Pultrusion Process Simulation

NDSU ME454 - Heat and Mass Transfer - Term Project Fall 2025

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ABSTRACT

This report investigates the heating of a fiberglass-resin composite during the pultrusion process. Pultrusion provides an efficient means of producing continuous fiberglass stock for large scale manufacturing, and the achievable production rate is governed by heat transfer through the die and the associated cure kinetics of the resin. Based on the thermal analysis developed in this work, the composite temperature and degree of cure are modeled as functions of position along the die, explicitly linking pull speed, platen heating, and inlet material properties to $T(x)$ and $\alpha(x)$ for possible use in process design and future optimization.

1 INTRODUCTION

The objective of this project is to analyze heat transfer within the die during the pultrusion process. Pultrusion is a continuous manufacturing method in which fiber reinforcements are pulled through a resin bath and then through a heated die to produce long composite profiles with a high strength-to-weight ratio. In this work, the goal is to take different parameters such as heating platen temperatures, pull speed, resin characteristics, and other relevant determinants of process performance to model the composite temperature and the resin's degree of cure as a function of its location along the die.

2 PROBLEM SETUP

The complete pultrusion line, shown schematically in Figure 1, consists of several key components, including the resin bath, heating die, mechanical puller, and cut-off saw. In the present analysis, only the heating-die stage is considered to simplify the downstream behavior and focus on the dominant heat-transfer mechanisms. Mainly, consideration of a smaller control volume also allows for more assumptions to be made.

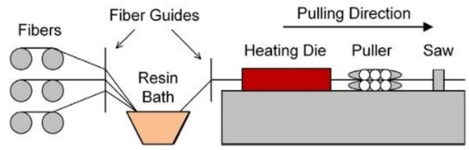


Figure 1. The schematic representation of a conventional pultrusion process *[1]*

Each of the four heating platen modeled in this simulation have a 250 mm length with a 50 mm square cross section[1] which sums to a total length of 1000 mm, or about 3.28 ft. A 50 mm × 50 mm square cavity in the die is filled with a composite matrix composed of continuous fiber reinforcements impregnated with resin and externally supported by a pack layer. To simplify the analysis, the fibers, resin, and pack layer are homogenized and treated as a single solid with uniform effective material properties and no voids. This rejects the need to model the saturation of the fibers and the necessary evaluation of different layers of composite material.

The resin enters the die as a liquid state. As the matrix is pulled through the die, the resin transitions from a liquid state to a gel state, followed by a solid state. During curing, the resin releases heat exothermically, and the rate of heat generation varies with the degree of cure and therefore with time and position. Indeed, the heat supplied by the platens is prescribed as a function of axial distance along the die for the present analysis.

Consequently, the pultrusion process is governed by the coupled effects of the resin cure reaction, the externally applied heating, the thermal properties of the die and composite, and the die length and pull speed. Effective process design requires these factors to be balanced so that the material reaches an adequate degree of cure by the time it exits the die.

3 PROCEDURE

To simplify the analysis, the following assumptions were made: 1-dimensional heat transfer, forced internal convection, uniform matrix cross-section temperature, uniform material properties, the absence of voids, and constant contact resistance at the die-composite interface. In observing the problem setup, an energy balance equation can be created, which is given by

$$\mathbf{0 = \dot{m}c_p[T(x) - T(x + dx)] + q'''_{platen}(x)Adx + q'''_{rxn}(x)Adx} \tag{1}$$

[1] Note that the widths of the platens, die, and the composite are the same for clarity in the 1-dimensional assumption

$$\mathbf{\dot{m}c_p[T(x+dx)-T(x)] = \left[q'''_{platen}(x)A + q'''_{rxn}(x)A\right]dx} \tag{2}$$

where the first term represents the net enthalpy change of the material as it moves along the die, the second term represents the heat flux from the heating platens, and the third term represents the heat generated from the resin exotherm occurring in the die. Dividing Equation 2 with dx, yields

$$\mathbf{\dot{m}c_p\frac{dT}{dx} = q'''_{platen}(x)A + q'''_{rxn}(x)A} \tag{3}$$

