Arnold Kompaniyets

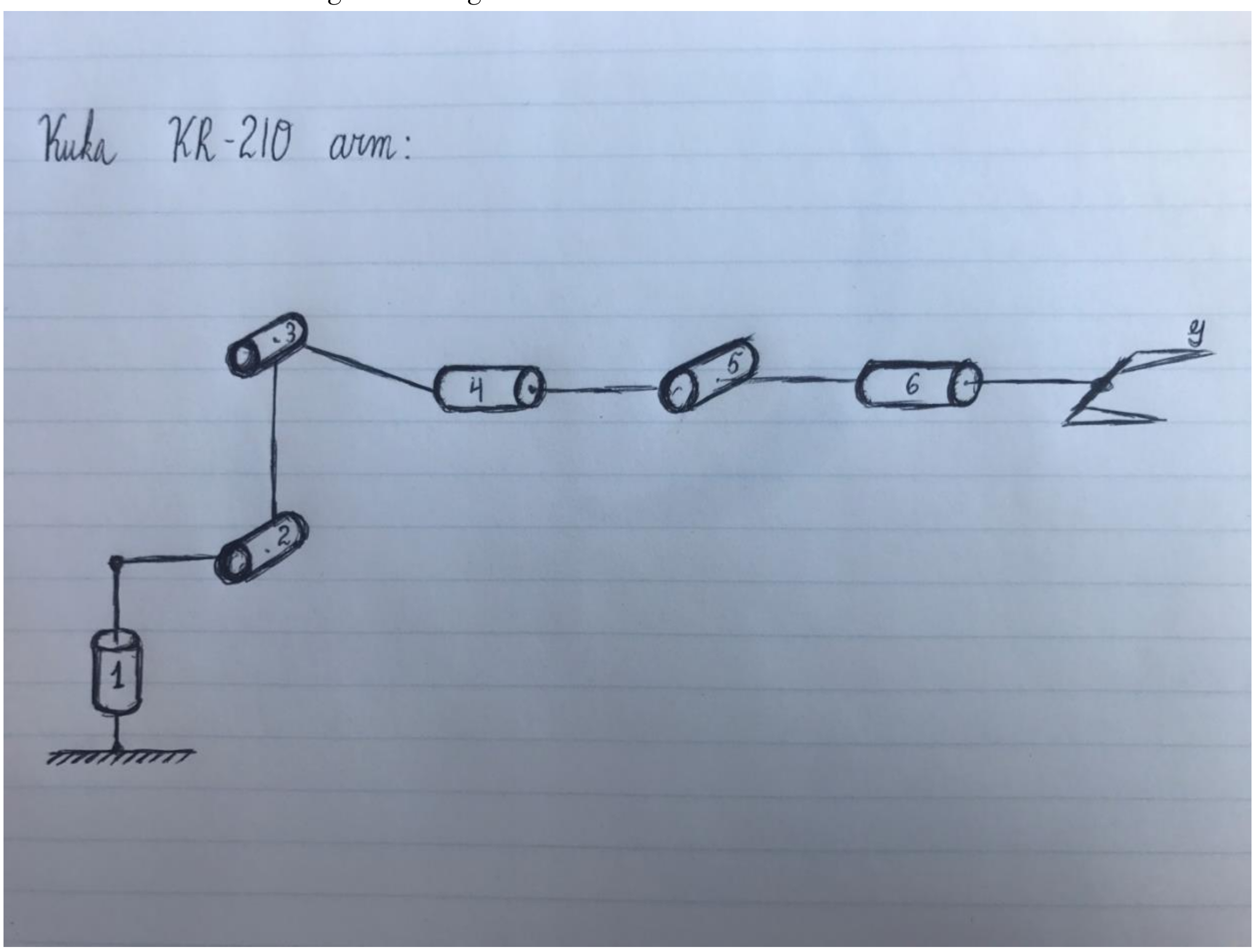
Robo ND (Term 1)
April 18, 2018

Robotic Arm: Pick and Place
Write-Up

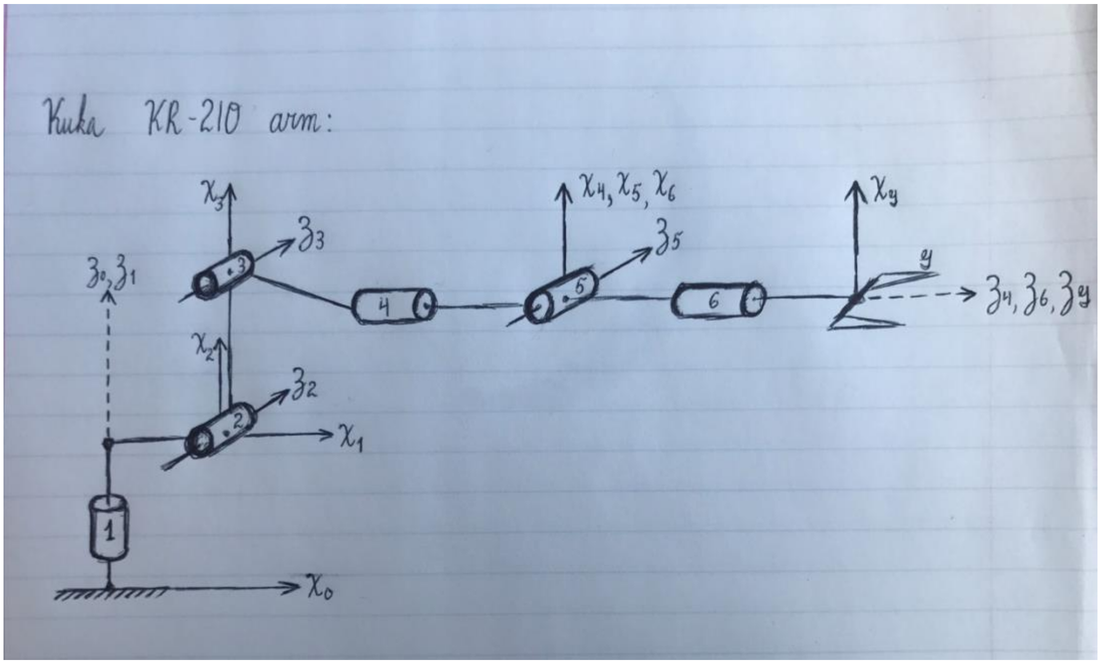
Kinematic Analysis:

Attaining the DH Parameters:

