HW 6

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1. A store sells clothes for men. It has 3 kinds of jackets, 7 kinds of shirts, and 5 kinds of pants. Find the number of ways a person can buy:

   (a) one of the items;
   3+7+5 = 15

   (b) one of each of the three kinds of clothes.
   3*7*5 = 105

2. A class has 10 male students and 8 female students. Find the number of ways the class can elect:

   (a) a class representative;
   10+8 = 18

   (b) 2 class representatives, one male and one female;
   10*8 = 80

   (c) a class president and a vice-president.
   P(18,2) = 18 * 17 = 306

3. Find the number of three-letter words using only the letters from {A,B,C,D,E,F,G} without repetition.

   P(7,3) = $\frac{7!}{7-3}! = \frac{7*6*5*4!}{4!} = 7*6*5 = 210$

4. Find the number of six-letter words that can be formed using the letters of the word **BANANA**.

   B, A happens 3 times, N happens 2 times = total 6

   $\frac{6!}{3!\times 2!} = \frac{720}{12} = 60$

5. A multiple-choice test contains 12 questions. There are five possible answers for each question.

   (a) In how many ways can a student answer the questions if they answer every question?
   5^{12}

(b) In how many ways can a student answer the questions if they can leave answers blank?

6^12 , 5 possible answers + 1 blank option

6. DNA molecules consist of two strands consisting of blocks known as nucleotides. Each nucleotide contains sub-components called bases. There are four types of bases: A, T, C, and G. How many 5-element DNA sequences:

(a) end with A?

first 4 can be any choice, 4 choices

last base must be A, - 1 choice

$4^4 = 256$

(b) start with T and end with G?

T and G, - 2 choices

$4^3 = 64$

(c) contain only A and T?

A and T, 2 choices

$2^5 = 32$

(d) do not contain C?

3 choices, A,T,G

$3^5 = 243$

7. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

1 governor, 2 senators = total options 3

3^50 total ways to form a committee

8. How many bit strings of length 10 contain:

(a) exactly four 1s?

choose 4 positions for 1s out of 10

$\binom{10}{4} = \frac{10!}{4!(10!-4!)} = \frac{10!}{4!(6!)} = 210$

(b) at most four 1s?

sum of cases where there are 0,1,2,3 or 4 ones:

$\binom{10}{1} + \binom{10}{2} + \binom{10}{3} + \binom{10}{4} = 1 + 10 + 45 + 120 + 210 = 386$

(c) at least four 1s?

2

(d) an equal number of 0s and 1s?

choose 5 positions for 1s, the remaining 5 will be 0s

$\binom{10}{5} = \frac{10!}{5!5!} = 252$

9. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

   (a) How many socks must he take out to be sure that he has at least two socks of the same color?

   12 brown and 12 black, worst case is picking

   (b) How many socks must he take out to be sure that he has at least two black socks?

10. Suppose that all students of FIU class 2027 were born in 2005. They went into one of the ten large classrooms for orientation in August 2023. What is the minimum number of students to guarantee that there were two students in some classroom born on exactly the same day?

11. (20 points extra credit) Consider the following expression:

$$5 \diamond 5 \diamond 5 \diamond 5 \diamond 5.$$

You can replace each of the four $\diamond$ symbols with either $+$ or $\times$ operator. You may also insert one or more pairs of parentheses to control the order of operations. How many different values can you get from such expressions? For example,

$$5 + 5 + 5 + 5 + 5 = 25,$$
$$5 + (5 + 5) \times 5 + 5 = 60.$$

So 25 and 60 are two different values you can get from such expressions.