

UNIT 5: 3D Viewing

3D Viewing-The three-dimensional transformations are extensions of two-dimensional transformation. In 2D two coordinates are used, i.e., x and y whereas in 3D three co-ordinates x , y , and z are used.

For three dimensional images and objects, three-dimensional transformations are needed. These are translations, scaling, and rotation. These are also called as basic transformations are represented using matrix. More complex transformations are handled using matrix in 3D.

The 2D can show two-dimensional objects. Like the Bar chart, pie chart, graphs. But some more natural objects can be represented using 3D. Using 3D, we can see different shapes of the object in different sections.

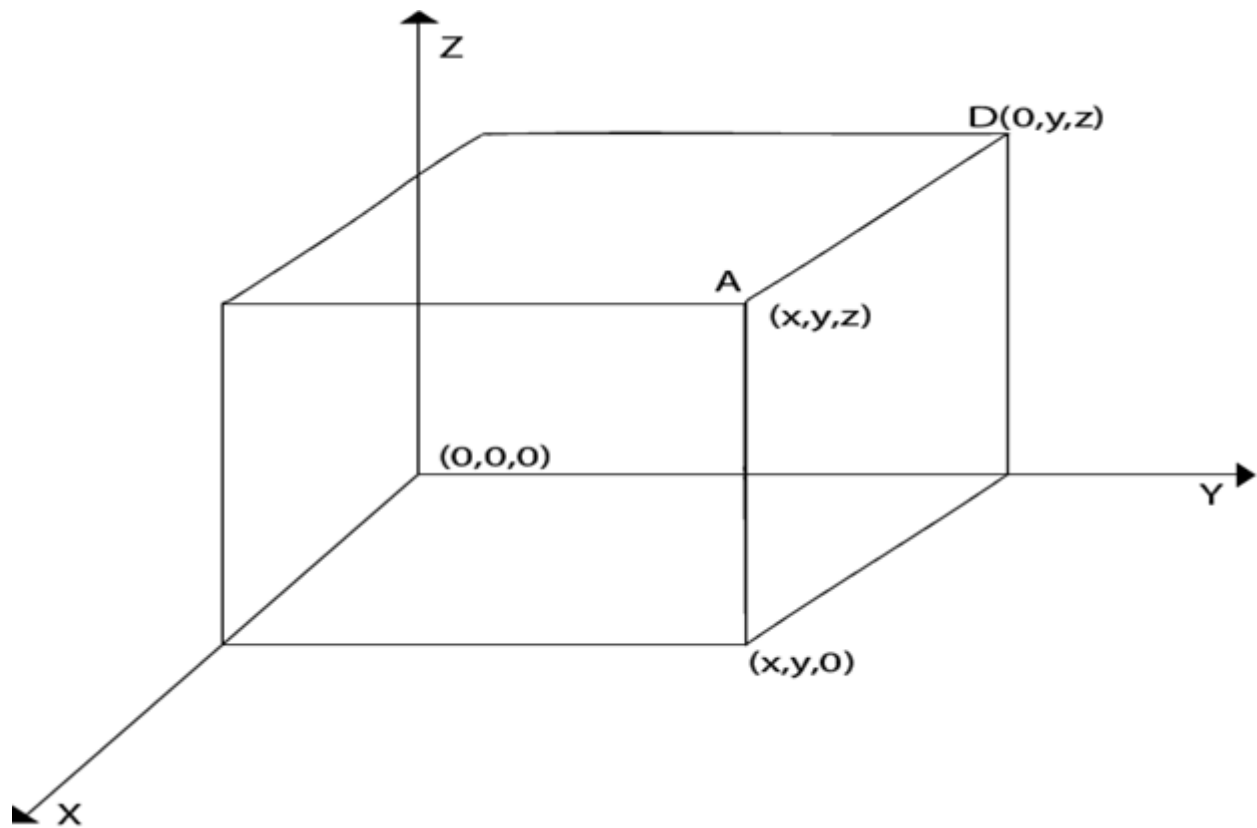
In 3D when a translation is done we need three factors for rotation also, it is a component of three rotations. Each can be performed along any three Cartesian axis. In 3D also we can represent a sequence of transformations as a single matrix.

