HW 2

Anthony Kolotov

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1. Write the following statements in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements. Let $m =$ "More people are moving into Miami" and $c =$ "the city gets more crowded."

   (a) More people are moving into Miami but the city does get more crowded.
   Symbolic form: $m \wedge c$

   (b) Neither more people are moving into Miami nor the city gets more crowded.
   Symbolic form: $\sim m \wedge \sim c$

2. Write the following statements in symbolic form using the symbols $\sim$, $\vee$, and $\wedge$ and the indicated letters to represent component statements. Let $H =$ "John is healthy," $S =$ "John is strong," and $W =$ "John is wise."

   (a) John is wise and healthy but not strong.
   Symbolic form: $W \wedge H \wedge \sim S$

   (b) John is not wise but he is healthy and strong.
   Symbolic form: $\sim W \wedge H \wedge S$

   (c) John is neither healthy, strong, nor wise.
   Symbolic form: $\sim H \wedge \sim S \wedge \sim W$

   (d) John is neither strong nor wise, but he is healthy.
   Symbolic form: $H \wedge \sim S \wedge \sim W$

   (e) John is wise, but he is not both healthy and strong.
   Symbolic form: $W \wedge \sim (H \wedge S)$

3. Write truth tables for the following statement forms (make sure you follow the right order of precedence to parse the logic formula).

   (a) $p \wedge \sim q$

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

(b) $\sim(p \wedge q) \vee (p \vee q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee q$	$\sim(p \wedge q) \vee (p \vee q)$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	F	T

(c) $p \wedge (q \wedge r)$

p	q	r	$q \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

(d) $p \wedge (\sim q \vee r)$

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

4. Use the truth table method to prove the following distributive laws.

(a) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

(b) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

5. Assume that x is a particular real number and use De Morgan's laws to write negations for the following statements.

   (a) $x \geq -10$
   Negation: $x < -10$

   (b) $-10 < x < 2$
   Negation: $x \leq -10 \vee x \geq 2$

   (c) $x \leq -10$ or $x > 2$
   Negation: $-10 < x \leq 2$

6. Use the truth tables method to establish which of the following statement forms are tautologies, which are contradictions, and which are neither.

   (a) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

p	q	$\sim p$	$p \wedge q$	$p \wedge \sim q$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	F	F	T
F	F	T	F	F	T

This is a tautology

   (b) $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

p	q	r	$\sim p$	$\sim q$	$(\sim p \wedge q)$	$(q \wedge r)$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$
T	T	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	T	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	F	F	F

This is a contradiction

(c) $(\sim p \vee q) \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \wedge \sim q$	$(\sim p \vee q) \vee (p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

This is a tautology

(d) $(p \to r) \leftrightarrow (q \to r)$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \leftrightarrow (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

This is neither

(e) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \to r$	$(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	F	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T

This is a neither

7. Write each of the following three statements in symbolic form and determine which pairs are logically equivalent. Make sure to include truth tables and a brief explanation.

(a) If it walks like a duck and it talks like a duck, then it is a duck.
Symbolic form: $(P \wedge Q) \to D$

(b) Either it does not walk like a duck or it does not talk like a duck, or it is a duck.
Symbolic form: $(\sim P \vee \sim Q) \vee D$

(c) If it does not walk like a duck and it does not talk like a duck, then it is not a duck.
Symbolic form: $(\sim P \wedge \sim Q) \to \sim D$

Equivalence: The first and third statements are logically equivalent because they both express the condition for a duck to exist based on walking and talking. The second statement is a different logical form but equivalent to the first in certain logical contexts.

8. Use the logical equivalence $p \to q \equiv \sim p \vee q$ and de Morgan's laws to rewrite the following statement forms using $\wedge$ and $\sim$ only (that is, you should eliminate all $\vee$, $\to$ and $\leftrightarrow$ symbols in your answer statement forms).

   (a) $p \wedge \sim q \to r$
   Rewritten form: $\sim (p \wedge \sim q) \vee r \equiv \sim p \vee q \vee r$

   (b) $p \vee \sim q \to r \vee q$
   Rewritten form: $\sim p \vee q \vee r \vee q \equiv \sim p \vee r$

   (c) $(p \to (q \to r)) \leftrightarrow ((p \wedge q) \to r)$
   Rewritten form: $\sim p \vee (q \vee r) \leftrightarrow \sim p \wedge q \vee r$

9. Rewrite the following statements which use "necessary condition" or "sufficient condition" form and turn them into statements using "if-then" form.

   (a) A necessary condition for Jon's team to win the championship is that it wins the rest of its games.

      i. If Jon's team wins the rest of its games, then it will be a necessary condition for winning the championship.

   (b) Winning this championship is a necessary condition for Andy to qualify for the Paris 2024 Olympics Games.

      i. If Andy wins this championship, then he will quality for the Paris 2024 Olympics games.