Your Name Goes Here
Math 387 Analysis I
Homework 10

Problem List
5.2 {1,2,4,5,7}

H/T: Last Names
Spring 2016
Due: Wednesday, April 13

Unassigned, but suggested: Problem 6 in Section 5.2

10.1 Problem 5.2.1

Let f be in $\mathcal{R}[a,b]$. Prove that $-f$ is in $\mathcal{R}[a,b]$ and

$$\int_a^b -f(x)\ dx = -\int_a^b f(x)\ dx.$$

Solution. $\square$

10.2 Problem 5.2.2

Let f and g be in $\mathcal{R}[a,b]$. Prove that $f+g$ is in $\mathcal{R}[a,b]$ and

$$\int_a^b f(x)+g(x)\ dx = \int_a^b f(x)\ dx + \int_a^b g(x)\ dx.$$

Hint: Use Proposition 5.1.7 to find a single partition P such that $U(P,f)-L(P,f) < \epsilon/2$ and $U(P,g)-L(P,g) < \epsilon/2$.

Solution. $\square$

10.3 Problem 5.2.4

Prove the *mean value theorem for integrals*. That is, prove that if $f\colon [a,b] \to \mathbb{R}$ is continuous, then there exists a $c \in [a,b]$ such that $\int_a^b f = f(c)(b-a)$.

Solution. $\square$

10.4 Problem 5.2.5

If $f\colon [a,b] \to \mathbb{R}$ is a continuous function such that $f(x) \geq 0$ for all $x \in [a,b]$ and $\int_a^b f = 0$. Prove that $f(x) = 0$ for all x.

Solution. $\square$

10.5 Problem 5.2.7

If $f\colon [a,b] \to \mathbb{R}$ and $g\colon [a,b] \to \mathbb{R}$ are continuous functions such that $\int_a^b f = \int_a^b g$. Then show that there exists a $c \in [a,b]$ such that $f(c) = g(c)$.

Solution. $\square$