Filter-Bank Structure of Multivariate EMD in multivariate Fractional Gaussian Noise

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1 Simulations Setup

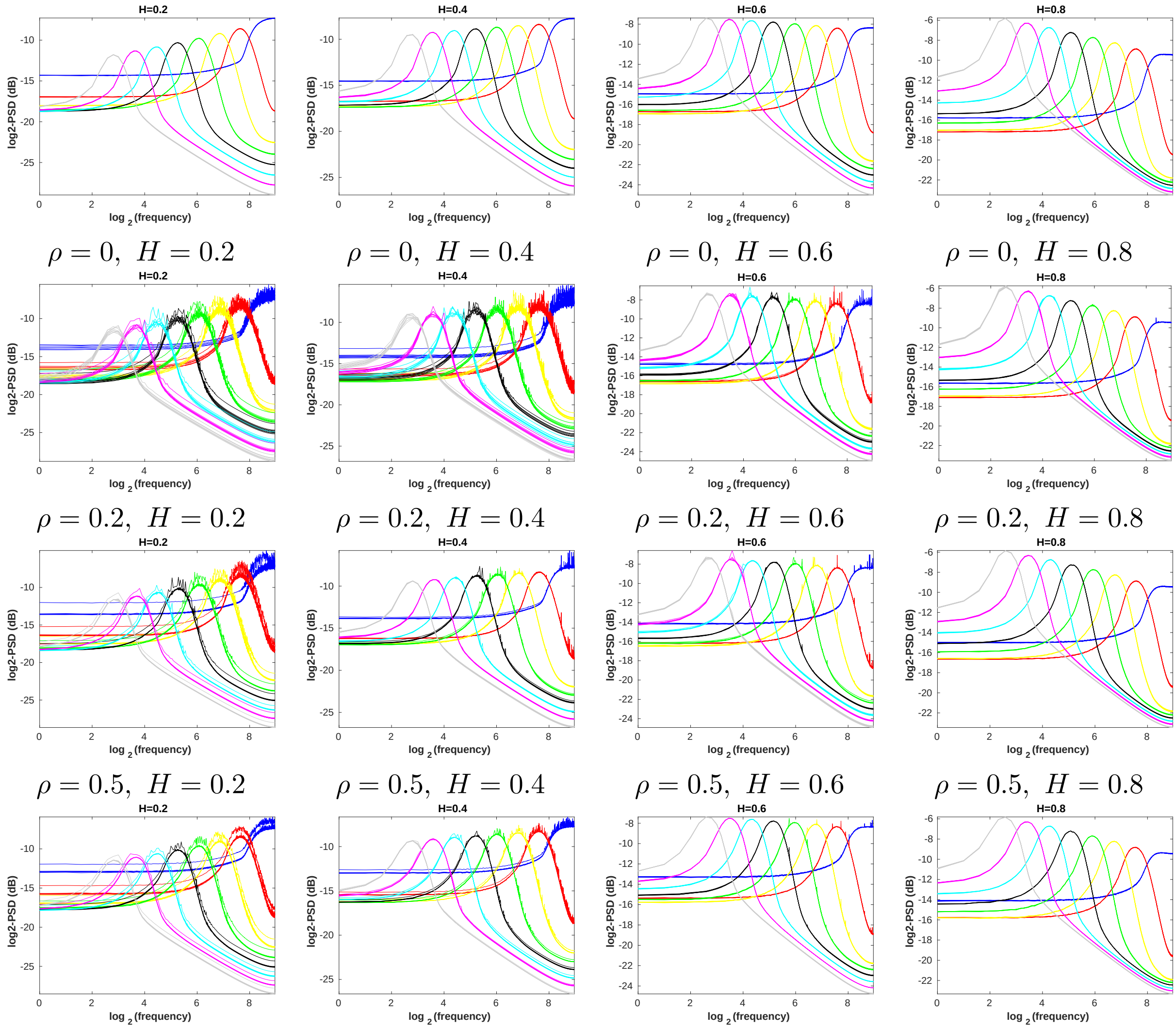
We used Wood&Chan's algorithm to simulate multivariate fractional Gaussian noise (mfGn) [1]. The resulting mfGn has the following:

- The number of channels is $p = 8$
- The correlation parameters used are $\rho \in \{0, 0.2, 0.5, 0.8\}$
- The Hurst exponent H takes all values in $\{0.20.40.60.8\}$. It should be noted that when $H = 0.5$, the fGn reduces to white Gaussian noise, a topic that was already studied in [2].
- the number of samples for each process is set to 1000.
- The number of Monte Carlo (MC) simulations is set to 5000 in order to be as precise as possible with average results.
- Because the number of obtained IMFs differs from simulation to another, we decided to keep the first 8 IMFs for all simulations.

