Spatial Fay-Herriot model

Fay-Herriot (FH) estimator with spatially correlated random area effects

One extension of the Fay-Herriot model is the estimation of small area indirect estimators under area level random effect models when only area level data are available and the random effects are correlated (Pratesi and Salvati 2008).

Load package and data

The emdi package loaded here is the one under development with the new area-level models. The installation is explained in installEmdiExtensions.

For the data, aggregted survey and population data is loaded and combined with function combined_data such that the indicators of interest (direct point and variance estimate) and the additional information are in one data frame.

```
# Load package emdi
library(emdi)
```

```
## Registered S3 method overwritten by 'MuMIn':
##   method         from
##   predict.merMod lme4
```

```
# Load aggregated data
data("eusilcA_popAgg")
data("eusilcA_smpAgg")

# Combine sample and population data
combined_data <- combine_data(pop_data = eusilcA_popAgg, pop_domains = "Domain",
                              smp_data = eusilcA_smpAgg, smp_domains = "Domain")
head(combined_data)
```

```
##               Domain     Mean     MTMED      Cash  Var_Mean    Var_MTMED
## 1          Amstetten 14768.57 0.2429907  9189.436  926167.4 0.0005730411
## 2              Baden 21995.72 0.7110553 12868.417  446534.3 0.0005162203
## 3            Bludenz 12069.59 0.1172840 10326.370 1243265.0 0.0006390644
## 4     Braunau am Inn 10770.53 0.1276596  6764.088 1029502.4 0.0003949029
## 5            Bregenz 35731.20 0.9053254 26558.063 4467316.4 0.0002535837
## 6 Bruck-MÃ¼rzzuschlag 23027.37 0.7622378 18816.987 1971664.0 0.0006336761
##   Var_Cash  n   eqsize      cash self_empl unempl_ben  age_ben  surv_ben
## 1  4147080 33 1.582866  8577.553  747.6235   303.6629 5557.927  68.33156
## 2  4070341 40 1.718342 13086.799 2014.7959   651.2849 7351.761  80.21344
## 3  5119468 17 1.664815  7927.413 1080.0148   634.2535 3593.826 133.12265
## 4  2192541 29 1.687589  7863.343 1182.2163   428.5239 4067.074 181.91383
## 5 13714562 34 1.628107 20708.773 3424.8690   403.6842 5947.884  61.29290
## 6 10084579 29 1.700350 15144.818 1752.0069   113.7574 5634.365 167.03164
##    sick_ben  dis_ben       rent fam_allow house_allow   cap_inv    tax_adj
## 1  31.82570 650.2229   78.03526  1344.615    54.41804  218.6198  -36.13875
## 2  19.66422 634.4199  348.43050  1772.672    57.56307  396.7049 -155.10809
## 3  35.38778 327.4857   29.22654  2497.553    83.11327  211.7293 -233.80512
## 4 150.16057 664.2563   71.55915  1908.017    83.36667  239.5027 -151.49656
## 5  69.47379 454.4688 3674.26633  2656.902   127.16695 1264.5426  131.10672
## 6 239.56916 827.9008  265.19710  1657.916    78.57378  291.2001 -183.24920
```

```
##   ratio_n
## 1 0.01284
## 2 0.01592
## 3 0.00648
## 4 0.01128
## 5 0.01352
## 6 0.01144
```

Create a spatial correlation matrix

For the estimation of the Fay-Herriot model with spatially correlated random effects, a spatial correlation matrix needs to be calculated. This matrix is one argument in the fh function.

