Inertia Wheel Inverted Pendulum

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***Abstract*—We explore the classic nonlinear controls problem, inverting a pendulum, using analyses learned in this class. Specifically, we look at using a flywheel to stabilize the pendulum. We build a physical system from scratch. In simulation, we derive the equations of motion and apply LQR and region of attraction analyses for our system. We additionally perform controllability analysis to the case where the full state may not be measurable, as happened in our physical system. In hardware, we successfully implement downward stabilization and swingup controls using both a PD controller an a bang-bang controller. We discuss the design decisions, design iterations, and present work toward implementing a controller for the inverted state.**

I. Introduction

Pendulum is inherently unstable around upright fixed point.

Example write-up:

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mds

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A. Todo

Control authority analysis – max torque output as function of motor and flywheel (torque constraints) Compare to mglsin theta – what is the maximum deviation angle we can recover from

1) Subsubsection Heading Here: Subsubsection text here.

II. Analyses

A. LQR

1) Equations of Motion: To derive the equations of motion (EOM), we use the Lagrangian method. Let L equal to the kinetic energy plus the potential energy of the system.

$$L = KE - PE \tag{1}$$

By Lagrange's method,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \sum_{i=0}^{n} F_i \tag{2}$$

for $i = 1, 2, 3...n$ forces.

Thus, we need to write out the KE, the PE, the derivative of L with respective to each state q, the derivative of L with respect to the (time) derivative of each state q, and then the time derivative of that last term.

Let us begin...

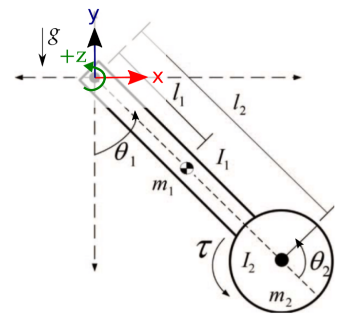

Fig. 1. Free body diagram

conservation of energy, we can conclude that for a closed system,

Defining our origin as the pivot point,

2) Linearization:

B. LQR on "System on Wheels"

For a brief detour (in order to demonstrate something other than just the pset) we imagine sticking the whole thing on wheels and redo the same analysis, although for sanity we run the calculations through sympy instead of by hand.

1) Controllability:

2) Equations of Motion:

3) Linearization:

III. Conclusion

The conclusion goes here.

Acknowledgment

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References

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.