BioE 101 Circuits Reference

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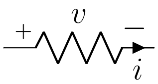
Circuit Components

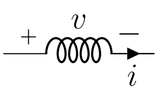
Resistors



Useful Resistor formulas

- **Ohm's Law**: $R = \frac{V}{I}$
- **Resistivity**: $R = \rho\frac{l}{A}$
- **Series:** $R_{series} = \sum_i R_i$
- **Parallel:** $R_{parallel} = (\sum_i \frac{1}{R_i})^{-1}$

Inductors



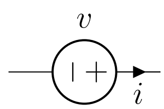
Useful Inductor formulas

- **Flux**: $\phi = LI$
- **Voltage**: $v = L\frac{dI}{dt}$
- **Series:** $L_{series} = \sum_i L_i$
- **Parallel:** $L_{parallel} = (\sum_i \frac{1}{L_i})^{-1}$

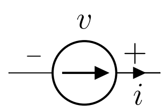
Sources

Reminder: these are *independent* sources, meaning that they are set constant values and don't depend on other values elsewhere.

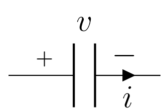
Voltage Source



Current Source



Capacitors



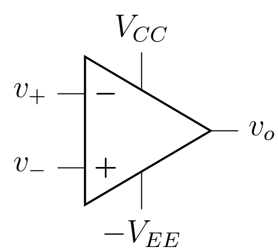
Useful Capacitor formulas

- **Charge**: $Q = CV$
- **Current**: $i = C\frac{dV}{dt}$
- **Series:** $C_{series} = (\sum_i \frac{1}{C_i})^{-1}$
- **Parallel:** $C_{parallel} = \sum_i C_i$

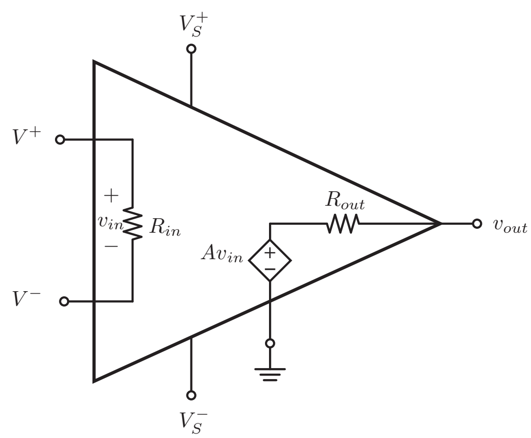
I didn't mention it above, but there are also *dependent* sources are marked with a diamond instead of a circle and there values are functions of values elsewhere in a circuit. A good example would be the model of an op-amp in the next section.

II. Operational Amplifiers

A very special component in circuit design is the operational amplifier. It is a circuit composed of transistors - take EE105 and EE140 to learn more about what's under the hood! Amplifier Diagram:



At an abstract level this is also what an opamp looks like:

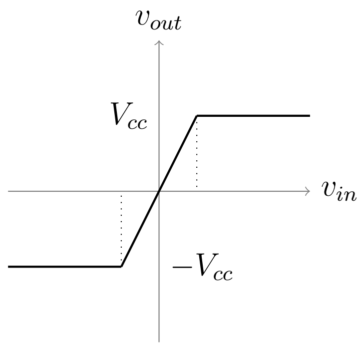


So ideally, the gain is $A = \infty$, input resistance $R_i = \infty\,\Omega$, output resistance $R_o = 0\,\Omega$. A high input resistance is so we can put in the full input voltage v_i. Zero output resistance is so we get the full output voltage v_o. Having a high voltage gain A is the main characteristic of an op amp. If the output voltage is represented as:

$$v_{out} = A(v_{in})$$

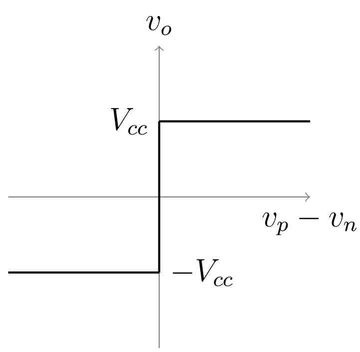
$$v_{in} = v^+ - v^n$$

This produces a linear response. But will the line go on forever? No. The limiting factor here will be the power supplies. The region that has a $v_{out} = V_{CC}$ is the positive saturation region and the region that has a $v_{out} = -V_{EE}$ is the negative saturation region.



