Numerical Methods I:
Tutorial I: Numerical experiments with randomness

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Battery problem

Wireless mouse requires two batteries with at least some charge to work. Assume that battery life times are exponentially distributed. Given average life time, μ, answer the following questions.

1. What is the average operation time of a mouse, $\langle T_M \rangle$? Assume that we change both batteries as soon as mouse stops working.
2. What is the distribution of T_M?
3. How much does the overall service (operation) time change if we would change just one battery, the one which has lost its charge?
4. What if the distribution of battery life times would be normal instead of exponential?

Dissecting the problem

- $T \sim \mathrm{Exp}(\frac{1}{\mu})$.
- $T_M \sim \min(T_i, T_j)$ with independent samples of T.
- Multiple observations of T_M.
- Inferring $\langle T_M \rangle$ (for Q1) and $p(T_M)$ (for Q2).
- Q3 will require us to rework the sampling of T_M.
- Q4: $T \sim \mathcal{N}(\mu, \sigma)$.

Here $\sim$ means that lhs and rhs follow the same distribution.

Analytical approach to “replace both” strategy

We can figure $p(T_M)$ and therefore $\langle T_M \rangle$. Just note that:

- $p(T_M)$ is the probability that one of the two independent observations of $T_i = T_M$ and the other $T_j > T_M$.

So:

$$P(T_M = x) = 2 \cdot P(T = x) \cdot P(T > x) = 2 \cdot \frac{1}{\mu} e^{\frac{x}{\mu}} \cdot e^{\frac{x}{\mu}} = \frac{2}{\mu} e^{\frac{2x}{\mu}}$$

We would expect that our numerical estimate of $\langle T_M \rangle$ should be close to $\frac{\mu}{2}$.

Reasoning about “replace bad” strategy

- $p(T_M)$ and $\langle T_M \rangle$ make little sense.
- Even if we redefine what T_M means, successive samples would not be i.i.d.
- What we actually care about is total time N batteries will serve us under both strategies:
  - For “replace both” total service time should be

$$t \approx \frac{\mu N}{4}.$$

  - For “replace bad” we can just make an educated guess. I would expect

$$t \approx \frac{\mu N}{2}.$$

What can be improved?

- What is the actual empirical distribution of battery life times? What are realistic μ and σ?
- For normally distributed life times, how does the difference between two strategies depend on σ?
- Total service (operation) time is also a random outcome. We haven't taken this into account. To do so we should run multiple total service time experiments (observations/samples) and then provide some discussion.

Until next time!