Computer Graphics uses CAD. CAD allows manipulation of machine components which are 3 Dimensional. It also provides automobile bodies, aircraft parts study. All these activities require realism. For realism 3D is required. In the production of a realistic 3D scene from 2D is tough. It require three dimension, i.e., depth.

3D Geometry

Three dimension system has three axis x , y , z . The orientation of a 3D coordinate system is of two types. Right-handed system and left-handed system.

In the right -handed system thumb of right- hand points to positive z -direction and left-hand system thumb point to negative two directions. Following figure show right-hand orientation of the cube.



Using right-handed system co-ordinates of corners A, B, C, D of the cube

Point	A	x,	y,	z
Point	B	x,	y,	0
Point	C	0,	y,	0
Point D	0, y, z			

Producing realism in 3D: The three-dimensional objects are made using computer graphics. The technique used for two Dimensional displays of three Dimensional objects is called projection. Several types of projection are available, i.e.,

1. Parallel Projection
2. Perspective Projection
3. Orthographic Projection

1. Parallel Projection: In this projection point on the screen is identified within a point in the three-dimensional object by a line perpendicular to the display screen. The architect Drawing, i.e., plan, front view, side view, elevation are nothing but lines of parallel projections.

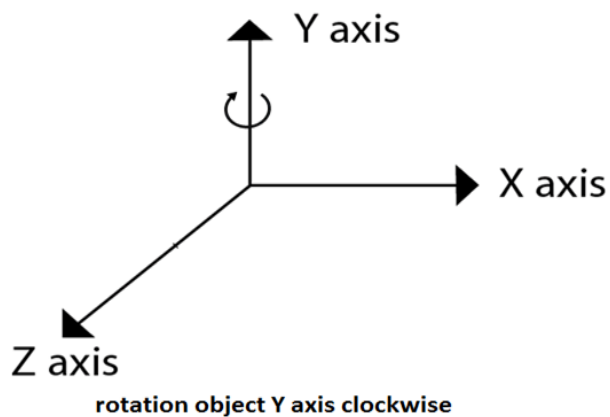
2. Perspective Projection: This projection has a property that it provides idea about depth. Farther the object from the viewer, smaller it will appear. All lines in perspective projection converge at a center point called as the center of projection.

3. Orthographic Projection: It is simplest kind of projection. In this, we take a top, bottom, side view of the object by extracting parallel lines from the object.

Specifying an Arbitrary 3D View

When the object is rotated about an axis that is not parallel to any one of co-ordinate axis, i.e., x, y, z. Then additional transformations are required. First of all, alignment is needed, and then the object is being back to the original position. Following steps are required

1. Translate the object to the origin
2. Rotate object so that axis of object coincide with any of coordinate axis.
3. Perform rotation about co-ordinate axis with whom coinciding is done.
4. Apply inverse rotation to bring rotation back to the original position.



Matrix for representing three-dimensional rotations about the Z axis

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

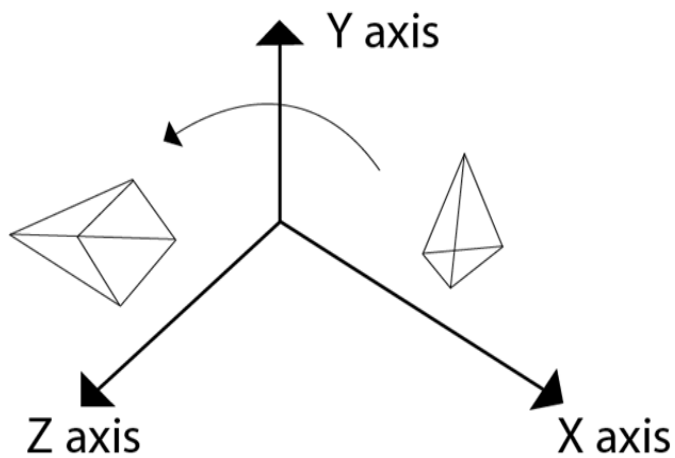
Matrix for representing three-dimensional rotations about the X axis