OpAmps as Comparators

If A is really high, then the slope of the linear response is also really high. Let $A = \infty$. The graph looks like this:



So if $v_p > v_n$, $v_o = V_{cc}$ and if $v_p < v_n$, $v_o = -V_{cc}$. This means we can use op amps to compare the input voltage to a reference (usually ground).

OpAmps in Negative Feedback

Negative feedback (NFB) is defined to be when the output of the opamp is fed back to the negative input in such a way the an increase in the input will create a decrease in the output and vice versa.

First Golden Rule (applies for all ideal opamps):

$$i_+ = i_- = 0$$

This is because of the opamp's high input resistance. That means no current flows into its input terminals.

Second Golden Rule: *** This rule only applies in negative feedback ***

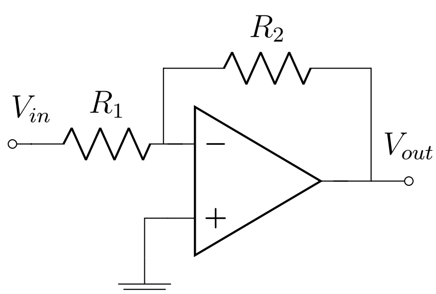
$$v_+ = v_-$$

In negative feedback, the an increase in the input will cause the output to change such that the difference between v_+ and v_- is minimized (roughly zero).

OpAmp Design Topologies

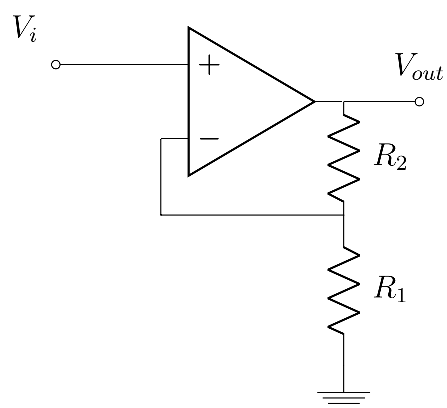
With the golden rules in mind, here are a few common topologies you might use for opamps in NFB. For each topology, keep in mind its purpose, the output as a function of the input, and the input and output impedances (what does the input see and what does the output see).

Inverting Amplifier



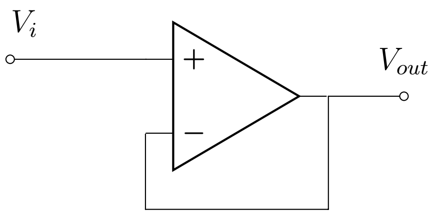
- Gain: $G = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$
- $R_{in} = R_1$
- $R_{out} = 0$
- Notes: used to invert (change sign) of your signal and boost it. Be careful that since R_{in} is finite, you might load the output of your previous stage.

Non-Inverting Amplifier



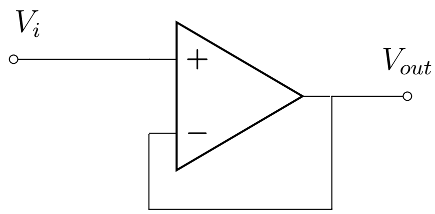
- Gain: $G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$
- $R_{in} = \infty$
- $R_{out} = 0$
- Notes: used to boost a signal by a positive factor. Has high input resistance and low output resistance.

