UNIT 4:

Transportation Problem

Transportation problem is a special kind of **Linear Programming Problem (LPP)** in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hitchcock problem.

Types of Transportation problems:

Balanced: When both supplies and demands are equal then the problem is said to be a balanced transportation problem.

Unbalanced: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

Methods to Solve:

To find the initial basic feasible solution there are three methods:

1. NorthWest Corner Cell Method.
2. Least Call Cell Method.
3. Vogel's Approximation Method (VAM).

Basic structure of transportation problem:

Destination

Source	D1	D2	D3	D4	Supply(s_i)
O1	C_{11}	C_{12}	C_{13}	C_{14}	s_1
O2	C_{21}	C_{22}	C_{23}	C_{24}	s_2
O3	C_{31}	C_{32}	C_{33}	C_{34}	s_3
O4	C_{41}	C_{42}	C_{43}	C_{44}	s_4
Demand (d_j):	d_1	d_2	d_3	d_4	

In the above table **D1**, **D2**, **D3** and **D4** are the destinations where the products/goods are to be delivered from different sources **S1**, **S2**, **S3** and **S4**. $\mathbf{S_i}$ is the supply from the source $\mathbf{O_i}$. $\mathbf{d_j}$ is the demand of the destination $\mathbf{D_j}$. $\mathbf{C_{ij}}$ is the cost when the product is delivered from source $\mathbf{S_i}$ to destination $\mathbf{D_j}$.

NorthWest Corner Method

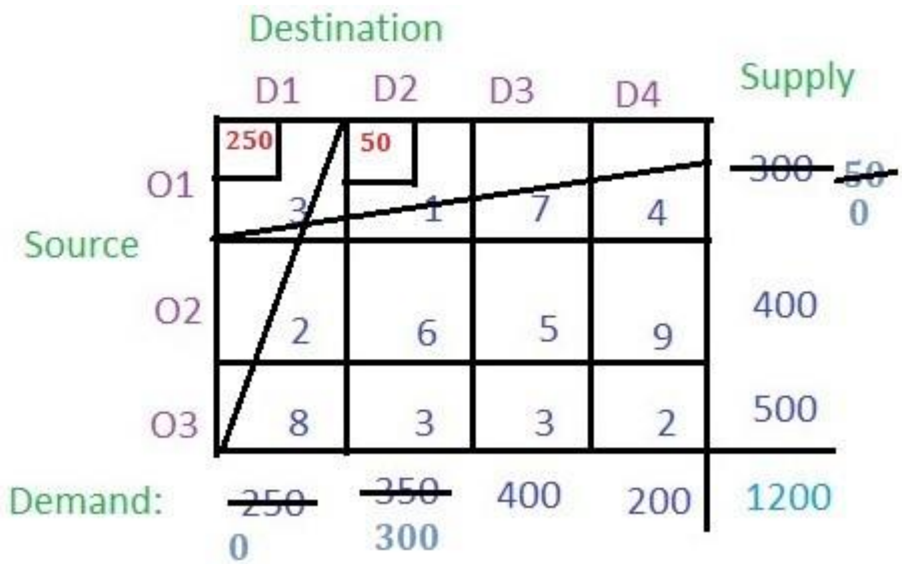
An introduction to Transportation problem has been discussed in the previous article, in this article, finding the initial basic feasible solution using the NorthWest Corner Cell Method will be discussed.

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	3	1	7	4	300
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		250	350	400	200	1200

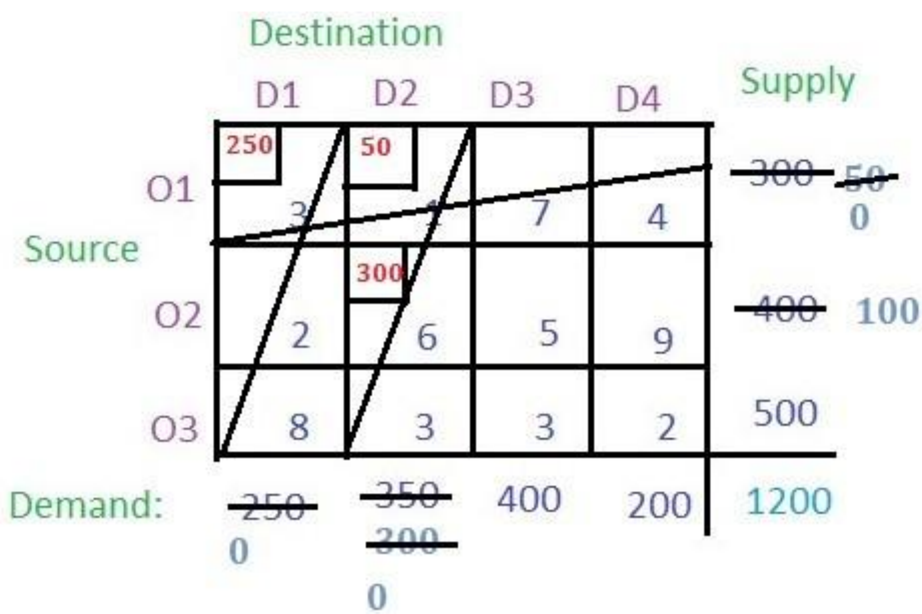
Explanation: Given three sources **O1**, **O2** and **O3** and four destinations **D1**, **D2**, **D3** and **D4**. For the sources **O1**, **O2** and **O3**, the supply is **300**, **400** and **500** respectively. The destinations **D1**, **D2**, **D3** and **D4** have demands **250**, **350**, **400** and **200** respectively.
Solution: According to North West Corner method, **(O1, D1)** has to be the starting point i.e. the north-west corner of the table. Each and every value in the cell is considered as the cost per transportation. Compare the demand for column **D1** and supply from the source **O1** and allocate the minimum of two to the cell **(O1, D1)** as shown in the figure.
The demand for Column **D1** is completed so the entire column **D1** will be canceled. The supply from the source **O1** remains **300 – 250 = 50**.