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Matrix for representing three-dimensional rotations about the Y axis

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

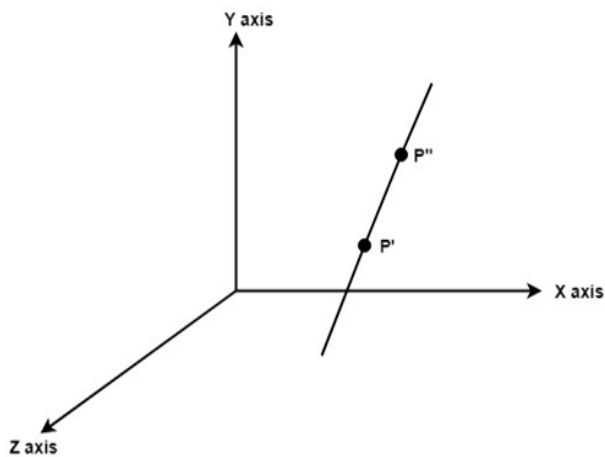
Following figure show the original position of object and position of object after rotation about the x-axis



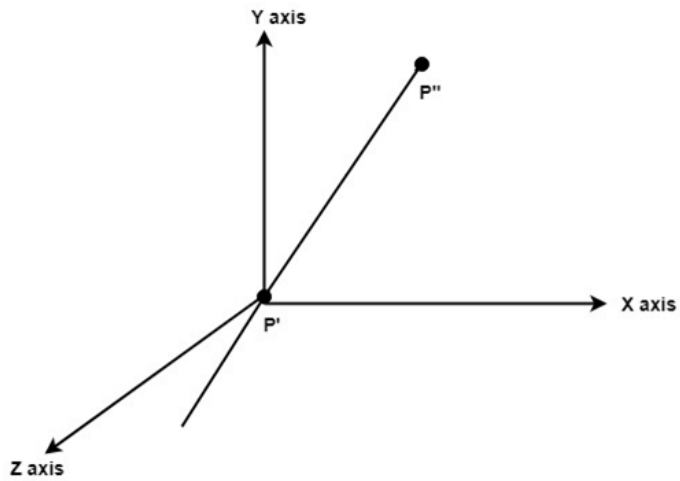
5. Apply inverse translation to bring rotation axis to the original position.

For such transformations, composite transformations are required. All the above steps are applied on points P' and P'' . Each step is explained using a separate figure.

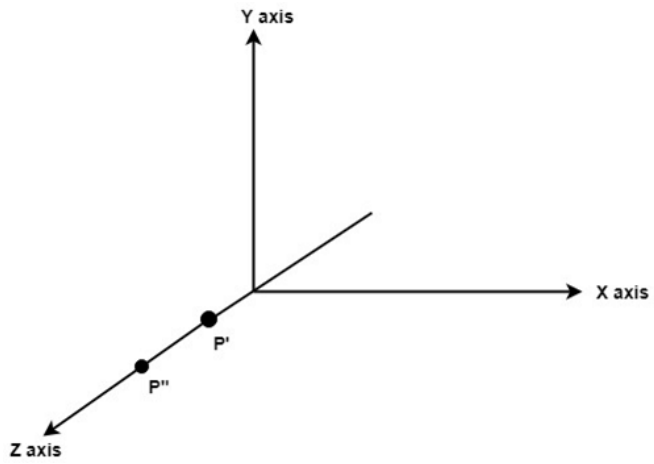
Step1: Initial position of P' and P'' is shown