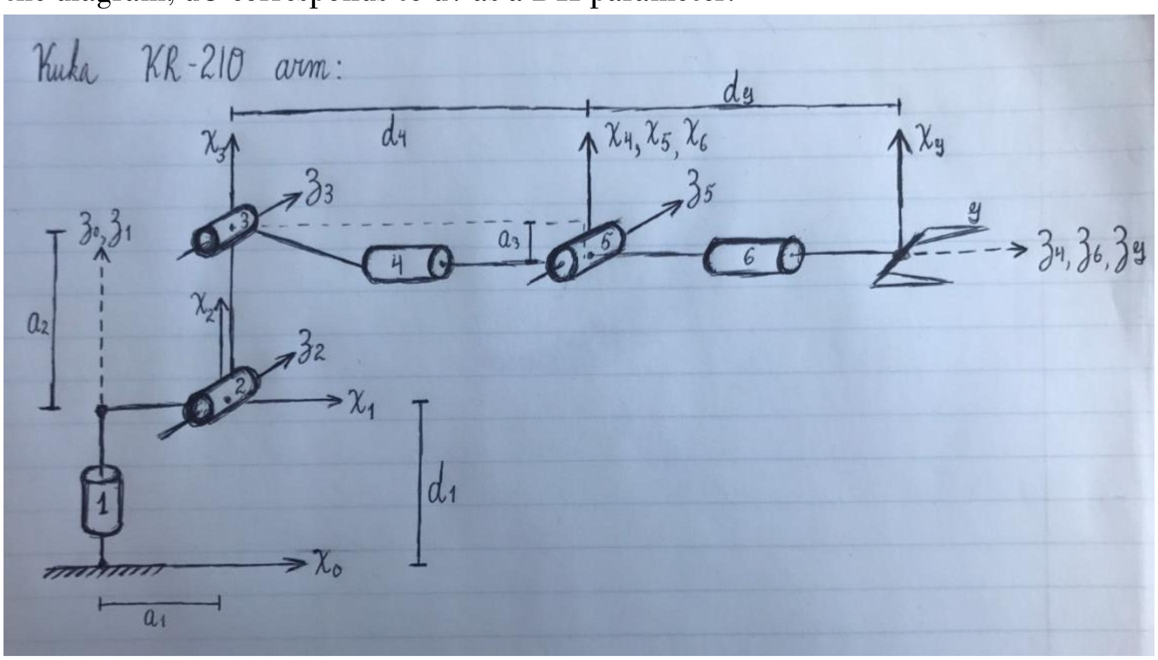
In order to derive all the necessary DH parameters, I first had to create an appropriate diagram of the robotic arm. With the help of included lessons on kinematics, I constructed the following basic diagram:



