TPA: Termination Proved Automatically

Adam Koprowski

Technical University Eindhoven
Department of Mathematics and Computer Science

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Overview & Motivation

TPA

TPA is a tool for proving (relative) termination.

There are many tools for proving termination. Why creating yet another one?

- Semantic labelling with natural numbers.
- Relative termination.
- CoLoR.

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Semantic labelling with natural numbers.

- Semantic labelling is an old and well-known transformational technique for proving termination of TRSs.
- Up to date if it was used in automatic termination provers it was used with finite model (2 or 3 elements).
- TPA was created to verify the conjecture that semantic labelling with infinite model can be automated.
- $\Longrightarrow$ Yes it can.

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IJCAR, Saturday, August 19th, 12:00
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Relative termination

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Termination, $SN(R)$: Is there an infinite $\rightarrow_R$ reduction sequence?

Relative termination, $SN(R/S)$: Is there an infinite $\rightarrow_R \cup \rightarrow_S$ reduction sequence containing infinitely many $\rightarrow_R$ steps?

- Relative termination is a natural generalization of termination.
- It turns out to be useful in verification for modelling fairness.

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CoLoR

CoLoR: a Coq library on rewriting and termination

`http://color.loria.fr`

Rainbow: a termination proof certification tool

Objectives:

- formalization of theory of term rewriting in the theorem prover Coq.
- certification of termination proofs produced by tools for proving termination of rewriting (Rainbow).

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TPA techniques

Techniques used by TPA:

- Polynomial interpretations,
- Semantic labelling with:
  - booleans,
  - natural numbers,
- RPO,
- Dependency pairs (weak variant),
- Dummy elimination,
- Reduction of right hand sides.

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Termination competition

An annual termination competition is being organized where different termination provers compete on a set of termination problems.

`http://www.lri.fr/~marche/termination-competition`

- In 2005 TPA was 3rd out of 6 participating tools (in the main TRS category).
- In 2006 TPA was 3rd out of 8 participating tools.
- In both cases it was able to prove termination of systems that no other tool could deal with, including the SUBST system.

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- Written in Objective Caml.
- Around 10,000 lines of code.
- Command line interface.
- Available:
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  TPA v.1.1
Result: TRS is terminating
[1] TRS loaded for the input file:
      (1) T(I(x), y) -> T(x, y)
      (2) T(x, y) ->= T(x, I(y))

[2] Label this TRS using the following interpretation
    over N\{0,1}:
     [T(x,y)] = 2
     [I(x)] = x + 1

    This interpretation is a model and yields the
    following TRS:
     (1) T{i+1, j}(I{i}(x), y) -> T{i,j}(x,y)
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