$$\dot{R_k} == \left(R_0 - 4\,\pi^2\,\text{Norm}[k]^2\right) R_k + \sum_{\substack{l \in \mathbb{Z}^2 + \frac{\text{Mod}[k,2]}{2} \\ 2\,l \neq \pm k}} \frac{2\,(2\,l + k).k}{\text{Norm}[2\,l + k]^2}\, R_{k-l}\, R_l$$

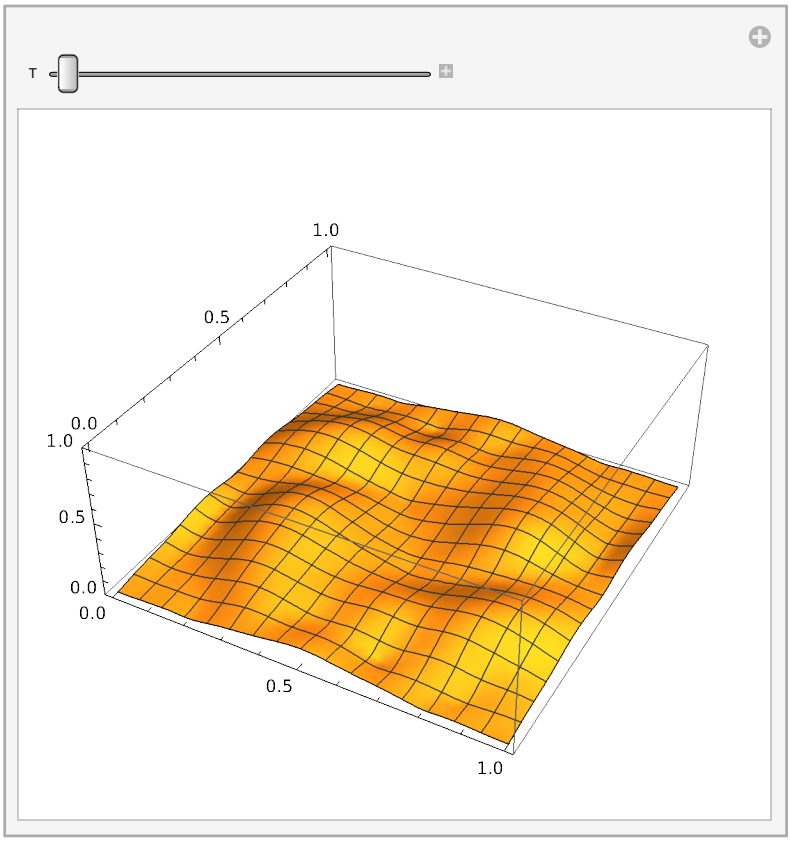
In[3]:=
```
reqn[k0_, k1_, sumbound_] :=
 With[{k = {k0, k1}, l = {l0, l1}}, D[R[k0, k1, t], t] == (R0 - 4 π^2 Norm[k]^2) R[k0, k1, t] +
    Sum[If[l0 == l1 == 0 ∨ l == k, 0, (l.k R[k0 - l0, k1 - l1, t] × R[l0, l1, t])/Norm[l]^2],
     {l0, -sumbound, sumbound}, {l1, -sumbound, sumbound}]]
```

In[4]:=
```
R0 = π
```

Out[4]= π

In[9]:=
```
func = With[{k = 3}, With[{odes = Table[reqn[k0, k1, k], {k0, -k, k}, {k1, -k, k}] /.
      (R[k0_, k1_, _] /; Max[Abs[k0], Abs[k1]] > k → 0)},
    With[{Rs = Union @ Cases[odes, R[x__] :> R[x], ∞]},
     Sum[E^(2 π I (k0 x0 + k1 x1)) R[k0, k1, t], {k0, -k, k}, {k1, -k, k}] /.
      First @ NDSolve[{odes, Table[R[k0, k1, 0] == RandomReal[{-.1, .1}] / (1 + Norm[{k0, k1}]^2),
        {k0, -k, k}, {k1, -k, k}]}, Rs, {t, 0, 1}]]]];
Manipulate[Plot3D[Norm[func /. t → T], {x0, 0, 1}, {x1, 0, 1}, PlotRange → {0, 1}], {T, 0, 1}]
```

Out[10]=



In[ ]:=
```
Table[R0 = Rzero;
 Table[Export["D:/Desktop/gifs/R0=" <> IntegerString[R0, 10, 2] <> "k=" <>
       ToString@k <> "randomR0propernorm " <> ".gif", #, "DisplayDurations " →
       Prepend[1/30 &/@ Rest@#, .5], AnimationRepetitions → ∞] &@With[{func =
       With[{odes = Table[reqn[k0, k1, k], {k0, -k, k}, {k1, -k, k}] /. (R[k0_, k1_, _] /; Max[
                Abs[k0], Abs[k1]] > k → 0)}, With[{Rs = Union@Cases[odes, R[x__] :> R[x], ∞]},
          Sum[E^(2 π I (k0 x0+k1 x1)) R[k0, k1, t], {k0, -k, k}, {k1, -k, k}] /. First@NDSolve[
              {#, Table[R[k0, k1, 0] == RandomReal[{-.1, .1}]/(1 + k0^2 + k1^2), {k0, -k, k}, {k1, -k, k}]},
              Rs, {t, 0, 1}] &@odes]]}, Table[Plot3D[Norm[func /. t → T], {x0, 0, 1},
       {x1, 0, 1}, PlotRange → {0, 1}], {T, 0, 1, 1/30}]], {k, 1, 4}], {Rzero, 0, 50, 5}]
```

NDSolve : At t == 0.659383881665856` , step size is effectively zero ; singularity or stiff system suspected .

