Your Name Goes Here	Problem List	H/T: Last Names
Math 387 Analysis I	**2.2** {3,6,9}	Spring 2017
Homework 4	**2.3** {1,3,5,9}	Due: Thursday, February 16

Unassigned, but suggested: Problems 7, 8 in Section 2.2
Unassigned, but suggested: Problems 2, 4 in Section 2.3

4.1 Problem 2.2.3

Prove that if $\{x_n\}$ is a convergent sequence, $k \in \mathbb{N}$, then

$$\lim_{n\to\infty} x_n^k = \left(\lim_{n\to\infty} x_n\right)^k.$$

Hint: Use induction.

Solution. □

4.2 Problem 2.2.6

Let $x_n := \frac{1}{n^2}$ and $y_n := \frac{1}{n}$. Define $z_n := \frac{x_n}{y_n}$ and $w_n := \frac{y_n}{x_n}$. Do $\{z_n\}$ and $\{w_n\}$ converge? What are the limits? Can you apply Proposition 2.2.5? Why or why not?

Solution. □

4.3 Problem 2.2.9

Suppose $\{x_n\}$ is a sequence and suppose for some $x \in \mathbb{R}$, the limit

$$L := \lim_{n\to\infty} \frac{|x_{n+1} - x|}{|x_n - x|}$$

exists and $L < 1$. Show that $\{x_n\}$ converges to x.

Solution. □

4.4 Problem 2.3.1

Suppose $\{x_n\}$ is a bounded sequence. Define a_n and b_n as in Definition 2.3.1. Show that $\{a_n\}$ and $\{b_n\}$ are bounded.

Solution. □

4.5 Problem 2.3.3

Finish the proof of Proposition 2.3.6. That is, suppose $\{x_n\}$ is a bounded sequence and $\{x_{n_k}\}$ is a subsequence. Prove $\liminf_{n\to\infty} x_n \le \liminf_{k\to\infty} x_{n_k}$.

Solution. □

4.6 Problem 2.3.5

a) Let $x_n := \dfrac{(-1)^n}{n}$, find $\limsup x_n$ and $\liminf x_n$.

b) Let $x_n := \dfrac{(n-1)(-1)^n}{n}$, find $\limsup x_n$ and $\liminf x_n$.

Solution. □

4.7 Problem 2.3.9

If $S \subset \mathbb{R}$ is a set, then $x \in \mathbb{R}$ is a *cluster point* if for every $\epsilon > 0$, the set $(x - \epsilon, x + \epsilon) \cap S \setminus \{x\}$ is not empty. That is, if there are points of S arbitrarily close to x. For example, $S := \{1/n : n \in \mathbb{N}\}$ has a unique (only one) cluster point 0, but $0 \notin S$. Prove the following version of the Bolzano-Weierstrass theorem:

Theorem. *Let $S \subset \mathbb{R}$ be a bounded infinite set, then there exists at least one cluster point of S.*

Hint: If S is infinite, then S contains a countably infinite subset. That is, there is a sequence $\{x_n\}$ of distinct numbers in S.

Solution. □