7.4 Completeness and Compactness

Def Let (X,d) be a metric space and $K \subset X$. The set K is <u>compact</u> if for any collection of open sets $\{U_\lambda\}_{\lambda \in I}$ such that $K \subset \bigcup_{\lambda \in I} U_\lambda$, there exists a finite subset $\{\lambda_1, \lambda_2, \dots, \lambda_k\} \subset I$ such that $K \subset \bigcup_{i=1}^{k} U_{\lambda_i}$.

Topological spaces

non metric spaces
?

Metric Spaces

K compact $\rightarrow$ K closed & bounded

K compact $\leftrightarrow$ Every seq. in K has a convergent subseq. in K with limit in K.

$(\mathbb{R}^n$, euclidean metric$)$

K compact $\leftrightarrow$ K closed & bounded (Heine-Borel)

metric spaces that are not $\mathbb{R}^n$
?
(see exercises about total boundedness)

It's possible for closed balls to be non compact.

Def Let (X, d_X) and (Y, d_Y) be metric spaces.
$f: X \to Y$ is <u>continuous at $c \in X$</u> if $\forall \varepsilon > 0$ $\exists \delta > 0$ so that whenever $x \in X$ and $d_X(x, c) < \delta$, then $d_Y(f(x), f(c)) < \varepsilon$

Prop $f: X \to Y$ is continuous at $c \in X$ if and only if $\forall$ open neighborhood U of $f(c)$ in Y, the set $f^{-1}(U)$ is an open neighborhood of c in X.