HW 1

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1. (a) Is $0 = 0$?

   - No, because 0 is a symbolic placeholder for the number, whereas {0} is a set that contains an element 0.

   (b) Is $\{0\} = \emptyset$?

   - No, since the set contains an element, and it does not equal nothingness

   (c) How many elements are there in the set {2, 3, 5, 7, 11, 2, 3, 5, 13}?

   - 6 elements

   (d) How many elements are there in the set {1, {1}, {1, {1}}}?

   - 3 elements

2. Which of the following sets are equal (recall that we use $\mathbb{Z}^+$ to denote the set of non-negative integers, and $\mathbb{Z}^{++}$ to denote the set of positive integers)? Be careful of the difference between discrete sets (finite sets) and infinite sets.

   $A = \{0, 1, 2, 3\}$
   $B = \{1, 2, 3\}$
   $C = \{x \in \mathbb{R} | 1 \leq x \leq 3\}$
   $D = \{x \in \mathbb{R} | 1 < x < 3\}$
   $E = \{x \in \mathbb{R}^+ | 1 < x^2 < 9\}$
   $F = \{x \in \mathbb{Z}^+ | -3 \leq x \leq 3\}$
   $G = \{x \in \mathbb{Z}^{++} | -1 < x < 4\}$

   - A is a finite set of {0,1,2,3}
   - B is a finite set of {1,2,3}
   - C is a continuous set $[1, 3]$, including decimal values
   - D is a continuous set (1,3), including decimal values
   - E, simplifying the equation, we get two conditions:
     x>1 (x can be restricted to positive values)
     x<3
     (1,3)
   - F, x in set of non-negative integers,
     {1,2,3}

- G, x in set of positive integers,
  {1,2,3}
- B = F = G = {1,2,3}

3. Let $A = \{a, b, c, d\}, B = \{a, b, f\},$ and $C = \{b, f\}$. Answer each of the following questions. Give reasons for your answers.
   (a) Is $B \subseteq A$?

   - No, because B contains an element that is not in A which is f

   (b) Is $C \subseteq A$?

   - No, because C contains an element that is not in A which is f

   (c) Is $C \subseteq C$?

   - Yes, because a set can be a *subset* of itself

   (d) Is C a proper subset of B?

   - Yes, because C contains elements that B has, *and* B has an element which isn't in C (a).

4. Let $A = \{x, y, z\}$ and $B = \{a, b\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

   (a) $A \times B$

   - {(x,a),(x,b),(y,a),(y,b),(z,a),(z,b)}

   (b) $B \times A$

   - {(a,x),(a,y), (a,z),(b,x),(b,y),(b,z)}

   (c) $A \times A$

   - {(x,x),(x,y),(x,z),(y,x),(y,y),(y,z),(z,x),(z,y),(z,z)}

   (d) $B \times B$

   - {(a,a),(a,b),(b,a),(b,b)}

5. Let $A = \{2, 3, 5\}$ and $B = \{6, 8, 10\}$ and define the relation R from A to B as follows: for all $(x, y) \in A \times B, (x, y) \in R$ if and only if $\frac{y}{x}$ is a *integer*.

   (a) Write R as a set of ordered pairs

   - 6/2 = 3
     8/2 = 4
     10/2 = 5
     6/3 = 2
     8/3 = 2.667
     10/3 = 3.33
     6/5 = 1.2
     8/5 = 1.6
     10/5 = 2

- R = {(6,2),(8,2),(10,2),(6,3),(10,5)}

(b) Write the domain and co-domain of R

- Domain = {2,3,5}
  Co-domain = {6,8,10}

(c) Draw an arrow Diagram for R

X		Y
2	$\rightarrow$	6,8,10
3	$\rightarrow$	6
5	$\rightarrow$	10

6. Can you modify the domain of the relation R in the previous question to turn R into a function? If so, how?

   - In order to make R into a function, the mapping of x would have to be restricted to a single y value. In this case $x = 2$ maps to multiple y values.

7. Let $A = \{0, 2\}$ and $B = \{1, 3, 5\}$ and define the relations U, V, W from A and B as follows:

$$(x, y) \in U \text{ if and only if } 4 < x + y < 6$$
$$(x, y) \in V \text{ if and only if } y - 1 = \tfrac{x}{2}$$
$$W = \{(0, 3), (2, 1), (0, 5)\}$$

(a) Following the definition of W, use the set-roster notation to enumerate all elements in U and V.

- for U:
  0+1 = 1, 4 < 1 < 6? No
  0+3 = 3, 4 < 3 < 6? No
  0+5 = 5, 4 < 5 < 6? Yes
  2+1 = 3, 4 < 3 < 6? No
  2+3 = 5,4 < 5 < 6? Yes
  2+5 = 7,4 < 7 < 6? No
- U = {(0,5),(2,3)}
- for V = { }

(b) Indicate whether any of the relations U, V , and W are functions from A to B. Justify your answers.

- W is not a function since 0 maps to two values, which breaks the rule of a function: one x value corresponds to one y value.

8. Define a relation T from $\mathbb{R}$ to $\mathbb{R}$ as follows: For all real numbers x and y, $(x, y) \in T$, if and only if, they satisfy the equation $y^2 - 2x^2 = 100$ is T a function? Briefly explain your answer.

   - We are given an equation $y^2 - 2x^2 = 100$. In order to find out if T is a function, x has to has to have only one corresponding y. Let's solve for y in $y^2 - 2x^2 = 100$:
     $y^2 = -2x^2 + 100$
     $y = \pm\sqrt{-2x + 100}$

since we are taking the square root, we have *two* possible x values corresponding a y value.
Therefore, T does not follow the definition of a function.