# Termination of Rewriting and Its Certification

Adam Koprowski

Eindhoven University of Technology

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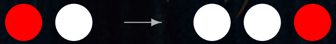
*Rewriting*: what is it all about...



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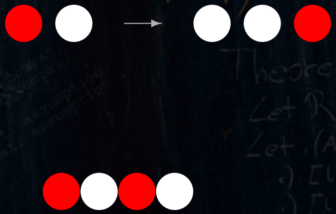

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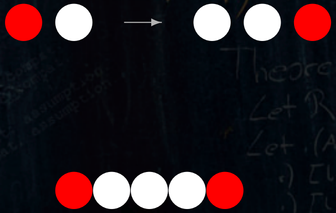

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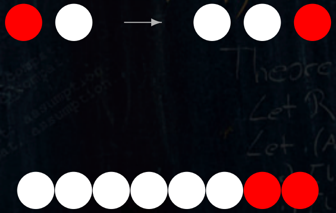

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*Term rewriting* is a specialization of rewriting where the objects under consideration are terms.

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**Example**

$$\begin{aligned} 0 + y &= y \\ s(x) + y &= s(x + y) \end{aligned}$$

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*Term rewriting* is a specialization of rewriting where the objects under consideration are terms.

**Example**

$$\begin{aligned} 0 + y &\to y \\ \mathsf{s}(x) + y &\to \mathsf{s}(x + y) \end{aligned}$$

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$$\begin{aligned} 0 + y &\rightarrow y \\ \mathsf{s}(x) + y &\rightarrow \mathsf{s}(x + y) \end{aligned}$$

Now let us do some math... how about $2 + 2$?

$$s(s(0)) + s(s(0)) \rightarrow$$

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Term rewriting is a model of computation.

Now let us do some math... how about 2 + 2?

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s(s(0)) + s(s(0)) \to s(s(0) + s(s(0))) \to s(s(0 + s(s(0)))) \to s(s(s(s(0))))
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How difficult is it to prove termination?

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**Example**

$$
\begin{aligned}
\mathsf{h\,1\,1} &\to \mathsf{1\,h} \\
\mathsf{1\,1\,h\,b} &\to \mathsf{1\,1\,s\,b} \\
\mathsf{1\,s} &\to \mathsf{s\,1} \\
\mathsf{b\,s} &\to \mathsf{b\,h} \\
\mathsf{h\,1\,b} &\to \mathsf{t\,1\,1\,b} \\
\mathsf{1\,t} &\to \mathsf{t\,1\,1\,1} \\
\mathsf{b\,t} &\to \mathsf{b\,h}
\end{aligned}
$$

# Termination of Rewriting and Its Certification

How difficult is it to prove termination?

**Example**

Collatz conjecture:

The function $f(n) = \begin{cases} n/2 & \text{if } n \text{ even} \\ 3n+1 & \text{if } n \text{ odd} \end{cases}$

converges to 1.

$$\mathsf{h}\ \mathsf{1}\ \mathsf{b} \longrightarrow \mathsf{t}\ \mathsf{1}\ \mathsf{1}\ \mathsf{b}$$

$$\mathsf{1}\ \mathsf{t} \longrightarrow \mathsf{t}\ \mathsf{1}\ \mathsf{1}\ \mathsf{1}$$

$$\mathsf{b}\ \mathsf{t} \longrightarrow \mathsf{b}\ \mathsf{h}$$

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**Open problem!**

$$h\ 1\ b \to t\ 1\ 1$$
$$1\ t \to t\ 1\ 1\ 1$$
$$b\ t \to b\ h$$

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converges to 1.

$\mathrm{h}\ 1\ \mathrm{b} \longrightarrow \mathrm{t}\ 1\ 1$

$1\ \mathrm{t} \longrightarrow \mathrm{t}\ 1\ 1\ 1$

**Open problem!**

**Termination is undecidable**

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1979 Polynomial interpretations

1982 Recursive path order

1995 Semantic labeling

2000 Dependency pairs

2006 Matrix interpretations

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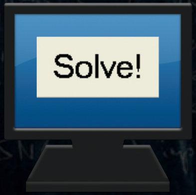

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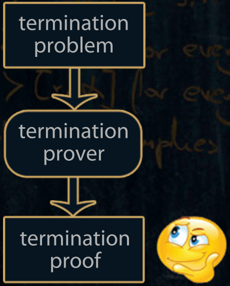

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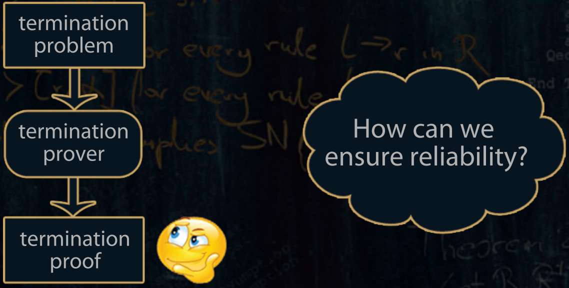