		Destination				
		D1	D2	D3	D4	Supply
Source	O1	$\boxed{250}$ 3	1	7	4	~~300~~ 50
	O2	2	6	5	9	400
	O3	8	3	3	2	500
Demand:		~~250~~ 0	350	400	200	1200

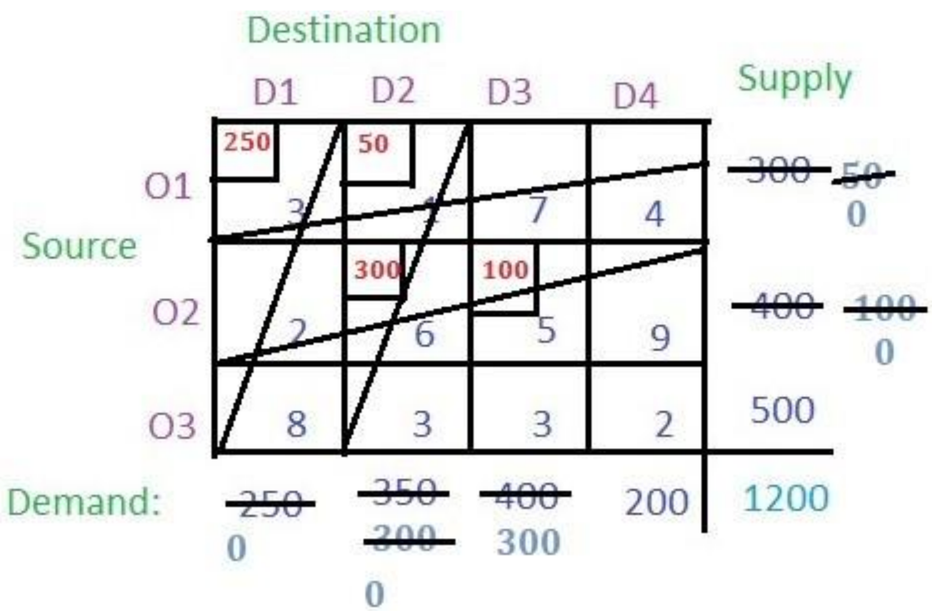
Now from the remaining table i.e. excluding column **D1**, check the north-west corner i.e. **(O1, D2)** and allocate the minimum among the supply for the respective column and the rows. The supply from **O1** is **50** which is less than the demand for **D2** (i.e. 350), so allocate **50** to the cell **(O1, D2)**. Since the supply from row **O1** is completed cancel the row **O1**. The demand for column **D2** remain **350 – 50 = 300**.



