Report on Adaptive Terminal Sliding Mode Control for Rigid Robotic Manipulators

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***Abstract*—This report explains the paper titled "Adaptive Terminal Sliding Mode Control for Rigid Robotic Manipulators" which addresses the limitations of applying the Terminal sliding mode control to robotic manipulator where it requires prior knowledge of upper bounds of uncertainties in parameters and external disturbances. But due to the unpredictable nature of uncertainties in robot dynamics determining these bounds is quite impractical. So, to overcome this limitation, this paper presents a novel robust Adaptive terminal sliding mode control approach for tracking problems in robotic manipulators. This adaptive controller eliminates the need for prior knowledge of upper bounds of uncertainties and disturbances. Additionally, the proposed controller also effectively eliminates the chattering effect while maintaining the robustness. Lyapunov theory was used to verify the stability of the control algorithm. Simulation results on a two-degree-of-freedom robot validate the effectiveness of the proposed controller in practical scenarios.**

***Index Terms*— Sliding mode control, Terminal sliding mode, sliding mode control, adaptive control of robot, robust control, Lyapunov method.**

I. INTRODUCTION

THE control of rigid robotic manipulators is a critical area of research with significant implications for both scientific advancements and industrial applications. Robotic manipulators contribute greatly to industrial processes by offering advantages such as reduced production costs, enhanced precision, improved quality, and increased productivity. However, controlling these manipulators is challenging due to their highly nonlinear, time varying, and coupled dynamic behavior. Additionally, there may be other uncertainties including external disturbances and parameter variations compounded with robotic systems which can lead to unstable performance.

Traditional control methods such as sliding mode control, have been widely applied due to their simplicity and robustness properties. Sliding mode control enables driving and maintaining the system trajectory on a sliding surface designed a priori in the state space, ensuring robustness to parameter variations and insensitivity to disturbances.
However, the sliding mode control ensures only asymptotic convergence, So the terminal sliding mode control was introduced for its rapid finite-time convergence, and it has been applied in control of robotic manipulators. This method requires prior knowledge of upper bounds of uncertainties in parameters and external disturbances. Unfortunately, because of the complex nature of uncertainties in the dynamics of robotic manipulators, acquiring such bounds will be challenging.

To address the limitations of conventional sliding mode control, particularly the need for precise prior knowledge of uncertainty bounds, this paper introduces a new approach – Adaptive Terminal Sliding Mode Control (ATSMC). Unlike Terminal sliding mode control, ATSMC does not rely on exact upper bounds for uncertainties and disturbances [13,14]. Instead, it uses adaptive control techniques to estimate and dynamically adjust for these uncertainties during the control process[17].
This paper introduces a new robust Adaptive sliding mode control for tracking of rigid robotic manipulators. This control method ensures a finite-time convergence of the error without requiring prior knowledge of parameter uncertainties and disturbances. The proposed controller is capable of estimating the upper bound of these uncertainties, eliminating the chattering effect while maintaining robustness and precision.

This paper is organized to present the robot model, discuss the continuous terminal sliding mode control, detail the development of the robust adaptive terminal sliding mode control, and validate the proposed approach through simulation results on a two-degree-of-freedom robot. The study concludes by summarizing the contributions and implications of the proposed ATSMC in advancing the control capabilities of rigid robotic manipulators.

II. ROBOT MODEL

This section introduces the dynamics of a robotic manipulator, specifically an n-link rigid manipulator. The dynamics are described by a second-order nonlinear vector differential equation. Dynamics of a n-link rigid robotic manipulator is described by the equation [17]

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = u + d(t) \quad (1)$$

The equation is derived using Lagrange equations, energy of n-DOF robotic manipulator is given by [1]

$$\varepsilon(q.\dot{q}) = K(q,\dot{q}) + U(q) \quad (1.a)$$

Where K is Kinetic energy and U is Potential Energy.
Kinetic energy equation for an n- DOF is given by.

$$K(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} \quad (1.b)$$

Lagrangian $L(q,\dot{q})$ equation for a n-DOF robotic manipulator is given by:

$$L(q,\dot{q}) = K(q,\dot{q}) - U(q) \tag{1.c}$$

The Lagrange equations of motion for a manipulator is given by:

$$\frac{d}{dt}\left[\frac{\partial L(q,\dot{q})}{\partial \dot{q}}\right] - \frac{\partial L(q,\dot{q})}{\partial q} = u \tag{1.d}$$

On substituting || the equation in || we get:

$$L(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} - U(q) \tag{1.e}$$

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right]\right] - \frac{\partial}{\partial q}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right] + \frac{\partial U(q)}{\partial q} =$$

$$u + d(t) \tag{1.f}$$

$$\frac{\partial}{\partial \dot{q}}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right] = M(q)\dot{q} \tag{1.g}$$

$$\frac{d}{dt}\left[\frac{\partial}{\partial \dot{q}}\left[\frac{1}{2}\dot{q}^T M(q)\dot{q}\right]\right] = M(q)\ddot{q} + \dot{M}(q)\dot{q} \tag{1.h}$$

$$M(q)\ddot{q} + \dot{M}(q)\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}[\dot{q}^T M(q)\dot{q}] + \frac{\partial U(q)}{\partial q} = u + d(t) \tag{1.i}$$

