Unassigned, but suggested: Problems 7,10 in Section 5.3

11.1 Problem 5.3.1

Compute $\frac{d}{dx}\left(\int_{-x}^{x} e^{s^2}\ ds\right)$.

Solution. □

11.2 Problem 5.3.2

Compute $\frac{d}{dx}\left(\int_{0}^{x^2} \sin(s^2)\ ds\right)$.

Solution. □

11.3 Problem 5.3.4

Let $f\colon [a,b] \to \mathbb{R}$ be a continuous function. Let $c \in [a,b]$ be arbitrary. Define

$$F(x) := \int_c^x f.$$

Prove that F is differentiable and that $F'(x) = f(x)$ for all $x \in [a,b]$.

Solution. □

11.4 Problem 5.3.5

Prove *integration by parts*. That is, suppose F and G are continuously differentiable functions on $[a,b]$. Then prove

$$\int_a^b F(x)G'(x)\ dx = F(b)G(b) - F(a)G(a) - \int_a^b F'(x)G(x)\ dx.$$

Solution. □

11.5 Problem 5.3.6

Suppose F and G are continuously differentiable functions defined on $[a,b]$ such that $F'(x) = G'(x)$ for all $x \in [a,b]$. Using the fundamental theorem of calculus, show that F and G differ by a constant. That is, show that there exists a $C \in \mathbb{R}$ such that $F(x) - G(x) = C$.

Solution. □