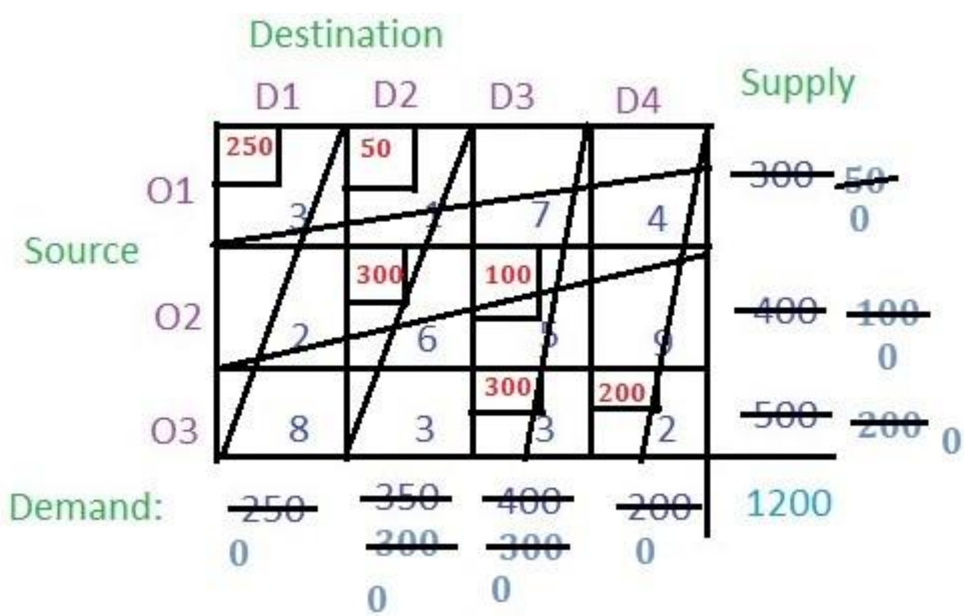
From the remaining table the north-west corner cell is **(O2, D2)**. The minimum among the supply from source **O2** (i.e 400) and demand for column **D2** (i.e 300) is **300**, so allocate **300** to the cell **(O2, D2)**. The demand for the column **D2** is completed so cancel the column and the remaining supply from source **O2** is **400 – 300 = 100**.



Now from remaining table find the north-west corner i.e. **(O2, D3)** and compare the **O2** supply (i.e. 100) and the demand for **D2** (i.e. 400) and allocate the smaller (i.e. 100) to the cell **(O2, D2)**. The supply from **O2** is completed so cancel the row **O2**. The remaining demand for column **D3** remains **400 – 100 = 300**.



Proceeding in the same way, the final values of the cells will be:

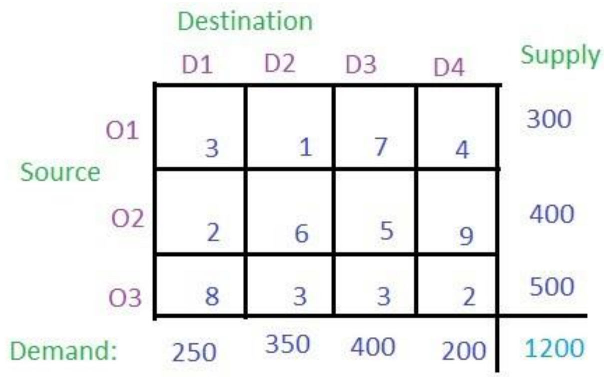


Note: In the last remaining cell the demand for the respective columns and rows are equal which was cell **(O3, D4)**. In this case, the supply from **O3** and the demand for **D4** was **200** which was allocated to this cell. At last, nothing remained for any row

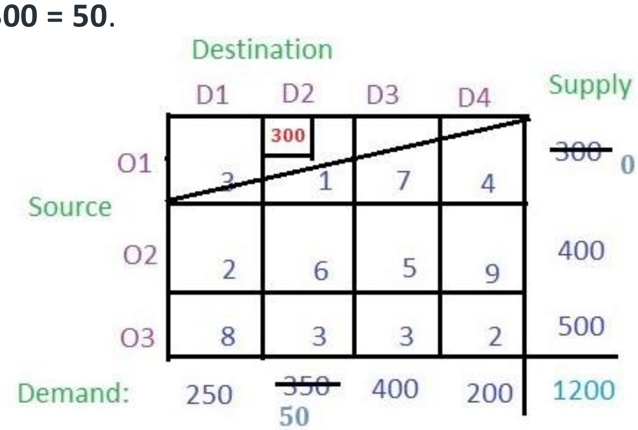
or column.
Now just multiply the allocated value with the respective cell value (i.e. the cost) and add all of them to get the basic solution i.e. **(250 * 3) + (50 * 1) + (300 * 6) + (100 * 5) + (300 * 3) + (200 * 2) = 4400**

Least Cost Cell Method

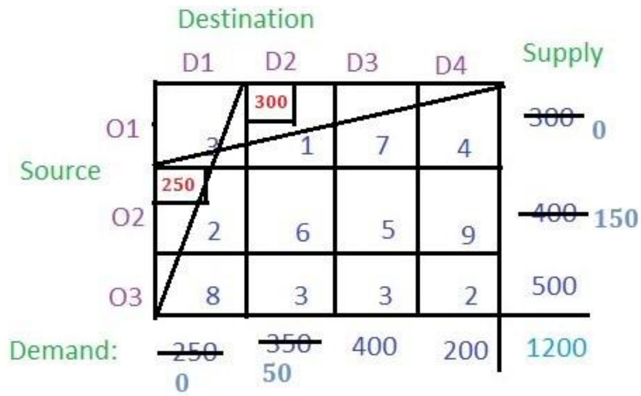
The **North-West Corner** method has been discussed in the previous article. In this article, the **Least Cost Cell** method will be discussed.



