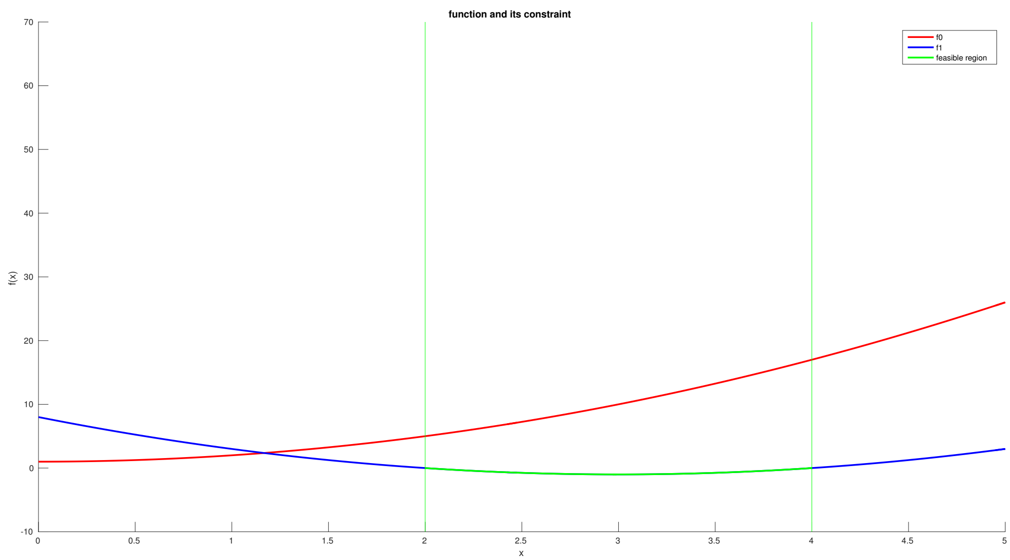
Machine Learnig Homework 8

Problem 5



As can be clearly seen in the above plot, $x^\star = 2$ is the optimal solution.

Problem 6

$$\begin{aligned} L(x,\alpha) &= f_0(x) + \alpha f_1(x) \\ &= x^2 + 1 + \alpha(x-2)(x-4) \\ &= (1+\alpha)x^2 + 6\alpha x + (1 + 8\alpha) \end{aligned} \tag{1}$$

The value of the objective function is given by the lagrangian $L(x,\alpha)$ evaluated at $\alpha = 0$, portrayed by the red curve in Fig. . Lagrangian is smaller than the objective function for $x \in (2,4)$ and greater than the objective function for $x \notin [2,4]$. Values at $x = 2$ and $x = 4$ are unaffected by α. The upper bound for $\min_x L(x,\alpha) = L(2,\alpha)$ for all $\alpha \geq 0$ is 5.

Problem 7

$$g(\alpha) = \min_x L(x,\alpha) \implies \frac{\partial}{\partial x} L(x,\alpha) = 0 \implies x^\star = \frac{3\alpha}{1+\alpha} \tag{2}$$

