SOAC Exercise 1

Simulation of the sea breeze at the coast

Andreas Kotilis
6641032

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This project is an effort to simulate the sea breeze at the Dutch coast of IJmuiden for the period of two days of May 1976. A simulation model was created, centered on the linearized horizontal wind momentum equations, which are imposed by a pressure gradient force, a surface frictional force and the Coriolis force. Under the conditions of the imposed forces, the quality of the model will be evaluated while the performance of the simulation will be measured through statistical analysis.

The linearized horizontal momentum equations used to build the model are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv \tag{1}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu \tag{2}$$

where it is assumed $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$. The horizontal wind velocities in the zonal and meridional direction are u and v respectively. The Coriolis parameter is defined as $f = 2\Omega sin\phi$, with the spin rate of Earth $\Omega = 7.2792 \cdot 10^{-5} s^{-1}$ and the latitude of IJmuiden $\phi = 52°$. Together with the air density $\rho = 1.25 kg/m^3$ they are considered constant in the simulation.

The meridional pressure gradient will be taken as zero, $\frac{\partial p}{\partial y} = 0$. The zonal pressure gradient, perpendicular to the coast, can be parametrised as:

$$\frac{\partial p}{\partial x} = Acos(\Omega t + \phi) + B \tag{3}$$

where A, ϕ and B are the constants that will be estimated from the best fit curve of the model to the data of the observed zonal pressure gradient(Figure 1). The values acquired are: $A = -8.17197 \cdot 10^{-4} Pa/m, \phi = 99.798°, B = -1.78855 \cdot 10^{-4} Pa/m$.

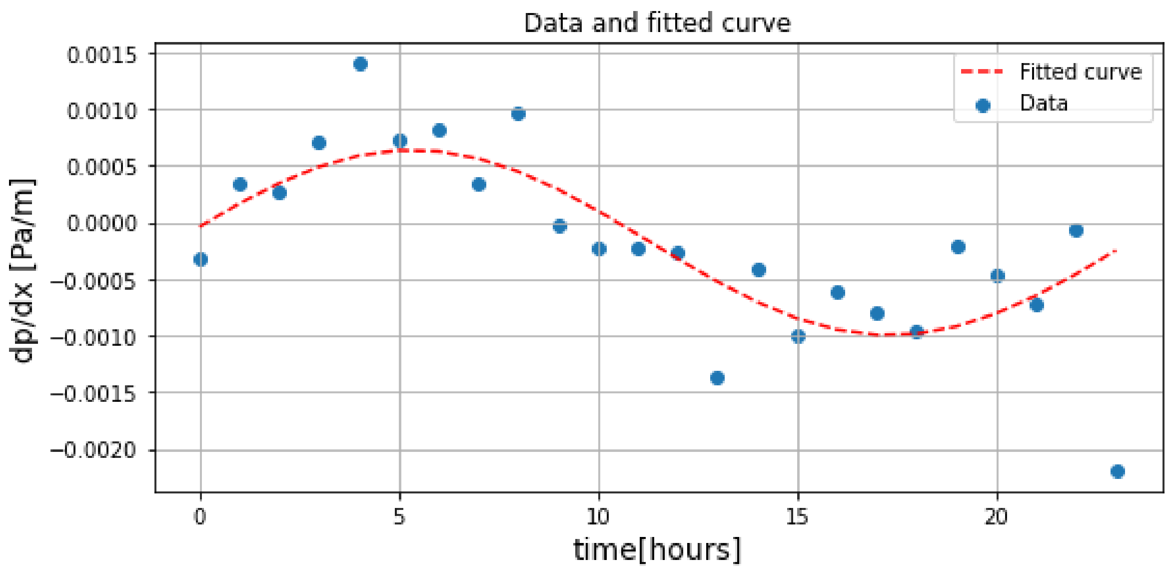


Figure 1: Modelled zonal pressure gradient and observational data.

