Certification of Termination Proofs for Term Rewriting

A short story of a long battle...

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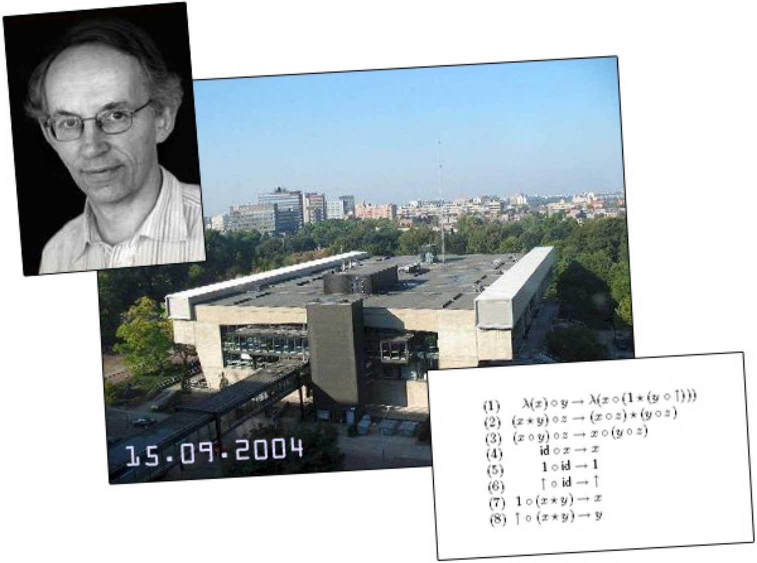
Who am I?

Who am I?



$$
\begin{array}{ll}
(1) & \lambda(x) \circ y \to \lambda(x \circ (1 \star (y \circ \uparrow))) \\
(2) & (x \star y) \circ z \to (x \circ z) \star (y \circ z) \\
(3) & (x \circ y) \circ z \to x \circ (y \circ z) \\
(4) & \mathsf{id} \circ x \to x \\
(5) & 1 \circ \mathsf{id} \to 1 \\
(6) & \uparrow \circ\, \mathsf{id} \to \uparrow \\
(7) & 1 \circ (x \star y) \to x \\
(8) & \uparrow \circ\, (x \star y) \to y
\end{array}
$$

Who am I?

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Termination of Rewriting and Its Certification

Who am I?

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Heat
$\dot{T} = 2$
$\dot{t} = 1$
$T \leq 10 \land t \leq 3$
$T \geq 9$
$t \geq 2 \to$
$t := 0$
$t \geq 0.5 \to$
$t := 0$
$T \leq 6 \to$
$t := 0$
Cool
$\dot{T} = -T$
$\dot{t} = 1$
$T \geq 5$
Check
$\dot{T} = -T/2$
$\dot{t} = 1$
$t \leq 1$
HUYGENSGEBOUW
01.12.2008

Outline

1. Background: termination of term rewriting

2. CoLoR project: certification of termination proofs
   - Why?... motivation
   - How?... CoLoR's approach to certification
   - When?... history of the project
   - What?... overview of the content
   - Related work
   - Certified competition

3. Conclusions... sort of

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Introduction to term rewriting

Example (Quick sort)

$$\begin{aligned}
\mathsf{qsort}(\mathsf{nil}) &\rightarrow \mathsf{nil}\\
\mathsf{qsort}(x :: xs) &\rightarrow \mathsf{append}(\mathsf{qsort}(\mathsf{filterLe}(x, xs)), x :: \mathsf{qsort}(\mathsf{filterGe}(x, xs)))
\end{aligned}$$

$$\begin{aligned}
\mathsf{append}(\mathsf{nil}, l) &\rightarrow l\\
\mathsf{append}(x :: xs, l) &\rightarrow x :: \mathsf{append}(xs, l)
\end{aligned}$$

$$\begin{aligned}
\mathsf{filterLe}(n, \mathsf{nil}) &\rightarrow \mathsf{nil}\\
\mathsf{filterLe}(n, x :: xs) &\rightarrow \mathsf{filter}(\mathsf{le}(x, n), x, \mathsf{filterLe}(n, xs))
\end{aligned}$$