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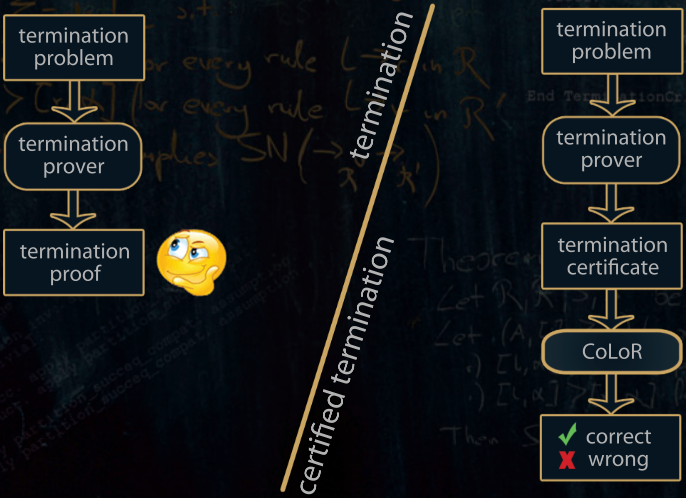

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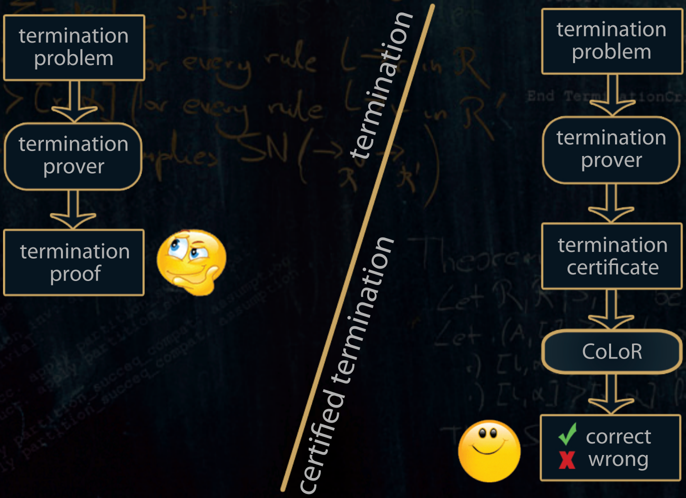

# Contributions

- New methods and refinements to existing methods for proving termination of rewriting.

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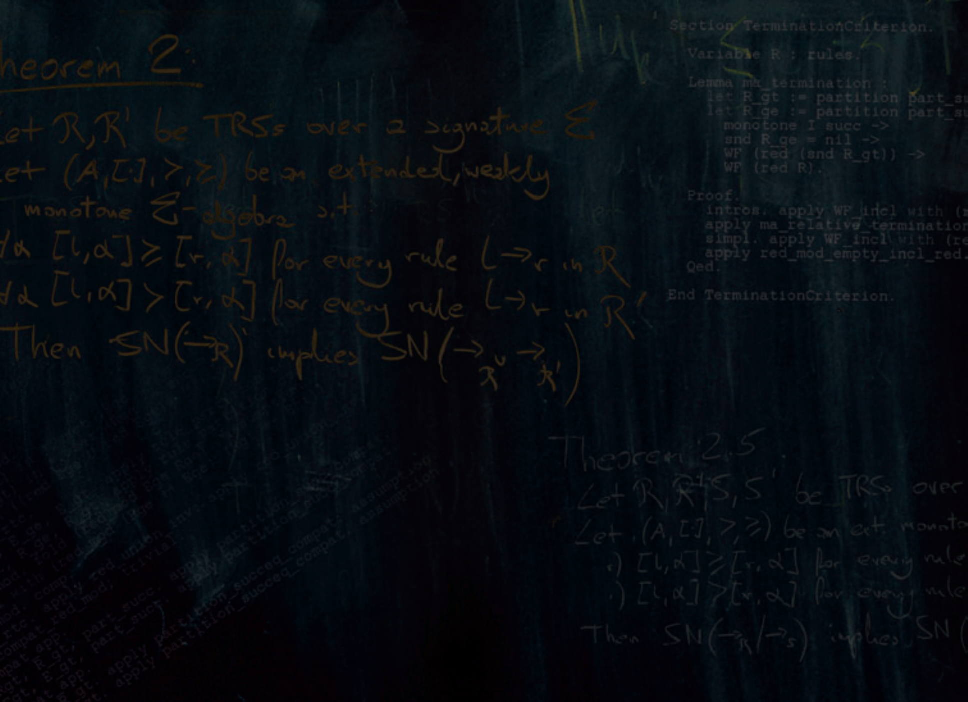
- New methods and refinements to existing methods for proving termination of rewriting.
- Contributions adding to the progress in the area of certification of termination proofs.


heorem 2:
et R, R' be TRSs over a signature Σ
et (A, [·], >, ≥) be an extended, weakly
monotone Σ-algebra s.t.
∀α [l, α] ≥ [r, α] for every rule l → r in R
∀α [l, α] > [r, α] for every rule l → r in R'
Then SN(→R) implies SN(→R ∪ →R')
Section TerminationCriterion.
Variable R : rules.
Lemma ma_termination :
let R_gt := partition part_su
let R_ge := partition part_su
monotone I succ ->
snd R_ge = nil ->
WF (red (snd R_gt)) ->
WF (red R).
Proof.
intros. apply WF_incl with (r
apply ma_relative_termination
simpl. apply WF_incl with (re
apply red_mod_empty_incl_red.
Qed.
End TerminationCriterion.
Theorem 2.5
Let R, R', S, S' be TRSs over
Let (A, [·], >, ≥) be an ext. monoto
·) [l, α] ≥ [r, α] for every rule
·) [l, α] > [r, α] for every rule
Then SN(→R / →S) implies SN(