

# HW 1

Anthony Kolotov

January 29, 2025

1. (a) Is  $0 = 0$ ?

- No, because 0 is a symbolic placeholder for the number, whereas  $\{0\}$  is a set that contains an element 0.

(b) Is  $\{0\} = \emptyset$ ?

- No, since the set contains an element, and it does not equal nothingness

(c) How many elements are there in the set  $\{2, 3, 5, 7, 11, 2, 3, 5, 13\}$ ?

- 6 elements

(d) How many elements are there in the set  $\{1, \{1\}, \{1, \{1\}\}\}$ ?

- 3 elements

2. Which of the following sets are equal (recall that we use  $\mathbb{Z}^+$  to denote the set of non-negative integers, and  $\mathbb{Z}^{++}$  to denote the set of positive integers)? Be careful of the difference between discrete sets (finite sets) and infinite sets.

$$A = \{0, 1, 2, 3\}$$

$$B = \{1, 2, 3\}$$

$$C = \{x \in \mathbb{R} \mid 1 \leq x \leq 3\}$$

$$D = \{x \in \mathbb{R} \mid 1 < x < 3\}$$

$$E = \{x \in \mathbb{R}^+ \mid 1 < x^2 < 9\}$$

$$F = \{x \in \mathbb{Z}^+ \mid -3 \leq x \leq 3\}$$

$$G = \{x \in \mathbb{Z}^{++} \mid -1 < x < 4\}$$

- A is a finite set of  $\{0, 1, 2, 3\}$
- B is a finite set of  $\{1, 2, 3\}$
- C is a continuous set  $[1, 3]$ ,
- D is a continuous set  $(1, 3)$
- E, simplifying the equation, we get two conditions:  
 $x > 1$  (x can be restricted to positive values)  
 $x < 3$   
 $(1, 3)$
- F, x in set of non-negative integers,  
 $\{1, 2, 3\}$

- $G, x$  in set of positive integers,  
 $\{1,2,3\}$

- $B = F = G = \{1,2,3\}$

- $D = E = (1,3)$

3. Let  $A = \{a, b, c, d\}$ ,  $B = \{a, b, f\}$ , and  $C = \{b, f\}$ . Answer each of the following questions. Give reasons for your answers.

(a) Is  $B \subseteq A$ ?

- No, because  $B$  contains an element that is not in  $A$  which is  $f$

(b) Is  $C \subseteq A$ ?

- No, because  $C$  contains an element that is not in  $A$  which is  $f$

(c) Is  $C \subseteq C$ ?

- Yes, because a set can be a *subset* of itself

(d) Is  $C$  a proper subset of  $B$ ?

- Yes, because  $C$  contains elements that  $B$  has, *and*  $B$  has an element which isn't in  $C$  ( $a$ ).

I do have a question regarding the difference between  $\subset$  and  $\subseteq$ . As I understand it,  $\subset$  is a subset, and  $\subseteq$  is a proper subset. If you could please correct in my understanding, professor.

4. Let  $A = \{x, y, z\}$  and  $B = \{a, b\}$ . Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

(a)  $A \times B$

- $\{(x,a),(x,b),(y,a),(y,b),(z,a),(z,b)\}$   
6 elements

(b)  $B \times A$

- $\{(a,x),(a,y),(a,z),(b,x),(b,y),(b,z)\}$   
6 elements

(c)  $A \times A$

- $\{(x,x),(x,y),(x,z),(y,x),(y,y),(y,z),(z,x),(z,y),(z,z)\}$   
9 elements

(d)  $B \times B$

- $\{(a,a),(a,b),(b,a),(b,b)\}$   
4 elements

5. Let  $A = \{2, 3, 5\}$  and  $B = \{6, 8, 10\}$  and define the relation  $R$  from  $A$  to  $B$  as follows: for all  $(x, y) \in A \times B$ ,  $(x, y) \in R$  if and only if  $\frac{y}{x}$  is a *integer*.

(a) Write  $R$  as a set of ordered pairs

- $6/2 = 3$   
 $8/2 = 4$   
 $10/2 = 5$   
 $6/3 = 2$   
 $8/3 = 2.667$   
 $10/3 = 3.33$   
 $6/5 = 1.2$   
 $8/5 = 1.6$   
 $10/5 = 2$
- $R = \{(2,6),(2,8),(2,10),(3,6),(5,10)\}$

(b) Write the domain and co-domain of  $R$

- Domain =  $\{2,3,5\}$   
Co-domain =  $\{6,8,10\}$

(c) Draw an arrow Diagram for  $R$

<b>X</b>		<b>Y</b>
2	→	6,8,10
3	→	6
5	→	10

6. Can you modify the domain of the relation  $R$  in the previous question to turn  $R$  into a function? If so, how?

- In order to make  $R$  into a function, the mapping of  $x$  would have to be restricted to a single  $y$  value. In this case  $x = 2$  maps to multiple  $y$  values. You would need to make that mapping of  $x$  to a single  $y$  value.

7. Let  $A = \{0, 2\}$  and  $B = \{1, 3, 5\}$  and define the relations  $U, V, W$  from  $A$  and  $B$  as follows:

$$\begin{aligned} (x, y) \in U & \text{ if and only if } 4 < x + y < 6 \\ (x, y) \in V & \text{ if and only if } y - 1 = \frac{x}{2} \\ W & = \{(0, 3), (2, 1), (0, 5)\} \end{aligned}$$

(a) Following the definition of  $W$ , use the set-roster notation to enumerate all elements in  $U$  and  $V$ .

- for  $U$ :  
 $0+1 = 1, 4 < 1 < 6?$  No  
 $0+3 = 3, 4 < 3 < 6?$  No  
 $0+5 = 5, 4 < 5 < 6?$  Yes  
 $2+1 = 3, 4 < 3 < 6?$  No  
 $2+3 = 5, 4 < 5 < 6?$  Yes  
 $2+5 = 7, 4 < 7 < 6?$  No
- $U = \{(0,5),(2,3)\}$
- for  $V = \{(0,1)\}$

(b) Indicate whether any of the relations  $U, V$ , and  $W$  are functions from  $A$  to  $B$ . Justify your answers.

- U is a function since it has one  $x$  value that maps to one  $y$  value.
- V is not a function because it does not map out all the elements in  $A$ . 0 is mapped to 1, but 2 has no mapping
- W is not a function since 0 maps to two values, and a function can only map one  $x$  value corresponds to one  $y$  value.

8. Define a relation  $T$  from  $\mathbb{R}$  to  $\mathbb{R}$  as follows: For all real numbers  $x$  and  $y$ ,  $(x, y) \in T$ , if and only if, they satisfy the equation  $y^2 - 2x^2 = 100$  is  $T$  a function? Briefly explain your answer.

- We are given an equation  $y^2 - 2x^2 = 100$ . In order to find out if  $T$  is a function,  $x$  has to have only one corresponding  $y$ .  
Let's solve for  $y$  in  $y^2 - 2x^2 = 100$ :  

$$y^2 = -2x^2 + 100$$

$$y = \pm\sqrt{-2x^2 + 100}$$
since we are taking the square root, we have *two* possible  $x$  values corresponding a  $y$  value.  
Therefore,  $T$  does not follow the definition of a function.

$\therefore$