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + d(t) \tag{1.j}$$

$$C(q,\dot{q})\dot{q} = \dot{M}(q)\dot{q} - \frac{1}{2}\frac{\partial}{\partial q}[\dot{q}^T M(q)\dot{q}] \tag{1.k}$$

$$G(q) = \frac{\partial U(q)}{\partial q} \tag{1.l}$$

In the above equations q is a n-dimensional vector of joint angles, $M(q)$ is $n \times n$ matrix of inertia, as shown in derivations $C(q,\dot{q})\dot{q}$ consists of Coriolis and Centrifugal terms, $G(q)$ is the result of gravitational torque, u is the n-dimension vector of input torque, and $d(t)$ is a bounded disturbance.

$$d_1 = \begin{cases} ||d(t)|| < d_1 \\ d_1 > 0 \end{cases} \tag{1.m}$$

A. Modeling Uncertainty:

Due to factors like modeling errors, parameter variations, and unknown loads, the dynamic model is assumed to have uncertainties. The inertia matrix $M(q)$, Coriolis and centrifugal terms $C(q,\dot{q})$, and gravitational torque $G(q)$ are modeled as the sum of their nominal values and uncertainties, denoted as $\Delta M(q)$, $\Delta C(q,\dot{q})$, and $\Delta G(q)$ respectively.

$$M(q) = M_0(q) + \Delta M(q) \tag{2.a}$$

$$C(q,\dot{q}) = C_0(q,q) + \dot{\Delta}C(q,\dot{q}) \tag{2.b}$$

$$G(q) = G_0(q) + \Delta G(q). \tag{2.c}$$

The differential equation (1) can then be rewritten to incorporate these uncertainties:

$$M_0(q)\ddot{q} + C_0(q,\dot{q}) + G_0(q) = u + \rho(t) \tag{3.a}$$

Here, ρ(t) is introduced to represent the combined effect of uncertainties and disturbances, defined as:

$$\rho(t) = -\Delta M(q) - \Delta C(q,\dot{q}) - \Delta G(q) + d(t) \tag{3.b}$$

B. Assumptions

Assumption 1:

The inertia matrix $M(q)$ is assumed to be upper bounded by a positive number α_0. Mathematically, this is expressed as:[17]

$$||M(q)|| < \alpha_0 \tag{4}$$

This assumption imposes a constraint on the norm of the inertia matrix. The inertia matrix characterizes the distribution of masses in the robotic manipulator and plays a crucial role in determining its dynamics. By assuming an upper bound α0 on the norm of $M(q)$, it is nothing but bounding the potential variations or uncertainties in the inertia matrix. This assumption is important for stability analysis and controller design, as it provides a limit on the variation of the inertia matrix that the control strategy must accommodate.

Assumption 2:

It is assumed that condition involving the Coriolis and centrifugal terms, expressed as: [17]

$$||C(q,q) + G(q)|| < \beta_0 + \beta_1||q|| + \beta_2||\dot{q}||^2$$

where β0, β1, and β2 are positive numbers (5.a)

This assumption is concerned with bounding the Coriolis and centrifugal terms in the dynamics of the robotic manipulator. The Coriolis and centrifugal terms are responsible for coupling effects and can contribute to the complexity of control strategies. By introducing the condition $||C(q,q) + G(q)|| < \beta_0 + \beta_1||q|| + \beta_2||\dot{q}||^2$, the authors are establishing a relationship between the velocity of the joints $||\dot{q}||$ and the associated Coriolis and centrifugal effects. This condition is crucial for ensuring that the control strategy can effectively handle the uncertainties and variations in these terms, contributing to the overall stability of the system.

C. Uncertainty Bounding:

In addition to the assumptions, the paper imposes further constraints on the uncertainties ρ(t)) to ensure they remain within certain bounds. The uncertainty term ρ(t) is defined as:

$$\left|\left|\rho(t)\right|\right| < b_0 + b_1\left|\left|q\right|\right| + b_2\left|\left|\dot{q}\right|\right|^2 \qquad (5.b)$$

This inequality places bound on the uncertainties arising from modeling errors, parameter variations, and external disturbances. The terms b_0, b_1, and b_2 are positive constants that determine the maximum allowable magnitude of uncertainties in the system. This bounding of uncertainties is essential for designing robust control strategies. It ensures that the control system is equipped to handle variations and disturbances within specific limits, contributing to the stability and performance of the robotic manipulator.

III. CONTINUOUS TERMINAL SLIDING MODE.

The sliding surface is a concept often used in control theory, particularly in the context of sliding mode control. It represents a specific mathematical relationship between the state variables of a system and is designed to facilitate the control of the system towards a desired state.

In sliding mode control, the goal is to drive the system's state onto a predefined sliding surface and then maintain it on that surface. The dynamics of the system on the sliding surface are often simpler and more controllable than in other regions of the state space.

The sliding surface is typically defined as a hyperplane or manifold in the state space. For a system with state variables represented by the vector x, the sliding surface can be expressed as:

$$S(x) = 0 \qquad (6)$$

Here, $S(x)$ is a function that defines the sliding surface. The key characteristic of the sliding surface is that, once the system trajectory reaches this surface, it remains on it for the rest of the evolution.

In continuous sliding mode control, the sliding surface is often designed to be an error function, representing the difference between the current state and the desired state. The control law is then designed to drive the system dynamics in such a way that this error function converges to zero.

