Your Name Goes Here
Math 387 Analysis I
Homework 8

Problem List
4.1 $\{1,3,6,9\}$
4.2 $\{1,2,5,8\}$

H/T: Last Names
Spring 2016
Due: Wednesday, March 30

Unassigned, but suggested: Problems 2,4,7,10 in Section 4.1
Unassigned, but suggested: Problems 3,4 in Section 4.2

8.1 Problem 4.1.1

Prove the product rule. Hint: Use $f(x)g(x) - f(c)g(c) = f(x)\big(g(x) - g(c)\big) + g(c)\big(f(x) - f(c)\big)$.

Solution. □

8.2 Problem 4.1.3

For $n \in \mathbb{Z}$, prove that x^n is differentiable and find the derivative, unless, of course, $n < 0$ and $x = 0$. Hint: Use the product rule.

Solution. □

8.3 Problem 4.1.6

Assume the inequality $|x - \sin(x)| \leq x^2$. Prove that sin is differentiable at 0, and find the derivative at 0.

Solution. □

8.4 Problem 4.1.9

Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a differentiable Lipschitz continuous function. Prove that f' is a bounded function.

Solution. □

8.5 Problem 4.2.1

Finish the proof of Proposition 4.2.6: Let $f : I \to \mathbb{R}$ be a differentiable function.

(i) f is increasing if and only if $f'(x) \geq 0$ for all $x \in I$.

(ii) f is decreasing if and only if $f'(x) \leq 0$ for all $x \in I$.

Note: Part (i) is proved in the book.

Solution. □

8.6 Problem 4.2.2

Finish the proof of Proposition 4.2.8: Let $f : (a, b) \to \mathbb{R}$ be continuous. Let $c \in (a, b)$ and suppose f is differentiable on (a, c) and (c, b).

(i) If $f'(x) \leq 0$ for $x \in (a, c)$ and $f'(x) \geq 0$ for $x \in (c, b)$, then f has an absolute minimum at c.

(ii) If $f'(x) \geq 0$ for $x \in (a, c)$ and $f'(x) \leq 0$ for $x \in (c, b)$, then f has an absolute maximum at c.

Note: Part (i) is proved in the book.

Solution. □

8.7 Problem 4.2.5

Suppose $f\colon \mathbb{R} \to \mathbb{R}$ is a function such that $|f(x) - f(y)| \leq |x - y|^2$ for all x and y. Show that $f(x) = C$ for some constant C. Hint: Show that f is differentiable at all points and compute the derivative.

Solution. ☐

8.8 Problem 4.2.8

Suppose $f\colon (a,b) \to \mathbb{R}$ and $g\colon (a,b) \to \mathbb{R}$ are differentiable functions such that $f'(x) = g'(x)$ for all $x \in (a,b)$, then show that there exists a constant C such that $f(x) = g(x) + C$.

Solution. ☐