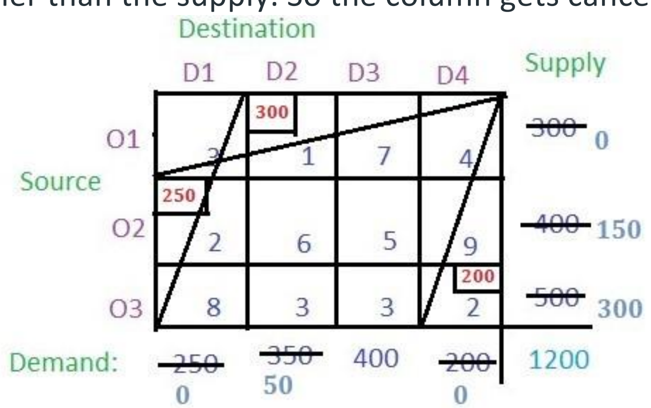
Solution: According to the Least Cost Cell method, the least cost among all the cells in the table has to be found which is **1** (i.e. cell **(O1, D2)**).
Now check the supply from the row **O1** and demand for column **D2** and allocate the smaller value to the cell. The smaller value is **300** so allocate this to the cell. The supply from **O1** is completed so cancel this row and the remaining demand for the column **D2** is **350 – 300 = 50**.



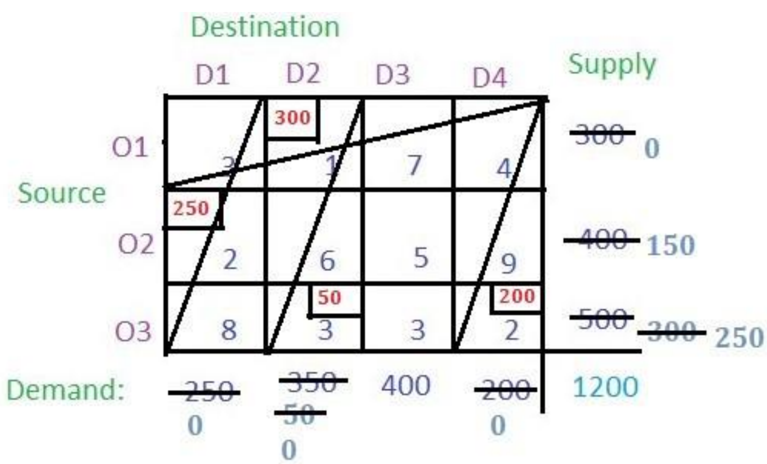
Now find the cell with the least cost among the remaining cells. There are two cells with the least cost i.e. **(O2, D1)** and **(O3, D4)** with cost **2**. Lets select **(O2, D1)**. Now find the demand and supply for the respective cell and allocate the minimum among them to the cell and cancel the row or column whose supply or demand becomes **0** after allocation.



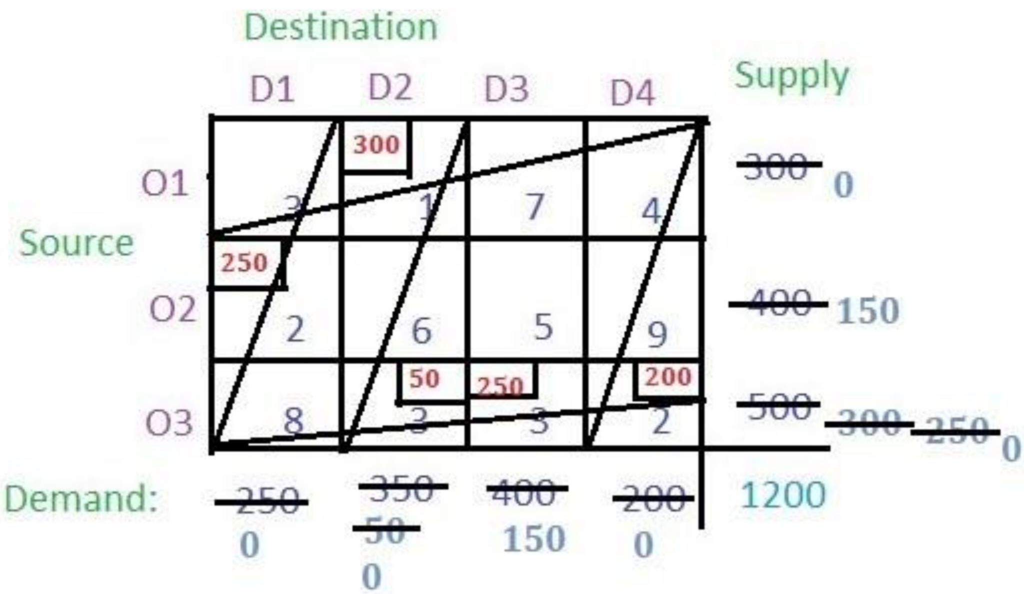
Now the cell with the least cost is **(O3, D4)** with cost **2**. Allocate this cell with **200** as the demand is smaller than the supply. So the column gets cancelled.