The spatial correlation matrix can be calculated based on a shape file and with the help of functions from package maptools and spdep.

```
library(maptools)
```

```
## Loading required package: sp
```

```
## Checking rgeos availability: TRUE
```

```
library(spdep)
```

```
## Loading required package: spData
```

```
## To access larger datasets in this package, install the spDataLarge
## package with: `install.packages('spDataLarge',
## repos='https://nowosad.github.io/drat/', type='source')`
```

```
## Loading required package: sf
```

```
## Linking to GEOS 3.6.1, GDAL 2.2.3, PROJ 4.9.3
```

```
# Load shapefile
load_shapeaustria()
austria_shape <- merge(shape_austria_dis, eusilcA_smpAgg, by.x = "PB",
                       by.y = "Domain", all.x = F)
rel <- poly2nb(austria_shape, row.names = austria_shape$PB)
euSilcA_prox <- nb2mat(rel, style = "W", zero.policy = TRUE)
head(euSilcA_prox)
```

```
##                          [,1]      [,2] [,3]      [,4]      [,5] [,6]
## Eisenstadt (Stadt)  0.0000000 0.0000000  1.0 0.0000000 0.0000000    0
## Rust (Stadt)        0.0000000 0.0000000  0.5 0.0000000 0.0000000    0
## Eisenstadt-Umgebung 0.1666667 0.1666667  0.0 0.0000000 0.0000000    0
## GÃ¼ssing             0.0000000 0.0000000  0.0 0.0000000 0.3333333    0
## Jennersdorf         0.0000000 0.0000000  0.0 0.3333333 0.0000000    0
## Mattersburg         0.0000000 0.0000000  0.0 0.0000000 0.0000000    0
##                          [,7]      [,8]      [,9] [,10] [,11] [,12] [,13]
## Eisenstadt (Stadt)  0.0000000 0.0000000 0.0000000     0     0     0     0
## Rust (Stadt)        0.5000000 0.0000000 0.0000000     0     0     0     0
## Eisenstadt-Umgebung 0.1666667 0.0000000 0.0000000     0     0     0     0
## GÃ¼ssing             0.0000000 0.0000000 0.3333333     0     0     0     0
## Jennersdorf         0.0000000 0.0000000 0.0000000     0     0     0     0
## Mattersburg         0.0000000 0.3333333 0.0000000     0     0     0     0
##                     [,14] [,15] [,16] [,17] [,18] [,19] [,20] [,21] [,22]
## Eisenstadt (Stadt)      0     0     0     0     0     0     0     0     0
```

```
## Rust (Stadt)            0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung     0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf             0     0     0     0     0     0     0     0     0
## Mattersburg             0     0     0     0     0     0     0     0     0
##                        [,23] [,24]     [,25]     [,26] [,27] [,28] [,29]
## Eisenstadt (Stadt)  0.0000000     0 0.0000000 0.0000000     0     0     0
## Rust (Stadt)        0.0000000     0 0.0000000 0.0000000     0     0     0
## Eisenstadt-Umgebung 0.0000000     0 0.1666667 0.1666667     0     0     0
## GÃ¼ssing              0.0000000     0 0.0000000 0.0000000     0     0     0
## Jennersdorf         0.0000000     0 0.0000000 0.0000000     0     0     0
## Mattersburg         0.3333333     0 0.0000000 0.0000000     0     0     0
##                       [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37] [,38]
## Eisenstadt (Stadt)      0     0     0     0     0     0     0     0     0
## Rust (Stadt)            0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung     0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf             0     0     0     0     0     0     0     0     0
## Mattersburg             0     0     0     0     0     0     0     0     0
##                       [,39] [,40] [,41]     [,42] [,43] [,44] [,45] [,46]
## Eisenstadt (Stadt)      0     0     0 0.0000000     0     0     0     0
## Rust (Stadt)            0     0     0 0.0000000     0     0     0     0
## Eisenstadt-Umgebung     0     0     0 0.1666667     0     0     0     0
## GÃ¼ssing                  0     0     0 0.0000000     0     0     0     0
## Jennersdorf             0     0     0 0.0000000     0     0     0     0
## Mattersburg             0     0     0 0.3333333     0     0     0     0
##                       [,47] [,48] [,49] [,50] [,51] [,52] [,53] [,54] [,55]
## Eisenstadt (Stadt)      0     0     0     0     0     0     0     0     0
## Rust (Stadt)            0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung     0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf             0     0     0     0     0     0     0     0     0
## Mattersburg             0     0     0     0     0     0     0     0     0
##                       [,56] [,57] [,58] [,59] [,60] [,61] [,62] [,63] [,64]
## Eisenstadt (Stadt)      0     0     0     0     0     0     0     0     0
## Rust (Stadt)            0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung     0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf             0     0     0     0     0     0     0     0     0
## Mattersburg             0     0     0     0     0     0     0     0     0
##                       [,65] [,66] [,67] [,68] [,69] [,70] [,71] [,72] [,73]
## Eisenstadt (Stadt)      0     0     0     0     0     0     0     0     0
## Rust (Stadt)            0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung     0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf             0     0     0     0     0     0     0     0     0
## Mattersburg             0     0     0     0     0     0     0     0     0
##                       [,74] [,75] [,76] [,77] [,78]     [,79]     [,80]
## Eisenstadt (Stadt)      0     0     0     0     0 0.0000000 0.0000000
## Rust (Stadt)            0     0     0     0     0 0.0000000 0.0000000
## Eisenstadt-Umgebung     0     0     0     0     0 0.0000000 0.0000000
## GÃ¼ssing                  0     0     0     0     0 0.3333333 0.0000000
## Jennersdorf             0     0     0     0     0 0.3333333 0.3333333
## Mattersburg             0     0     0     0     0 0.0000000 0.0000000
```

```
##                       [,81] [,82] [,83] [,84] [,85] [,86] [,87] [,88] [,89]
## Eisenstadt (Stadt)       0     0     0     0     0     0     0     0     0
## Rust (Stadt)             0     0     0     0     0     0     0     0     0
## Eisenstadt-Umgebung      0     0     0     0     0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0     0     0     0     0
## Jennersdorf              0     0     0     0     0     0     0     0     0
## Mattersburg              0     0     0     0     0     0     0     0     0
##                       [,90] [,91] [,92] [,93] [,94]
## Eisenstadt (Stadt)       0     0     0     0     0
## Rust (Stadt)             0     0     0     0     0
## Eisenstadt-Umgebung      0     0     0     0     0
## GÃ¼ssing                  0     0     0     0     0
## Jennersdorf              0     0     0     0     0
## Mattersburg              0     0     0     0     0
```