The pressure gradient is now integrated in the parametrisation and the horizontal momentum equations 1 and 2 are written as:

$$\frac{\partial u}{\partial t} = fv - \frac{A}{\rho}cos(\Omega t + \phi) - \frac{B}{\rho} \tag{4}$$

$$\frac{\partial v}{\partial t} = -fu \tag{5}$$

which can be simplified to

$$\frac{\partial^2 u}{\partial t^2} + f^2 u = \frac{A\omega}{\rho}sin(\Omega t + \phi) \tag{6}$$

and has the following analytical solution:

$$u(t) = C_1 sin(ft) + C_2 cos(ft) + \frac{A\Omega}{\rho(f^2 - \Omega^2)}sin(\Omega t + \phi) \tag{7}$$

Assuming that $\phi = 0$ and that the boundary conditions are $u = v = 0$ at $t = 0$, yields:

$$C_2 = 0$$

$$C_1 = -\frac{Af}{\rho(f^2 - \Omega^2)}$$

The zonal and the meridional velocity now have an analytical solution:

$$u(t) = \frac{A}{\rho(f^2 - \Omega^2)}(\Omega sin(\Omega t + \phi) - fsin(ft)) \tag{8}$$

$$v(t) = \frac{1}{f}\frac{\partial u}{\partial t} + \frac{A}{f\rho}cos(\Omega t) + \frac{B}{f\rho} = \frac{A}{\rho(f^2 - \Omega^2)}(\Omega cos(\Omega t + \phi) - fcos(ft)) + \frac{B}{f\rho} \tag{9}$$

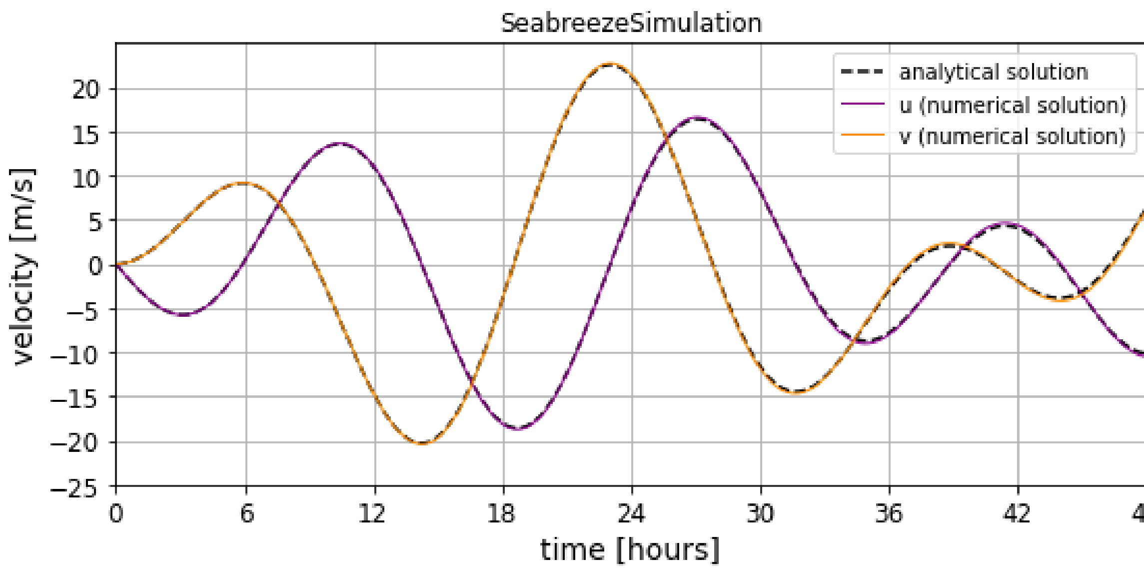


Figure 2: Comparison between the analytical and numerical solutions of the wind velocity.