$$\begin{aligned}
\mathsf{filterGe}(n, \mathsf{nil}) &\rightarrow \mathsf{nil}\\
\mathsf{filterGe}(n, x :: xs) &\rightarrow \mathsf{filter}(\mathsf{ge}(x, n), x, \mathsf{filterGe}(n, xs))
\end{aligned}$$

$$\begin{aligned}
\mathsf{filter}(\mathsf{false}, x, xs) &\rightarrow xs\\
\mathsf{filter}(\mathsf{true}, x, xs) &\rightarrow x :: xs
\end{aligned}$$

$$\begin{aligned}
\mathsf{ge}(x, y) &\rightarrow \mathsf{le}(y, x)\\
\mathsf{le}(0, y) &\rightarrow \mathsf{true}\\
\mathsf{le}(\mathsf{s}(x), 0) &\rightarrow \mathsf{false}\\
\mathsf{le}(\mathsf{s}(x), \mathsf{s}(y)) &\rightarrow \mathsf{le}(x, y)
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$$\mathrm{append}(\mathrm{nil}, l) \to l$$

$$\mathrm{ge}(x, y) \to \mathrm{le}(y, x)$$

$$\begin{aligned}
\mathrm{filter}(\mathrm{false}, x, xs) &\to xs & \mathrm{le}(0, y) &\to \mathrm{true} \\
\mathrm{filter}(\mathrm{true}, x, xs) &\to x :: xs & \mathrm{le}(\mathrm{s}(x), 0) &\to \mathrm{false} \\
& & \mathrm{le}(\mathrm{s}(x), \mathrm{s}(y)) &\to \mathrm{le}(x, y)
\end{aligned}$$

Example (Collatz conjecture)

$$\begin{aligned}
\mathrm{collatz}(\mathrm{s}(\mathrm{s}(x))) &\to \mathrm{f}(\mathrm{even}(x), \mathrm{s}(\mathrm{s}(x))) & \mathrm{even}(0) &\to \mathrm{true} \\
\mathrm{f}(\mathit{true}, x) &\to \mathrm{collatz}(\mathrm{half}(x)) & \mathrm{even}(\mathrm{s}(0)) &\to \mathrm{false} \\
\mathrm{f}(\mathit{false}, x) &\to \mathrm{collatz}(\mathrm{s}(\mathrm{triple}(x))) & \mathrm{even}(\mathrm{s}(\mathrm{s}(x))) &\to \mathrm{even}(x)
\end{aligned}$$

$$\begin{aligned}
\mathrm{half}(0) &\to 0 & \mathrm{triple}(0) &\to 0 \\
\mathrm{half}(\mathrm{s}(\mathrm{s}(x))) &\to \mathrm{s}(\mathrm{half}(x)) & \mathrm{triple}(\mathrm{s}(x)) &\to \mathrm{s}(\mathrm{s}(\mathrm{s}(\mathrm{triple}(x))))
\end{aligned}$$

Introduction to term rewriting

Example (Quick sort)

$$\mathsf{qsort}(\mathsf{nil}) \to \mathsf{nil}$$

$$\mathsf{qsort}(x :: xs) \to \mathsf{append}(\mathsf{qsort}(\mathsf{filterLe}(x, xs)), x :: \mathsf{qsort}(\mathsf{filterGe}(x, xs)))$$

$$\mathsf{append}(\mathsf{nil}, l) \to l$$

Example (Collatz conjecture)

$\mathsf{collatz}(\mathsf{s}(\mathsf{s}(x))) \to \mathsf{f}(\mathsf{even}(x), \mathsf{s}(\mathsf{s}(x)))$ $\qquad$ $\mathsf{even}(0) \to \mathsf{true}$

$\mathsf{f}(\mathit{tru}$

$\mathsf{f}(\mathit{fals}$

Definition

A TRS is terminating iff it does not admit infinite reductions.