Estimate the FH estimates with spatially correlated random area effects

In the following, the various options for adding spatially correlated random area effects in package emdi are presented: correlation = "spatial".

Standard

One way to add spatially correlated random area effects follows Pratesi and Salvati (2008). In this specification, following estimation methods are possible:

- Variance estimation:
  - Maximum likelihood: method = "ml"
  - Restricted maximum likelihood: method = "reml"
- MSE estimation:
  - Analytical: mse_type = "analytical" (default)
  - Naive parametric bootstrap: mse_type = "spatialparboot"
  - Bias corrected parametric bootstrap: mse_type = "spatialparbootbc"
  - Naive nonparametric bootstrap: mse_type = "spatialnonparboot"
  - Bias corrected nonparametric bootstrap: mse_type = "spatialnonparbootbc"

```
fh_spatial <- fh(fixed = Mean ~ cash + self_empl, vardir = "Var_Mean",
                 tol = 0.00000001, maxit = 2000, combined_data = combined_data,
                 domains = "Domain", method = "reml", correlation = "spatial",
                 corMatrix = eusilcA_proxmat, MSE = TRUE)
fh_spatial
```

```
## Empirical Best Linear Unbiased Prediction (Fay-Herriot)
##
## Out-of-sample domains:  0
## In-sample domains:  94
##
## Variance and MSE estimation:
## Variance estimation method:  reml
## Estimated variance component(s):  spatial correlation assumed
## Variance of random effects:  1440040
## Spatial correlation parameter:  -0.1616871
## MSE method:  prasad-rao-Singh
##
## Transformation: No transformation
```

Robust

The package also offers a robust estimation of the FH model with spatially correlated random area effects following Warnholz (2016). In this specification, following estimation methods are possible:

- Variance estimation:
  - Robustified maximum likelihood with robust eblup prediction: method = "reblup"
  - Robustified maximum likelihood with robust and bias-corrected eblup prediction: method = "reblupbc"
- MSE estimation:
  - Bootstrap: mse_type = "boot"
  - Pseudo linearisation: mse_type = "pseudo"

For an application, the bootstrap iterations should be increased but due to computational reasons B is set to 2.

```
fh_robustSpatial <- fh(fixed = Mean ~ cash + self_empl, vardir = "Var_Mean",
                 tol = 0.00000001, maxit = 2000, combined_data = combined_data,
                 domains = "Domain", method = "reblupbc", correlation = "spatial",
                 corMatrix = eusilcA_proxmat, MSE = TRUE, mse_type = "boot", B = 2)
fh_robustSpatial
```

```
## Empirical Best Linear Unbiased Prediction (Fay-Herriot)
##
## Out-of-sample domains:  0
## In-sample domains:  94
##
## Variance and MSE estimation:
## Variance estimation method: robustified ml, reblupbc
## k =  , c =
## Estimated variance component(s):  spatial correlation assumed
## Variance of random effects:  1289304
## Spatial correlation parameter:  -0.1504612
## MSE method:  bootstrap
##
## Transformation: No transformation
```

Pratesi, M., and N. Salvati. 2008. "Small Area Estimation: The EBLUP Estimator Based on Spatially Correlated Random Area Effects." *Statistical Methods and Applications* 17: 113–41.

Warnholz, Sebastian. 2016. "Small Area Estimation Using Robust Extensions to Area Level Models." PhD thesis, Freie Universität Berlin.