The numerical solution of Equations 4 and 5 can be approximated using the Euler-forward scheme:

$$\frac{u(t+\Delta t)-u(t)}{\Delta t} = fv(t) - \frac{A}{\rho}cos(\Omega t+\phi) - \frac{B}{\rho} \tag{10}$$

$$\frac{v(t+\Delta t)-v(t)}{\Delta t} = -fu(t) \tag{11}$$

Figure 2 represents the outcomes of the analytical and the numerical solutions, assuming that $\phi = 0$, $u = v = 0$ and $A = 0.001Pa/m$. The chosen time step was $\Delta t = 30s$ providing accurate results as the numerical approximation was in total agreement with the analytical solution.

Sea Breeze model results and discussion

The model was initialised with the observed values of u and v at IJmuiden at 00 UTC on 7 May 1976. Equations 4 and 5 were used and parameters A and ϕ took the values calculated from the best fit curve of the previous section. Parameter B equals to zero for the simulations where the geostrophic wind is not included. To compare the outcome with the hourly observations of u and v, the correlation coefficient r, standard deviation σ, root-mean-square (RMS) difference E and centred pattern RMS E' were computed.

Figure 3 demonstrates the results of the simulation of a no-friction case together with the observations for u and v. The values of the statistical parameters are visible in Table 1. It can be seen that there is no correlation between the horizontal velocities and the observed wind speeds. Also, the high values of the standard deviation, the RMS difference and the centred pattern RMS is an indication of the inaccuracy of the model which possibly rises from the fact that friction is not included.

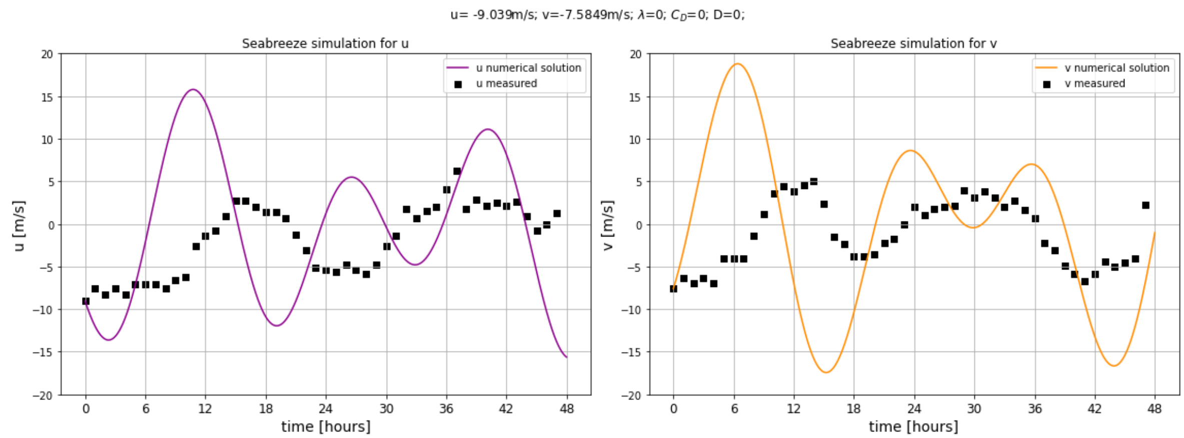


Figure 3: Simulation of the sea breeze with no friction and the observed data.

Parameter	Value
r_u	0.104701
r_v	0.0456135
σ_u	8.53996
σ_v	9.93905
E_u	9.51775
E_v	10.7031
E'_u	9.07869
E'_v	10.4817

Table 1: Statistical parameters for the no frictional case.

In the following simulation friction is included and represented by the Rayleigh damping coefficient λ. The horizontal momentum equations become:

$$\frac{\partial u}{\partial t} = fv - \frac{A}{\rho} cos(\Omega t + \phi) - \lambda u \tag{12}$$

$$\frac{\partial v}{\partial t} = -fu - \lambda v \tag{13}$$

Simulations for different values of λ took place before concluding into the best statistical results. For the value of $\lambda = 6 \cdot 10^{-5}$ the model correlates better with the observations. The outcomes of the simulation can be seen in Figure 4 and the statistical parameters in Table 2. It is evident that the accuracy of the model has increased as the correlation coefficients have increased relative to the no-friction case while the standard deviation, the RMS difference and the centred pattern RMS values have decreased.