$\mathsf{half}(0) \to 0$ $\qquad$ $\mathsf{triple}(0) \to 0$

$\mathsf{half}(\mathsf{s}(\mathsf{s}(x))) \to \mathsf{s}(\mathsf{half}(x))$ $\qquad$ $\mathsf{triple}(\mathsf{s}(x)) \to \mathsf{s}(\mathsf{s}(\mathsf{s}(\mathsf{triple}(x))))$

$\mathsf{ge}(x, y) \to \mathsf{le}(y, x)$

$\mathsf{filter}(\mathsf{false}, x, xs) \to xs$ $\qquad$ $\mathsf{le}(0, y) \to \mathsf{true}$

$\mathsf{filter}(\mathsf{true}, x, xs) \to x :: xs$ $\qquad$ $\mathsf{le}(\mathsf{s}(x), 0) \to \mathsf{false}$

$\mathsf{le}(\mathsf{s}(x), \mathsf{s}(y)) \to \mathsf{le}(x, y)$

Termination of rewriting

Termination of rewriting:

- Is undecidable.
- Is an important topic in term rewriting.
- Many methods exist and new ones are constantly being developed.
- Recently the emphasis is on automation.
- There exists a number of tools for proving termination.
- Stimulated by the termination competition.
- Tools (and proofs that they produce) are getting more and more complex, so reliability is an issue (tools disqualifications in the competition).
- In 2007 a new category of certified termination introduced in the competition.

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Motivation

CoLoR

http://color.loria.fr

CoLoR: Coq Library on Rewriting and Termination.
Goal: certification of termination proofs produced by various termination provers.

- Increasing reliability of termination provers.
- Common proof format for termination provers:
  - common tools (proof presentation, manipulation, . . .),
  - control language for provers (integration of tools)
- Extension of proof assistance kernels.

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CoLoR's approach to certification

How to certify termination results?

- Possibility: certification of tools source code.
  ⇒ difficult, tool dependent, extra work with every change, . . .
- CoLoR's approach:
  - TPG: common format for termination proofs.
  - Tools output proofs in TPG format.
  - CoLoR: a Coq library of results on termination.
  - Rainbow: a tool for translation from proofs in TPG format to Coq proofs, using results from CoLoR.

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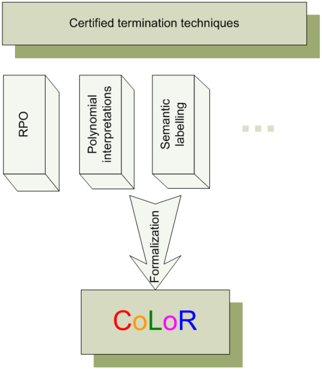

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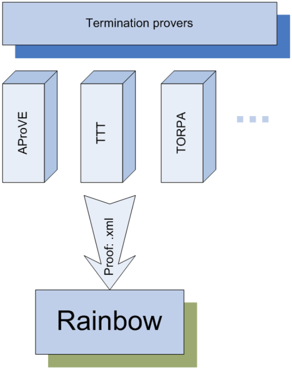

