Unassigned, but suggested: Problems 2,9 in Section 1.2

2.1 Problem 1.2.1

Prove that if $t > 0$ ($t \in \mathbb{R}$), then there exists an $n \in \mathbb{N}$ such that $\frac{1}{n^2} < t$.

Solution. □

2.2 Problem 1.2.7

Prove the *arithmetic-geometric mean inequality*. That is, for two positive real numbers x, y we have

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

Furthermore, equality occurs if and only if $x = y$.

Solution. □

2.3 Problem 1.2.8

Show that for any two real numbers x and y such that $x < y$, there exists an irrational number s such that $x < s < y$. Hint: Apply the density of $\mathbb{Q}$ to $\frac{x}{\sqrt{2}}$ and $\frac{y}{\sqrt{2}}$.

Solution. □

2.4 Problem 1.2.10

Let A and B be two nonempty bounded sets of nonnegative real numbers. Define the set $C := \{ab : a \in A, b \in B\}$. Show that C is a bounded set and that

$$\sup C = (\sup A)(\sup B) \qquad \text{and} \qquad \inf C = (\inf A)(\inf B).$$

The proof can be a bit long, so just prove C is bounded and the sup equality.

Solution. □