B. Equivalent Control Method:

The equivalent control method is a technique used in sliding mode control to determine the system trajectory on the sliding surface. It involves decomposing the total control signal into two components: equivalent control (ueq) and a high-frequency control (Δu).

$$u = u_{eq} + \Delta u$$

The equivalent control is a low-frequency component of the total control signal. It is responsible for maintaining the system's movement on the sliding surface. In the absence of disturbances and uncertainties, the equivalent control can be determined to keep the system state on the sliding surface.

The high-frequency control is a discontinuous component of the total control signal. Its primary role is to drive the system trajectory to reach the sliding surface. It is activated when the system is not on the sliding surface, and its discontinuous nature helps in achieving fast convergence.

The control objective is tracking a given trajectory with a manipulator output such that the tracking error converges to zero in finite time. The desired trajectory is denoted as q_d and the tracking error is defined as $e_1 = q - q_d$.

To achieve this goal, the concept of a terminal sliding surface is introduced. The sliding surface is defined as: [16-21]

$$\mathrm{S} = \mathrm{e}_2 + \mathrm{Ce}_1^{\frac{a}{b}} \qquad (7)$$

e_2 is the difference between the actual velocity q and the desired velocity q_d.

C is a diagonal matrix with elements $diag\{c1, \dots, c2\}$

a and b are odd integers satisfying 0 < a < b.

1. Dynamic Error Equations:

The dynamic error corresponding to the robotic manipulator's equation of motion (previously defined in (3.a)) is given by a system of first-order differential equations:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -\ddot{q}_d - M_0(q)^{-1}\left(C_0(q,\dot{q}) + G_0(q)\right) + \\ \quad M_0(q)^{-1}u + M_0(q)^{-1}\rho(t). \end{cases} \qquad (8)$$

After selecting the sliding surface, the subsequent step involves identifying a control law that meets the condition for sliding, expressed as

$$S^T\dot{S} < 0. \qquad (9)$$

And we use equivalent control method for this goal. In the paper the components of control signal, (ueq) and (Δu) are defined as [23]

$$u_{eq} = M_0(q)\left(\ddot{q} - \frac{a}{b}Cdiag\left\{e_1^{\frac{a}{b}-1}\right\}\right) + C_0(q,\dot{q}) + G_0(q). \quad (10)$$

$$\Delta u = -\frac{\left(S^T M_0(q)^{-1}\right)^T}{\left|\left|S^T M_0(q)^{-1}\right|\right|^2} \times \left[\left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|\left(b_0 + b_1\left|\left|q\right|\right| + b_2\left|\left|\dot{q}\right|\right|^2\right)\right] \qquad (11)$$

But in the above control their exists chattering problem arises due to the presence of a discontinuous control term Δu. Chattering refers to the rapid and undesired switching of the control signal between extreme values near the sliding surface. This discontinuity can lead to high-frequency oscillations and excessive control efforts, which are undesirable in practical control applications. The paper attempts to address this chattering issue by introducing a modification in the form of Δu which is intended to eliminate the chattering phenomenon. This modification involves replacing the original Δu with a smoother term that depends on the sliding mode and a parameter δ which can be considered as a boundary layer.

2) Boundary Layer:

A boundary layer is a region where a variable or a function changes rapidly compared to its behavior in the surrounding regions. In control systems, the introduction of a boundary layer is a technique used to smooth out rapid transitions or discontinuities in the control signal, reducing chattering effects. In this case, δ serves a similar purpose by controlling the rate at which the modified control term Δu_1 change. It provides a buffer or a layer around the sliding surface, aiming to reduce the chattering phenomenon associated with abrupt changes in the control signal. The choice of δ affects the width of this boundary layer and, consequently, the trade-off between chattering reduction and the robustness of the control system.

So, Δu can be replaced by the following expression. [23]

$$\Delta u_1 = \begin{cases} -\dfrac{(S^{\mathrm{T}} M_0(q)^{-1})^{\mathrm{T}}}{\left\|S^{\mathrm{T}} M_0(q)^{-1}\right\|^2}[\left\|S\right\|\left\|M_0(q)^{-1}\right\| \times \\ (b_0 + b_1\left\|q\right\| + b_2\left\|\dot{q}\right\|^2)], \quad \text{if } \left\|S^{\mathrm{T}} M_0(q)^{-1}\right\| \geq \delta \\ -\dfrac{(S^{\mathrm{T}} M_0(q)^{-1})^{\mathrm{T}}}{\delta^2}[\left\|S\right\|\left\|M_0(q)^{-1}\right\| \times \\ (b_0 + b_1\left\|q\right\| + b_2\left\|\dot{q}\right\|^2)], \quad \text{if } \left\|S^{\mathrm{T}} M_0(q)^{-1}\right\| < \delta \end{cases}$$

Where $\delta > 0$.

A narrower boundary layer may reduce chattering, it can potentially compromise robustness by making the system more reactive to uncertainties. On the other hand, a wider boundary layer, achieved with a larger δ, provides a smoother transition in the control signal, making the system less sensitive to rapid variations and uncertainties. Moreover, effectiveness of this controller relies on determining the upper limit of uncertainties and disturbances which are impractical to determine. So, we need a robust adaptive terminal sliding mode control method in order to address these two issues.