Next, I added X and Z-axes for each labeled joint, including the gripper (once again labeled as 'G'):



Following, the non-zero distance 'a' and 'd' were labeled, where 'a' represents the distance between axes Z_{i+1} and Z_i, and 'd' represents the distance between X_{i-1} and X_i. Since axes Z_0/Z_1 and Z_6/Z_G were concurrent and Z_4/Z_5 along with Z_5/Z_6 shared the same origin point, a0, a4, a5, and a5 were zero. Additionally, since axes X_2/X_3, X_4/X_5, and X_5/X_6 were concurrent, with axes X_1/X_2 sharing the same origin point, d2, d3, d5, and d6 were zero. In the diagram, dG corresponds to d7 as a DH parameter:



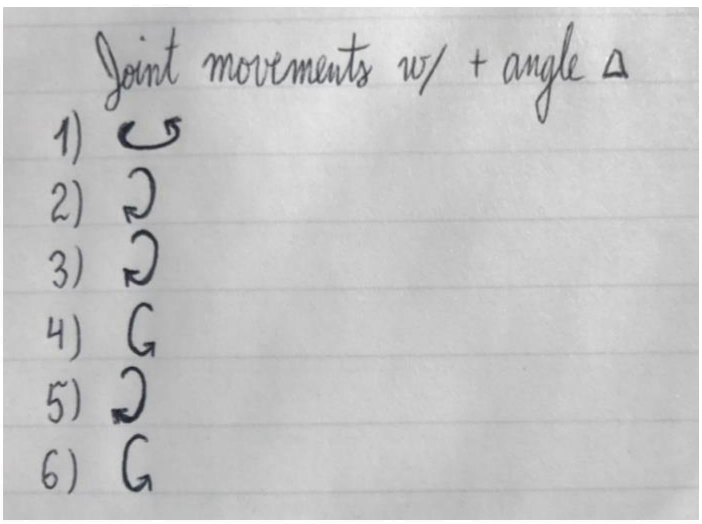
In order to attain the actual values for the non-zero 'a' and 'd' parameters, I had to reference the provided URDF file. Within this file, specifically in the joint and link definitions, I was able to reconstruct the robot arm diagram as follows:

- a_1 = x displacement between joints 1 and 2, or 0.35 meters.
- a_2 = z displacement between joints 2 and 3, or 1.25 meters.
- a_3 = z displacement between joints 3 and 4, or -0.054 meters (negative because joint 4 sits lower on the z-axis than joint 3).
- d_1 = z displacement between joint 2 and the ground; this is the sum of the z displacement between joint 1/ the ground and joint1/joint 2, or 0.33 + 0.42 = 0.75 meters.
- d_4 = x displacement between joints 3 and 5; this is the sum of x displacement between joint 3/4 and joint 4/5, or 0.96 + 0.54 = 1.50 meters.
- d_7 = x displacement between joint 5 and the end-effector, or 0.193 + 0.11 = 0.303 meters.

```
<!-- joints -->
<joint name="fixed_base_joint" type="fixed">
  <parent link="base_footprint"/>
  <child link="base_link"/>
  <origin xyz="0 0 0" rpy="0 0 0"/>
</joint>
<joint name="joint_1" type="revolute">
  <origin xyz="0 0 0.33" rpy="0 0 0"/>
  <parent link="base_link"/>
  <child link="link_1"/>
  <axis xyz="0 0 1"/>
  <limit lower="${-185*deg}" upper="${185*deg}" effort="300" velocity="${123*deg}"/>
</joint>
<joint name="joint_2" type="revolute">
  <origin xyz="0.35 0 0.42" rpy="0 0 0"/>
  <parent link="link_1"/>
  <child link="link_2"/>
  <axis xyz="0 1 0"/>
  <limit lower="${-45*deg}" upper="${85*deg}" effort="300" velocity="${115*deg}"/>
</joint>
<joint name="joint_3" type="revolute">
  <origin xyz="0 0 1.25" rpy="0 0 0"/>
  <parent link="link_2"/>
  <child link="link_3"/>
  <axis xyz="0 1 0"/>
  <limit lower="${-210*deg}" upper="${(155-90)*deg}" effort="300" velocity="${112*deg}"/>
</joint>
<joint name="joint_4" type="revolute">
  <origin xyz="0.96 0 -0.054" rpy="0 0 0"/>
  <parent link="link_3"/>
  <child link="link_4"/>
  <axis xyz="1 0 0"/>
  <limit lower="${-350*deg}" upper="${350*deg}" effort="300" velocity="${179*deg}"/>
</joint>
<joint name="joint_5" type="revolute">
  <origin xyz="0.54 0 0" rpy="0 0 0"/>
  <parent link="link_4"/>
  <child link="link_5"/>
  <axis xyz="0 1 0"/>
  <limit lower="${-125*deg}" upper="${125*deg}" effort="300" velocity="${172*deg}"/>
</joint>
<joint name="joint_6" type="revolute">
  <origin xyz="0.193 0 0" rpy="0 0 0"/>
  <parent link="link_5"/>
  <child link="link_6"/>
  <axis xyz="1 0 0"/>
</joint>
<joint name="gripper_joint" type="fixed">
  <parent link="link_6"/>
  <child link="gripper_link"/>
  <origin xyz="0.11 0 0" rpy="0 0 0"/><!--0.087-->
  <axis xyz="0 1 0" />
</joint>
```

To derive the twist angle, or α, defined as the angle between Z_{i+1} and Z_i measured about X_i, I needed to use the right hand rule. My understanding of this was to have the thumb represent X_i, the index finger Z_i, and the middle finger Z_{i+1}. The resulting angle between the index and middle fingers would represent the twist angle. As a side-note, with this method of noting the angle, if the index finger was closer to the palm, the twist angle is <0°, and if the middle finger is closer to the palm, the twist angle is >0°.

After solving for this, the only parameter left was theta, or θ, defined as the angle between X_{i-1} and X_i measured about Z_i. I would also need to implement the right hand rule for this; however, since these angles will vary depending on the position of the arm, I have to leave them as empty variables for the time being. The variable chosen were q1-6, for ease of programming later on. In the diagram made above, all the joints are drawn with their theta set to zero, except for one. Following the lessons provided, it was noted that joint 2 starts out drawn with a -90° deficit, so θ_2 would need to be defined as $-90 + \theta_2$. Using the forward kinematics demo, I was able to see how each joint would move with a positive angle change and documented it as such:



Since the end-effector, or the gripper itself, is locked in place, its theta (or θ_7) will always be zero.

With the four necessary DH parameters in place for each joint, I was able to construct the DH Table:

DH Table

Twist angle		'a'		'd'		Theta	
α_0:	0°	a_0:	0	d_1:	0.75	θ_1:	q1
α_1:	-90°	a_1:	0.35	d_2:	0	θ_2:	-π/2 + q2
α_2:	0°	a_2:	1.25	d_3:	0	θ_3:	q3
α_3:	-90°	a_3:	-0.054	d_4:	1.5	θ_4:	q4
α_4:	90°	a_4:	0	d_5:	0	θ_5:	q5
α_5:	-90°	a_5:	0	d_6:	0	θ_6:	q6
α_6:	0°	a_6:	0	d_1:	0.303	θ_7:	0

Transformation Matrices:

Now that the DH parameters have been established, they must be properly placed into transformation matrices and combined into one homogenous transformation matrix. As explained in the lessons, this can be accomplished by simply multiplying the individual transform matrices. This is illustrated in the first line of the image below:

$${}^0_G T = {}^0_1 T * {}^1_2 T * {}^2_3 T * {}^3_4 T * {}^4_5 T * {}^5_6 T * {}^6_G T$$

$${}^{i-1}_{i} T = R_x(\alpha_{i-1})\, D_x(a_{i-1})\, R_z(\theta_i)\, D_z(d_i)$$

$${}^{i-1}_{i} T = \left[\begin{array}{ccc|c} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i\, c\alpha_{i-1} & c\theta_i\, c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}\, d_i \\ s\theta_i\, s\alpha_{i-1} & c\theta_i\, s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}\, d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R_T & & P_T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$c = \cos$ $\quad s = \sin$

R_T = rotational transform
P_T = positional transform

In the following line, the structure of each individual transform matrix is defined, being a combination of four matrices (using each of the four parameters from the DH parameter table). When combined, the complete transformation matrix looks as in the image above, with the top left 3 X 3 matrix representing the rotational transform and the top right 3 X 1 matrix representing the positional transform.

With each of the seven transformation matrices, the structure is identical to that illustrated above, only with unique substitutions for 'i-1' and 'i'.

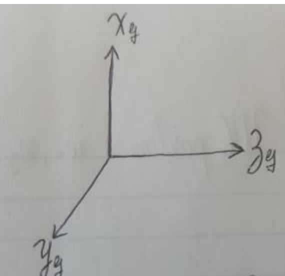
Inverse Kinematics:

Position:

In considering the IK problem of position, it is the thetas of the first three joints that are important relative to the wrist center (or the middle of joints 4,5, and 6, since those three joints are responsible for the ending rotational position of the end-effector). Therefore, the first task in identifying joint angles 1,2, and 3 is defining the location of the wrist center relative to the end-effector position. The EF position is provided by the ROS interface, specifically from the location of the randomly placed blue cylinder.

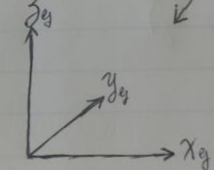
From the first diagram in this write-up, it is evident that the displacement between the EF and the wrist center, or WC, is purely in the x displacement between joints 5/6 and joints 6/G. This has already been identified as 'd_G' (or 'd_7') = 0.303 meters. However, while it would be simple to subtract this distance from the z-coordinate of the EF, this would only suffice with all thetas equal to zero. Since the final position of the EF can vary widely, it is important to identify the x, y, and z components of the aforementioned distance. This can be done by first determining the roll, pitch, yaw rotational matrix of the EF and selecting for the vector that points to the new z-axis. The roll, pitch, and yaw rotations correlate to x, y, and z rotations described in previous lessons. More specifically, these are the final rotation coordinates of the EF, relative to the initial external reference frame, and are also provided by the ROS interface. Furthermore, two additional rotations have to be added, since the final orientation of the EF needs to match URDF axes specifications, as opposed to the DH axes parameters defined above (a correction matrix defined as R_{corr}):

Gripper frame: in DH ⇒



180° about z-axis
-90° about y-axis
(intrinsic)

in URDF ⇒



$R_{corr} = R_z(\pi) * R_y(-\pi/2)$

Elementary matrices:

Rotation about x-axis: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ = Rot_x

y-axis: $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ = Rot_y = R_y

z-axis: $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = Rot_z = R_z

$$R_{rpy} = \underbrace{Rot\_z(yaw) * Rot\_y(pitch) * Rot\_x(roll) * R_{corr}}_{\text{extrinsic rotation}}$$

$$= \begin{bmatrix} l_x & m_x & \boxed{n_x} & p_x \\ l_y & m_y & \boxed{n_y} & p_y \\ l_z & m_z & \boxed{n_z} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