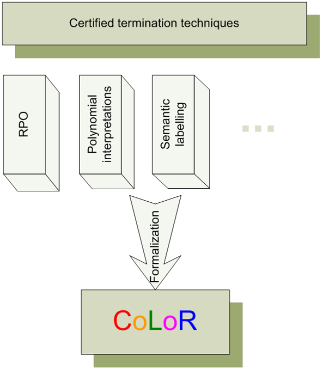

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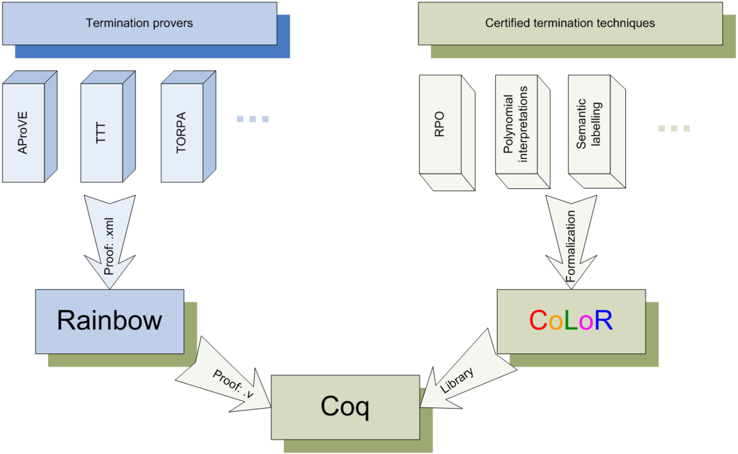

History

Project started (Blanqui)	March 2004
First release	March 2005
First certified proofs	July 2006
First certification workshop	May 2007
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  - recursive path ordering [Coupet-Grimal, Delobel]
  - higher-order recursive path ordering [Koprowski]
  - dependency graph cycles [Blanqui]
  - **matrix interpretations** [Koprowski, Zantema]
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  - varyadic terms [Blanqui]
  - algebraic terms with symbols of fixed arity [Hinderer, Blanqui]
- General libraries and algorithms:
  - matrices [Koprowski]
  - semi-rings [Koprowski,Zantema]
  - finite multisets [Koprowski]
  - integer polynomials with multiple variables [Hinderer]
  - computation of strongly connected components (SCCs) [Ducas]
  - lists, vectors, relations, etc.

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In total:

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- Only 20% of that is the code for actual termination methods!

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- C-CoRN
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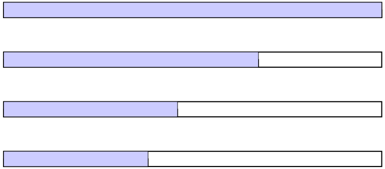

Related work

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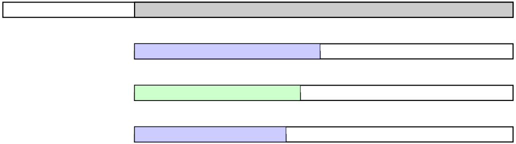

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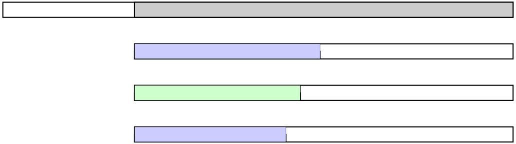

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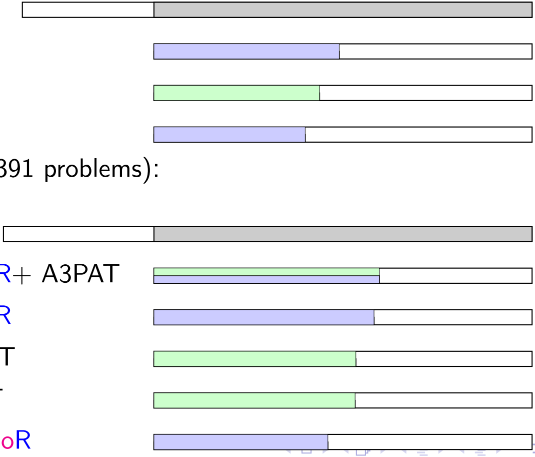

Outline

1. Background: termination of term rewriting

2. CoLoR project: certification of termination proofs
   - Why?... motivation
   - How?... CoLoR's approach to certification
   - When?... history of the project
   - What?... overview of the content
   - Related work
   - Certified competition

3. Conclusions... sort of

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  If it is possible do (involved) computations/reasoning in an unsafe setting and verify the results in Coq a posteriori.

  That requires some notion of a certificate.

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  We see that even in theorem provers — proof checking VS proof searching.

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The end

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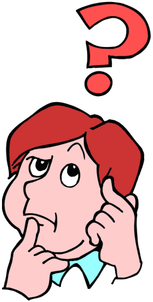

Thank you for your attention.