IV. ROBUST ADAPTIVE TERMINAL SLIDING MODE CONTROL

To address the above-mentioned issues, the paper proposes a novel robust adaptive terminal sliding mode controller for manipulators described by the system dynamics, where the goal is to estimate the bounds of uncertainties and external disturbances online. The proposed adaptive terminal sliding mode control, defined by equation, is expressed as the sum of the equivalent control (u_{eq}) and a term Δu_2 as follows:

$$u = u_{eq} + \Delta u_2 \tag{12}$$

where u_{eq} is same as defined as before, but Δu2 is defined as below:

$$\Delta u_2 = \begin{cases} -\dfrac{(S^{\mathrm{T}} M_0(q)^{-1})^{\mathrm{T}}}{\left\|S^{\mathrm{T}} M_0(q)^{-1}\right\|^2}[\left\|S\right\|\left\|M_0(q)^{-1}\right\| \times \\ (\hat{b}_0 + \hat{b}_1\left\|q\right\| + \hat{b}_2\left\|\dot{q}\right\|^2)], \quad \text{if } \left\|S^{\mathrm{T}} M_0(q)^{-1}\right\| \geq \delta \\ -\dfrac{(S^{\mathrm{T}} M_0(q)^{-1})^{\mathrm{T}}}{\delta^2}[\left\|S\right\|\left\|M_0(q)^{-1}\right\| \times \\ (\hat{b}_0 + \hat{b}_1\left\|q\right\| + \hat{b}_2\left\|\dot{q}\right\|^2)], \quad \text{if } \left\|S^{\mathrm{T}} M_0(q)^{-1}\right\| < \delta \end{cases}$$

where $\hat{b}_0$, $\hat{b}_1$, and $\hat{b}_2$ are the adaptive variables for b_0, b_1, and b_2 defined in (5.b). The adaptation laws are:

A. Adaptation laws.

$$\hat{b}_0 = x_0\left\|S\right\|\left\|M_0(q)^{-1}\right\| \tag{13.a}$$

This equation governs the rate of change of the adaptive variable $\hat{b}_0$. The term x_0 is an arbitrary parameter, and $\left\|S\right\|\left\|M_0(q)^{-1}\right\|$ represents the inverse of the sliding surface matrix.

$$\hat{b}_1 = x_1\left\|S\right\|\left\|M_0(q)^{-1}\right\|\left\|q\right\| \tag{13.b}$$

Similar to the first adaptation law, this equation dictates how $\hat{b}_1$ adapts over time. The term x_1 is again an arbitrary parameter, and the inclusion of q introduces a dependency on the joint angles.

$$\hat{b}_2 = x_2\left\|S\right\|\left\|M_0(q)^{-1}\right\|\left\|\dot{q}\right\|^2 \tag{13.c}$$

This equation defines the evolution of the adaptive variable $\hat{b}_2$ with respect to time. As in the previous laws x_2 is an arbitrary parameter, and the dependence on $\dot{q}$ introduces a quadratic dependence on the joint velocities.

These adaptation laws play a crucial role in the proposed robust adaptive terminal sliding mode controller. They determine how the adaptive variables $\hat{b}_0$, $\hat{b}_1$, $\hat{b}_2$ evolve over time based on the current state of the system represented by q. The arbitrary parameters x_0, x_1, and x_2 are introduced to facilitate flexibility and tuning in the adaptation process. The adaptation laws contribute to the controller's ability to estimate the bounds of uncertainties and external disturbances online, enhancing its performance in dealing with complex and unpredictable manipulator structures.

Theorem 1

The theorem aims to establish that the application of the proposed control law (12), with the defined sliding surface (7) and adaptation laws, ensures the finite-time convergence of the tracking error to zero for a nonlinear uncertain system (8).

To proceed first we need to define a Lyapunov function

Lyapunov Function:

A Lyapunov function is a mathematical function used in the analysis of the stability of a system. It helps determine whether the trajectories of a dynamic system approach a stable equilibrium point, remain bounded, or exhibit some other desirable behavior. The primary idea is to find a function whose derivative along the system trajectories provides information about the system's behavior.

The following are the properties of Lyapunov function:

The Lyapunov function must be positive definite, meaning it is greater than zero for all nonzero states and is equal to zero only at the equilibrium point.

$$V(x) > 0, \forall x \neq 0, \text{and } V(0) = 0$$

The derivative of the Lyapunov function with respect to time should be negative or non-positive along the trajectories of the system, except at the equilibrium point.

$$\dot{V}(x) < 0, \forall x \neq 0. \, and \; V(0) = 0$$

In this case, the Lyapunov function is chosen as:

$$V = \frac{1}{2} S^{\mathrm{T}} S + \frac{1}{2} \sum_{i=0}^{2} x_i^{-1} \tilde{b}_i^2$$

Here's why this particular Lyapunov function is selected:

The Lyapunov function is constructed to be positive definite, meaning that V is is always positive for any non-zero state error S adaptation error $\tilde{b}_i$. This property is essential for ensuring that the Lyapunov function provides a meaningful measure of the system's energy.

The Lyapunov function is decomposed into two terms. The first term, $\frac{1}{2} S^{\mathrm{T}} S$ is related to the sliding surface and ensures that the system stays on the sliding surface during operation. The second term involves the adaptation errors $\tilde{b}_i$ and it contributes to measuring the adaptation performance.

The Lyapunov function includes the term $x_i^{-1} \tilde{b}_i^2$ which represents the squared adaptation errors normalized by positive constants x_i. This allows the Lyapunov function to capture the impact of adaptation errors on system stability.