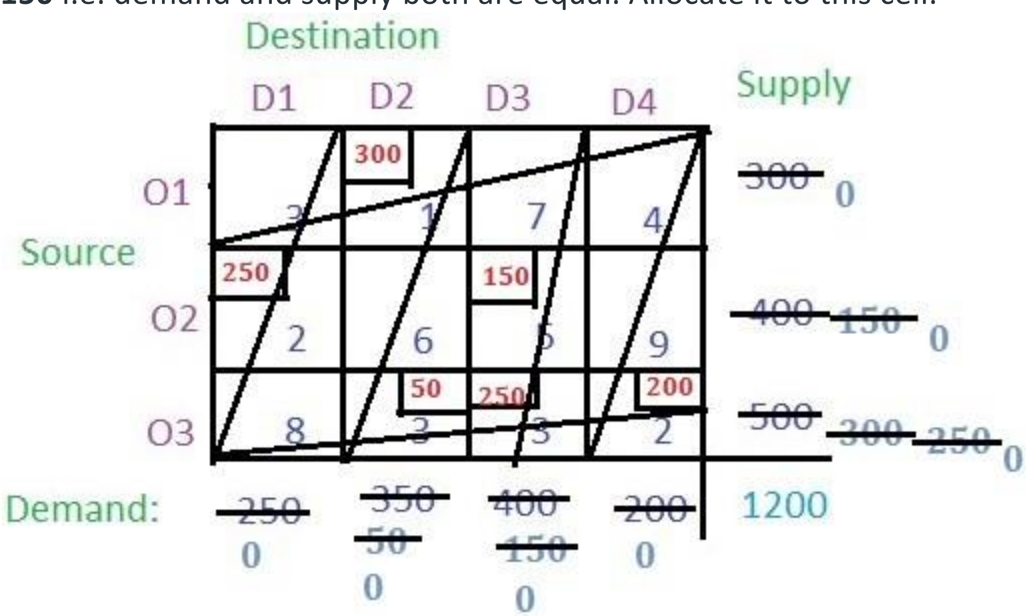
There are two cells among the unallocated cells that have the least cost. Choose any at random say **(O3, D2)**. Allocate this cell with a minimum among the supply from the respective row and the demand of the respective column. Cancel the row or column with zero value.



Now the cell with the least cost is **(O3, D3)**. Allocate the minimum of supply and demand and cancel the row or column with zero value.



The only remaining cell is **(O2, D3)** with cost **5** and its supply is **150** and demand is **150** i.e. demand and supply both are equal. Allocate it to this cell.



Now just multiply the cost of the cell with their respective allocated values and add all of them to get the basic solution i.e. **(300 * 1) + (250 * 2) + (150 * 5) + (50 * 3) + (250 * 3) + (200 * 2) = 2850**

Vogel’s Approximation Method

The **North-West Corner** method and the **Least Cost Cell** method has been discussed in the previous articles. In this article, the **Vogel’s Approximation** method will be discussed.

Source \ Destination	D1	D2	D3	D4	Supply
O1	3	1	7	4	300
O2	2	6	5	9	400
O3	8	3	3	2	500
Demand:	250	350	400	200	1200

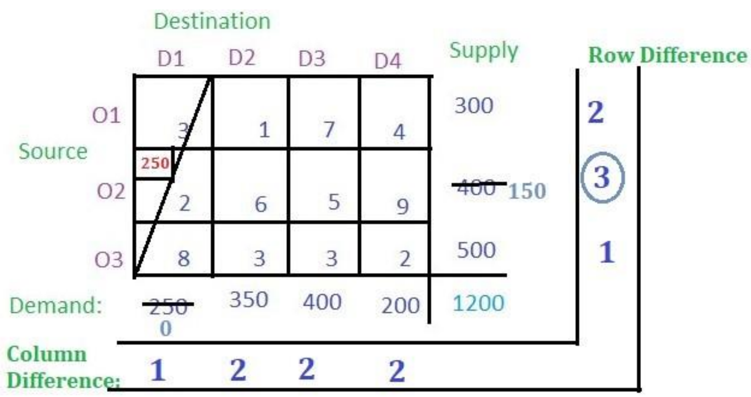
Solution:

- For each row find the least value and then the second least value and take the absolute difference of these two least values and write it in the corresponding row difference as shown in the image below. In row **O1**, **1** is the least value and **3** is the second least value and their absolute difference is **2**. Similarly, for row **O2** and **O3**, the absolute differences are **3** and **1** respectively.