2 IMFs Power-Spectra

In figure 1, we show the power spectrum of the obtained IMFs for different parameter combination (ρ, H). These results are the mean over 5000 MC runs. Figure 2 shows the normalized power spectrum, the normalisation is performed according to the following equation:

$$S_{m,p,H}(f) = \rho_H^{a(m-l)} S_{l,p,H}(\rho_H^{m-l} f) \tag{1}$$



$\rho = 0,\ \ H = 0.2$ $\rho = 0,\ \ H = 0.4$ $\rho = 0,\ \ H = 0.6$ $\rho = 0,\ \ H = 0.8$

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$\rho = 0.8,\ \ H = 0.2$ $\rho = 0.8,\ \ H = 0.4$ $\rho = 0.8,\ \ H = 0.6$ $\rho = 0.8,\ \ H = 0.8$

Fig. 1: The power spectra of IMFs as a function of $\log_2(f)$, for $H \in \{0.2, 0.4, 0.6, 0.8\}$ and $\rho \in \{0, 0.2, 0.5, 0.8\}$. Overlapping of the frequency bands corresponds to the same-index IMFs, showing the mode alignment. We notice some leak in some values of (ρ, H), this could be due to the correlation between channels.

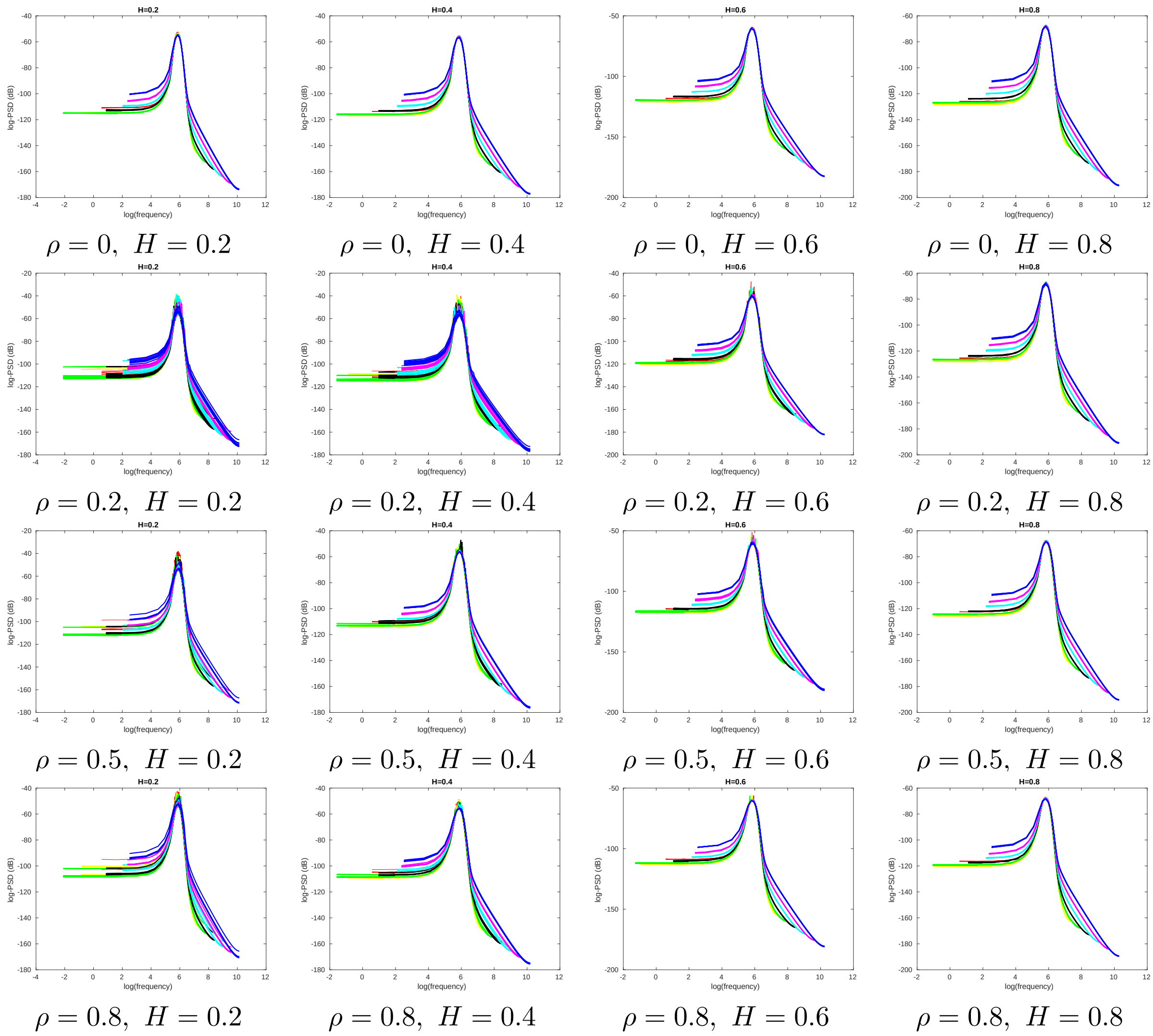
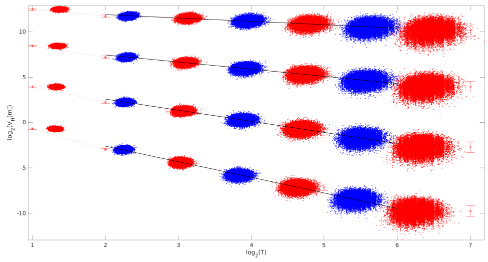
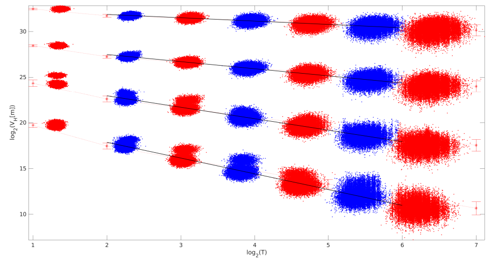