The constants x_i are arbitrary positive constants. The choice of these constants allows flexibility in shaping the Lyapunov function to meet specific stability and convergence requirements. The analysis shows that the Lyapunov function still holds and ensures stability regardless of the specific values chosen for these constants.

This property is essential for ensuring that the Lyapunov function provides a meaningful measure of the system's energy, hence its stability.

$$V = \frac{1}{2} S^{\mathrm{T}} S + \frac{1}{2} \sum_{i=0}^{2} x_i^{-1} \tilde{b}_i^2$$

where $\tilde{b}_i = b_i - \hat{b}_i, i \in \{0,1,2\}$.

Differentiating V with respect to time and using (12) for $\left|\left|S^{\mathrm{T}} M_0(q)^{-1}\right|\right| \geq \delta$ which gives

$$\dot{V} = S^{\mathrm{T}}[-\ddot{q}_d - M_0(q)^{-1}\left(C_0(q,\dot{q}) + G_0(q)\right) + M_0(q)^{-1}\left(u_{eq} + \Delta u_2\right) + M_0(q)^{-1}\rho(t) + \frac{a}{b} C \mathrm{diag}\left\{e_1^{\frac{a}{b}-1}\right\} e_2] - \sum_{i=0}^{2} x_i^{-1} \tilde{b}_i \dot{\hat{b}}_i$$

$$\dot{V} = S^{\mathrm{T}} M_0(q)^{-1}\rho(t) - [S^{T} M_0(q)^{-1} \frac{(S^{\mathrm{T}} M_0(q)^{-1})^{\mathrm{T}}}{\left|\left|S^{\mathrm{T}} M_0(q)^{-1}\right|\right|^2} \times \left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|(\hat{b}_0 + \hat{b}_1\left|\left|q\right|\right| + \hat{b}_2\left|\left|\dot{q}\right|\right|^2)] - \sum_{i=0}^{2} x_i^{-1} \tilde{b}_i \dot{\hat{b}}_i$$

Simplifying and substituting $\dot{\hat{b}}_i$ by the expression defined by (13.a) -(13.c), we get

$$\dot{V} = S^{\mathrm{T}} M_0^{-1}\rho(t) - \left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|\left(\hat{b}_0 + \hat{b}_1\left|\left|q\right|\right| + \hat{b}_2\left|\left|\dot{q}\right|\right|^2\right) - \left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|\left(\tilde{b}_0 + \tilde{b}_1\left|\left|q\right|\right| + \tilde{b}_2\left|\left|\dot{q}\right|\right|^2\right) = S^{\mathrm{T}} M_0(q)^{-1}\rho(t) - \left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|\left(b_0 + b_1\left|\left|q\right|\right| + b_2\left|\left|\dot{q}\right|\right|^2\right) \leq \left|\left|S\right|\right|\left|\left|M_0(q)^{-1}\right|\right|(\left|\left|\rho(t)\right|\right| - \left(b_0 + b_1\left|\left|q\right|\right| + b_2\left|\left|\dot{q}\right|\right|^2\right) < 0$$

The time derivative of V is shown to be negative definite, indicating a systematic decrease in system energy. This decrease in energy ensures that the tracking error approaches zero within a finite time, demonstrating the finite-time convergence property of the proposed control scheme.

V.SIMULATION RESULTS

The effectiveness of the proposed controller is evaluated through simulation using a two-degrees-of-freedom robot described by the following model. [17]

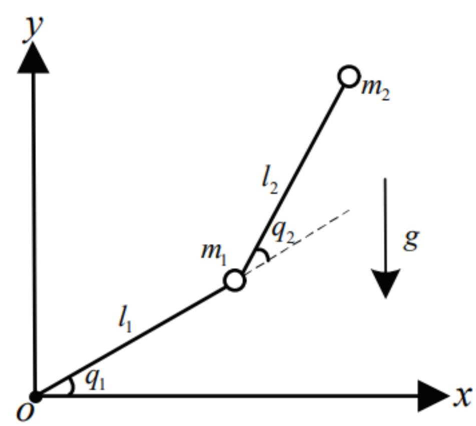

Fig. 1. Two DOF robotic manipulator

$$\begin{pmatrix} M_{11}(q) & M_{12}(q) \\ M_{12}(q) & M_{22}(q) \end{pmatrix} \begin{pmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{pmatrix} + \begin{pmatrix} C_1(q,\dot{q}) \\ C_2(q,\dot{q}) \end{pmatrix} + \begin{pmatrix} G_1(q) \\ G_2(q) \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} d_1(t) \\ d_2(t) \end{pmatrix}$$

Where,

$$M_{11}(q) = (m_1 + m_2)L_1^2 + m_2L_2^2 + 2m_2L_1L_2\cos(q_2) + J_1$$

$$M_{12}(q) = m_2L_2^2 + m_2L_1L_2\cos(q_2)$$

$$M_{22}(q) = m_2L_2^2 + J_2$$

$$C_1(q,\dot{q}) = -m_2L_1L_2\sin(q_2)\,\dot{q}_1^2 - 2m_2L_1L_2\sin(q_2)\ \dot{q}_1\dot{q}_2$$

$$C_2(q,\dot{q}) = m_2L_1L_2\sin(q_2)\ \dot{q}_2$$

$$G_1(q) = (m_1 + m_2)L_1\cos(q_2) + m_2L_2\cos(q_1 + q_2)$$

$$G_2(q) = m_2L_2\cos(q_1 + q_2).$$

The nominal values of m_1 and m_2 for the calculation considered by the author are given below.

