Arctic Termination ... Below Zero

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15 July 2008
RTA, Hagenberg

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$\Longrightarrow$ This talk concerns a new method for proving termination, its automation and certification.

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Given TRSs $\mathcal{R}$ and $\mathcal{S}$ define:

- *top rewrite relation:* $t \xrightarrow{\text{top}}_{\mathcal{R}} u$ if and only if there is a rewrite rule $\ell \to r \in \mathcal{R}$ and a substitution $\sigma : \mathcal{V} \to \mathcal{T}(\Sigma, \mathcal{V})$ such that $t = \ell\sigma$ and $u = r\sigma$.

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- *relative top termination:* $\mathrm{SN}(\overset{\text{top}}{\to}_{\mathcal{R}} / \to_{\mathcal{S}})$ (important in the dependency pairs setting).

Monotone algebras

Definition (Monotonicity)

An operation $[f] : A \times \cdots \times A \to A$ is *monotone* with respect to a binary relation $\rhd$ on A if
$a_i \rhd a_i' \implies [f](a_1, \ldots, a_i, \ldots a_n) \rhd [f](a_1, \ldots, a_i', \ldots, a_n)$.

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Definition (Monotone Σ-algebras)

A *weakly monotone Σ-algebra* $(A, [\cdot], >, \gtrsim)$ is a Σ-algebra $(A, [\cdot])$ equipped with two binary relations $>, \gtrsim$ on A such that

- $>$ is well-founded;
- $> \cdot \gtrsim \subseteq >$;
- for every $f \in \Sigma$ the operation $[f]$ is monotone with respect to $\gtrsim$.

An *extended monotone Σ-algebra* $(A, [\cdot], >, \gtrsim)$ is a weakly monotone Σ-algebra $(A, [\cdot], >, \gtrsim)$ in which moreover for every $f \in \Sigma$ the operation $[f]$ is monotone with respect to $>$.

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Theorem

Let $\mathcal{R}, \mathcal{R}', \mathcal{S}, \mathcal{S}'$ *be TRSs over a signature* Σ*,* $(A, [\cdot], >, \gtrsim)$ *be an extended monotone* Σ*-algebra such that:*

- $\forall_\alpha\ [\ell]_\alpha \gtrsim [r]_\alpha$ *for every rule* $\ell \to r$ *in* $\mathcal{R} \cup \mathcal{S}$ *and*
- $\forall_\alpha\ [\ell]_\alpha > [r]_\alpha$ *for every rule* $\ell \to r$ *in* $\mathcal{R}' \cup \mathcal{S}'$

Then $\mathrm{SN}(\to_{\mathcal{R}} / \to_{\mathcal{S}})$ *implies* $\mathrm{SN}(\to_{\mathcal{R}} \cup \to_{\mathcal{R}'} \ / \ \to_{\mathcal{S}} \cup \to_{\mathcal{S}'})$.

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$$[x * (y + z)] = 2x + 2(y + z + 2) + 2x(y + z + 2) + 1$$
$$[x * y + x * z] = (2x + 2y + 2xy + 1) + (2x + 2z + 2xz + 1) + 2$$

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$$[x * (y + z)] = 2x + 2y + 2z + 4 + 2xy + 2xz + 4x + 1$$
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- To obtain strict monotonicity we require that for every interpretation $[f(x_1, \ldots, x_n)]$, $\forall_i \, \exists_{c>0} \, cx_i \in [f(x_1, \ldots, x_n)]$.

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- Strict monotonicity ensured if for every interpretation $[f(x_1, \ldots, x_n)] = F_1 x_1 + \ldots F_n x_n + \vec{f}$ we have $\forall_i\ (F_i)_{1,1} > 0$.

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- Now we compute in the $\langle \mathbb{A}_\mathbb{N}, max, + \rangle$ semi-ring.
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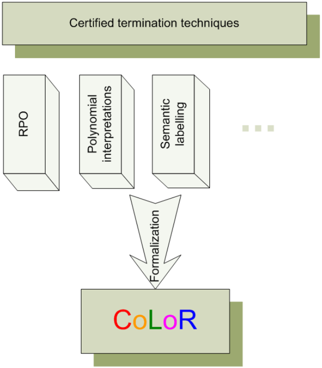
- Rainbow: a tool for translation from proofs in TPG format to Coq proofs, using results from CoLoR.

CoLoR's architecture overview



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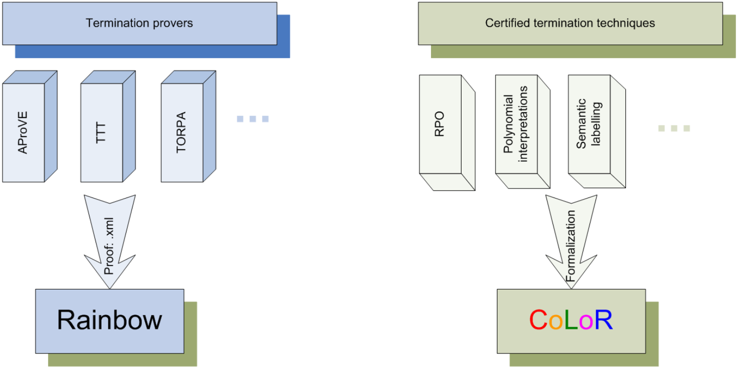

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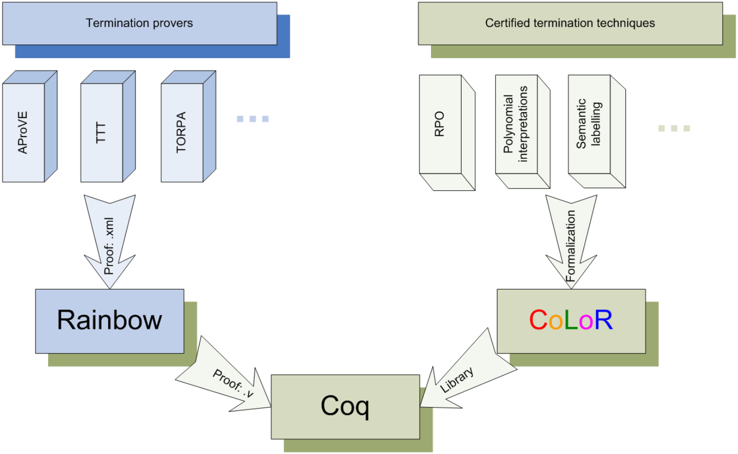

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problem set	time	s	sa	sz	saz	2007 winner
975 TRS	1 min	361	376	388	389	TPA: 354
	10 min	365	381	393	394	
517 SRS	1 min	178	312	298	320	Matchbox: 337
	10 min	185	349	323	354	

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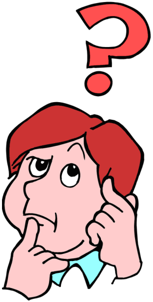
The end



Thank you for your attention.