Buffer, Voltage Follower



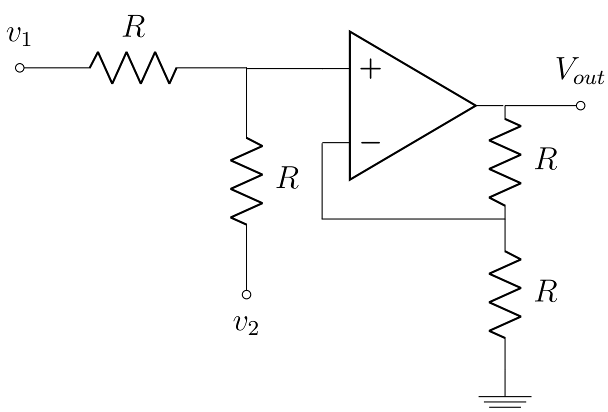
- Gain: $G = \frac{V_{out}}{V_{in}} = 1$
- $R_{in} = \infty$
- $R_{out} = 0$
- Notes: used to "drive" a signal without loading the input and output. It separates the load from the source since the input resistance is high and output resistance is low.

Inverting Summing Amplifier



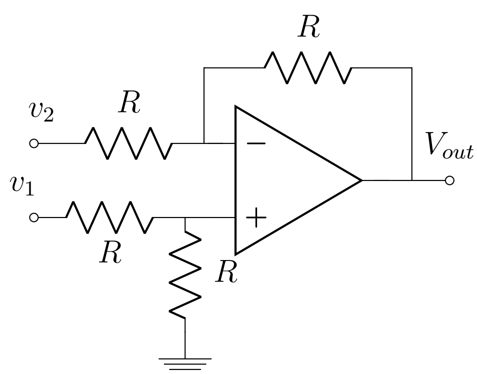
- Gain: $G = \frac{V_{out}}{V_{in}} = 1$
- $R_{in} = \infty$
- $R_{out} = 0$
- Notes: used to "drive" a signal without loading the input and output. It separates the load from the source since the input resistance is high and output resistance is low.

Non-inverting Amplifier



- Gain: $G = \frac{V_{out}}{V_{in}} = 1$
- $R_{in} = \infty$
- $R_{out} = 0$
- Notes: used to "drive" a signal without loading the input and output. It separates the load from the source since the input resistance is high and output resistance is low.

Difference Amplifier



- Gain: $G = \frac{V_{out}}{V_{in}} = 1$
- $R_{in} = \infty$
- $R_{out} = 0$
- Notes: used to "drive" a signal without loading the input and output. It separates the load from the source since the input resistance is high and output resistance is low.

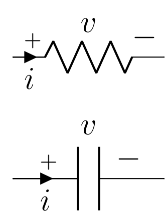
III. Circuit Analysis

There are some key things to always remember:

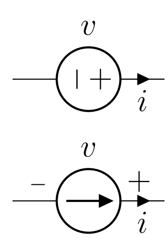
- **Voltages across parellel branches are the same**
- **Current in a loop is the same**
- **Passive Sign Convention**:

  Disclaimer: passive sign convention is a convention, and some people might prefer the complete opposite of what I put here, but as long as you are consistent then your analysis should be correct.

  Passive Components have a voltage drop across them with currents flowing from the positive terminal to the negative terminal. Passive Components include resistors, capacitors, and in 16B, inductors.



  Active Components have a voltage increase across them with currents flowing from the negative terminal to the positive terminal. Active Components include voltage sources and current sources.

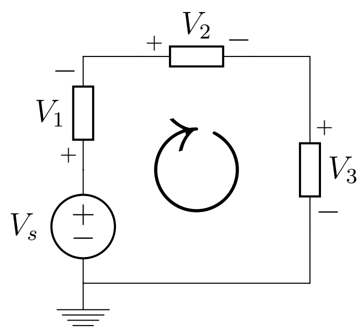


  Note 1: It's best to define the direction of current as the direction of the current source. Remember that you can arbitrarily set the direction of current for analysis as long as you are consistent with your sign convention.

  Note 2: Yes, a current source can have any voltage across it, and any current flows through a voltage source. All that is required of a current source is that it sets the current in that branch and all that is required of a voltage source is that it sets a voltage potential across its two nodes.