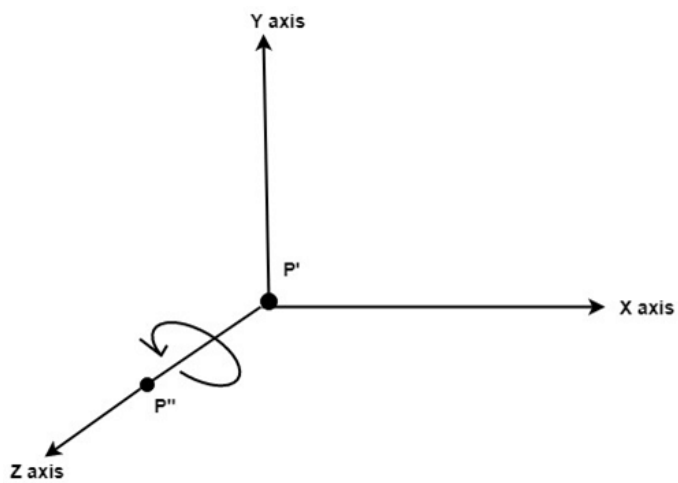
Step2: Translate object P' to origin



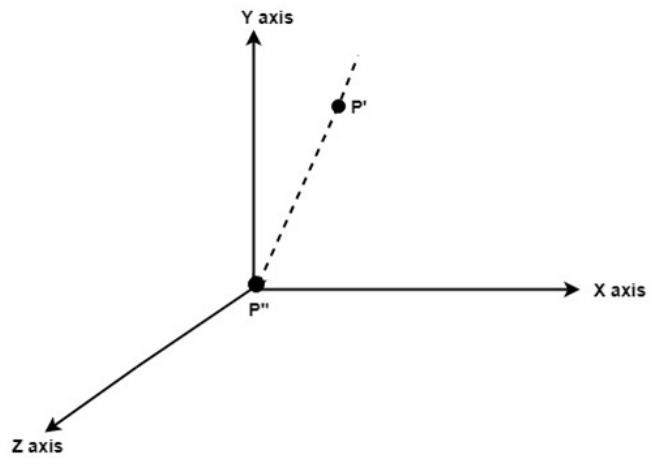
Step3: Rotate P'' to z axis so that it aligns along the z-axis



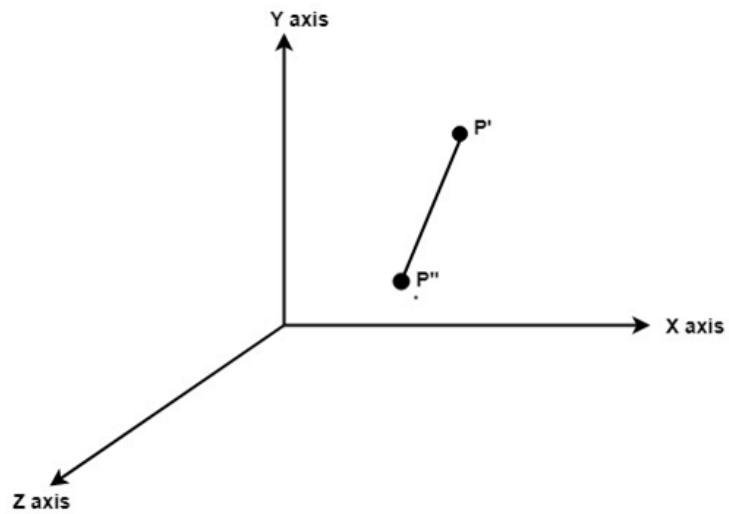
Step4: Rotate about around z- axis



Step5: Rotate axis to the original position



Step6: Translate axis to the original position.



Right handed viewing coordinate system

In computer graphics, a right-handed viewing coordinate system is used to represent the three-dimensional space in a way that is consistent with how we perceive objects in the real world. This system is widely employed in 3D graphics rendering, modeling, and animation software.

Here are the key components of the right-handed viewing coordinate system in computer graphics:

1. **X-Axis:** The X-axis points to the right. In screen space, increasing X values move objects to the right.
2. **Y-Axis:** The Y-axis points upwards. In screen space, increasing Y values move objects upwards.
3. **Z-Axis:** The Z-axis points out of the screen towards the viewer. In screen space, increasing Z values move objects closer to the viewer (towards the screen) and decreasing Z values move objects farther away.

When setting up a 3D scene in computer graphics using this coordinate system, the camera is typically positioned at the origin (0, 0, 0) and points towards the negative Z-axis. The field of view is defined by a viewing frustum that determines what objects are visible within the camera's view.

To help visualize the right-handed viewing coordinate system in computer graphics, consider the following analogy:

Imagine yourself standing in a 3D virtual space, looking at a monitor. The screen is your "viewing plane," and it is parallel to the XY plane. The camera is your viewpoint, located at the origin, looking towards the negative Z direction.