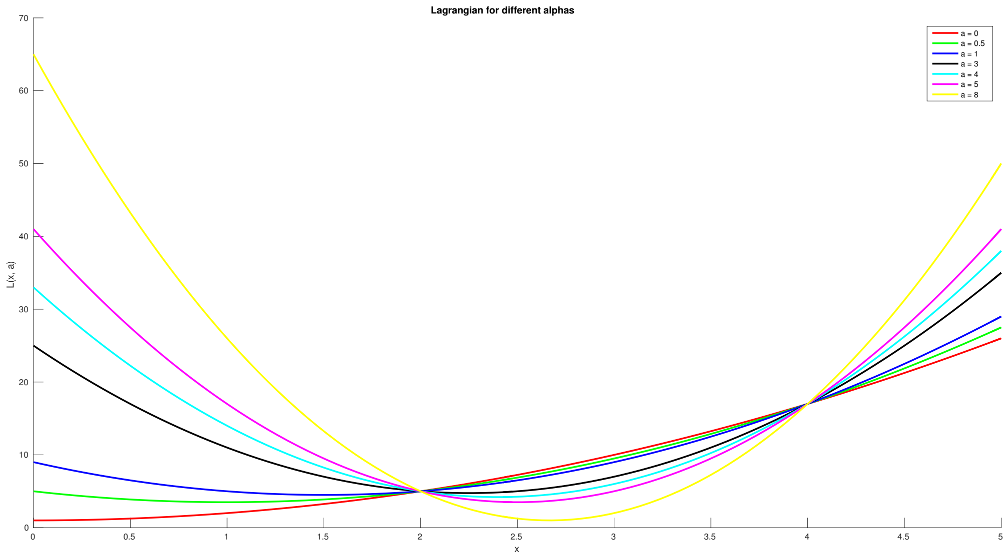


Figure 1: Lagrangian $L(x, \alpha)$ for different values of α

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$$g(\alpha) = \frac{-\alpha^2 + 9\alpha + 1}{1 + \alpha} \tag{3}$$

The dual problem is plotted in Fig. and it is given by

$$\max_{\alpha} g(\alpha) \\ \text{subject to } \alpha \geq 0 \tag{4}$$

Problem 8

$$\alpha^\star = \arg\max_{\alpha} g(\alpha) \tag{5}$$

$$0 = \frac{\partial}{\partial \alpha} g(\alpha) = \frac{-\alpha^2 - 2\alpha + 8}{(1 + \alpha)^2} \text{ and } \alpha \geq 0 \tag{6}$$

The dual optimal solution $\alpha^\star = 2$ and $g(\alpha^\star) = 5$.

Problem 9

$$x^\star = \frac{3\alpha^\star}{1 + \alpha^\star} = \frac{6}{1 + 2} = 2 \tag{7}$$

The optimal solution is given by $f_0(x^\star) = 5$, which is equal to the dual optimal value.

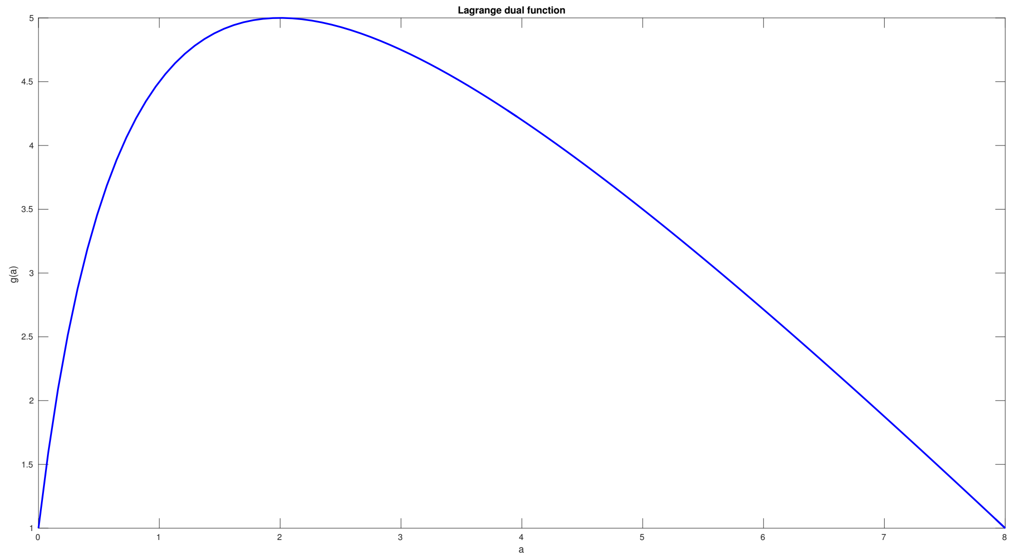

Figure 2: Lagrange dual function $g(\alpha)$.

Problem 10

The constraint f_1 is active. We can see it also on the plot of the primal problem in Fig. , since the feasible region is bounded by the zero-crossings of this constraint and the objective function attains its minimum at one of them.