A Study in Disaster

The Titanic Survivors

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Abstract

This research aims to construct a logistic regression model to predict the survival outcome of passenger on the Titanic based on their various demographic and socioeconomic factors, such as age, gender, number of spouses and siblings, passenger class, etc. The findings of this research can be generalized to most shipwreck where the ship is considered big (having capacity of at least 1000 passengers) and the route is through Atlantic Ocean in the early $20^{\rm th}$ century. We used logistic regression in tandem with backward feature selection to obtain several prediction models. We evaluate each obtained model using the Hosmer-Lemeshow goodness-of-fit test and then conduct a more vigorous feature selection process to come up with the best models. Despite the different approaches, all of the obtained models suggest a common trend that children, female and first class passengers are among the ones with highest chance of survival.

Introduction

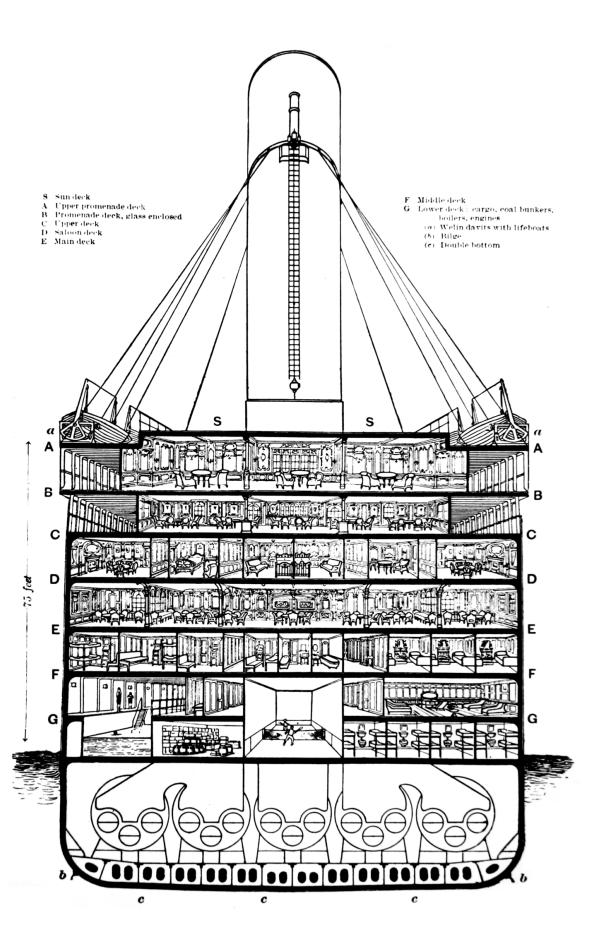
On that fateful night April 15, 1912, the great Titanic carrying 2,200 people struck an iceberg and sank, brought down with it 1,500 poor souls. Such tragedy never fails to leave all of us unsettled, but as statisticians, we are naturally drawn to ask ourselves the question: "How do demographic and socioeconomic factors affect chance of survival in disaster?" Perhaps, based on just the titanic data set, such general question cannot be answered, since our findings are probably just generelizable to a specific type of disaster, which is shipwreck and to a time-frame limited to the early 20th century. Also, we have to consider the fact that Titanic is not just another ship... It is (even till today) one of the most luxurious ship ever been built (see figure below - the ship even had swimming pool and tennis court at the lowest level!). As such, we think the findings we obtain using this data set can only be generalized to big ships, i.e. ships whose capacity is at least 1000, crossing the Atlantic ocean. Note that this number 1000 seems rather arbitrary, but we found that nowadays, on average, cruise ships with tonnage of around 40,000 usually have capacity of 1,000 passengers¹ and most of the bigger ships in early 20th century were around 40,000 or more in tonnage². For this reason, we formulate our research question as follows:

"How do various demographic and socioeconomic factors affect chance of survival in shipwreck where the ship's capacity is at least 1000 passegners and the route is through the Atlantic Ocean in the early 20th century?"

In simpler words, we will attempt to create a model to be able to predict the survival outcome of a passenger, given his/her relevant background information. We constructed several different models with 2 major approaches, one using backward selection and one using a more rigorous feature selection process. Despite this, the models we obtain show a common trend that female, little kids and first-class passengers are more likely to survive the shipwreck. Following this, we will step by step present our data preparation and modelling process, as well as our findings.