$$m_{10} = 0.4 \text{ kg}, \quad m_{20} = 1.2 \text{ kg}$$

They have considered an uncertainty of masses by an order of $\pm 10\%$ (see Figs. 1 and 2). Other parameters assumed are:

$$L_1 = 1 \text{ m}, \quad L_2 = 0.8 \text{ m}$$
$$J_1 = 5 \text{ kg.m}, \quad J_2 = 5 \text{ kg.}$$

And the disturbance vector considered in the paper are $d(t) = [d_1(t) \quad d_2(t)]^{\mathrm{T}}$

$$d_1(t) = 0.2\sin(3t) + 0.02\sin(26\pi t)$$

$$d_2(t) = 0.1\sin(2t) + 0.01\sin(26\pi t)$$

The initial values of the system that were given as input for the ODE45 equation in MATLAB are obtained form:

$$[q_1(0) \quad q_2(0)]^{\mathrm{T}} = [0.8 \quad 0.9]^{\mathrm{T}}$$

$$[\dot{q}_1(0) \quad q_2\dot{(0)}]^T = [0 \quad 0]^{\mathrm{T}}.$$

Desired angular positions for this manipulator are:

$$q_{d1} = 1.25 - \frac{7}{5}\exp(-t) + \frac{7}{20}\exp(-4t)$$

$$q_{d2} = 1.4 - \frac{7}{5}\exp(-t) + \frac{7}{20}\exp(-4t)$$

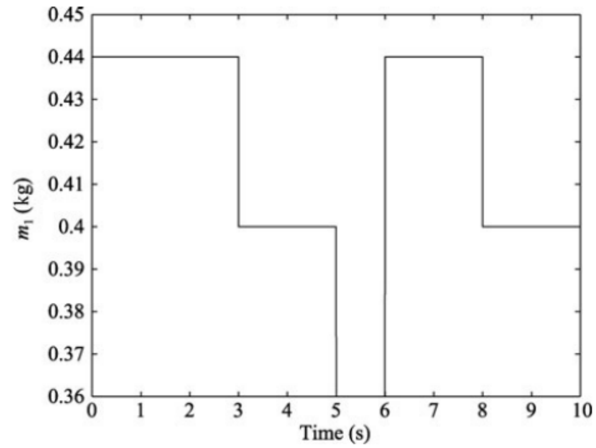

Fig. 2. Variation of mass m1

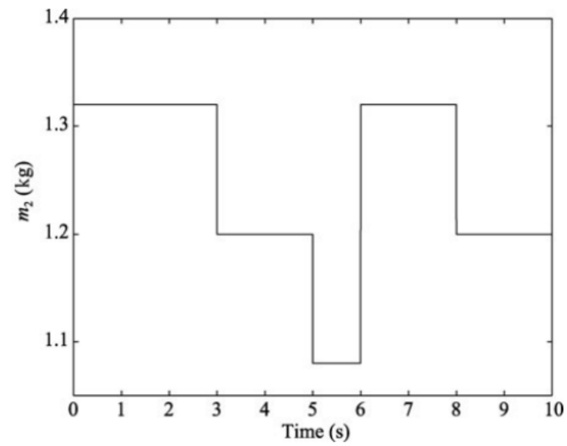