As you move the camera around or rotate it, objects in the scene respond accordingly. For example, if you move the camera to the right, objects will appear to move to the left on the screen because of the right-handed nature of the coordinate system.

This coordinate system simplifies many calculations and rendering operations, as it is consistent with our natural way of observing the world and how we interact with objects in space. It also facilitates working with cross products, lighting calculations, and camera transformations. As a result, the right-handed viewing coordinate system has become the standard in the field of computer graphics.

View reference point

In computer graphics, a view reference point (VRP) is a critical concept used in the process of rendering a three-dimensional scene onto a two-dimensional display, such as a computer screen. The VRP represents the observer's viewpoint or camera position within the 3D scene. It defines the location in world coordinates from which the scene is viewed or "observed."

When rendering a 3D scene, the graphics pipeline processes each object's vertices and transforms them into view coordinates based on the camera's position (VRP), orientation, and other camera parameters. The transformed view coordinates are then projected onto a 2D viewing plane, which creates the final 2D image that is displayed on the screen.

The view reference point, along with other camera parameters, allows the artist or developer to control what portion of the 3D scene is visible to the viewer and how the objects appear from that viewpoint. Different VRP positions or camera settings can result in entirely different perspectives of the same 3D scene.

Here are some of the essential camera parameters that are influenced by the view reference point:

1. VRP (View Reference Point): The 3D world coordinates that represent the camera's position or viewpoint.
2. VPN (View Plane Normal): The vector that represents the direction the camera is looking. It is perpendicular to the view plane.
3. VUP (View Up Vector): A vector that defines the camera's "up" direction. It helps determine the camera's orientation.
4. Field of View (FOV): The angle that represents the extent of the scene that is visible from the camera's viewpoint.
5. Aspect Ratio: The ratio of the width to the height of the viewing plane. It affects the distortion of the rendered scene.
6. Near and Far Clipping Planes: The minimum and maximum distances from the VRP at which objects are visible. Objects outside this range are clipped and not rendered.

By adjusting these camera parameters, including the VRP, developers can create various camera angles, perspectives, and views to present the 3D scene in different ways, enhancing the visual experience for the viewer.

View plane normal vector

In computer graphics, the view plane normal vector (VPN) is a fundamental concept used in the process of rendering a three-dimensional scene onto a two-dimensional display, such as a computer screen. The VPN defines the direction in which the camera is looking or the orientation of the camera's view plane.

The view plane normal vector is a 3D vector that is perpendicular to the view plane. The view plane itself is an imaginary flat surface located at a certain distance from the camera's viewpoint (VRP - View Reference Point). When rendering the 3D scene, the camera projects the scene onto this view plane.

The VPN is crucial because it determines the direction in which the camera is pointed and thus controls the perspective of the rendered scene. Objects that are visible to the camera are those that lie within the viewing frustum, which is defined by the camera's position, orientation (determined by the VPN and VUP - View Up Vector), and the field of view (FOV).

To set up the VPN, you need to define the target point or the point towards which the camera is directed. The VPN is then calculated as the normalized vector pointing from the camera's VRP to the target point. Normalizing the vector means converting it to a unit vector with a length of 1, which preserves its direction but scales it down to have a magnitude of 1.

Mathematically, if the camera's VRP is given by the coordinates (VRPx, VRPy, VRPz), and the target point is given by the coordinates (targetx, targety, targetz), then the VPN (Vx, Vy, Vz) is calculated as follows:

$$Vx = targetx - VRPx \quad Vy = targety - VRPy \quad Vz = targetz - VRPz$$

Next, the VPN is normalized to create a unit vector (Vnx, Vny, Vnz):

$$length = \sqrt{Vx^2 + Vy^2 + Vz^2} \quad Vnx = Vx / length \quad Vny = Vy / length \quad Vnz = Vz / length$$

Once the VPN is determined, it, together with the VRP and the field of view (FOV), defines the camera's orientation and what part of the 3D scene will be visible in the final rendered image.