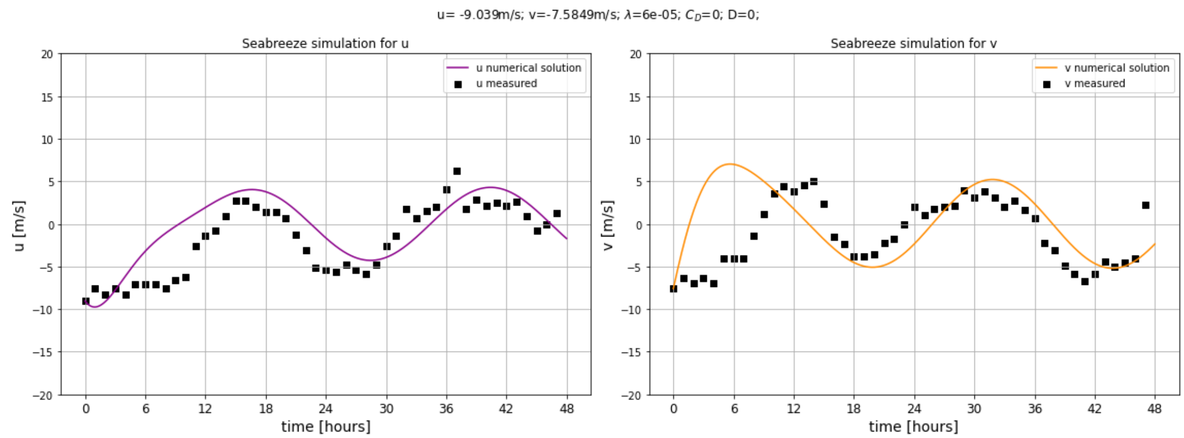


Figure 4: Simulation of the sea breeze including the Rayleigh damping coefficient and the observed data.

Parameter	Value
r_u	0.802329
r_v	0.377468
σ_u	3.75054
σ_v	4.06523
E_u	5.7978
E_v	5.78337
E'_u	2.49119
E'_v	4.40025

Table 2: Statistical parameters including the Rayleigh damping coefficient.

For the next case the model was initialised with the observed values of u and v at IJmuiden at 12 UTC on 6 May 1976 ($u = +3.9m/s$; $v = -3.3m/s$). Figure 5 represents the model output and the observational values and Table 3 the produced statistical parameters. It is obvious from Figure 5 that the simulated values of the horizontal wind velocities are in line with the observations. Regarding the statistical parameters, higher correlation coefficient is retrieved for v and smaller for u, with their sum being larger than this of the previous simulation. Similarly, the sums of RMS difference and centred pattern RMS is smaller while the standard deviation for both u and v give smaller values compared to the previous case. It can be concluded that initialising the model with the observed values at 12 UTC on 6 May 1976 results in a better agreement between the observed and the numerical velocities and the model's accuracy is increased.