The heat flux from the heating platens can be modeled with

$$\mathbf{q'''_{platen}(x) = \frac{d\dot{Q}}{dV} = \frac{q''_{platen}(x)Pdx}{Adx} = \frac{P}{A}q''(x) = \frac{P}{A}\frac{T_{platen}(x) - T_{composite}(x)}{\frac{t_{steel}}{k_{steel}} + R''_{tc}}} \tag{4}$$

Which includes both conduction resistance and contact resistance. The heat flux from the internal exothermic reaction can be expressed as

$$\mathbf{q'''_{rxn}(x) = \rho\Delta H_r\frac{d\alpha}{dt}} \tag{5}$$

where $\frac{d\alpha}{dt}$ is the rate of cure[2], which can be expressed as a function of Arrhenius parameters

$$\mathbf{\frac{d\alpha}{dt} = \frac{A_o}{V}\exp\left(-\frac{E}{R\left(T(x)\right)}\right)f(\alpha(x))} \tag{6}$$

Combing Equations (*3*(4(5(6, and substituting for the inlet mass flow rate, yields,

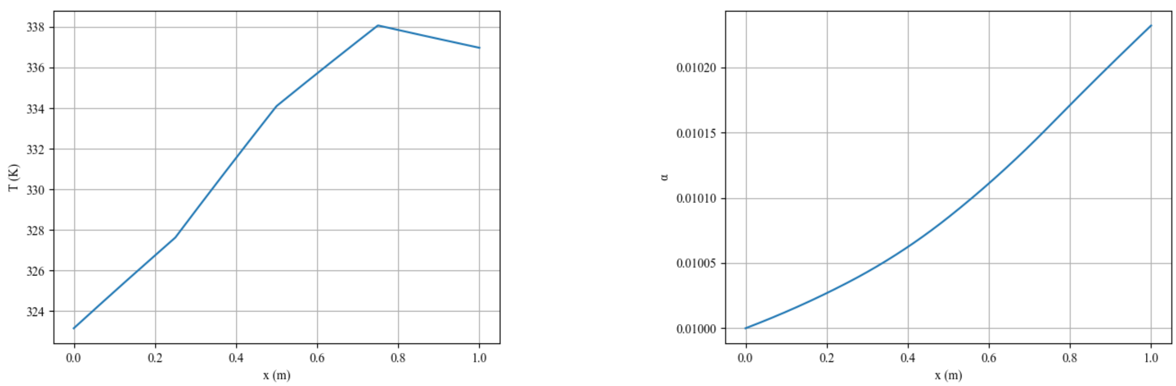
$$\mathbf{\rho_{eff}Vc_p\frac{\mathit{dT}}{\mathit{dx}} = \frac{P}{A}\frac{T_{platen}(x) - T_{composite}(x)}{\frac{t_{steel}}{k_{steel}} + R''_{tc}} + \rho_{resin}\Delta H_r\frac{A_o}{V}\exp\left(-\frac{E}{R\left(T(x)\right)}\right)f(\alpha(x))} \tag{7}$$

[2] The full derivations related to dα/dt are provided in Appendix A.

Equation (*7* represents the governing equation for the defined control volume within the die. For this derived relation, note that the composite temperature $T_{\mathbf{composite}}(x)$ and the degree of cure $\alpha(x)$ are the two unknowns, which can be modelled with respect to previously defined constant parameters.

4 RESULTS AND DISCUSSION

Reasonable case parameters are provided in Appendix B as obtained from [4]-[9] and corresponding unknowns $T_{\mathbf{composite}}(x)$ and $\alpha(x)$ were solved by using Python's SciPy library[3], as shown in Figure 2a and 2b.



(a) (b)

Figure 2. Temperature of the composite and degree of cure as a function of temperature from the base case provided in Appendix B

Figure 2a shows an apparent discontinuous derivative at the platen boundary locations. Here, the first limitation of the model surfaces as the analysis assumes the platen temperatures to be constant along specified lengths. To remedy this action, a more continuous temperature gradient must be modeled for the die-composite interface. A simpler conduction model can be presumed by treating the platen-die combination to be its own control volume, independent of the composite. However, note that the composite in the die contains its own heat generation due to the concurrent exothermic reactions. A more accurate model would be to acknowledge the coupled nature of the platen-die-composite combination. The energy balance along the die can be expressed as