¹CruiserMapper, Cruise Ship Passenger Capacity

²Wikipedia, List of largest passenger ships



1. Data Exploration

1.1 Summary

The data set contains information about 1306 passengers on the Titanic. Recall that there were around 2,200 people on the ship so with this sample size, clearly the data set is a good representation of the population on the Titanic. There are 11 variables in the original data set, listed as follows:

Type	Unit	Meaning	Value/Range
nominal		previous name of county	0 = No, 1 = Yes
nominal		passenger's name	
ordinal		ticket class	1 = 1st, 2 = 2nd, 3 =
nominal		passenger's gender	m = male, f = female
numerical	year	passenger's age	[0,80]
numerical	person	number of siblings/spouses aboard the Titanic	[0,8]
numerical	person	number of parents/children aboard the Titanic	[0,6]
nominal		ticket number	
numerical	pound	ticket fare	[0, 93.5]
nominal		cabin number	
nominal		port of embarkation	C = Cherbourg, Q = C
	nominal nominal ordinal nominal numerical numerical numerical nominal numerical	nominal nominal ordinal nominal numerical year numerical person numerical person nominal numerical pound nominal	nominal previous name of county nominal passenger's name ordinal ticket class nominal passenger's gender numerical year passenger's age numerical person number of siblings/spouses aboard the Titanic numerical person number of parents/children aboard the Titanic nominal ticket number numerical pound ticket fare nominal cabin number

1.2 Variable Selection

Our response variable is the nominal variable survival (or boolean as its outcome is 0 or 1). As for explanatory variables, we have to make some pre-selection beforehand. We decide to take out several variables which we consider only as identifier variables, such as name and ticket number. Also we did not include the cabin number since this information is missing for too many passengers. The only value it brings to the model is when we can actually compute the shortest distance from each cabin to the rescue area. However, again as mentioned, there are a lot of missing data and also, if distance is our only concern, then

passenger class is quite sufficient for this purpose (refer to the cross-section plot of the Titanic in previous section).

1.3 Data Preparation

We notice that age is missing for many passenger, but age is such a crucial variable that we cannot simply take away. As such, we discretize age into several bins: missing, 0-5, 6-18, 19-55, and 56 and above. Also, we rename the variables for readability. So the final model we obtained has the following variables:

Name	Type	Unit	Value/Range
has_survived passenger_class gender age number_of_siblings_and_spouses	nominal ordinal nominal numerical	year person	0 = No, 1 = Yes 1 = 1st, 2 = 2nd, 3 = 3rd male, female [0,5], [6,18], [19,55], [56 and above], missing [0,8]
number_of_parents_and_children fare embarked_from	numerical numerical nominal	-	[0,6] $[0, 93.5]$ $C = Cherbourg, Q = Queenstown, S = South$

1.4 Final Data Set Summary

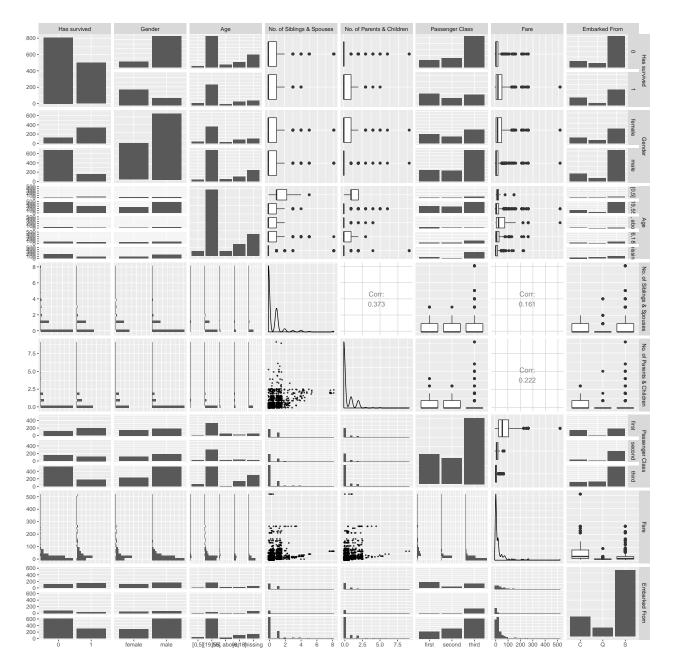
Following is the summary of the final data set:

```
## has_survived
##
     0
         1
## 808 498
## gender
## female
             male
      464
              842
##
## age
##
          [0,5]
                     [19,55] [56, above)
                                                [6,18]
                                                            missing
##
             56
                         790
                                       57
                                                   140
                                                                263
##
    number_of_siblings_and_spouses number_of_parents_and_children
            :0.0
##
    Min.
                                              :0.0000
                                      Min.
    1st Qu.:0.0
                                      1st Qu.:0.0000
##
    Median:0.0
                                      Median :0.0000
##
```

```
:0.5
                                           :0.3859
##
   Mean
                                   Mean
##
    3rd Qu.:1.0
                                   3rd Qu.:0.0000
           :8.0
                                           :9.0000
##
   Max.
                                   Max.
##
         fare
         : 0.00
##
   Min.
    1st Qu.: 7.90
##
##
   Median : 14.45
##
   Mean
         : 33.22
   3rd Qu.: 31.27
##
   Max.
          :512.33
##
## passenger_class
##
   first second
                  third
      321
                    708
##
             277
## embarked_from
    С
         Q
##
             S
## 270 123 913
```