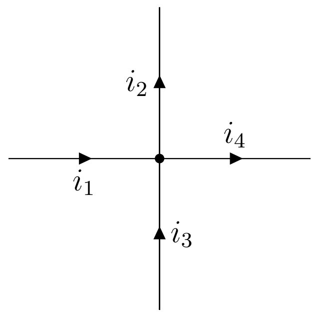
Kirchoff's Voltage and Current Laws (KVL, KCL)

- **KVL**: the increase in voltages in a loop has to equal the decrease in voltages in a loop



$$V_s = V_1 + V_2 + V_3$$

- **KCL**: all the current entering a node has to equal the current exiting a node



$$i_1 + i_3 = i_2 + i_4$$

Superposition

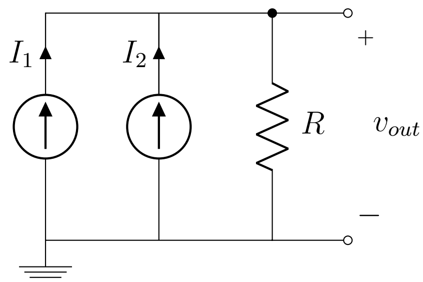
Nearly all the circuits you'll encounter in EE16A/B are what we call "linear." Because these circuits are linear, we can calculate voltages and currents across passive components by considering only one active component at a time while the other active components are "turned off" and then adding up their contributions. This property is called superposition.

Key things to remember:

- **Turning off voltage sources means you SHORT it**; this is the same as adding a replacing it with a resistor $R = 0$ so that there is no voltage contribution from it while you analyze the other active components.

- **Turning off current sources means you OPEN it**; this is the same as adding a replacing it with a resistor $R = \infty$ so that there is no current contribution from it while you analyze the other active components.

Here's an example:



What is the voltage and current across R?

Due to I_1:

$$v_{out1} = I_1 R$$
$$i_{out1} = I_1$$

Due to I_2:

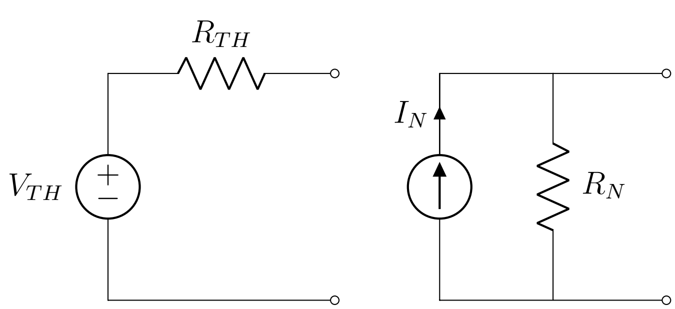
$$v_{out2} = I_2 R$$
$$i_{out2} = I_2$$

By superposition:

$$v_{out} = (I_1 + I_2)R$$
$$i_{out} = I_1 + I_2$$

Thevenin/Norton Equivalence

We can treat any portion of a circuit with an active component as a black box that has some source voltage and series source resistance (Thevenin voltage and resistance) or some source current and parallel source resistance (Norton current and resistance).



It turns out that $V_{TH} = V_{OC}$ and $I_N = I_{SC}$, where OC means open circuit and SC means short circuit.

Key things to remember when finding open circuit voltage and short circuit current of a circuit that you're probing:

- **For** V_{oc}: find the voltage across the component(s) you're probing using regular analysis techniques.
- **For** I_{sc}: **SHORT** your probes! Then consider that short to be part of the circuit, then find the current going through that short.
- **For** R_{TH}: Use Ohm's Law, $R_{TH} = \frac{V_{oc}}{I_{sc}}$

IV. AC Analysis

From lecture, you should now know that any time-varying function can be decomposed into a linear combination of complex sinusoids via the Fourier Transform. This means that a time-varying voltage can be done the same way!

That is, any voltage $v(t)$ can be written as:

$$v(t) = \sum_{\infty} V_n \cdot e^{j\omega nt}$$

For simplicity let's work with a voltage that only contains one frequency - a simple sinusoid. Let

$$v(t) = V_{amp}e^{j\omega t}$$

This mans that