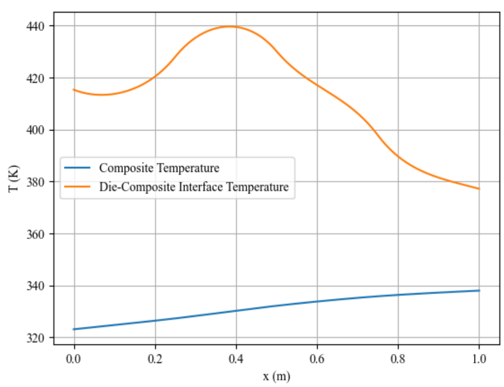
$$\frac{\mathbf{d}}{\mathbf{dx}}\mathbf{q_x(x)} + \mathbf{q'_{platen}(x)} - \mathbf{q'_{to\ composite}(x)} - \mathbf{q'_{to\ ambient}(x)} = 0 \qquad (8)$$

[3] Code can be found in Appendix C

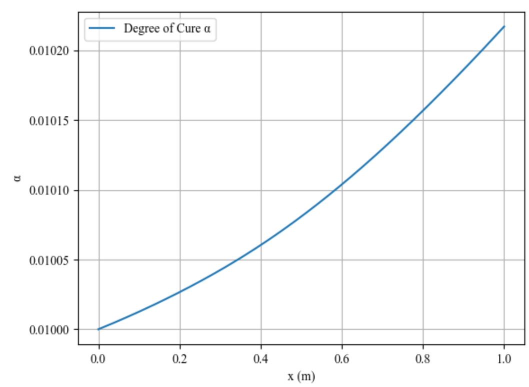
$$\frac{d}{dx}\left(-k_s A_s \frac{dT_{int}}{dx}\right) + q'_{platen}(x) - \frac{P}{\frac{t_{steel}}{k_{steel}} + R''_{tc}}\left[T_{interface}(x) - T_{composite}(x)\right] = 0 \tag{9}$$

which represents the governing expression for the coupled platen-die-composite interface. Note that for Equation (***9***), heat loss from the die to the air is only considered at the inlet and outlet through boundary conditions, hence, $q'_{to\ ambient}(x) = 0$ along the entire interior of the die, which does not appear explicitly in the expression. In addition, rather than defining the temperatures explicitly, uniform heat input is instead assumed along given lengths[4].

This new expression was included in the code, and the same procedure was performed with Python, which yielded the outputs shown in Figure 3a and 3b.



(a)



(b)

Figure 3. Temperature of the composite and degree of cure as a function of temperature with platen-die-composite coupling via heat input as opposed to constant platen temperatures

The outputs from the new expression follow a smoother transition between the platens after denying the constant temperature assumptions for the heaters. Though the model abides by many limiting assumptions which will be expanded later, it is a much more accurate approximation than the first iteration.

Note that the first platen and third platens were set to 1000 W, the second to 1500 W, and the last platen was set to 500 W. This follows the nature of a true pultrusion process wherein the last portion of the die acts to cool the composite as it exits.

5 CONCLUSION AND LIMITATIONS

[4] Additional parameter test cases for the new expression can be found in Appendix B

In the preceding analysis of the pultrusion process, different process parameters were chosen to model the temperature of the composite and the die-composite interface through heat transfer principles. The die component of the pultrusion process serves as a device to compact, form, and cure the resin impregnated fiber matrix. The heat generated by the exothermic reaction requires balance between the numerous process parameters in order to predict the matrix temperature as it progresses through the die.

In this analysis, the length, cross sectional area, die temperature, and homogenous matrix properties were specified to examine the resulting temperature and degree of cure. The resulting data could be used as a basis for further pultrusion analysis or implementation. As the matrix progresses through the die, the temperature increases as a result of the heat generated by the resin and the heat transferred from the die. The degree of cure plot did not reach a value of unity which infers that the resin did not fully cure, which would require parameter alteration.

In addition, the findings of the present analysis are subject to certain limitations. The matrix properties were assumed to be homogenous throughout the cross section. In reality, the response would be altered by the different layer properties of the composite and the saturation of the fiberglass. The contact resistance between the die and the matrix was assumed to be constant, which would realistically change as the resin cures and viscosity approaches infinity. Residual stresses were also not considered, which would introduce a large impeding force opposite the direction of pull. In addition, the analysis was largely 1-dimensional and did not consider heat transfer via the sides of the die. Nevertheless, the results of this analysis concur with the results of external sources. Figure 4 below shows the results of a similar (numerical) analysis, which closely resembles the rising and falling action shown in Figure 4.

