialized with an entrance node to the SUI, as well as a destination node and speed. They then traverse the nodes in the shortest path toward their destination node. Once they have reached a node with an *exit* NodeType, the `SetEgressComplete()` method is called for the pedestrian with an argument of `true`, alerting the simulation that the pedestrian has finished the simulation. Once there are no pedestrians in the simulation remaining with the `EgressComplete` attribute set to `false`, the driver program breaks from the main simulation loop and the program exits.

4.2.3 Node Class Interfaces

We define a *Node* class as shown in Table 5. Nodes are the primary abstraction for the resource entities in the simulation, which are sidewalks and roads. Nodes have several key attributes defined that can be retrieved and set. Among them, the `Neighbors` attribute specifies all the nodes that are one degree away from the node object. The `Neighbors` attribute is then used to populate the `Paths` attribute for the node, which is a hash map that gives the next node in the shortest path to each possible destination node in the simulation. We precompute the `Paths` attribute for every node in the graph prior to beginning the simulation. This precomputation step needs only be repeated when a parameter change occurs that alters the underlying node structure or availability.

The `GetNextNode()` method of the *Node* class provides the next node in the shortest path for a given destination. When a pedestrian lands on a

Method	Description
Move(NewNode)	Takes one argument: a <i>Node</i> object. When called, marks the node set to <i>Current</i> as unavailable, sets the <i>Current</i> attribute to NewNode, and marks the NewNode object as unavailable.
GetCurrentNode() SetCurrentNode(CurrentNode)	Returns or sets the current node that the pedestrian is occupying.
GetDestinationNode() SetDestinationNode(DestNode)	Returns or sets the destination node for the pedestrian.
GetTargetNext() SetTargetNext(TargetNext)	Returns or sets the target next node for the pedestrian in the shortest path to his destination.
GetEgressComplete() SetEgressComplete(Complete)	Returns or sets the current egress status of the pedestrian. True once the pedestrian has exited the SUI, and false otherwise.

Table 4: Methods of the *Pedestrian* class.

given node, this method is utilized to then provide the pedestrian with his next location in the following time step.

4.2.4 Intersection Class Interfaces

Within our simulation, we utilize a number of intersections that are parametrically closed and opened. Thus, we define key methods for the *Intersection* class in Table 6.

Intersections are used to limit the rate of flow of pedestrians through the SUI, increasing the reality of the simulation. As an intersection contains one or more nodes, when an intersection is closed, all the nodes within it are closed to additional pedestrian traffic. Pedestrians currently in the intersection as it closes are able to exit the intersection and continue walking, but no new pedestrian traffic is allowed into the intersection.

In our initial simulation model, we have not yet explored the effects of the intersections, but nevertheless present code for our intersection abstraction in *intersection.py*.

5 Final Notes

It should be noted that all of the authors contributed equally to this work. From a testing perspective, our simulation model has been tested primarily in a Mac OS X environment - specifically, OS X 10.11.3 with Python 2.7.

Thanks are due to the TAs and instructor for their assistance with our many questions.

Method	Description
GetAvailable() SetAvailable(Bool)	Gets or sets the current value of the <i>Available</i> attribute for the node. If the setter, must be a boolean true or false.
GetNextNode(DestinationNode)	Takes one argument: destination <i>Node</i> object. Returns the next <i>Node</i> object for the pedestrian to advance to based on the pedestrian's final destination.
GetPaths() SetPaths(Paths)	Returns or sets the Paths attribute of the node, which is a hash map that gives the next node in the shortest path to each possible destination node in the simulation.
GetNeighborNodes() SetNeighborNodes(ListNodes)	Returns or sets the Neighbors attribute of the node, corresponding to the nodes one edge length away. If the setter, takes one argument: list of <i>Node</i> objects.
GetNodeType()	Returns the NodeType Attribute of the node.
GetNodeID()	Returns the NodeID Attribute of the node.

Table 5: Methods of the *Node* class.

Method	Description
OpenMe()	When called, sets the <i>Available</i> attribute to true for every node in its <i>Nodes</i> array. Sets the <i>Open</i> attribute for the intersection to true.
CloseMe()	When called, sets the <i>Available</i> attribute to false for every node in its <i>Nodes</i> array. Sets the <i>Open</i> attribute for the intersection to false.
GetNodes()	Returns the list of <i>Node</i> objects that are part of the intersection.
SetNodes(ListNodes)	Takes one argument: list of <i>Node</i> objects. Sets the <i>Node</i> objects that make up the intersection.

Table 6: Methods of the *Intersection* class.

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