1.5 Correlation Investigation

Following, we create a plot matrix to investigate potential collinearity between variables in our data set.



As seen above, it seems that there might be some collinearity between <code>gender</code> and <code>passenger_class</code> as female and male groups have very different first class to third class ratio. Also, notice that <code>passenger_class</code> and <code>embarked_from</code> seem to have some strong association, for example, there seems to be more third class passenger embarking from Southampton than that of other embarkation ports. As for now, we will take note of these observations; we will use them later for a more vigorous feature selection process as we attempt to improve our model.

2. Methodology

2.1 Backward Selection

For backward selection, we start out by including all of the variables we have as explanatory variables and construct a regression model from that. We then take out one of the insignificant predictor (predictor whose p-value is greater than the significance level of 0.05 and whose p-value is the greatest among all variables), and then construct another model using the remaining variables. Repeat the process until we obtain a model with all significant predictors.

2.2 Logistic Regression

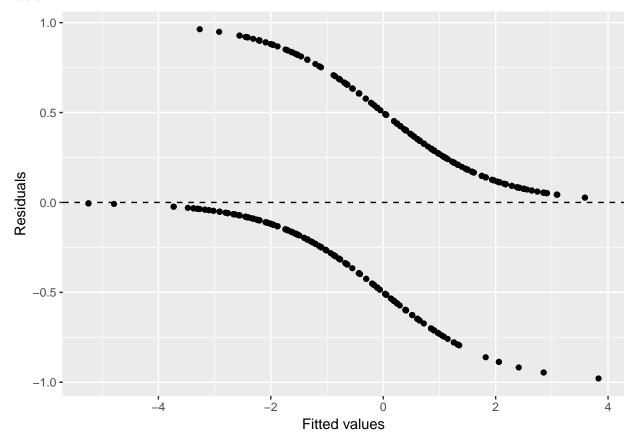
In our project, since our response variable is a binary variable, we are going to use logistic regression for the model. Logistic regression is a type of generalized linear model. Its equation is in the form of $ln(\frac{p}{1-p}) = \ln(odds) = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$, where p, in this context, is the probability of survival–1 means the passenger survived and 0 otherwise, and x_1, x_2, \ldots, x_n are explanatory variables. The right hand side of this equation is the same as in a linear model, hence it is called generalized linear model. The left-hand side is the natural log of odds, where odds is a representation of probability, and it equals to $\frac{p}{1-p}$ Once we fit a model, we can predict the probability of success p by $p = \frac{1}{1+e^{-RHS}}$, where $RHS = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n$.

To perform logistic regression, we can use glm() function, which stands for generalized linear model, in R, and specify the parameter family = binomial(link = "logit").

2.3 Homser-Lemeshow Goodness-Of-Fit Test

Since we decided to use logistic regression for our model, the assumptions we need to verify are not the same as in a linear regression model. For example, we cannot plot a residual plot to see whether the residuals have a pattern because however good or bad a model is, the residuals will always show a pattern, as they follow 2 curves: positive residuals follow

the curve $1 - \frac{1}{1 + e^{-predicted}}$ and the negative residuals follow the curve $0 - \frac{1}{1 + e^{-predicted}}$ as seen below:



To examine how good a logistic regression model is, we can use Homser-Lemeshow goodness-of-fit test. The idea of this test is to divide the sample into several groups according to their predicted values, and compare the expected proportion of success to the observed proportion of success in each group to see whether there is a significant difference between the expected and the observed proportion. The null hypothesis of this test is that there is no difference between the expected and the observed proportion of success. In other words, if the p-value of this test is too low, it means that we have strong evidence that the fit is not good enough.