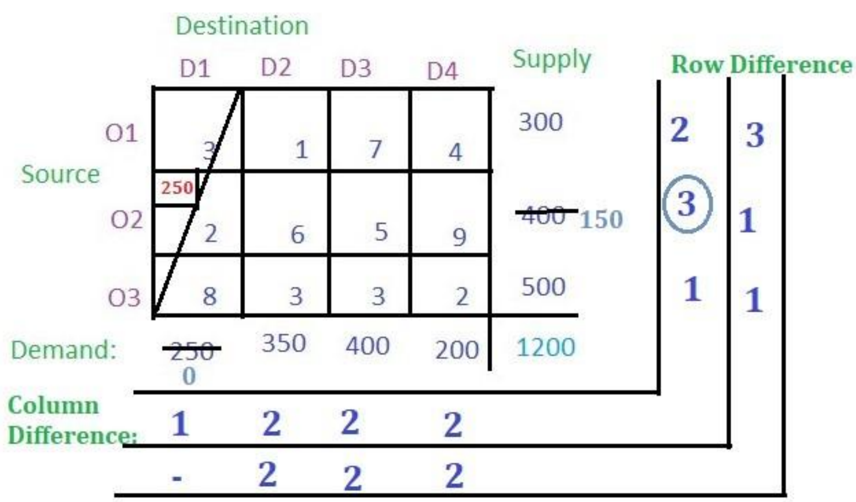
- For each column find the least value and then the second least value and take the absolute difference of these two least values then write it in the corresponding column difference as shown in the figure. In column **D1**, **2** is the least value and **3** is the second least value and their absolute difference is **1**. Similarly, for column **D2**, **D3** and **D3**, the absolute differences are **2**, **2** and **2** respectively.

Source \ Destination	D1	D2	D3	D4	Supply	Row Difference
O1	3	1	7	4	300	2
O2	2	6	5	9	400	3
O3	8	3	3	2	500	1
Demand:	250	350	400	200	1200	
Column Difference:	1	2	2	2		

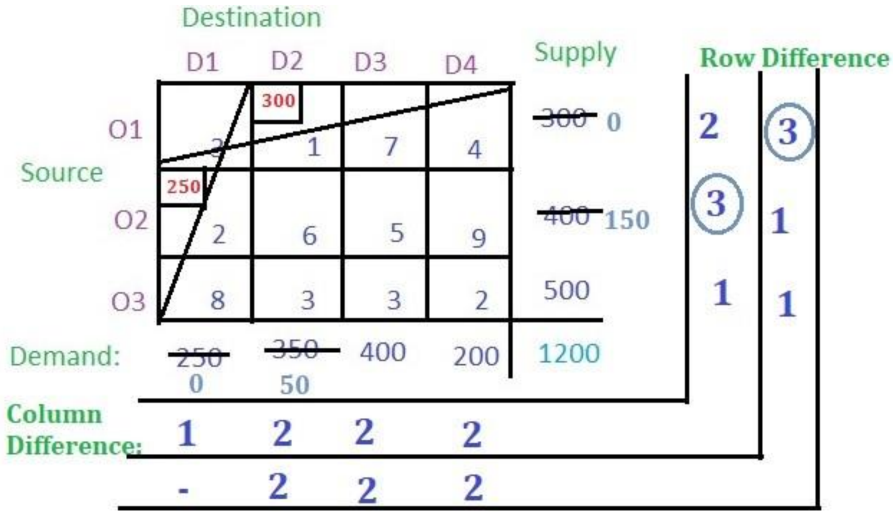
- These value of row difference and column difference are also called as penalty. Now select the maximum penalty. The maximum penalty is **3** i.e. row **O2**. Now find the cell with the least cost in row **O2** and allocate the minimum among the supply of the respective row and the demand of the respective column. Demand is smaller than the supply so allocate the column's demand i.e. **250** to the cell. Then cancel the column **D1**.



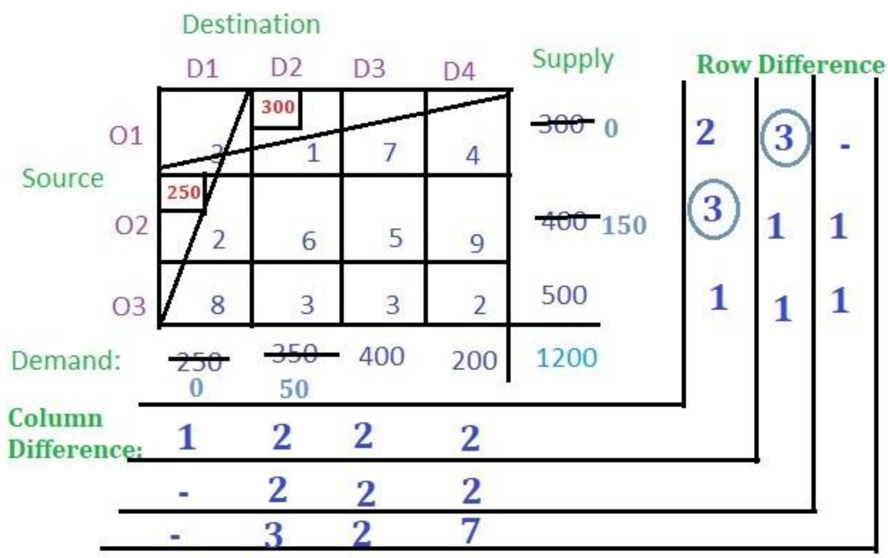
- From the remaining cells, find out the row difference and column difference.