Fig. 3. Variation of mass m2

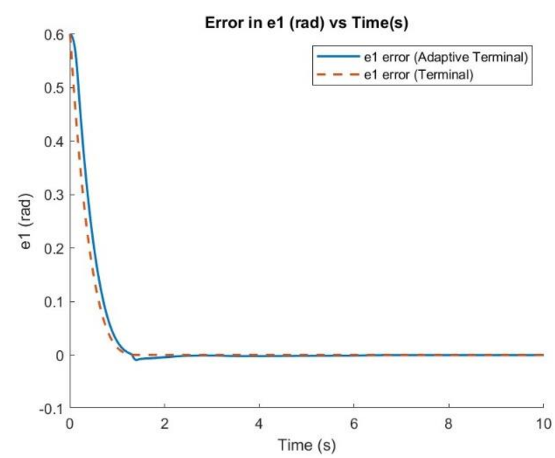

Fig. 4. Error 1(rad) vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

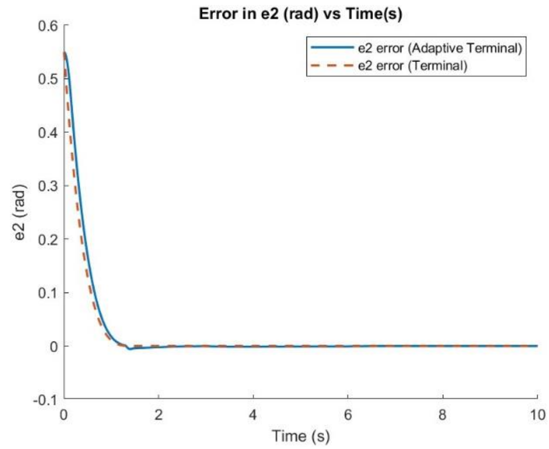

Fig. 5. Error 2(rad) vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

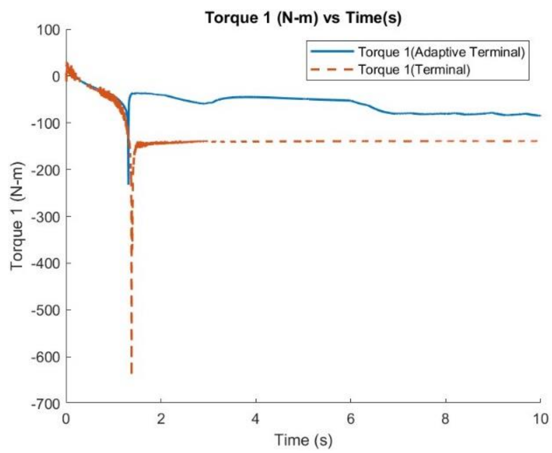

Fig. 6. Torque 1(N-m) vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

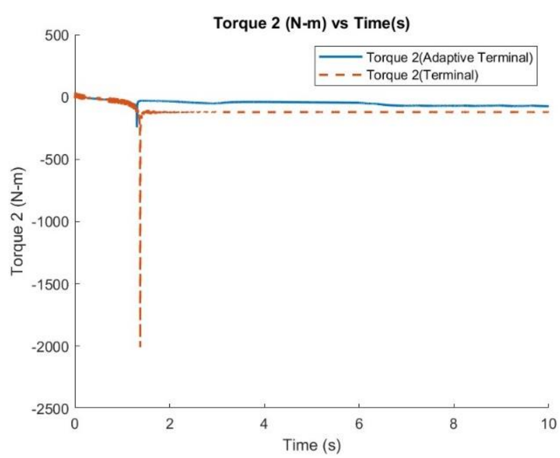

Fig. 7. Torque 2(N-m) vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

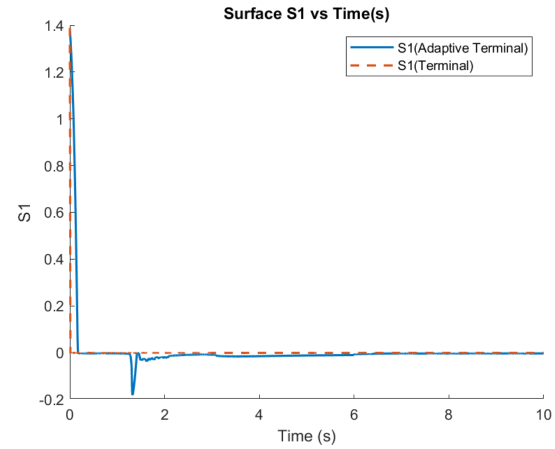

Fig. 8. Surface1 vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

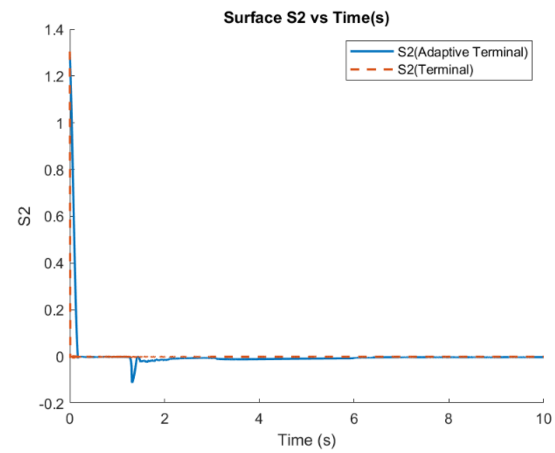

Fig. 9. Surface2 vs Time Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

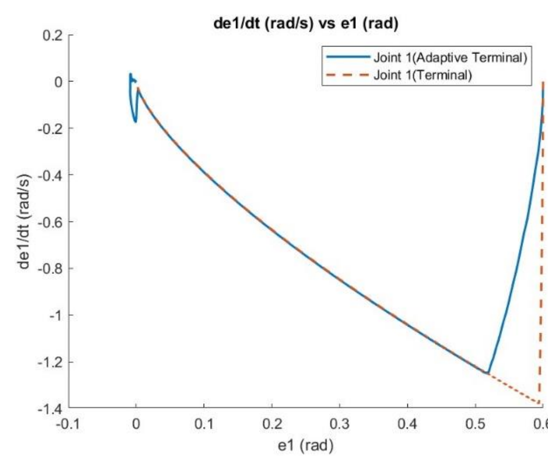

Fig. 10. $\frac{de_1}{dt}$ vs e_1 Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

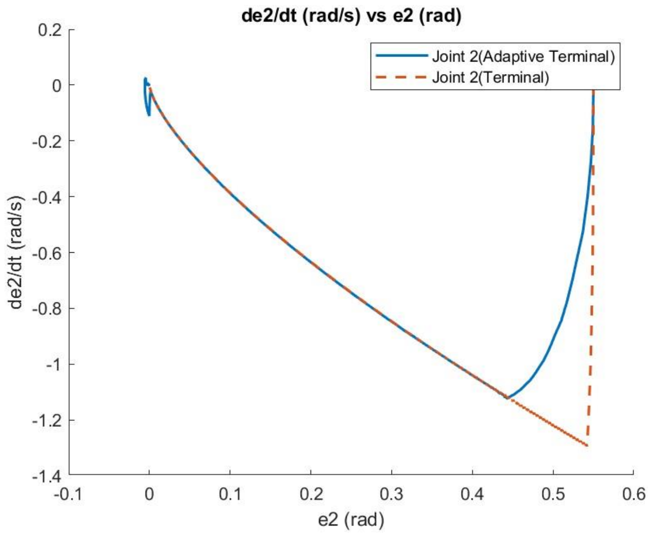

Fig. 11. $\frac{de_2}{dt}$ vs e_2 Graph for Adaptive Terminal Sliding Mode Control and Terminal Sliding Mode Control