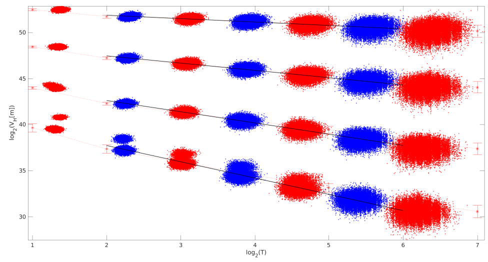
Fig. 2: The normalized power spectra of IMFs as a function of $\log_2(f)$, for $H \in \{0.2, 0.4, 0.6, 0.8\}$ and $\rho \in \{0, 0.2, 0.5, 0.8\}$. Overlapping of the frequency bands is obtained using (1).



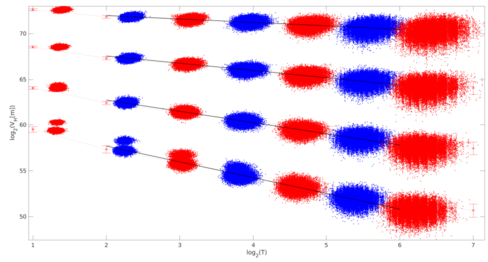
$\rho = 0$



$\rho = 0.2$

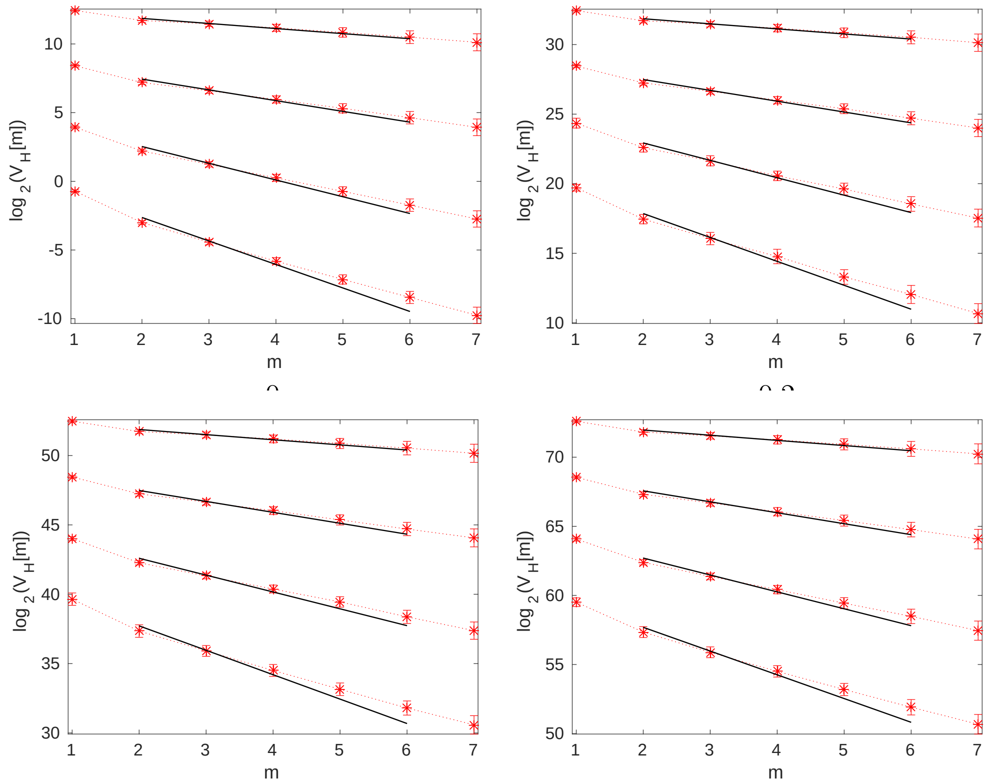


$\rho = 0.5$



$\rho = 0.8$

Fig. 3: Relationship between $V_H[m]$ and $log_2(T)$ for different values of ρ.
The grouped blue and red dots from the upper left to the lower right are the mean energy density (estimation of the variance) as a function of the spectrum-weighted mean period for IMFs $1-7$ over all realizations. The solid black line cutting through the clouds is the weighted linear fit within the IMF indices range $m = 2$ to $m = 6$. For the sake of clarity, all curves have been shifted upwards to avoid overlapping.



$\rho = 0.5$ $\rho = 0.8$

Fig. 4: Relationship between $V_H[m]$ and m for different values of ρ. The values of the empirical variance estimate are given by red dotted lines for different values of H. The error bars correspond to the standard deviation associated with the realizations. The black thin line is the weighted linear fit within the IMF indices range $m = 2$ to $m = 6$.