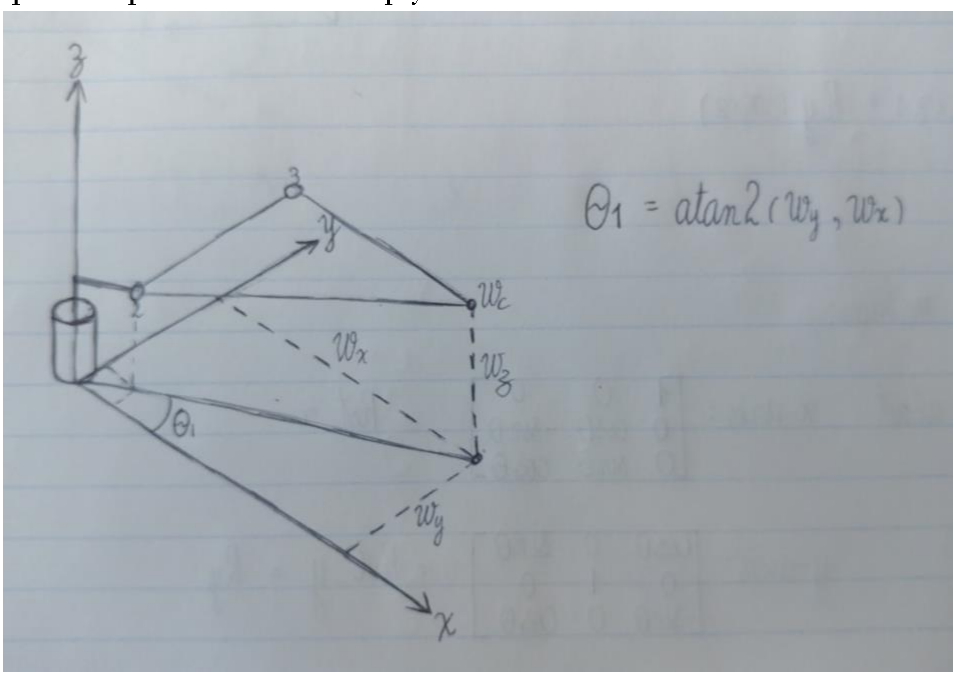
→ new z-axis

Now that the vector pointing to the new z-axis is identified, these x,y, and z distances can be used to identify the coordinates of the WC:

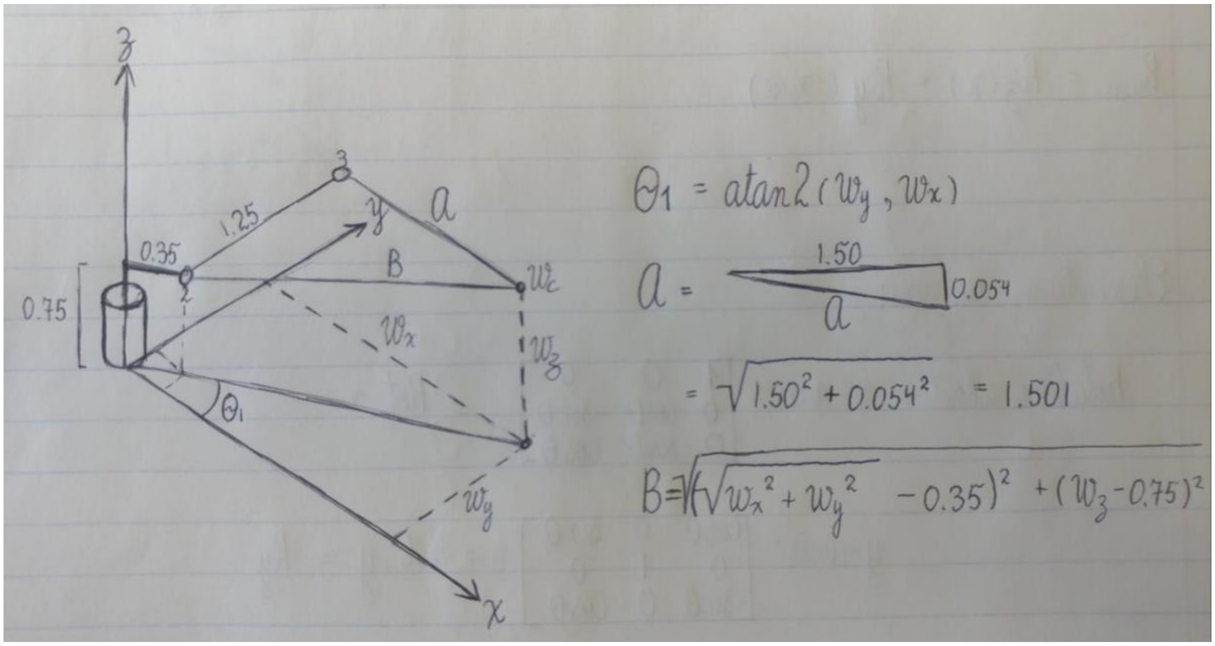
EF position: p_x, p_y, p_z WC position: w_x, w_y, w_z

$$w_x = p_x - (d_g \cdot n_x)$$
$$w_y = p_y - (d_g \cdot n_y)$$
$$w_z = p_z - (d_g \cdot n_z)$$