Transformation from World Coordinate to Viewing Coordinates

Transforming from world coordinates to viewing coordinates involves a series of mathematical operations that allow you to convert 3D points in the world space (also known as world coordinates) to 3D points relative to the camera or viewer's perspective (viewing coordinates). This transformation is a crucial step in computer graphics to render 3D scenes onto a 2D viewing plane, such as a computer screen.

The transformation involves three main steps:

1. Translation: Move the world coordinates to a new coordinate system, centered at the viewpoint or camera position. This step is necessary because the camera can be placed at an arbitrary location in the world space, and we want to make it the origin of our viewing coordinate system.
2. Rotation: Align the viewing coordinate system (camera) with the world coordinate system. This step is achieved by specifying the orientation of the camera, which typically includes the view plane normal (VPN) vector and the view-up vector (VUP).
3. Projection: Project the 3D points onto the 2D viewing plane using the camera's parameters, such as field of view (FOV) and aspect ratio.

Here's a more detailed explanation of each step:

1. Translation: Let (VRP_x, VRP_y, VRP_z) be the coordinates of the camera's viewpoint (VRP) in the world coordinate system. To translate a 3D point (X_w, Y_w, Z_w) from world coordinates to the new viewing coordinate system (X_v, Y_v, Z_v) , perform the translation as follows:

$$X_v = X_w - VRP_x \quad Y_v = Y_w - VRP_y \quad Z_v = Z_w - VRP_z$$

2. Rotation: The viewing coordinate system's orientation is defined by the view plane normal (VPN) and the view-up vector (VUP). These vectors determine the camera's orientation relative to the world space. You can use these vectors to compute a

transformation matrix that aligns the camera's viewing direction with the negative Z-axis (assuming a right-handed viewing coordinate system).

Let (V_x, V_y, V_z) be the normalized view plane normal (VPN) vector, and (UP_x, UP_y, UP_z) be the normalized view-up vector (VUP). Then, the rotation matrix can be constructed as follows:

$\text{Right} = \text{cross}(V_x, UP_x, UP_y, UP_z)$ // Right is the cross product of V_x and VUP
 $Up = \text{cross}(\text{Right}, V_x)$ // Up is the cross product of Right and V_x

The rotation matrix would be:

$$\begin{bmatrix} \text{Right}.x & \text{Right}.y & \text{Right}.z & 0 \\ Up.x & Up.y & Up.z & 0 \\ -V_x.x & -V_x.y & -V_x.z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Projection: The final step is to project the 3D points in the viewing coordinate system onto the 2D viewing plane. This process is typically accomplished using perspective projection, which takes into account the field of view (FOV) and aspect ratio.

Let (X_p, Y_p) be the coordinates of the projected point on the viewing plane. The equations for perspective projection are as follows:

$$X_p = (d * X_v) / -Z_v \quad Y_p = (d * Y_v) / -Z_v$$

where 'd' is the distance from the camera to the viewing plane. The value of 'd' depends on the specific camera setup and the desired perspective effect.

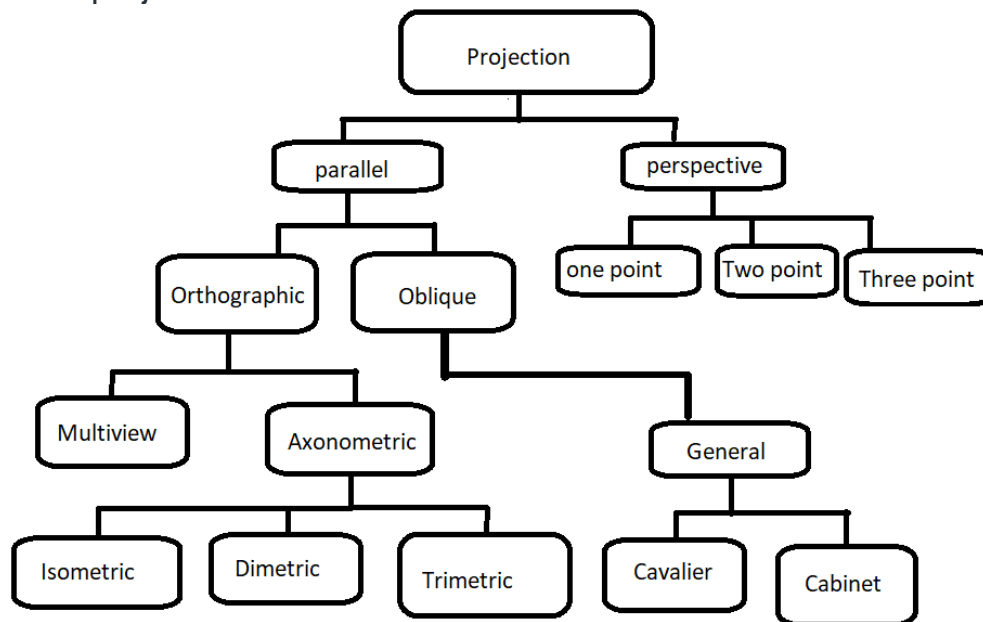
After performing these three transformation steps, you will have converted a 3D point from world coordinates to viewing coordinates, which can then be used to generate the final 2D image that represents the 3D scene from the camera's perspective.