(†) scale alignment is almost satisfied, it is highly dependent on the p-dimensional envelopes of the original signal. However, in NA-MEMD, the scale misalignment is reduced.

2.1 Variance distributions

3 Correlograms

We choose to compare all the channels corresponding to the n^{th} IMF. We report in figures 5, 6, 7, 8 alignment results. These figures shows the inter-channel correlations, that is, the correlation matrix $Corr$. $Corr$ is the correlation matrix between $IMF_i^1, IMF_i^2, \ldots, IMF_i^P$, where $i \in 1, 2, ..., N$ is the IMF index and P is the number of channels

$$Corr_{H,\rho} = \begin{pmatrix} 1 & c_i^{1,2} & \cdots & c_i^{1,P} \\ c_i^{1,2} & 1 & \cdots & c_i^{2,P} \\ \vdots & \vdots & \ddots & \vdots \\ c_i^{P,1} & c_i^{P,2} & \cdots & 1 \end{pmatrix}$$

Where $c_i^{p,l}$ is correlation coefficient between IMF_i^p and IMF_i^l. In other words, it is the correlation coefficient between the i^{th} IMF of channel p and the same i^{th} IMF of channel l. So $Corr_{H,\rho}$ is a measure of inter-channel correlation.

We notice that when the correlation parameter ρ is different from zero, the correlation between channels is seen clearly, therefore MEMD conserves the inter-channel correlation of the mfGn process.

