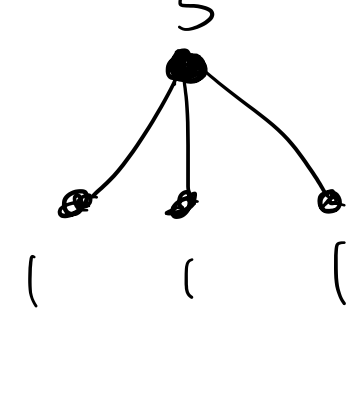


$$\forall_i \exists_{j,k,c} \quad v_i + v_j = v_k + v_c$$

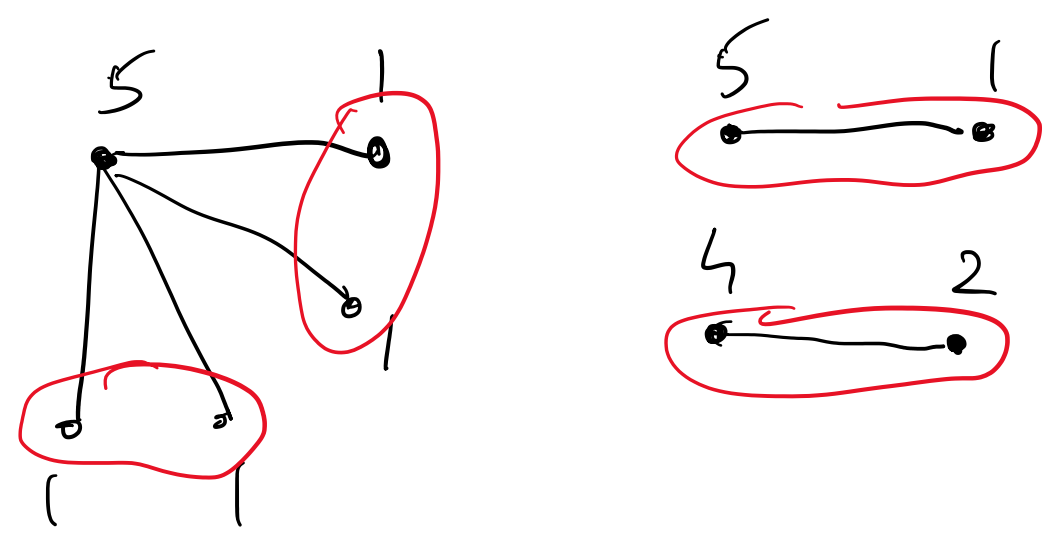
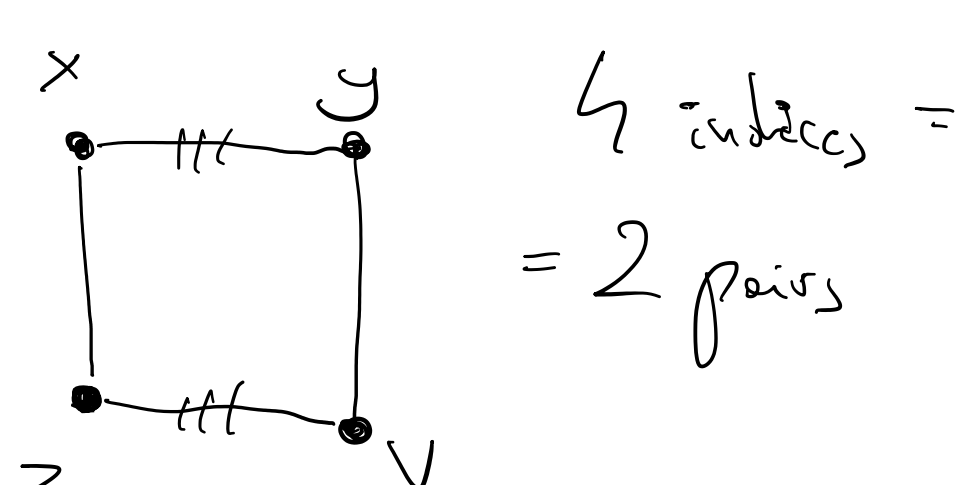