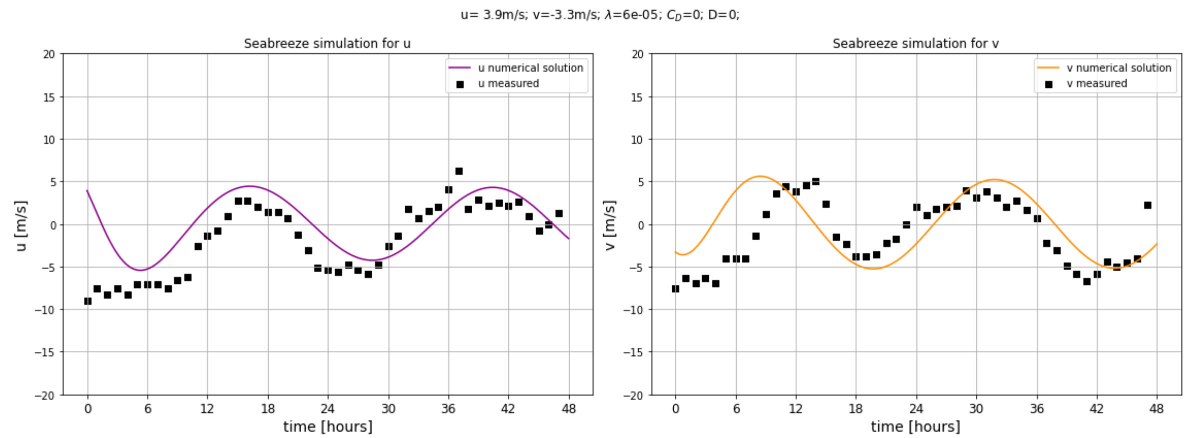


Figure 5: Simulation of the sea breeze initialised at 12 UTC on 6 May 1976 including the Rayleigh damping coefficient and the observed data.

Parameter	Value
r_u	0.69457
r_v	0.591879
σ_u	3.1601
σ_v	3.74138
E_u	5.56175
E_v	5.42584
E'_u	2.9675
E'_v	3.41306

Table 3: Statistical parameters including the Rayleigh damping coefficient. Simulation initialised at 12 UTC on 6 May 1976.

In the next simulation, a different representation of the surface drag is tested which depends on the square of the velocity. Including this non-linear friction term the horizontal momentum equations become:

$$\frac{\partial u}{\partial t} = fv - \frac{A}{\rho} cos(\Omega t + \phi) - C_D |u| u \tag{14}$$

$$\frac{\partial v}{\partial t} = -fu - C_D |v| v \tag{15}$$

The model was initialised at 12 UTC on 6 May 1976($u = +3.9m/s$; $v = -3.3m/s$). Figure 6 demonstrates the output of the simulation and Table 4 the statistical parameters produced. The model corresponds very well with the observations and it delivers better correlation coefficients for both u and v relative to the simulation of the Rayleigh damping coefficient initialised at the same period. However, it could be said that it performs relatively poor when looking at the standard deviation, the RMS difference and the centred RMS.

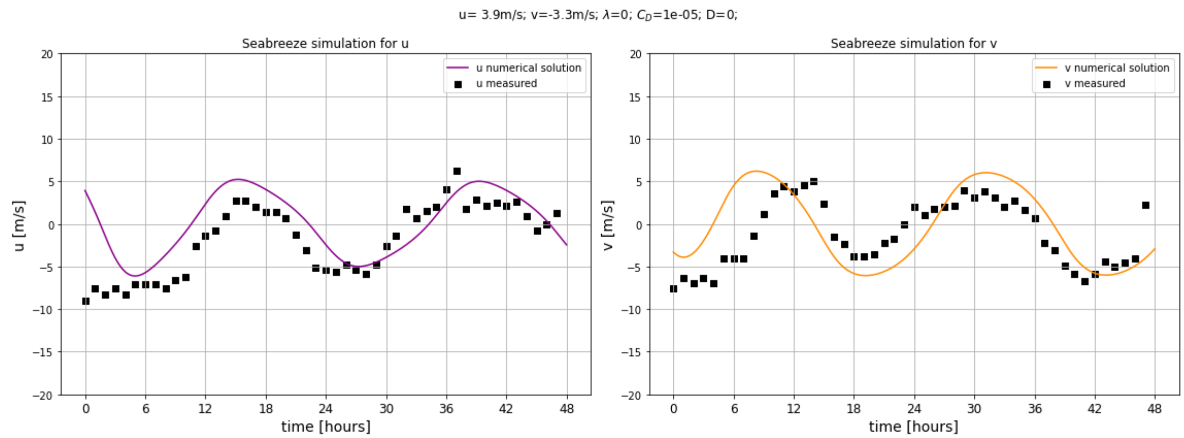


Figure 6: Simulation of the sea breeze initialised at 12 UTC on 6 May 1976 including the non-linear friction term, C_D, and the observed data.