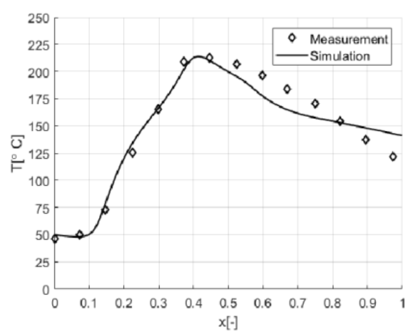


Figure 4. Reference pultrusion simulation output

It is plausible for future analyses to address the limitations of the present. In addition, further investigation may focus on using the preceding simulation in investigating the variation of different input parameters, and optimization sequences may be applied to obtain favorable conditions such as the maximum attainable pull speed.

APPENDIX [A]

Arrhenius Parameters

$$\frac{d\alpha}{dt} = A_o exp\left(\frac{-E}{RT}\right) f(\alpha)$$

$$t = \frac{x}{V}$$

$$\frac{d\alpha}{dt} = V\frac{d\alpha}{dx}$$

$$\frac{d\alpha}{dx} = \frac{A_o}{V} exp(-\frac{E}{R(T(x)})f(\alpha(x))$$

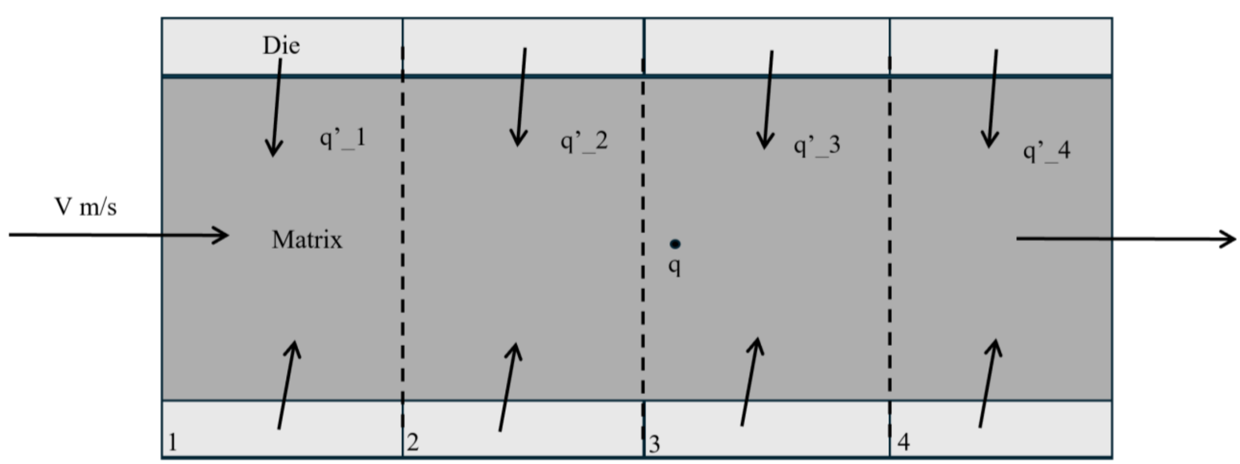
Energy Balance for Die

$$\frac{d}{dx}\alpha_x(x) + q'_{plat}(x) - q'_{into\ composite}$$

$$\frac{d}{dx}\left(-k_s A_s \frac{dT_w}{dx}\right) + q'_{plat}(x) - \frac{P}{R_{tot}}[(T_w(k) - T(x)]$$

$$\frac{dT_w}{dt} = q$$

$$\frac{dq}{dx} = \frac{[\frac{P}{R_{tot}}(T_w - T) - q'_{plat}(x)]}{k_s A_s}$$



APPENDIX [B]

Parameter	*Value*
Kinetic heating rate	173e3
Kinetic Exponent	1.2
Gas Constant (R)	8.314
A0	1e4
e	60e3
Heating Zone Length (x4)	0.250m
Cross Sectional Area	0.05 x0.05m
Steel Thickness	0.125m
Side	0.05m
Steel k	20.0
Contact Resistance	0.01
Pull Speed	0.01667m/s
Q1	1000w
Q2	1500w
Q3	1000w
Q4	500w
Temperature in	50c
α in	0.01
Temperature surrounding	20c
hi=hl	10