$$v_k + v_c - v_j = v_i$$

1 2 3 4 5

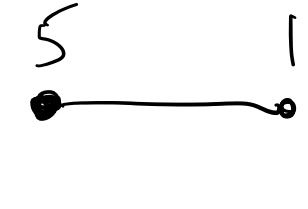
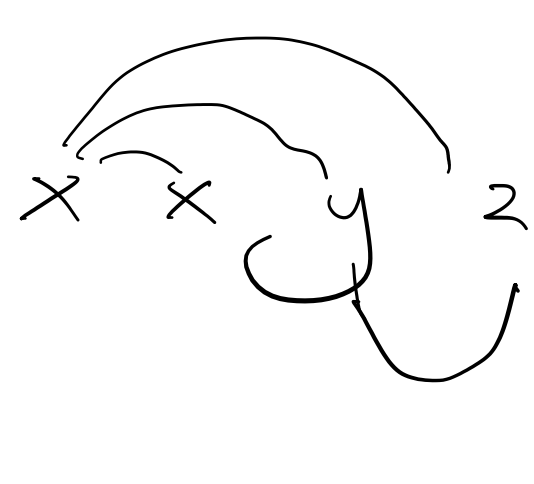
how many different sums?

$$S = \begin{matrix} v_1 + v_2 \\ v_3 + v_4 \\ v_5 + v_6 \\ v_7 + v_8 \end{matrix} \Rightarrow \exists x, y, z \quad v + x = y + z$$



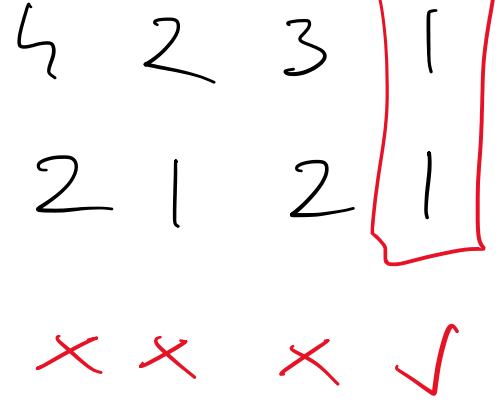
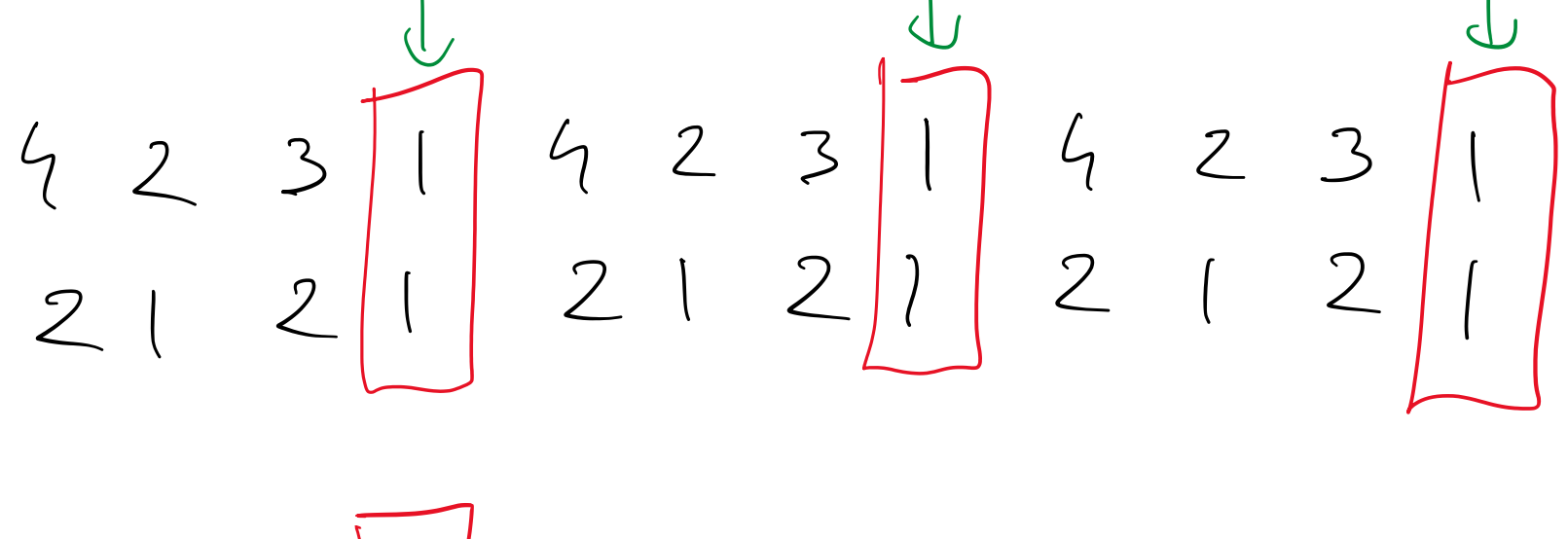
4 equal elements

OR 2 different pairs?