Parameter	Value
r_u	0.716405
r_v	0.608476
σ_u	3.59432
σ_v	4.39906
E_u	5.82555
E_v	5.88373
E'_u	2.9361
E'_v	3.67119

Table 4: Statistical parameters including the non-linear friction term, C_D. Simulation initialised at 12 UTC on 6 May 1976.

For the final simulation, the model was initialised at 12 UTC on 6 May 1976 with geostrophic wind and the Rayleigh damping coefficient was used as friction. The horizontal momentum equations become:

$$\frac{\partial u}{\partial t} = fv - \frac{A}{\rho} cos(\Omega t + \phi) - \frac{B}{\rho} - \lambda u \tag{16}$$

$$\frac{\partial v}{\partial t} = -fu - \frac{D}{\rho} - \lambda u \tag{17}$$

where $D = \partial p / \partial y$ is the pressure gradient parallel to the coast, its value is assumed to be constant. This simulation utilises the mean of the observed values. The value of B is derived from the parametrisation of the zonal pressure gradient and the best fit curve. The outcome of the simulation is visible in Figure 7 and the statistical parameters can be seen in Table 5. It can be seen that the correlation coefficient for the horizontal wind speeds has decreased along with the standard deviation and the RMS difference relative to the previous simulation run. Regarding the centred pattern RMS, an increase can be distinguished for u and a decrease for v.

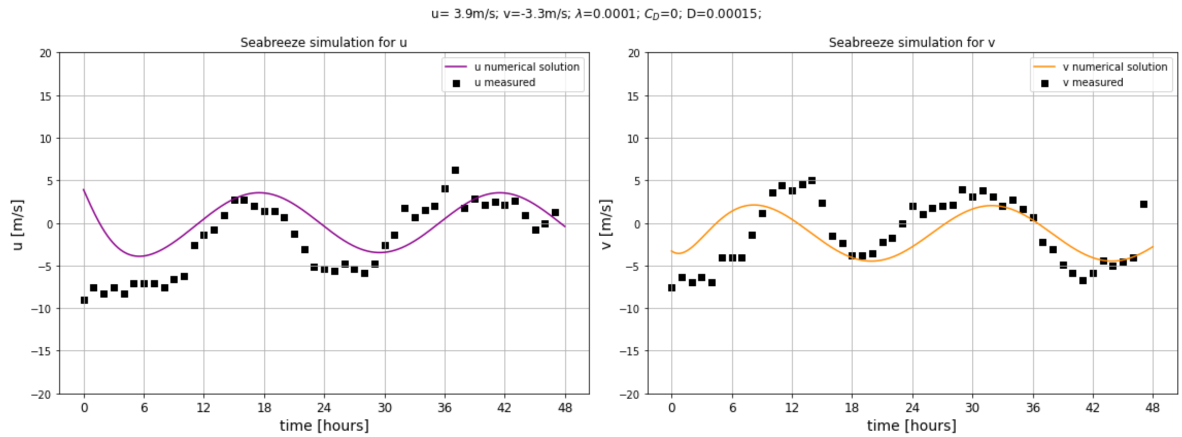


Figure 7: Simulation of the sea breeze initialised at 12 UTC on 6 May 1976 with geostrophic wind including the Rayleigh damping coefficint, λ, and the observed data.

