DSC 511 Assignment 01

Adithya KR
20241MCS11023

Q1) Total number of chairs = 12

Number of committee members = 8

no: of combinations that 8 persons can sit in 12 chairs = $nCr = 12C_8$

$$= \frac{12!}{8!(12-8)!} = \frac{12!}{8!4!}$$

$$= \frac{9 \times 10 \times 11 \times 12}{1 \times 2 \times 3 \times 4} = 9 \times 5 \times 11$$

$$= 495$$

Q2) Black cell phones = 20

White cell phones = 30

Total cell phones = 20+30 = 50

no: of Phones selected in random = 10.

a) Probability of Exactly 4 phones selected.

$$= \frac{\text{No: of favorable outcomes}}{\text{Total outcomes.}} = \frac{20C_4 \cdot 30C_6}{50C_{10}}.$$

$$= \frac{\frac{20!}{4!16!} * \frac{30!}{6!24!}}{\frac{50!}{10!40!}}$$

$$\frac{4845 * 593775}{1.0272 \times 10^{10}}$$

$$= 0.28$$

Q2b) probability of getting less than 3 black phone.

$$= \frac{\text{no of favorable outcomes}}{\text{Total outcomes}} = \frac{{}^{20}C_0 \cdot {}^{30}C_{10} + {}^{20}C_1 \cdot {}^{30}C_9 + {}^{20}C_2 \cdot {}^{30}C_8}{{}^{50}C_{10}}$$

$$\frac{\left(1 \cdot \frac{30!}{10!20!}\right) + \left(20 * \frac{30!}{9!21!}\right) + \left(\frac{20!}{2! \cdot 18!} \cdot \frac{30!}{8!22!}\right)}{\frac{50!}{10!\,40!}}$$

$$\frac{3.0045e^7 + (20 * 1.4307e^7) + (190 * 5.8529e^6)}{1.0272e^{10}} = 0.139$$

Q3) * Let A be the event where the hand contains Exactly 2 aces.

* Let B be the event where the hand contains at least 1 aces.

* We need to find the probability that the hand contains exactly 2 aces given that atleast one ace is present

ie $P(A/B)$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}.$$

* Since B = no of aces ≥ 1 = B

A = no of aces = 2

* A is a subset of B

$\therefore P(A \cap B) = P(A)$.

$\therefore P(A/B) = \frac{P(A)}{P(B)}$

$P(A)$ = Probability of 2 aces present 5 cards randomly selected from a deck of cards.

Total no: of cards in a Deck = 52

number of cards picked = 5.

Total number of possible combinations of 5 cards $= {}^{52}C_5$

$P(A) = \frac{\text{no: of favorable outcomes}}{\text{Total number of outcomes}} = \frac{{}^4C_2 \cdot {}^{48}C_3}{{}^{52}C_5} \Rightarrow (1)$

$P(B)$ = at least one ace being present in 5 cards randomly selected from a deck of cards.

$P(B) = 1 - P(\text{No aces being present})$.

$P(\text{no aces being present}) = \frac{\text{No of favorable outcomes}}{\text{Total outcomes}} = \frac{{}^4C_0 \cdot {}^{48}C_5}{{}^{52}C_5}$

$\therefore P(B) = 1 - \frac{{}^4C_0 \times {}^{48}C_5}{{}^{52}C_5} = 1 - \frac{{}^{48}C_5}{{}^{52}C_5} \Rightarrow (2)$

$\frac{P(A)}{P(B)} = (1)/(2) = \frac{\frac{{}^4C_2 \times {}^{48}C_3}{{}^{52}C_5}}{\frac{{}^{52}C_5 - {}^{48}C_5}{{}^{52}C_5}} = \frac{({}^4C_2)\cdot({}^{48}C_3)}{{}^{52}C_5 - {}^{48}C_5}$

$= 0.117$

Q4) Total students = 50.

number of student chosen at random = 15

P(you are chosen) = P(A) = $15/50$ = 0.3

P(Joe is chosen) = P(B) = $15/50$ = 0.3.

P(A or B) = P(you or Joe chosen)

P(A or B) = P(A) + P(B) − P(A and B)

= 0.3 + 0.3 − (0.3 × 0.3)

= 0.6 − 0.9 = 0.5

Q5). Let S be event of spam Email

Let R be event of email containing "refiano"

let given P(S) = $50/100$ = 0.5

P(spam email contains 'referance') = P(R/S)

= 0.01

P(non spam email contains 'refiano') = P(R/S^c)

$$P(S/R) = \frac{P(R/S)\cdot P(S)}{P(R/S)\cdot P(S) + P(R/S^c)P(S^c)}$$

= $0.001/100$ = 0.0000

$$= \frac{0.01 \times 0.5}{(0.01 \times 0.5) + (0.000001 \cdot 0.5)} = \frac{0.005}{0.005 + 5\times 10}$$

= 0.997