References

[1] J. F. Coeurjolly, P. O. Amblard and S. Achard *On multivariate fractional brownian motion and multivariate fractional Gaussian noise.* Signal Processing Conference, 2010 18th European, Aalborg, 2010, pp. 1567-1571.

[2] N. ur Rehman and D. P. Mandic, *Filter Bank Property of Multivariate Empirical Mode Decomposition.* in IEEE Transactions on Signal Processing, vol. 59, no. 5, pp. 2421-2426, May 2011.

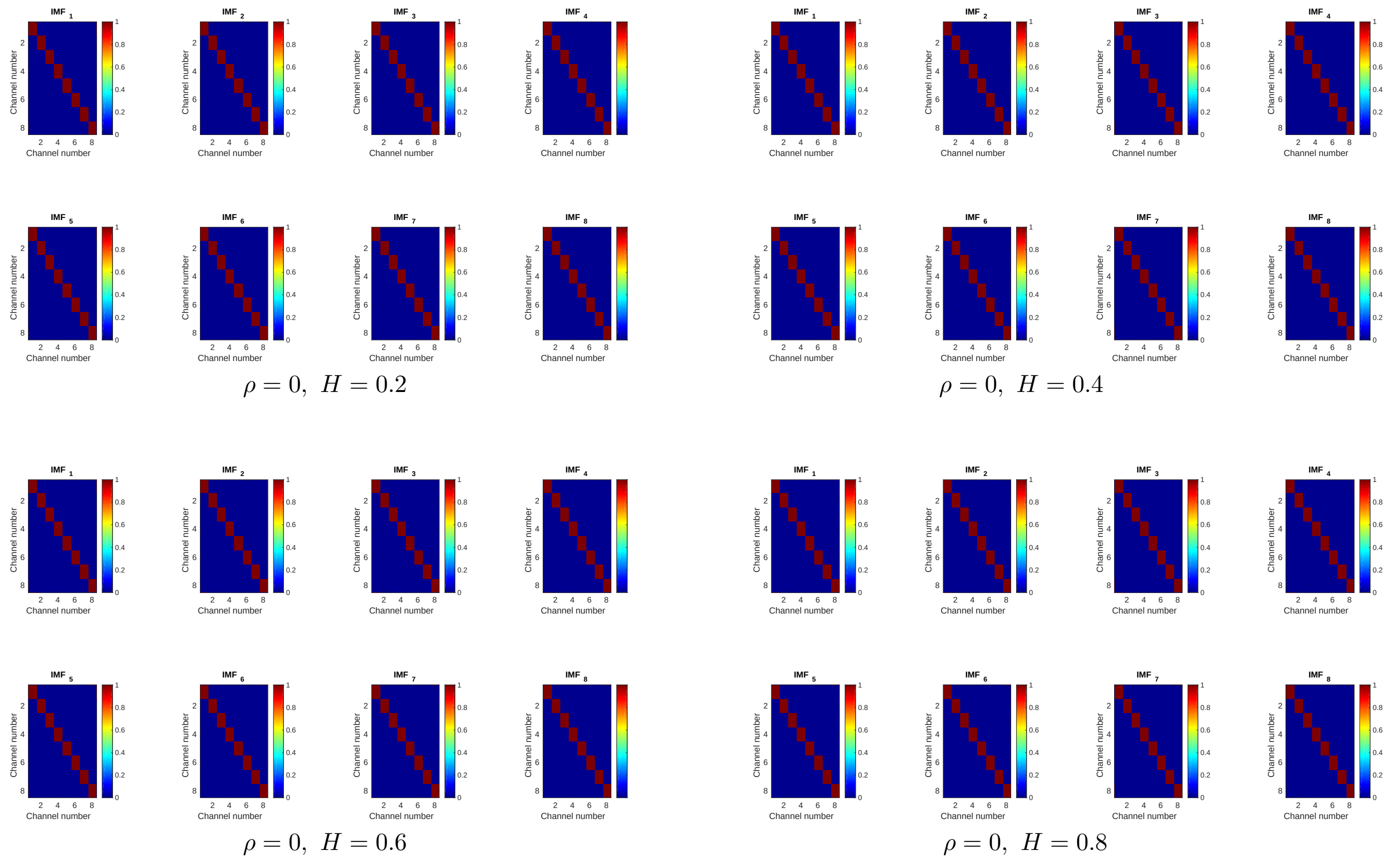


Fig. 5: Cross-correlations of IMFs for different values of parameters $(\rho, H) \in \{(0, 0.2), (0, 0.4), (0, 0.6), (0, 0.8)\}$.