InterpolatingFunction : Input value {2/3} lies outside the range of data in the interpolating function . Extrapolation will be used .

InterpolatingFunction : Input value {2/3} lies outside the range of data in the interpolating function . Extrapolation will be used .

InterpolatingFunction : Input value {2/3} lies outside the range of data in the interpolating function . Extrapolation will be used .

General : Further output of InterpolatingFunction ::dmval will be suppressed during this calculation .

NDSolve : At t == 0.6607879954066508` , step size is effectively zero ; singularity or stiff system suspected .

NDSolve : At t == 0.7386223514962316` , step size is effectively zero ; singularity or stiff system suspected .

General : Further output of NDSolve ::ndsz will be suppressed during this calculation .

Out[ ]=
```
{{D:/Desktop/gifs/R0=00k=1randomR0propernorm .gif,
  D:/Desktop/gifs/R0=00k=2randomR0propernorm .gif,
  D:/Desktop/gifs/R0=00k=3randomR0propernorm .gif,
  D:/Desktop/gifs/R0=00k=4randomR0propernorm .gif},
 {D:/Desktop/gifs/R0=05k=1randomR0propernorm .gif,
  D:/Desktop/gifs/R0=05k=2randomR0propernorm .gif,
  D:/Desktop/gifs/R0=05k=3randomR0propernorm .gif,
  D:/Desktop/gifs/R0=05k=4randomR0propernorm .gif},
 {D:/Desktop/gifs/R0=10k=1randomR0propernorm .gif,
  D:/Desktop/gifs/R0=10k=2randomR0propernorm .gif,
  D:/Desktop/gifs/R0=10k=3randomR0propernorm .gif,
  D:/Desktop/gifs/R0=10k=4randomR0propernorm .gif},
 {D:/Desktop/gifs/R0=15k=1randomR0propernorm .gif,
  D:/Desktop/gifs/R0=15k=2randomR0propernorm .gif,
  D:/Desktop/gifs/R0=15k=3randomR0propernorm .gif,
```

```
 D:/Desktop/gifs/R0=15k=4randomR0propernorm .gif},
{D:/Desktop/gifs/R0=20k=1randomR0propernorm .gif,
 D:/Desktop/gifs/R0=20k=2randomR0propernorm .gif,
 D:/Desktop/gifs/R0=20k=3randomR0propernorm .gif,
 D:/Desktop/gifs/R0=20k=4randomR0propernorm .gif},
{D:/Desktop/gifs/R0=25k=1randomR0propernorm .gif,
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{D:/Desktop/gifs/R0=35k=1randomR0propernorm .gif,
 D:/Desktop/gifs/R0=35k=2randomR0propernorm .gif,
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 D:/Desktop/gifs/R0=35k=4randomR0propernorm .gif},
{D:/Desktop/gifs/R0=40k=1randomR0propernorm .gif,
 D:/Desktop/gifs/R0=40k=2randomR0propernorm .gif,
 D:/Desktop/gifs/R0=40k=3randomR0propernorm .gif,
 D:/Desktop/gifs/R0=40k=4randomR0propernorm .gif},
{D:/Desktop/gifs/R0=45k=1randomR0propernorm .gif,
 D:/Desktop/gifs/R0=45k=2randomR0propernorm .gif,
 D:/Desktop/gifs/R0=45k=3randomR0propernorm .gif,
 D:/Desktop/gifs/R0=45k=4randomR0propernorm .gif},
{D:/Desktop/gifs/R0=50k=1randomR0propernorm .gif,
 D:/Desktop/gifs/R0=50k=2randomR0propernorm .gif,
 D:/Desktop/gifs/R0=50k=3randomR0propernorm .gif,
 D:/Desktop/gifs/R0=50k=4randomR0propernorm .gif}}
```

In[ ]:=
```
Export["D:/Desktop/gifs/R0onnormksquared .gif",
 MapThread[ImageAssemble @ Partition[{##}, 11] &,
  Flatten[Table[Import["D:/Desktop/gifs/R0=" <> IntegerString[Rzero, 10, 2] <> "k=" <>
      ToString @ k <> "randomR0onnormksquared .gif"], {k, 1, 4}, {Rzero, 0, 50, 5}], 1]]
```

Out[ ]=
```
D:/Desktop/gifs/R0onnormksquared .gif
```