However, there are some restrictions for this test as well. One of which is that the choice of number of groups may affect the p-value significantly, and there is no clear rule for how to choose the most proper number of groups. Also, it does not take overfitting into consideration. Because of these problems, we will only use Homser-Lemeshow goodness-of-fit test to give us a rough sense of how good a model is, but not necessarily to reject that model altogether.

To perform Homser-Lemeshow goodness-of-fit test, we can use hoslem.test() function in ResourceSelection package in R.

2.4 Akaike Information Criterion (AIC)

We will also use Akaike information criterion to evaluate our model. This criterion is a method to compare several regression models together. It is a function of the goodness-of-fit of a model and the number of parameters it takes. It gives reward to better goodness-of-fit and gives penalty to increasing number of explanatory variables The advantage of this criterion is that it will give penalty to over-fitting. However, the limitation of this method is that there is no natural scale to compare with, so it is possible that all of our models have poor fit, and we just choose the relatively best one among these. In general, the lower the AIC value, the better the model is.

This is very convenient in R. It is included in the **summary()** function and we can also call it using AIC() function.

3. Model

3.1 Model Obtained Using Backward Selection

The first model we obtained is by backward selection. We eliminated number_of_siblings_and_spouses and fares before we obtained a model whose p-value of each variable is smaller than 0.05. The model we obtained and its summary are shown below:

```
##
## Call:
## glm(formula = has_survived ~ gender + age + number_of_siblings_and_spouses +
       passenger_class + embarked_from, family = binomial(link = "logit"),
##
##
       data = titanic)
##
## Deviance Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
                                           Max
## -2.7755 -0.6791 -0.4560
                               0.7074
                                        2.5697
##
## Coefficients:
##
                                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                   4.86097
                                              0.47163 10.307 < 2e-16 ***
## gendermale
                                  -2.60736
                                              0.15798 -16.505 < 2e-16 ***
## age[19,55]
                                  -2.00712
                                              0.38995 -5.147 2.65e-07 ***
## age[56, above)
                                  -3.10763
                                              0.53229 -5.838 5.27e-09 ***
## age[6,18]
                                  -1.76429
                                              0.43159 -4.088 4.35e-05 ***
## agemissing
                                  -2.21886
                                              0.42028 -5.280 1.30e-07 ***
## number of siblings and spouses -0.35361
                                              0.09297 -3.803 0.000143 ***
                                              0.21489 -4.277 1.90e-05 ***
## passenger classsecond
                                  -0.91904
## passenger_classthird
                                  -1.77251
                                              0.19438
                                                      -9.119 < 2e-16 ***
## embarked_fromQ
                                  -0.47350
                                              0.30504 -1.552 0.120601
## embarked fromS
                                  -0.67719
                                              0.18791 -3.604 0.000314 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

```
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 1736.2 on 1305 degrees of freedom
## Residual deviance: 1191.1 on 1295 degrees of freedom
## AIC: 1213.1
##
## Number of Fisher Scoring iterations: 5
```

As can be seen from the summary, each variable has a p-value that is much lower than 0.05 except emabrked_fromQ. However, since the p-value of emabrked_fromS is low enough, we do not need to eliminate embarked_from variable. Also, there is no huge change of coefficients while we eliminate number_of_siblings_and_spouses and fares, so we do not need to worry about collinearity for these two variables; otherwise, we have to study whether the low p-value is caused by collinearity. The formula of this model is

$$ln(\frac{\hat{p}}{1-p}) = 4.86097 - 2.60736 \times gendermale - 2.00712 \times age[19, 55] - 3.10763 \times age[56, above] - 1.76429 \times age[66, above]$$

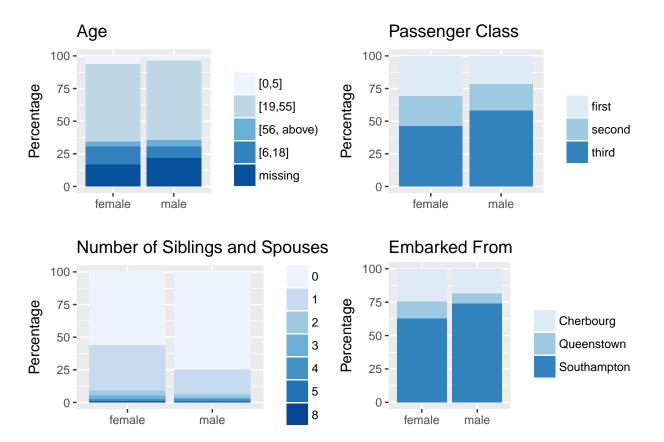
Now, we shall evaluate this model using Hosmer-Lemeshow goodness-of-fit test, and the outcome is as shown below:

```
##
## Hosmer and Lemeshow goodness of fit (GOF) test
##
## data: titanic$has_survived, fitted(m_best)
## X-squared = 22.308, df = 6, p-value = 0.001065
```

The p-value is 0.001065, which is much lower than 0.05; this suggests there is strong evidence that this model has a lack of fit. Hence, the model obtained using purely backward selection may not be the best and thus, we should conduct further feature selection steps to improve our model.

3.2 The Final Models

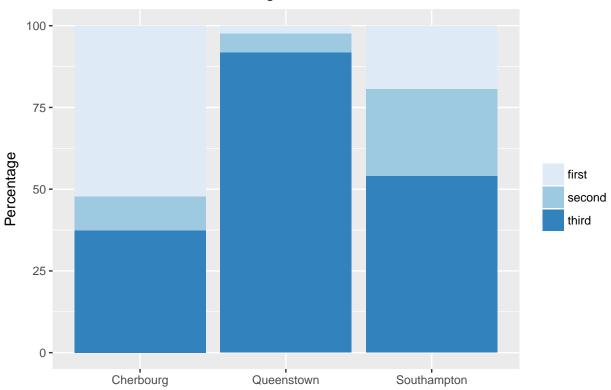
To get our final model, we first checked our suspicion of collinearity mentioned in the Data Exploration section. We are going to examine the correlation of gender with other variables through the plots below:



It seems that there is no significant correlation between age and gender, or embarked_from and gender. There is, however, a significant difference in the proportion of between the passenger classes for each gender. The number of siblings and spouses also seems to be correlated with gender. Because of this, we considered making two models, one with gender and one without gender.

Also, we looked at the correlation of embarked_from and passenger_class; the plot is shown below:

Embarked From vs. Passenger Class



As can be seen from this graph, there is a very significant correlation between passenger_class and embarked_from. For example, more than half of the passengers from Cherbourg are first class passengers, and only a few from Queenstown are first class. This suggests that we should be very careful when include both passenger_class and embarked_from in the same model.

Next, we created the following models. The one without gender takes passenger_class, age and number_of_siblings_and_spouses into account. We tried several models, and despite the fact that we will get a better p-value from Hosmer-Lemeshow goodness-of-fit test by including embarked_from, we still think it might be problematic because of its strong association with passenger_class. The summary of the best model (without gender) found and the result from the Hosmer-Lemeshow goodness-of-fit test is as shown:

```
##
## Call:
## glm(formula = has_survived ~ passenger_class + age + number_of_siblings_and_spouses,
##
       family = binomial(link = "logit"), data = titanic)
##
## Deviance Residuals:
##
                      Median
                                    3Q
       Min
                 1Q
                                            Max
## -2.1791 -0.8806 -0.7015
                                0.9754
                                         2.1284
##
```