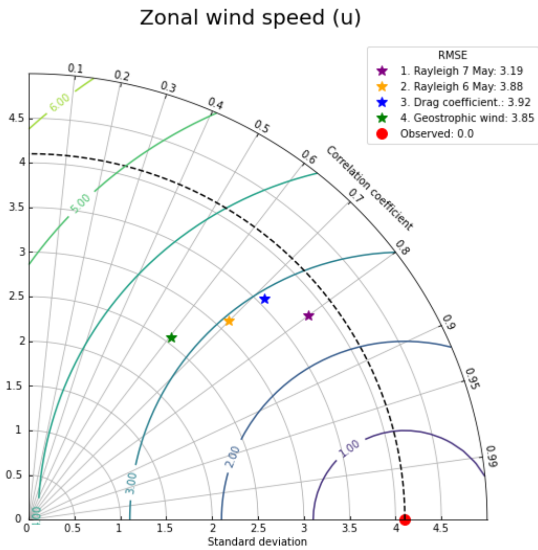
Parameter	Value
r_u	0.602207
r_v	0.570281
σ_u	2.53232
σ_v	2.31886
E_u	5.24247
E_v	4.46728
E'_u	3.275471
E'_v	3.13538

Table 5: Statistical parameters including the Rayleigh damping coefficient, λ. Simulation initialised at 12 UTC on 6 May 1976 with geostrophic wind.

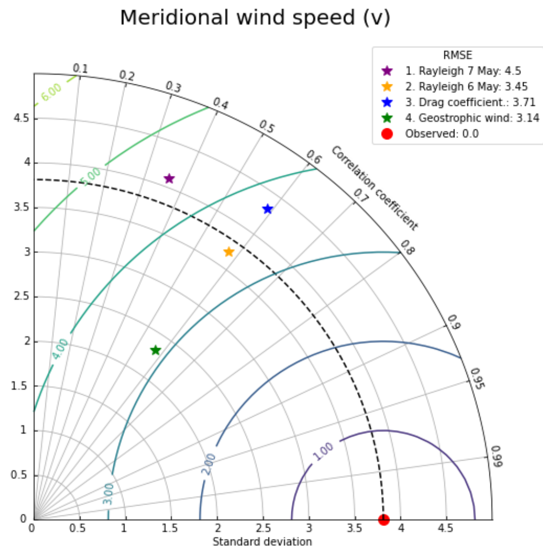
Measuring model performance with Taylor diagram

Overall, there is difficulty in defining the best performing simulation relying solely on statistical parameters. Therefore, the last section of this project will inspect the degree of correspondence between the simulations and the observed data. For this reason, a Taylor diagram is constructed for both wind components demonstrating the performance of each simulation. The Taylor diagram describes the statistical correlation between the simulation and the observed data.

Figures 8a and 8b visualise the Taylor diagrams for the zonal and the meridional wind speed respectively. They include the statistics of the simulations carried out in the previous section. The no-friction case is excluded since its correspondence exceeds the borders of the diagrams. *Rayleigh 7 May* and *Rayleigh 6 May* resemble the simulations of the model including the Rayleigh damping coefficient λ, initialised at 12 UTC on 7 May 1976 and 12 UTC on 6 May 1976 respectively. *Drag coefficient* stands for the simulation of the model including the drag coefficient C_D and *Geostrophic wind* the simulation initialised with geostrophic wind. *Observed* corresponds to the observational data.



(a) Zonal wind



(b) Meridional wind

Figure 8: Taylor diagrams for zonal wind speed (a) and meridional wind speed (b). The position of the stars measure how closely the model's simulation matches the observations. The dashed arc represents the standard deviation of the observed data which are indicated with the red dot.

From Figure 8a, it is evident that *Rayleigh 7 May* simulation performs better regarding the zonal wind speed. It delivers the highest correlation coefficient with the smaller RMS error and a standard deviation close to reality. On the subject of the meridional wind speed, Figure 8b, regarding the correlation coefficient, the green, yellow and blue star all take similar values. In between those three models, *Geostrophic Wind* and *Rayleigh 6 May* appear more prominent. While *Geostrophic Wind* stands out for performing with lower RMS error relative to the rest, *Rayleigh 6 May* simulation corresponds better to the observed values with its standard deviation being the closest to the reality. To conclude, these two model simulations have the most promising results.