APPENDIX [C]

Repository can be found in the following link:
https://github.com/akositerrence/pultrusion/blob/main/pultrusion_b.py

```
import numpy as np from scipy.integrate import solve_bvp import
matplotlib.pyplot as plt
plt.rcParams["font.family"] = "serif" plt.rcParams["font.serif"] = ["Times
New Roman"]
effective_composite_density = 1800 effective_resin_density = 1200
cp_effective = 900
kinetic_heating_rate = 175e3
a0 = 1e4 e = 60e3 r_gas = 8.314 kinetic_exponent_n = 1.2
side = 0.05 cross_sectional_area = side*2 perimeter = side4 steel_thickness =
0.125 steel_k = 20.0 thermal_contact_resistance = 0.01 pull_speed = 0.01667
L1 = 0.25 L2 = 0.25 L3 = 0.25 L4 = 0.25 T_platen_2 = 2 + 273.15 L_total = L1
+ L2 + L3 + L4 steel_as = side * steel_thickness
t_in = 50 + 273.15 alpha_in = 0.01
t_inf = 20 + 273.15 hi, hl = 10.0, 10.0 a_end = side * steel_thickness
r_tot = (steel_thickness/steel_k) + thermal_contact_resistance
def q_heater(x): q1 = 1000.0 q2 = 1500.0 q3 = 1000.0 q4 = 500.0
x = np.asarray(x)
out = np.empty_like(x)
```

```
for i, xi in enumerate(x):
    if xi < L1:
        out[i] = q1
    elif xi < L1 + L2:
        out[i] = q2
    elif xi < L1 + L2 + L3:
        out[i] = q3
    else:
        out[i] = q4

return out

def f_alpha(alpha): # KINETIC MODEL alpha = np.clip(alpha, 0.0, 0.999999) y =
(1.0-alpha)**kinetic_exponent_n return y
def ode_right(x, y): T = y[0, :] alpha = y[1, :] Tint = y[2, :] q = y[3, :]
# KINETICS
da_dx = ( (a0/pull_speed)* np.exp(-e/(r_gas*T)) ) * (f_alpha(alpha))

# COMPOSITE
q_ppp_wall = (perimeter/ cross_sectional_area) * (Tint - T) / r_tot
q_ppp_reaction =
(effective_resin_density*kinetic_heating_rate*pull_speed*da_dx)
dt_dx = (q_ppp_wall + q_ppp_reaction) / (effective_composite_density *
cp_effective * pull_speed)

# WALL
dTint_dx = q
dq_dx = ((perimeter/r_tot) * (Tint - T) - q_heater(x) ) / (steel_k *
steel_as)

return np.vstack([dt_dx, da_dx, dTint_dx, dq_dx])

def bc(y_i, y_l): T_i, a_i, Tint_i, q_i = y_i T_l, a_l, Tint_l, q_l = y_l bc1
= T_i - t_in bc2 = a_i - alpha_in bc3 = -steel_k * steel_as * q_i - hi *
a_end * (Tint_i - t_inf) bc4 = -steel_k * steel_as * q_l - hl * a_end *
(Tint_l - t_inf)
return np.array([bc1, bc2, bc3, bc4])

x_stuff = np.linspace(0.0, L_total, 200) y_guess = np.zeros((4,
x_stuff.size)) y_guess[0, :] = t_in y_guess[1, :] = alpha_in y_guess[2, :] =
T_platen_2 y_guess[3, :] = 0.0
solution = solve_bvp(ode_right, bc, x_stuff, y_guess) x = solution.x
temperatures = solution.y[0, :] alphas = solution.y[1, :]
temperatures_interface = solution.y[2, :]
plt.figure() plt.plot(x, temperatures, label="Composite Temperature")
plt.plot(x, temperatures_interface, label="Die-Composite Interface
Temperature") plt.grid(True) plt.xlabel("x (m)") plt.ylabel("T (K)")
plt.legend() plt.show()
plt.figure() plt.plot(x, alphas, label="Degree of Cure α") plt.grid(True)
plt.xlabel("x (m)") plt.ylabel("α") plt.legend() plt.show()
```

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