```
## Coefficients:
##
                                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                    2.43417
                                               0.35008
                                                         6.953 3.57e-12 ***
## passenger classsecond
                                  -0.97894
                                               0.17462
                                                       -5.606 2.07e-08 ***
## passenger classthird
                                  -1.78063
                                               0.15635 -11.389 < 2e-16 ***
## age[19,55]
                                   -1.78069
                                               0.32160
                                                       -5.537 3.08e-08 ***
## age[56, above)
                                  -2.80888
                                               0.44150
                                                       -6.362 1.99e-10 ***
## age[6,18]
                                                        -3.558 0.000373 ***
                                   -1.24306
                                               0.34932
## agemissing
                                   -1.93037
                                               0.34351
                                                        -5.620 1.91e-08 ***
## number of siblings and spouses -0.15772
                                                        -2.177 0.029468 *
                                               0.07244
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 1736.2
##
                              on 1305
                                        degrees of freedom
## Residual deviance: 1557.0
                              on 1298
                                        degrees of freedom
## AIC: 1573
##
## Number of Fisher Scoring iterations: 4
##
##
    Hosmer and Lemeshow goodness of fit (GOF) test
##
         titanic$has_survived, fitted(m_best_8)
## X-squared = 9.1521, df = 6, p-value = 0.1652
```

As we mentioned before, this p-value is not the best that could be achieved, but it is still good enough given that it is greater than 0.05. The equation of this regression model is:

The negative coefficients of all variables suggest that the reference group is the group with highest survival rate. That is the group of children (age smaller than 5) in first class without any siblings and spouses are the most likely to survive; and the natural log of odds is expected to be 2.43. This means that their chance of survival is approximately 0.9190865. There are many other interesting interpretation that can be extracted from this model. For example, it suggests that first class passengers are expected to have a 0.98 higher natural log of odds than second class passengers, all else held constant. Also, we can expect a decrease of 0.16 in natural log of odds for every one more increase in number of siblings and spouses, all else held constant.

This model seems good as it matches with our instinct that children and first class passengers

will have better chance of survival in a shipwreck. This could be due to their proximity to the rescue site, or perhaps, first class passengers were more likely to know how to swim given their background. However, since **gender** is such an important variable, we would also like to examine a model with **gender** as one of the explanatory variables.

We made a set of models and compared them according to their meanings and their results from Hosmer-Lemeshow goodness-of-fit test. It turns out that all the model with gender variable fails the Hosmer-Lemeshow goodness-of-fit test since they all have really really low p-value, but arguably, we could conclude that the best model (with gender) is the one that takes gender, age, passenger_class, and number_of_siblings_and_spouses because it contains the most crucial information of a passenger, and its AIC value is the lowest among all the models including gender as an explanatory variable. The summary of this model is as shown below:

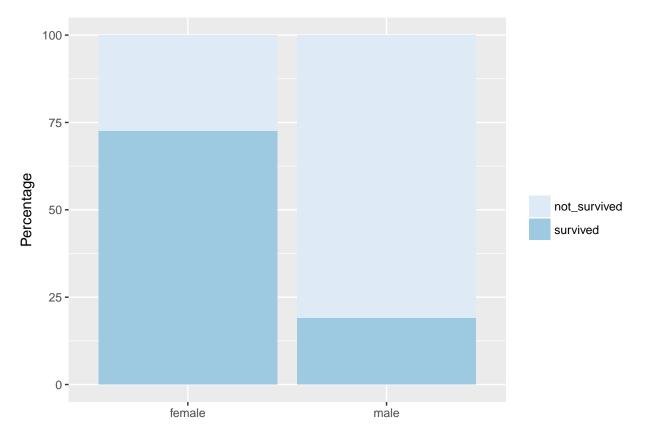
```
##
## Call:
## glm(formula = has survived ~ gender + age + number of siblings and spouses +
       passenger class, family = binomial(link = "logit"), data = titanic)
##
##
## Deviance Residuals:
##
       Min
                 10
                      Median
                                    30
                                            Max
## -2.8653
           -0.6894 -0.4535
                                0.6856
                                         2.5462
##
## Coefficients:
##
                                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                                    4.46068
                                               0.44952
                                                         9.923
                                                                < 2e-16 ***
## gendermale
                                   -2.61030
                                               0.15445 -16.901
                                                                < 2e-16 ***
## age[19,55]
                                   -2.01284
                                               0.38663
                                                        -5.206 1.93e-07 ***
## age[56, above)
                                   -3.08942
                                               0.52946
                                                        -5.835 5.38e-09 ***
## age[6,18]
                                                        -4.004 6.22e-05 ***
                                   -1.70756
                                               0.42644
                                                        -5.217 1.82e-07 ***
## agemissing
                                   -2.15090
                                               0.41233
                                                        -4.058 4.95e-05 ***
## number_of_siblings_and_spouses -0.37239
                                               0.09176
## passenger classsecond
                                   -1.11954
                                               0.20723 -5.402 6.57e-08 ***
## passenger classthird
                                   -1.92213
                                               0.18536 -10.369 < 2e-16 ***
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 1736.2
                              on 1305
                                        degrees of freedom
## Residual deviance: 1204.1
                             on 1297
                                        degrees of freedom
```

