Your Name Goes Here	<u>Problem List</u>	H/T: Last Names
Math 387 Analysis I	**2.4** {1,2,4}	Spring 2016
Homework 5	**2.5** {1,2}	Due: Wednesday, March 2

Unassigned, but suggested: Problems 3,5 in Section 2.4
Unassigned, but suggested: Problems 4,5,6,11 in Section 2.5

5.1 Problem 2.4.1

Prove that $\{\frac{n^2-1}{n^2}\}$ is Cauchy using directly the definition of Cauchy sequences.

Solution. □

5.2 Problem 2.4.2

Let $\{x_n\}$ be a sequence such that there exists a $0 < C < 1$ such that

$$|x_{n+1} - x_n| \leq C\,|x_n - x_{n-1}|\,.$$

Prove that $\{x_n\}$ is Cauchy. Hint: You can freely use the formula (for $C \neq 1$)

$$1 + C + C^2 + \cdots + C^n = \frac{1 - C^{n+1}}{1 - C}.$$

Solution. □

5.3 Problem 2.4.4

Let $\{x_n\}$ and $\{y_n\}$ be sequences such that $\lim y_n = 0$. Suppose that for all $k \in \mathbb{N}$ and for all $m \geq k$ we have

$$|x_m - x_k| \leq y_k.$$

Show that $\{x_n\}$ is Cauchy.

Solution. □

5.4 Problem 2.5.1

For $r \neq 1$, prove

$$\sum_{k=0}^{n-1} r^k = \frac{1 - r^n}{1 - r}.$$

Hint: Let $s := \sum_{k=0}^{n-1} r^k$, then compute $s(1-r) = s - rs$, and solve for s.

Solution. □

5.5 Problem 2.5.2

Prove that for $-1 < r < 1$ we have

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}.$$

Hint: Use the previous exercise.

Solution. □