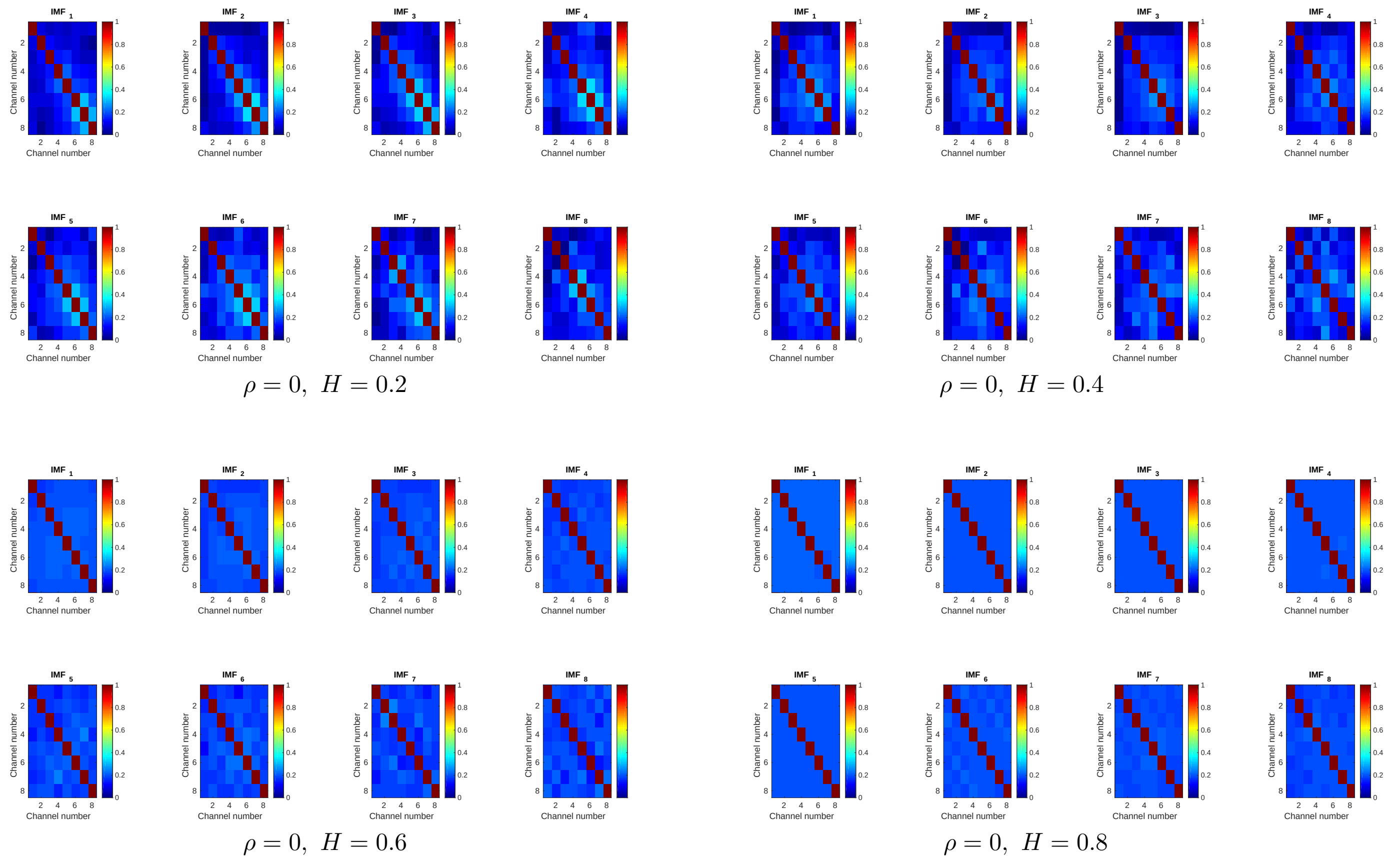


Fig. 6: Cross-correlations of IMFs for different values of parameters $(\rho, H) \in \{(0.2, 0.2), (0.2, 0.4), (0.2, 0.6), (0.2, 0.8)\}$.