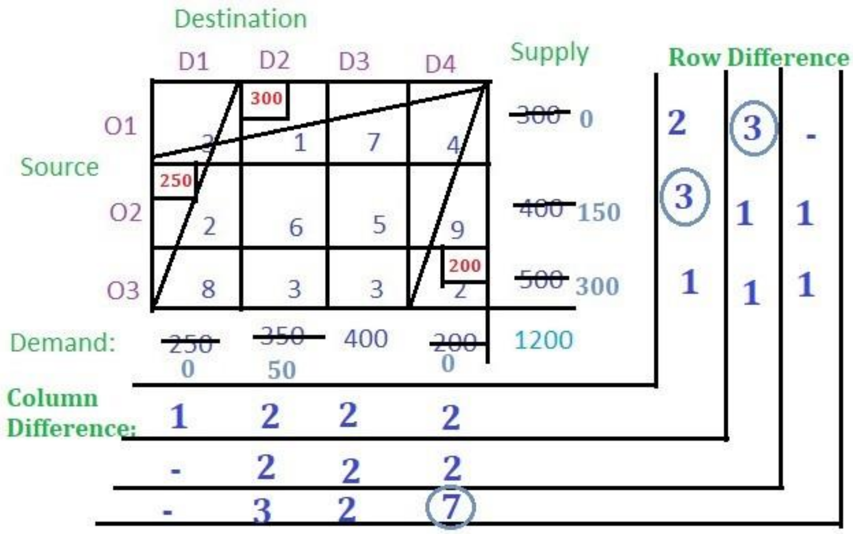
- Again select the maximum penalty which is **3** corresponding to row **O1**. The least-cost cell in row **O1** is **(O1, D2)** with cost **1**. Allocate the minimum among supply and demand from the respective row and column to the cell. Cancel the row or column with zero value.



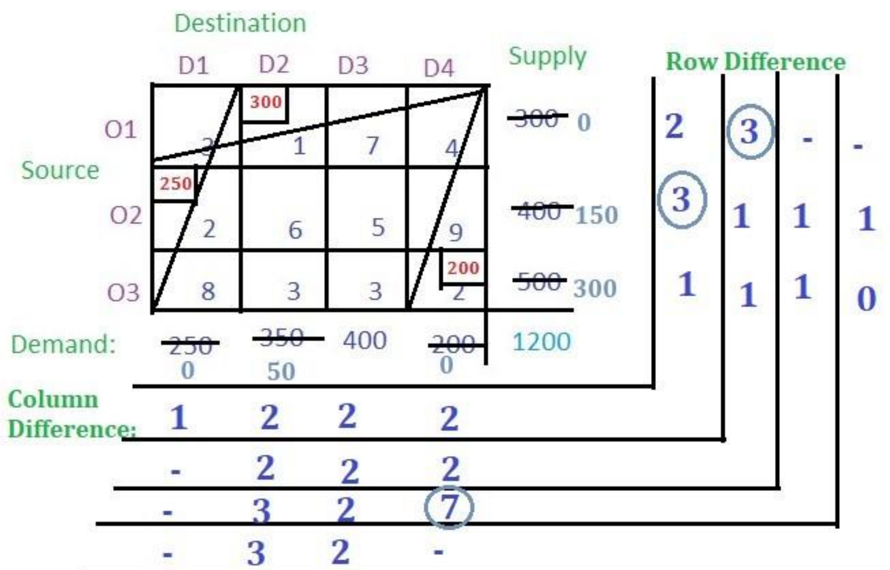
- Now find the row difference and column difference from the remaining cells.



