

HW 1

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January 13, 2025

1. (a) Is $0 = 0$?

- No, because 0 is a symbolic placeholder for the number, whereas $\{0\}$ is a set that contains an element 0.

(b) Is $\{0\} = \emptyset$?

- No, since the set contains an element, and it does not equal nothingness

(c) How many elements are there in the set $\{2, 3, 5, 7, 11, 2, 3, 5, 13\}$?

- 6 elements

(d) How many elements are there in the set $\{1, \{1\}, \{1, \{1\}\}\}$?

- 3 elements

2. Which of the following sets are equal (recall that we use \mathbb{Z}^+ to denote the set of non-negative integers, and \mathbb{Z}^{++} to denote the set of positive integers)? Be careful of the difference between discrete sets (finite sets) and infinite sets.

$$A = \{0, 1, 2, 3\}$$

$$B = \{1, 2, 3\}$$

$$C = \{x \in \mathbb{R} \mid 1 \leq x \leq 3\}$$

$$D = \{x \in \mathbb{R} \mid 1 < x < 3\}$$

$$E = \{x \in \mathbb{R}^+ \mid 1 < x^2 < 9\}$$

$$F = \{x \in \mathbb{Z}^+ \mid -3 \leq x \leq 3\}$$

$$G = \{x \in \mathbb{Z}^{++} \mid -1 < x < 4\}$$

- A is a finite set of $\{0, 1, 2, 3\}$
- B is a finite set of $\{1, 2, 3\}$
- C is a continuous set $[1, 3]$,
- D is a continuous set $(1, 3)$
- E, simplifying the equation, we get two conditions:
 $x > 1$ (x can be restricted to positive values)
 $x < 3$
 $(1, 3)$
- F, x in set of non-negative integers,
 $\{1, 2, 3\}$

- G, x in set of positive integers,
 $\{1,2,3\}$

- $B = F = G = \{1,2,3\}$

- $D = E = (1,3)$

3. Let $A = \{a, b, c, d\}$, $B = \{a, b, f\}$, and $C = \{b, f\}$. Answer each of the following questions. Give reasons for your answers.

(a) Is $B \subseteq A$?

- No, because B contains an element that is not in A which is f

(b) Is $C \subseteq A$?

- No, because C contains an element that is not in A which is f

(c) Is $C \subseteq C$?

- Yes, because a set can be a *subset* of itself

(d) Is C a proper subset of B ?

- Yes, because C contains elements that B has, *and* B has an element which isn't in C (a).

I do have a question regarding the difference between \subset and \subseteq . As I understand it, \subset is a subset, and \subseteq is a proper subset. If you could please correct in my understanding, professor.

4. Let $A = \{x, y, z\}$ and $B = \{a, b\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

(a) $A \times B$

- $\{(x,a),(x,b),(y,a),(y,b),(z,a),(z,b)\}$
6 elements

(b) $B \times A$

- $\{(a,x),(a,y), (a,z),(b,x),(b,y),(b,z)\}$
6 elements

(c) $A \times A$

- $\{(x,x),(x,y),(x,z),(y,x),(y,y),(y,z),(z,x),(z,y),(z,z)\}$
9 elements

(d) $B \times B$

- $\{(a,a),(a,b),(b,a),(b,b)\}$
4 elements

5. Let $A = \{2, 3, 5\}$ and $B = \{6, 8, 10\}$ and define the relation R from A to B as follows: for all $(x, y) \in A \times B$, $(x, y) \in R$ if and only if $\frac{y}{x}$ is a *integer*.

(a) Write R as a set of ordered pairs

- $6/2 = 3$
 $8/2 = 4$
 $10/2 = 5$
 $6/3 = 2$
 $8/3 = 2.667$
 $10/3 = 3.33$
 $6/5 = 1.2$
 $8/5 = 1.6$
 $10/5 = 2$
- $R = \{(2,6),(2,8),(2,10),(3,6),(5,10)\}$

(b) Write the domain and co-domain of R

- Domain = $\{2,3,5\}$
Co-domain = $\{6,8,10\}$

(c) Draw an arrow Diagram for R

X		Y
2	→	6,8,10
3	→	6
5	→	10

6. Can you modify the domain of the relation R in the previous question to turn R into a function? If so, how?

- In order to make R into a function, the mapping of x would have to be restricted to a single y value. In this case $x = 2$ maps to multiple y values. You would need to make that mapping of x to a single y value.

7. Let $A = \{0, 2\}$ and $B = \{1, 3, 5\}$ and define the relations U, V, W from A and B as follows:

$$\begin{aligned} (x, y) \in U & \text{ if and only if } 4 < x + y < 6 \\ (x, y) \in V & \text{ if and only if } y - 1 = \frac{x}{2} \\ W & = \{(0, 3), (2, 1), (0, 5)\} \end{aligned}$$

(a) Following the definition of W , use the set-roster notation to enumerate all elements in U and V .

- for U :
 $0+1 = 1, 4 < 1 < 6?$ No
 $0+3 = 3, 4 < 3 < 6?$ No
 $0+5 = 5, 4 < 5 < 6?$ Yes
 $2+1 = 3, 4 < 3 < 6?$ No
 $2+3 = 5, 4 < 5 < 6?$ Yes
 $2+5 = 7, 4 < 7 < 6?$ No
- $U = \{(0,5),(2,3)\}$
- for $V = \{(0,1)\}$

(b) Indicate whether any of the relations U, V , and W are functions from A to B . Justify your answers.

- U is a function since it has one x value that maps to one y value.
- V is not a function because it does not map out all the elements in A . 0 is mapped to 1, but 2 has no mapping
- W is not a function since 0 maps to two values, and a function can only map one x value corresponds to one y value.

8. Define a relation T from \mathbb{R} to \mathbb{R} as follows: For all real numbers x and y , $(x, y) \in T$, if and only if, they satisfy the equation $y^2 - 2x^2 = 100$ is T a function? Briefly explain your answer.

- We are given an equation $y^2 - 2x^2 = 100$. In order to find out if T is a function, x has to have only one corresponding y .
Let's solve for y in $y^2 - 2x^2 = 100$:
 $y^2 = -2x^2 + 100$
 $y = \pm\sqrt{-2x^2 + 100}$
since we are taking the square root, we have *two* possible x values corresponding a y value.
Therefore, T does not follow the definition of a function.