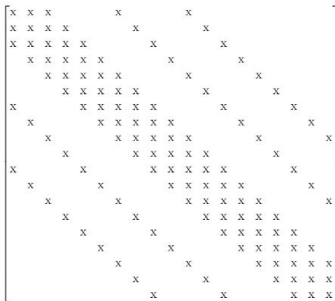


Parallelism study of iterative linear solvers using Regent and CUDA

EE382N-20 - Parallelism and Locality

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Presentation Outline

- Problem motivation and background
- CUDA implementation results
- Regent implementation results, and comparison to CPU performance
- Results discussion and conclusions

Sparse Linear Systems



- Linear systems arise naturally when discretizing PDEs to solve numerically
- Systems are usually quite sparse in nature, and follow certain patterns
- Direct solvers are generally of complexity $O(n^3)$, which can be cost prohibitive for large systems
- Additionally, direct solve storage requirements also become an issue, as factors of a sparse matrix are generally non-sparse
- Iterative solvers are crucial to handling these types of problems

$$\begin{bmatrix} -2 & 1 & & & & & & & & & \\ 1 & -2 & 1 & & & & & & & & \\ & 1 & -2 & 1 & & & & & & & \\ & & 1 & -2 & 1 & & & & & & \\ & & & 1 & -2 & 1 & & & & & \\ & & & & 1 & -2 & 1 & & & & \\ & & & & & 1 & -2 & 1 & & & \\ & & & & & & 1 & -2 & 1 & & \\ & & & & & & & 1 & -2 & 1 & \\ & & & & & & & & 1 & -2 & 1 \\ & & & & & & & & & 1 & -2 \end{bmatrix}$$

[illegible]

Sparse Linear Systems

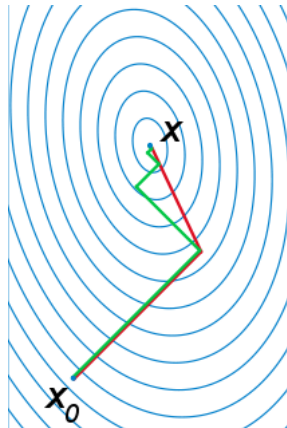
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$$\begin{bmatrix} -4 & 1 & & & & & & & & & \\ 1 & -4 & 1 & & & & & & & & \\ & 1 & -4 & 1 & & & & & & & \\ & & 1 & -4 & 1 & & & & & & \\ & & & 1 & -4 & 1 & & & & & \\ & & & & 1 & -4 & 1 & & & & \\ & & & & & 1 & -4 & 1 & & & \\ & & & & & & 1 & -4 & 1 & & \\ & & & & & & & 1 & -4 & 1 & \\ & & & & & & & & 1 & -4 & 1 \\ & & & & & & & & & 1 & -4 \end{bmatrix}$$

Conjugate Gradient Method

- Common iterative method used for solving sparse linear systems
- Designed to work on SPD matrices, but can be expanded to work on more general cases
- Can be thought of as solving a minimization problem $q(x) = \frac{1}{2} \cdot Ax - x \cdot b$
- search directions and an optimal step size are chosen at each iteration, until $\|Ax - b\|_2$ falls below some threshold



CUDA Results

Regent Implementation



Conclusions