Your Name Goes Here | Problem List | H/T: Last Names
Math 387 Analysis I | **3.3** {1,5,7} | Spring 2016
Homework 7 | **3.4** {2,3,7} | Due: Wednesday, March 23

Unassigned, but suggested: Problems 3,6,8 in Section 3.3
Unassigned, but suggested: Problems 1,4,5,6 in Section 3.4

7.1 Problem 3.3.1

Find an example of a discontinuous function $f\colon [0,1] \to \mathbb{R}$ where the intermediate value theorem fails.

Solution. □

7.2 Problem 3.3.5

Suppose $g(x)$ is a polynomial of odd degree d such that

$$g(x) = x^d + b_{d-1}x^{d-1} + \cdots + b_1 x + b_0,$$

for some real numbers $b_0, b_1, \ldots, b_{d-1}$. Show that there exists a $K \in \mathbb{N}$ such that $g(-K) < 0$. Hint: Make sure to use the fact that d is odd. You will have to use that $(-n)^d = -(n^d)$.

Solution. □

7.3 Problem 3.3.7

Suppose $f\colon [a,b] \to \mathbb{R}$ is a continuous function. Prove that the direct image $f([a,b])$ is a closed and bounded interval or a single number.

Solution. □

7.4 Problem 3.4.2

Let $f\colon (a,b) \to \mathbb{R}$ be a uniformly continuous function. Finish the proof of Theorem 3.4.6 by showing that the limit $\lim\limits_{x\to b} f(x)$ exists.

Solution. □

7.5 Problem 3.4.3

Show that $f\colon (c,\infty) \to \mathbb{R}$ for some $c > 0$ and defined by $f(x) := 1/x$ is Lipschitz continuous.

Solution. □

7.6 Problem 3.4.7

Let $f\colon (0,1) \to \mathbb{R}$ be a bounded continuous function. Show that the function $g(x) := x(1-x)f(x)$ is uniformly continuous.

Solution. □