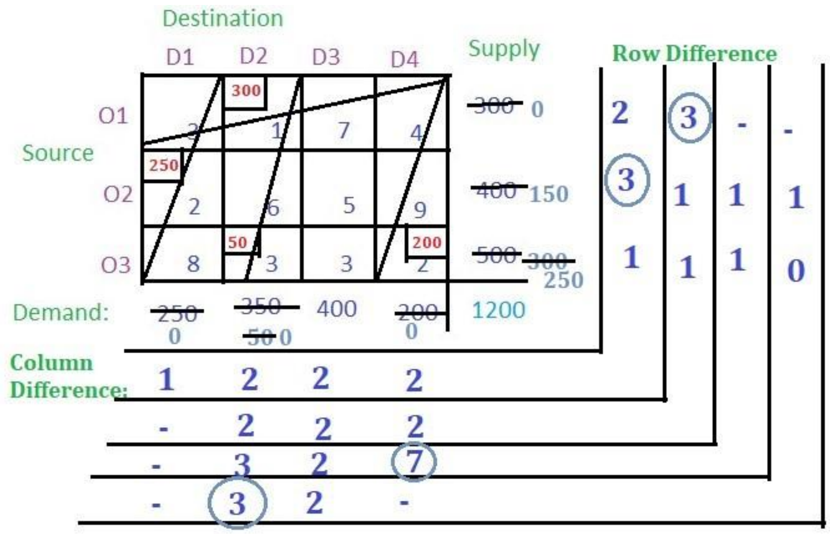
- Now select the maximum penalty which is **7** corresponding to column **D4**. The least cost cell in column **D4** is **(O3, D4)** with cost **2**. The demand is smaller than the supply for cell **(O3, D4)**. Allocate **200** to the cell and cancel the column.



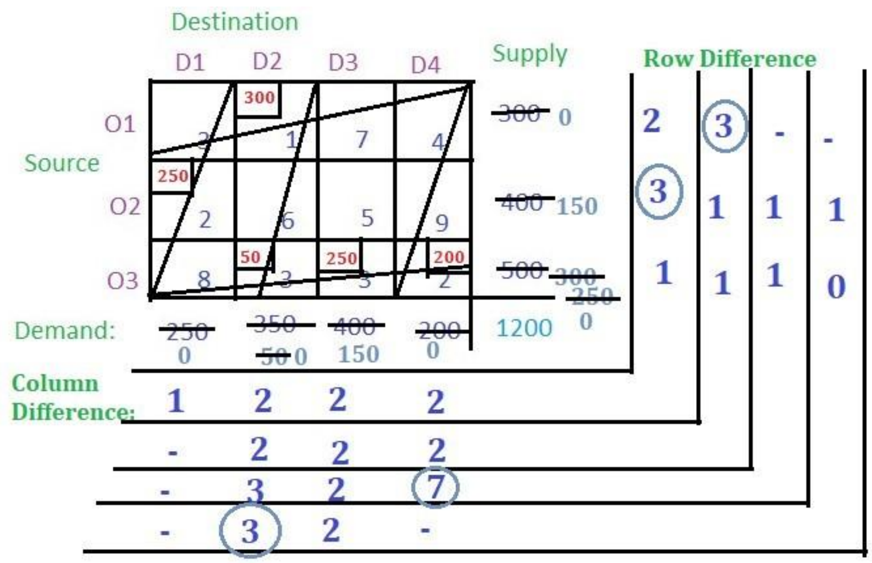
- Find the row difference and the column difference from the remaining cells.



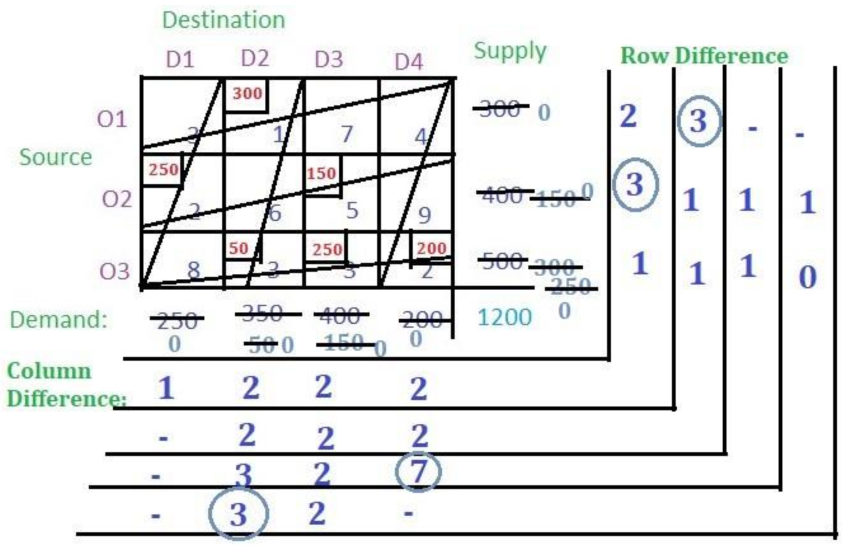
- Now the maximum penalty is **3** corresponding to the column **D2**. The cell with the least value in **D2** is **(O3, D2)**. Allocate the minimum of supply and demand and cancel the column.



- Now there is only one column so select the cell with the least cost and allocate the value.



- Now there is only one cell so allocate the remaining demand or supply to the cell



- No balance remains. So multiply the allocated value of the cells with their corresponding cell cost and add all to get the final cost i.e. **(300 * 1) + (250 * 2) + (50 * 3) + (250 * 3) + (200 * 2) + (150 * 5) = 2850**