```
## AIC: 1222.1
##
## Number of Fisher Scoring iterations: 5
##
                      y0
                          у1
                                 yhat0
                                             yhat1
  [0.00548,0.0977]
                          22 199.42682
                     195
                                         17.573180
## (0.0977,0.111]
                     205
                          42 219.77092
                                         27.229080
## (0.111,0.144]
                      26
                           6
                              27.56544
                                          4.434556
## (0.144,0.225]
                          18 132.71261
                                         33.287390
                     148
                                         75.080540
## (0.225,0.459]
                     122
                          63 109.91946
## (0.459,0.628]
                      81
                          87
                              70.01437
                                         97.985627
## (0.628,0.798]
                      21 108
                              32.54474
                                         96.455263
## (0.798,0.984]
                              16.04564 145.954365
                      10 152
##
   Hosmer and Lemeshow goodness of fit (GOF) test
##
##
## data: titanic$has survived, fitted(m best 12)
## X-squared = 33.874, df = 6, p-value = 7.114e-06
```

As we can see from the summary, the equation of this regression model is

Since all the coefficients are negative, the reference group is expected to be the group of people with highest chance of survival, similar to the previous model. That is, little girls (age under 5) in first class without siblings and spouses are ones with highest survival chance; and the natural log of odds is expected to be 4.46, which means that their chance of survival is approximately 0.9885698. Also, the coefficients suggest how each variable affect the chance of survival. For example, males are expected to have a 2.61 lower natural log of odds than females, all else held constant. We can also see that having one more siblings or spouses is associated with 0.37 decrease in natural log of odds, all else held constant.

As for the best model between these two, we chose the latter one, i.e. the model with gender variable. We find it interesting to look at the correlation between gender and has survived.



From the graph, we can see that a majority of female survived while much lower percentage of male survived, so there is indeed, a very strong correlation between <code>gender</code> and <code>has_survived</code>. Thus, we think it makes more sense to have <code>gender</code> as one of the explanatory variables in the final model, even if including it causes a decrease in the p-value of Hosmer-Lemeshow goodness-of-fit test result. Also, the AIC decreases significantly after we added <code>gender</code> in the model. This suggests that including <code>gender</code> will make our model a better fit even if we take penalty of adding new variables. Hence, we conclude our best model as:

$$ln(\frac{\hat{p}}{1-p}) = 4.46068 - 2.61030 \times gendermale - 2.01284 \times age[19, 55] - 3.08942 \times age[56, above] - 1.70756 \times age[66, above]$$

4. Conclusion

In this research, we attempted to answer the question "How do various demographic and socioeconomic factors affect chance of survival in shipwreck where the ship's capacity is at least 1000 passegners and the route is through the Atlantic Ocean in the early 20th century?" and we have found a not-so-surprising result that women, children, and first class passengers have better chance of survival. Besides, there is an interesting finding we found that people with more siblings and spouses are less likely to survive.

As mentioned, the Hosmer-Lemeshow goodness-of-fit test p-value for the final model is very low (7.114e-06); this suggests a lack of fit in the model. This might also hint on the fact that generalized linear model might not be the best approach to answer the research question, and thus, we should try out different modelling methods. Again, as mentioned before, AIC value only suggests if a model is relatively better than another but does not provide an absolute scale to estimate how well a model performs. Also, among the explanatory variables used in the model, age has approximately 20% missing data; this adds unnecessary complexity in the model and thus could have affected the performance of the model. Another limitation is that we only build the model but did not really test it against real data; what we could have done is to split the data into training and testing groups, however that is out of the scope of the class and the project. If we can run the model against test samples, we will have a much better sense of how the model perform.

Another the point to note is that we only have access to information of 1300/2200 passengers on the Titanic, hence our data might not have reflected the true demographics and socioeconomic settings of passengers on the ship. We are not so sure why there are many people missing. If these people are majorly male labor and third-class passengers, our findings might not been affected much. But if many of these are rich people who tried to keep their identity hidden for example or they all did not survived the shipwreck (so that we do not have any information about them), then definitely our model could not generalize well.

We also consider some opportunities for further studies, which includes splitting the data set by gender and come up with models for each part of the data to see whether we can come up with models with better diagnostic results for each gender. We are considering splitting the children group (age 0-5) based on their passenger classes to see if first class children have higher rate of survival. The only problem is that the sample size might be too small and also there are a lot of missing data for age.

To conclude, out of curiosity, we attempt to reimagine the night of the disaster, putting ourselves in the shoes of those tragic passengers. As one-score-and-two male who are properly only rich enough to get a third-class ticket on the Titanic, we realize our inevitable fate. We will go down with the ship, as the saying goes...



Bibliography