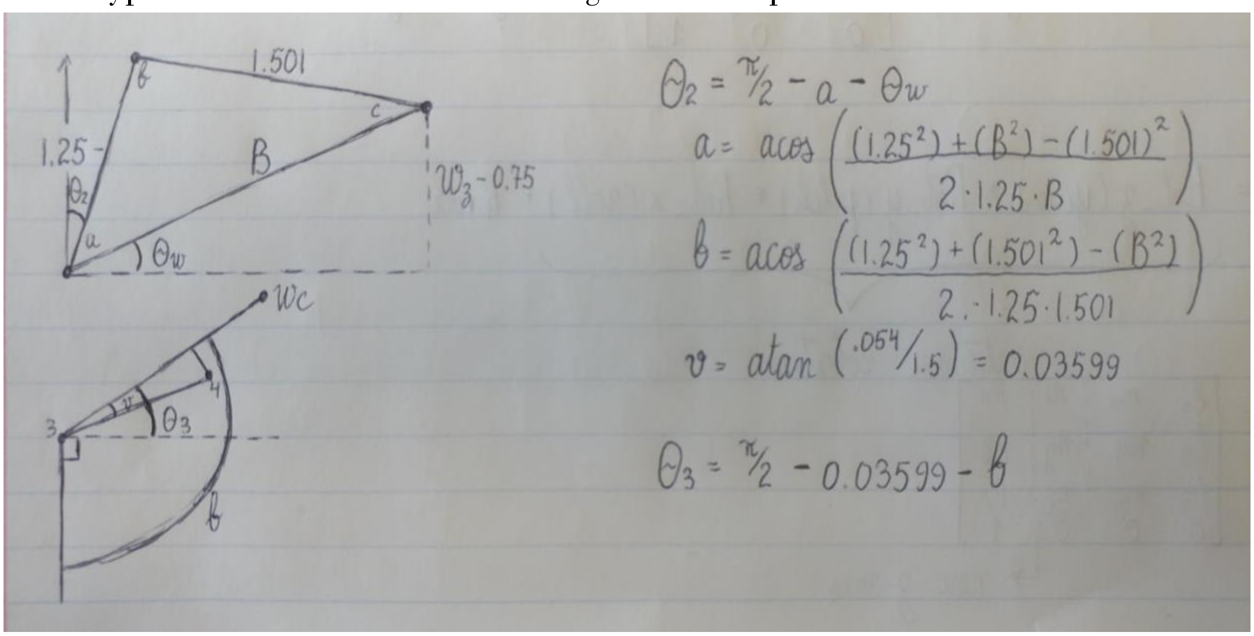
With the WC location found, the first three thetas can be found. The first theta is quite simple to find: it is simply the deviation of the WC from the x-axis:



The resulting triangle formed between joints 2, 3, and the WC will be used to find thetas 2 and 3. Before this triangle can be redrawn in its own two-dimensional plane, lengths A and B need to be found (using the law of cosines):



The distances between joint 1 and 2 are defined in the URDF file (0.75 and 0.35 meters), as well as between joint 2 and 3 (1.25 meters). Using the distances between joint 3 and 5 (the WC), length A can be found. Length B can be found by forming a triangle, with B as the hypotenuse. Now to redraw the triangle in its own plane:



The second theta can be found by subtracting angles ‘a’ and ‘θ_w’ from 90° (both found using the law of cosines). The third theta can be found by subtracting ‘b’ from 90°; however, this situation is a bit more complicated. Since joint 4 sits lower on the z-axis in its initial state, this extra amount needs to be taken into account (angle ‘v’). Angle ‘v’ is found using the law of cosines on the triangle used to find length ‘A’.