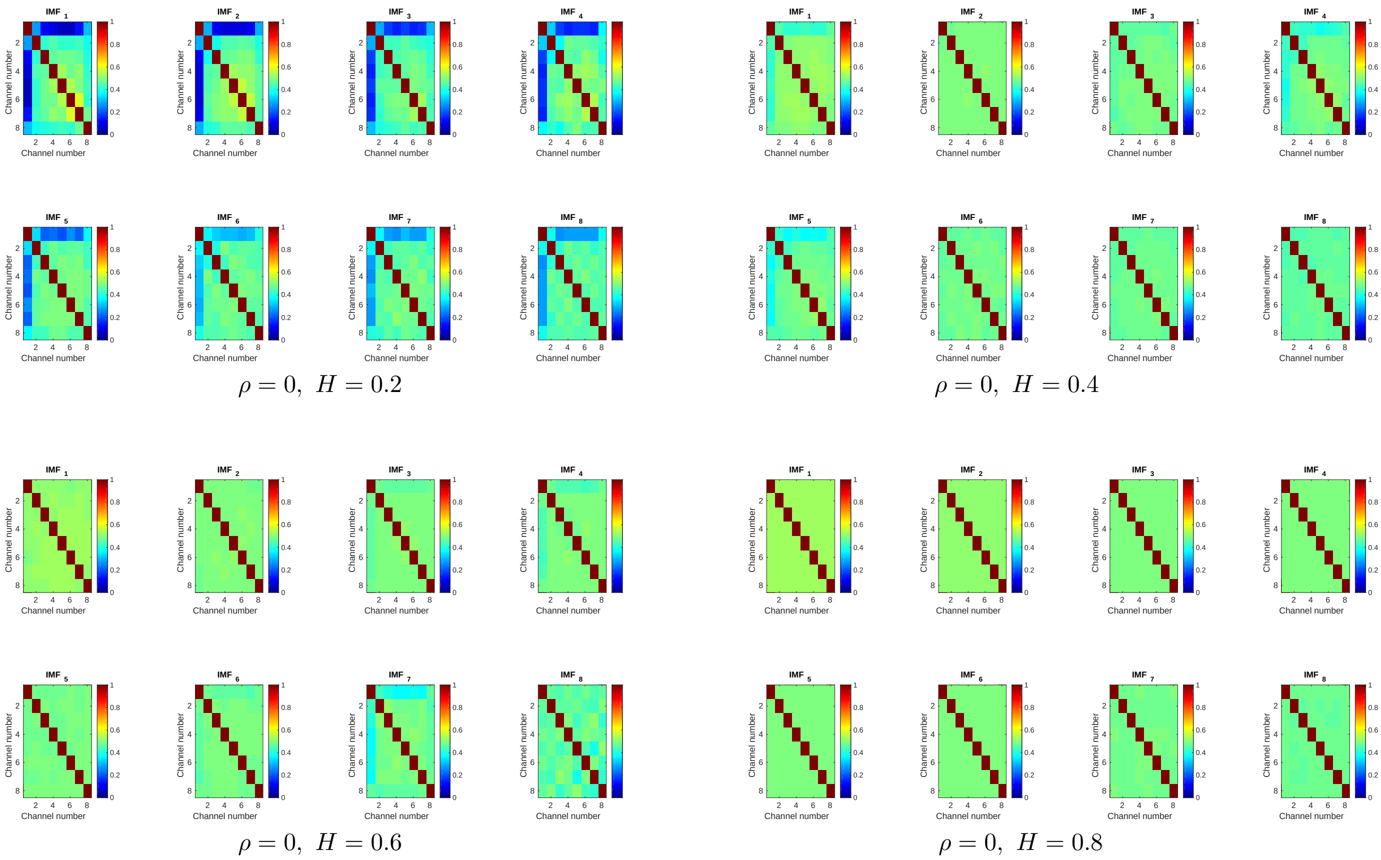


Fig. 7: Cross-correlations of IMFs for different values of parameters $(\rho, H) \in \{(0.5, 0.2), (0.5, 0.4), (0.5, 0.6), (0.5, 0.8)\}$.