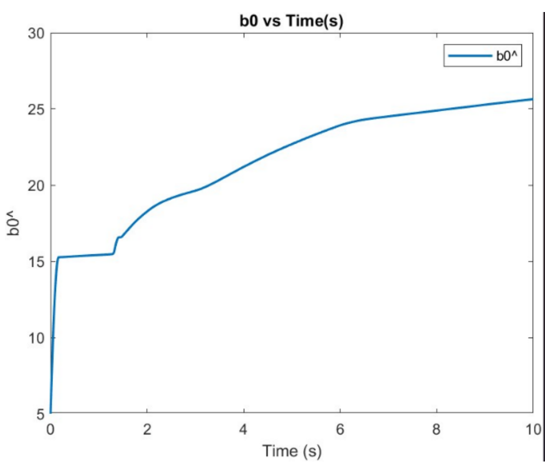

Fig. 12. Estimated parameter $\hat{b}_0$

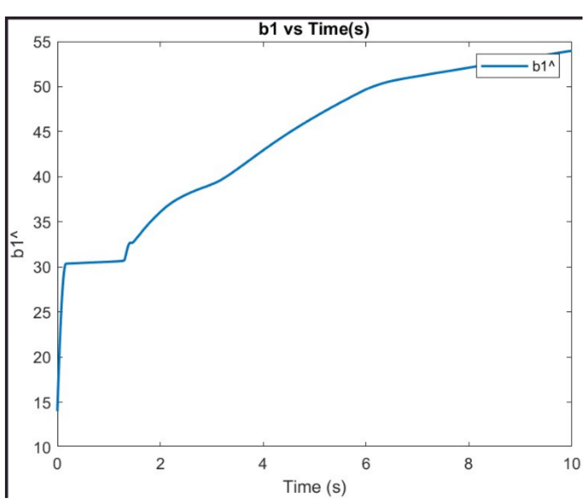

Fig. 13. Estimated parameter $\hat{b}_1$

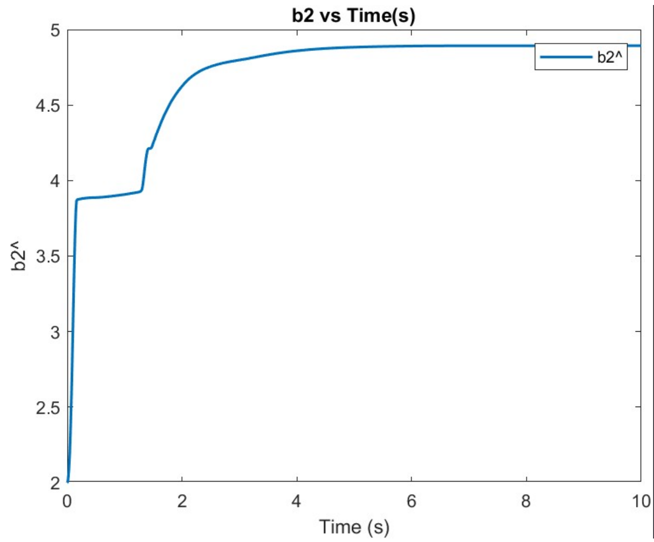

Fig. 14. Estimated parameter $\hat{b}_2$

To summarize, the study investigates continuous terminal sliding mode control and adaptive terminal sliding mode control using specific surface parameters and initial uncertainty conditions ($a = 5, b = 7, c_1 = c_2 = 2, b_{00} = 5, b_{10} = 14, b_{20} = 2$). The continuous control approach, while incorporating a small boundary layer ($\delta = 0.0005$) for system robustness, exhibits drawbacks, including non-continuous control and undesirable high-frequency commutation at 1 second. Additionally, this method relies on known upper bounds of uncertainties.

Addressing these issues, the proposed adaptive terminal sliding mode control maintains continuity, robustness, and conserves the chosen parameter δ. This adaptive strategy successfully estimates and converges the upper bound parameters online, resolving the dependency on a priori knowledge of uncertainty bounds. Figs. 4-11. illustrate the finite-time convergence of these parameters. There is slight variation in the graphs compared to the ones on paper. This is because of the arbitrary coefficients which were not available in the paper, we can take any random positive number, but we have to keep in mind to get correct graphs the values should be positive and $x_0 = b_0, x_1 = b_1, and\ x_2 = b_2.$ The convergence time is reduce as we increase arbitrary constants but we need further more data and study.

To solve the ODE equations, we used ODE45 in MATLAB which uses the Runge Kutta method, to solve the ODE equation by splitting the time interval given into very small steps about 10^{-4} each step size. This has resulted in longer times to have precise graph we tried to store all the data received at each iteration this resulted system crash several times. This is resolved by only considering a small sample data from the array of results, which results in less accuracy in results but the level of computation vs accuracy supports our idea. Which reduced the computation time.

V. CONCLUSION

Moreover, the controls show great performance in simulations on a two-degree-of-freedom robot manipulator showing its ability to achieve precise and robust control. Furthermore, the elimination of the chattering phenomenon, a common problem in sliding mode control, without compromising robustness, adds to the practicality and efficiency of the proposed approach.

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