Projections

Representing an n-dimensional object into an n-1 dimension is known as projection. It is process of converting a 3D object into 2D object, we represent a 3D object on a 2D plane $\{(x,y,z) \rightarrow (x,y)\}$. It is also defined as mapping or transforming of the object in projection plane or view plane. When geometric objects are formed by the intersection of lines with a plane, the plane is called the projection plane and the lines are called projections.

Types of Projections:

1. Parallel projections
2. Perspective projections



Center of Projection:

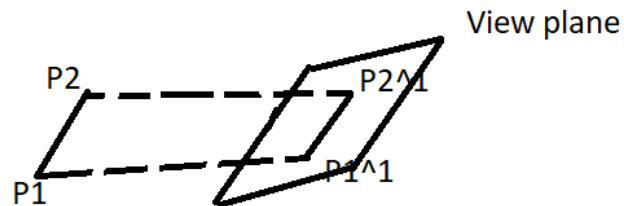
It is an arbitrary point from where the lines are drawn on each point of an object.

- If cop is located at a finite point in 3D space , Perspective projection is the result
- If the cop is located at infinity, all the lines are parallel and the result is a parallel projection.

Parallel Projection:

A parallel projection is formed by extending parallel lines from each vertex of object until they intersect plane of screen. Parallel projection transforms object to the view plane along parallel lines. A projection is said to be parallel, if center

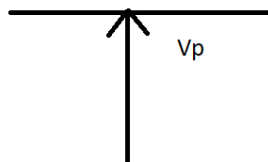
of projection is at an infinite distance from the projected plane. A parallel projection preserves relative proportion of objects, accurate views of the various sides of an object are obtained with a parallel projection. The projection lines are parallel to each other and extended from the object and intersect the view plane. It preserves relative proportions of objects, and it is used in drafting to produce scale drawings of 3D objects. This is not a realistic representation, the point of intersection is the projection of the vertex.



Parallel projection is divided into two parts and these two parts sub divided into many.

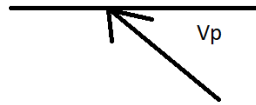
Orthographic Projections:

In orthographic projection the direction of projection is normal to the projection of the plane. In orthographic lines are parallel to each other making an angle 90 with view plane. Orthographic parallel projections are done by projecting points along parallel lines that are perpendicular to the projection line. Orthographic projections are most often used to procedure the front, side, and top views of an object are called evaluations. Engineering and architectural drawings commonly employ these orthographic projections. Transformation equations for an orthographic parallel projection as straight forward. Some special orthographic parallel projections involve plan view, side elevations. We can also perform orthographic projections that display more than one phase of an object, such views are called monometric orthographic projections.