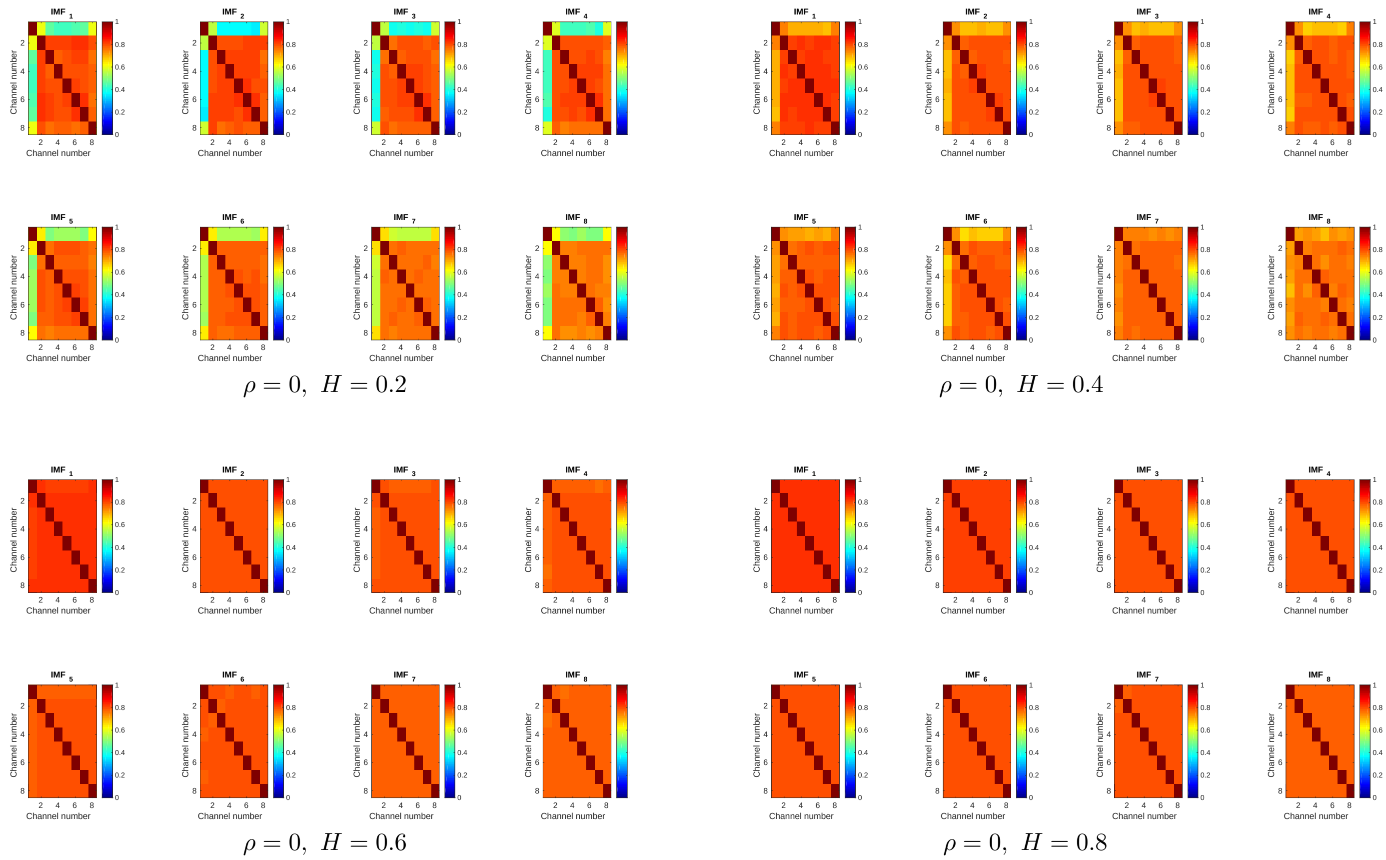


Fig. 8: Cross-correlations of IMFs for different values of parameters $(\rho, H) \in \{(0.8, 0.2), (0.8, 0.4), (0.8, 0.6), (0.8, 0.8)\}$.