**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Answers**. The probability that the service manager cannot meet his commitment is **B.** (0.2676) By using Z score value is 60-55/8=0.625 and 1-stats.norm.cdf(0.625).

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answers.**

1. Mean = 38

Standard deviation = 6

Z score = (Value - Mean)/SD

Z score for 44  = (44 - 38)/6  = 1  =>  84.13 %

=> People above 44 age = 100 - 84.13 =  15.87%  ≈  63    out of 400

Z score for 38  = (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44  age = 84.13 - 50 = 34.13 % ≈  137 out of 400

Hence More employees at the processing center are older than 44 than between 38 and 44. is F**ALSE.**

1. Z score for 30  = (30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the center would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** As we know that if X ~ N (µ1, σ1^2), and Y ~ N (µ2, σ2^2) are two

independent random variables then,

X + Y ~ N (µ1 + µ2, σ1^2 + σ2^2), and

X − Y ~ N (µ1 − µ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is

linear combination of X and Y then,

Z ~ N (aµ1 + bµ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2µ,4 σ^2) and

X1+X2 ~ N (µ + µ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4µ,6 σ^2)

Normal Distribution parameters are same.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: D.**

The Probability of getting value between a and b should be 0.99.

So, the Probability of going wrong, or the Probability outside the a and b area

is 0.01 (i.e., 1-0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).

The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the

random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X

values.

Z= (X- μ) / σ

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Z \* σ + μ = X

Z (-0.005) \*20+100 = -(-2.57) \*20+100 = 151.5

Z (+0.005) \*20+100 = (-2.57) \*20+100 = 48.5.

So, the option ‘D’ is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**A.** Specify a Rupee range (centred on the mean) such that it contains

95% probability for the annual profit of the company.

**Ans.** Mean Profit is Rs 540 million.

Standard Deviation is Rs 225.0 million

Range is Rs (99.00810347848784, 980.9918965215122) in Millions.

1. Specify the 5th percentile of profit (in Rupees) for the company

**Ans.** 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans.** Making 1 loss: - Confidence Interval: 0.0477903522728147.

Making 2 loss: Confidence Interval: 0.040059156863817086.

Probability of Division 1 making a loss in a given year is more than Division (profit 2).