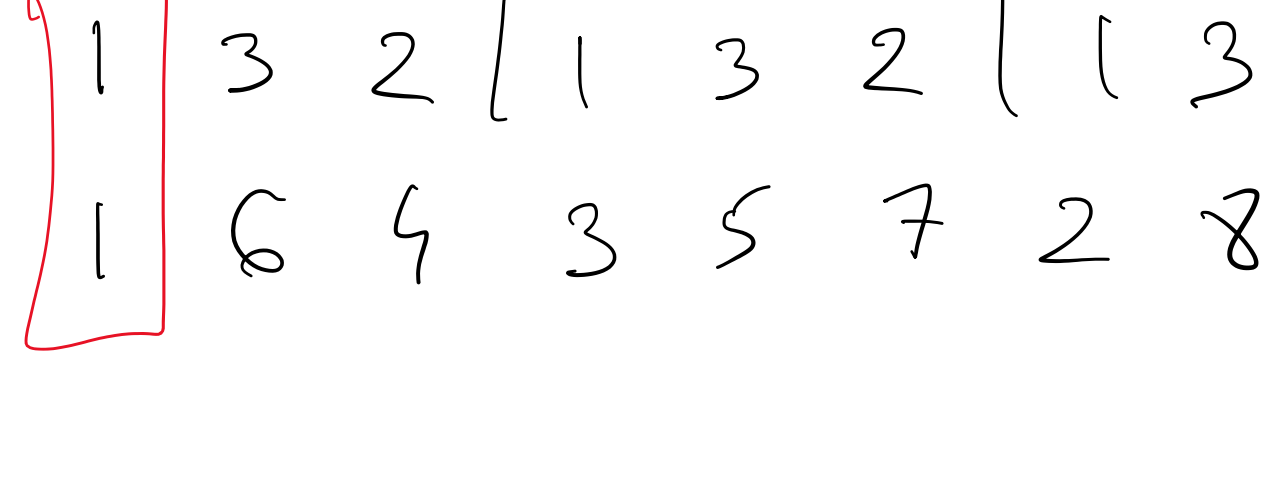


4 2 3 1

2 1



x x x ✓



4 2 3 1

2 1

always same??

offset/indices:

| | | | |
|---|---------|---------|---|
| 0 | 4 2 3 1 | 2 1 2 1 | 1 |
| 1 | 2 3 1 4 | 2 1 2 1 | 1 |
| 2 | 3 1 4 2 | 2 1 2 1 | 1 |
| 3 | 1 4 2 3 | 2 1 2 1 | 1 |

$$-2 \% 3 = 1$$

$$-7 \% 3 =$$

1 6 4 3 5 7 2 8

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | 1 | 3 | 2 | 1 | 3 | 1 |
| 3 | 2 | 1 | 3 | 2 | 1 | 3 | 2 | 1 |
| 2 | 1 | 3 | 2 | 1 | 3 | 2 | 1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 4 | 1 | 1 |
| 3 | 2 | 1 | 2 | 1 | 1 |
| 2 | 1 | 3 | 2 | 1 | 1 |

$$0 \% 3 = 0$$

$$8 \% 3 = 2$$

$$16 \% 3 = 1$$

| | | | | |
|---|---|---|---|---|
| 4 | 2 | 3 | 1 | 1 |
| 2 | 1 | 2 | 1 | 1 |
| 1 | 2 | 1 | 2 | 1 |

| | |
|------|-----|
| left | res |
| 4 | 0 |
| 1 | 4 |
| 0 | 5 |

left res

4 1 0

20 24

13 32

6 40

46

21 in 24 full

7 in 8 i=0

7 in 8 i=1

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 1 |
| 3 | 5 | 1 | 3 | 5 | 1 |
| 5 | 1 | 3 | 5 | 1 | 1 |

1 → ... → 4

(a_i, b_i)
↑
arrived
↓
departure

f_m

f_m

2 10 2

4 12

0 2