Unassigned, but suggested: Problems 3,5 in Section 5.1

9.1 Problem 5.1.1

Let $f\colon [0,1] \to \mathbb{R}$ be defined by $f(x) := x^3$ and let $P := \{0, 0.1, 0.4, 1\}$. Compute $L(P,f)$ and $U(P,f)$.

Solution. □

9.2 Problem 5.1.2

Let $f\colon [0,1] \to \mathbb{R}$ be defined by $f(x) := x$. Show that $f \in \mathcal{R}[0,1]$ and compute $\int_0^1 f$ using the definition of the integral (but feel free to use the propositions of this section).

Solution. □

9.3 Problem 5.1.7

Suppose $f\colon [a,b] \to \mathbb{R}$ is Riemann integrable. Let $\epsilon > 0$ be given. Then show that there exists a partition $P = \{x_0, x_1, \ldots, x_n\}$ such that if we pick any set of numbers $\{c_1, c_2, \ldots, c_n\}$ with $c_k \in [x_{k-1}, x_k]$ for all k, then

$$\left| \int_a^b f - \sum_{k=1}^{n} f(c_k)\Delta x_k \right| < \epsilon.$$

Solution. □

9.4 Problem 5.1.10

Let $f\colon [0,1] \to \mathbb{R}$ be a bounded function. Let $P_n = \{x_0, x_1, \ldots, x_n\}$ be a uniform partition of $[0,1]$, that is, $x_j := {}^j\!/\!{}_n$. Is $\{L(P_n, f)\}_{n=1}^{\infty}$ always monotone? Yes/No: Prove or find a counterexample.

Solution. □

9.5 Problem 5.1.11

For a bounded function $f\colon [0,1] \to \mathbb{R}$ let $R_n := ({}^1\!/\!{}_n) \sum_{j=1}^{n} f({}^j\!/\!{}_n)$ (the uniform right hand rule). a) If f is Riemann integrable show $\int_0^1 f = \lim R_n$. b) Find an f that is not Riemann integrable, but $\lim R_n$ exists.

Solution. □