Oblique Projections:

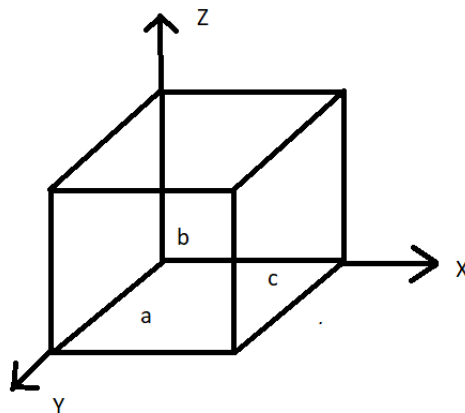
Oblique projections are obtained by projectors along parallel lines that are not perpendicular to the projection plane. An oblique projection shows the front and top surfaces that include the three dimensions of height, width and depth. The front or principal surface of an object is parallel to the plane of projection. Effective in pictorial representation.



- **Isometric Projections:** Orthographic projections that show more than one side of an object are called axonometric orthographic projections. The most common axonometric projection is an isometric projection. In this projection parallelism of lines are preserved but angles are not preserved.
- **Dimetric projections:** In these two projectors have equal angles with respect to two principal axis.
- **Trimetric projections:** The direction of projection makes unequal angle with their principal axis.

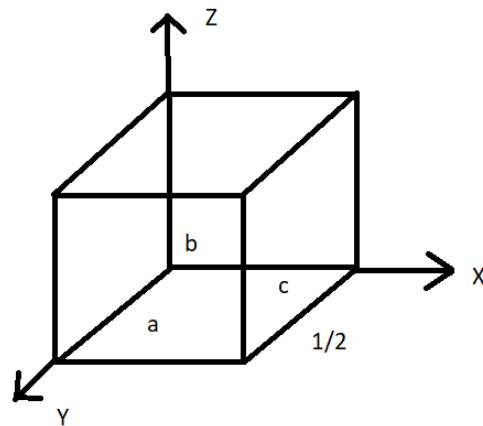
Cavalier Projections:

All lines perpendicular to the projection plane are projected with no change in length. If the projected line making an angle 45 degrees with the projected plane, as a result the line of the object length will not change.



Cabinet Projections:

All lines perpendicular to the projection plane are projected to one half of their length. This gives a realistic appearance of object. It makes 63.4 degrees angle with the projection plane. Here lines perpendicular to the viewing surface are projected at half their actual length.



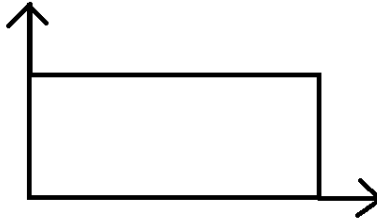
Perspective Projections:

- A perspective projection is the one produced by straight lines radiating from a common point and passing through point on the sphere to the plane of projection.
- Perspective projection is a geometric technique used to produce a three dimensional graphic image on a plane, corresponding to what person sees.
- Any set of parallel lines of object that are not parallel to the projection plane are projected into converging lines. A different set of parallel lines will have a separate vanishing point.
- Coordinate positions are transferred to the view plane along lines that converge to a point called projection reference point.
- The distance and angles are not preserved and parallel lines do not remain parallel. Instead, they all converge at a single point called center of projection there are 3 types of perspective projections.

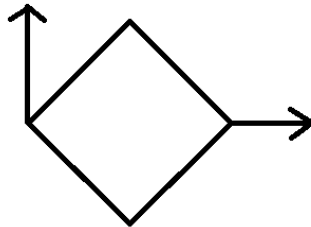
Two characteristics of perspective are vanishing point and perspective force shortening. Due to force shortening objects and lengths appear smaller from the center of projections. The projections are not parallel and we specify a center of projection cop.

Different types of perspective projections:

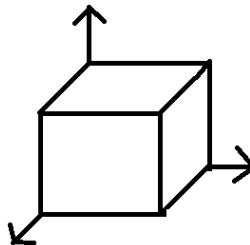
- **One point perspective projections:** In this, principal axis has a finite vanishing point. Perspective projection is simple to draw.



- **Two point perspective projections:** Exactly 2 principals have vanishing points. Perspective projection gives better impression of depth.



- **Three point perspective projections:** All the three principal axes have finite vanishing point. Perspective projection is most difficult to draw.



Perspective fore shortening:

The size of the perspective projection of the object varies inversely with distance of the object from the center of projection.