Orientation:

With the joint angles necessary to reach the EF's position determined, the joint angles necessary attain the EF's orientation can be found. The total rotation matrix to the EF has already been found as R_{rpy}, which makes it equivalent to the combined rotation matrix from the ground to joint 6. With the first 3 thetas known, the last three can be isolated as such:

$${}^0_6R = {}^0_1R * {}^1_2R * {}^2_3R * {}^3_4R * {}^4_5R * {}^5_6R$$

$${}^0_6R = R_{rpy} \quad \Rightarrow \quad {}^0_3R^{-1} * {}^0_6R = R_{rpy} * {}^0_3R^{-1}$$

$$\Downarrow$$

$$\underline{{}^3_6R = {}^0_3R^{-1} * R_{rpy}}$$

The rotation matrices above are simply the R_T matrices described in the transformation matrix section. In order to compile the inverse matrix needed, the first three twist angles and thetas are needed, which have already been found. The resulting matrix can now be broken down to find the last three thetas. Using the first equation listed in the figure above, R3_6 is the product of three separate matrices. As stated before, these matrices are identical in structure to R_T:

$${}^{i-1}_{i}R_T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} \end{bmatrix} \qquad \begin{matrix} c = \cos \\ s = \sin \end{matrix}$$

However, this time the twist angle values can be substituted, according to the DH Parameter table above. The resulting matrices can then be post-multiplied together (since the three successive joint rotations are intrinsic rotations) to form the R3_6 matrix:

$$
{}^3_4R = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 \\ 0 & 0 & 1 \\ -s\theta_4 & -c\theta_4 & 0 \end{bmatrix} \quad {}^4_5R = \begin{bmatrix} c\theta_5 & -s\theta_5 & 0 \\ 0 & 0 & -1 \\ s\theta_5 & c\theta_5 & 0 \end{bmatrix} \quad {}^5_6R = \begin{bmatrix} c\theta_6 & -s\theta_6 & 0 \\ 0 & 0 & 1 \\ -s\theta_6 & -c\theta_6 & 0 \end{bmatrix}
$$

$$
{}^3_5R = {}^3_4R * {}^4_5R = \begin{bmatrix} c\theta_4 c\theta_5 & -c\theta_4 s\theta_5 & s\theta_4 \\ s\theta_5 & c\theta_5 & 0 \\ -s\theta_4 c\theta_5 & s\theta_4 s\theta_5 & c\theta_4 \end{bmatrix}
$$

$$
{}^3_6R = {}^3_5R * {}^5_6R = \begin{bmatrix} c\theta_4 c\theta_5 c\theta_6 - s\theta_4 s\theta_6 & -c\theta_4 c\theta_5 s\theta_6 - s\theta_4 c\theta_6 & -c\theta_4 s\theta_5 \\ s\theta_5 c\theta_6 & -s\theta_5 s\theta_6 & c\theta_5 \\ -s\theta_4 c\theta_5 c\theta_6 - c\theta_4 s\theta_6 & s\theta_4 c\theta_5 s\theta_6 - c\theta_4 c\theta_6 & s\theta_4 s\theta_5 \end{bmatrix}
$$

With only the desired thetas left, each can be solved using $\tan^{-1}$:

$$
\theta_4 = \operatorname{atan2}\left({}^3_6R[2,2],\ -{}^3_6R[0,2]\right)
$$

$$
\theta_5 = \operatorname{atan2}\left(\sqrt{({}^3_6R[2,2])^2 + ({}^3_6R[0,2])^2},\ {}^3_6R[1,2]\right)
$$

$$
\theta_6 = \operatorname{atan2}\left(-{}^3_6R[1,1],\ {}^3_6R[1,0]\right)
$$

The tangent of theta 4 would equal the ratio of R3_6[2,2] and -R3_6[0,2], since the theta 5 values would cancel out, leaving only the sin/cos ratio that is necessary for the tangent function. This same logic is applied in finding theta 6, only with the sin/cos ratio of theta 6. Theta 5 poses a slightly higher difficulty in finding the sin/cos ratio, specifically for the numerator. Luckily, both of the other ratios used contained sine of theta 5. Keeping in mind that $\sqrt{\cos^2 + \sin^2} = 1$, the variables associated with theta 4 or 6 can cancel out, leaving only the sine of theta 5. For this example, I chose to use the ratio used for finding theta 4.