Unassigned, but suggested: Problem 4 in Section 7.3
Unassigned, but suggested: Problem 4 in Section 7.4

14.1 Problem 7.3.1

Let (X, d) be a metric space and let $A \subset X$. Let E be the set of all $x \in X$ such that there exists a sequence $\{x_n\}$ in A that converges to x. Show $E = \overline{A}$.

Solution. □

14.2 Problem 7.3.2

a) Show that $d(x, y) := \min\{1, |x - y|\}$ defines a metric on $\mathbb{R}$. b) Show that a sequence converges in $(\mathbb{R}, d)$ if and only if it converges in the standard metric. c) Find a bounded sequence in $(\mathbb{R}, d)$ that contains no convergent subsequence.

Solution. □

14.3 Problem 7.3.3

Prove Proposition 7.3.4 which says a convergent sequence in a metric space is bounded.

Solution. □

14.4 Problem 7.4.1

Let (X, d) be a metric space and A a finite subset of X. Show that A is compact.

Solution. □

14.5 Problem 7.4.2

Let $A = \{1/n : n \in \mathbb{N}\} \subset \mathbb{R}$. a) Show that A is not compact directly using the definition. b) Show that $A \cup \{0\}$ is compact directly using the definition.

Solution. □