UNIT 1:

Set theory, Relations and Functions

Set - Definition

A set is an unordered collection of different elements. A set can be written explicitly by listing its elements using set bracket. If the order of the elements is changed or any element of a set is repeated, it does not make any changes in the set.

Some Example of Sets

- A set of all positive integers
- A set of all the planets in the solar system
- A set of all the states in India
- A set of all the lowercase letters of the alphabet

Representation of a Set

Sets can be represented in two ways —

- Roster or Tabular Form
- Set Builder Notation

Roster or Tabular Form

The set is represented by listing all the elements comprising it. The elements are enclosed within braces and separated by commas.

Example 1 — Set of vowels in English alphabet, A$= \{a, e, i, o, u\}$

Example 2 — Set of odd numbers less than 10, B$= \{1, 3, 5, 7, 9\}$

Set Builder Notation

The set is defined by specifying a property that elements of the set have in common. The set is described as A = {x : p(œ)}

Example 1 - The set {a, e, i, o, u} is written as —

A — {c : x is a vowel in English alphabet}

Example 2 — The set {1, 3, 5, 7, 9} is written as —

B {c : 1 < x < 10 and (c%2) 0}

If an element x is a member of any set S, it is denoted by c e s and if an element y is not a member of set S, it is denoted by y

Example — If S = {1, 1.2, 1.7, 2}, l e s but 1.5$\notin S$

Some Important Sets

N — the set of all natural numbers —$1, 2, 3, 4, \ldots\ldots\}$

Z — the set of all integers = $\{\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$

Z+ — the set of all positive integers Q — the set of all rational numbers R — the set of all real numbers

W — the set of all whole numbers

Cardinality of a Set

Cardinality of a set S, denoted by ISI , is the number of elements of the set. The number is also

referred as the cardinal number. If a set has an infinite number of elements, its cardinality is oo .

Example —$1, 4, 3, 5\}| = 4, |\{1, 2, 3, 4, 5, \dots\}| = \infty$

If there are two sets X and Y,

- IXI = I YI denotes two sets X and Y having same cardinality. It occurs when the number of elements in X is exactly equal to the number of elements in Y. In this case, there exists a bijective function 'f' from X to Y.

- IXI < I YI denotes that set X's cardinality is less than or equal to set Y's cardinality. It occurs when number of elements in X is less than or equal to that of Y. Here, there exists an injective function 'f' from X to Y.

- IXI < I YI denotes that set X's cardinality is less than set Y 's cardinality. It occurs when number of elements in X is less than that of Y. Here, the function 'f' from X to Y is injective function but not bijective.

- If IXI < I YI and IXI > I YI then XI YI . The sets X and Y are commonly referred as equivalent sets.

Types of Sets

Sets can be classified into many types. Some of which are finite, infinite, subset, universal, proper, singleton set, etc.

Finite Set

A set which contains a definite number of elements is called a finite set.

Example — S = {x I c e N and 70 > x > 50}

Infinite Set

A set which contains infinite number of elements is called an infinite set.

Example - S$= \{x \mid x \in N$and > 10}

Subset

A set X is a subset of set Y (Written as X C Y) if every element of X is an element of set Y.

Example 1 — Let, X = {1, 2, 3, 4, 5, 6} and Y = {1, 2} . Here set Y is a subset of set X as

all the elements of set Y is in set X. Hence, we can write Y C X .

Example 2 — Let, X = {1, 2, 3} and Y = {1, 2, 3} . Here set Y is a subset (Not a proper

subset) of set X as all the elements of set Y is in set X. Hence, we can write Y C X .

Proper Subset

The term "proper subset" can be defined as "subset of but not equal to". A Set X is a proper subset of set Y (Written as X C Y) if every element of X is an element of set Y and IXI < I YI .

Example Let, X = {1, 2, 3, 4, 5, 6} and Y {1, 2} . Here set Y c X since all elements in Y are contained in X too and X has at least one element is more than set

Universal Set

It is a collection of all elements in a particular context or application. All the sets in that context or application are essentially subsets of this universal set. Universal sets are represented as U .

Example — We may define U as the set of all animals on earth. In this case, set of all mammals is a subset of U set of all fishes is a subset of U , set of all insects is a subset of U , and so on.

Empty Set or Null Set

An empty set contains no elements. It is denoted by Ø . As the number of elements in an empty set is finite, empty set is a finite set. The cardinality of empty set or null set is zero.

Example — S = {c I c e N and 7 < 8}= ∅

Singleton Set or Unit Set

Singleton set or unit set contains only one element. A singleton set is denoted by s} .

Example — S = {c I c e N, 7 < < 9} {8}

Equal Set

If two sets contain the same elements they are said to be equal.

> Example — If A = {1, 2, 6} and B = {6, 1, 2} , they are equal as every element of set A is an element of set B and every element of set B is an element of set A.

Equivalent Set

If the cardinalities of two sets are same, they are called equivalent sets.

> Example — If A = {1, 2, 6} and B — {16, 17, 22} , they are equivalent as cardinality of A is equal to the cardinality of B. i.e. 1B| $= 3$

Overlapping Set

Two sets that have at least one common element are called overlapping sets.

In case of overlapping sets —

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$$

$$n(A) = n(A - B) + n(A \cap B)$$

$$n(B) = n(B - A) + n(A \cap B)$$

Example — Let, A $= \{1, 2, 6\}$ and B {6, 12, 42} There is a common element '6', hence these sets are overlapping sets.

Disjoint Set

Two sets A and B are called disjoint sets if they do not have even one element in common.

Therefore, disjoint sets have the following properties —

$$n(A \cap B) = \emptyset$$

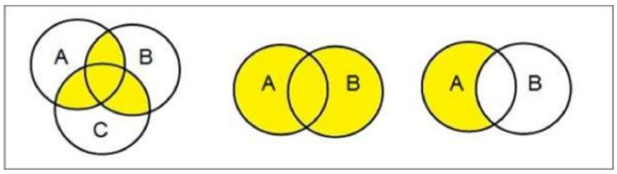
$$n(A \cup B) = n(A) + n(B)$$

Example - Let, $A = \{1, 2, 6\}$ and B {7, 9, 14} there is not a single common element, hence these sets are overlapping sets.

Venn Diagrams

Venn diagram, invented in 1880 by John Venn, is a schematic diagram that shows all possible logical relations between different mathematical sets.

Examples



Set Operations

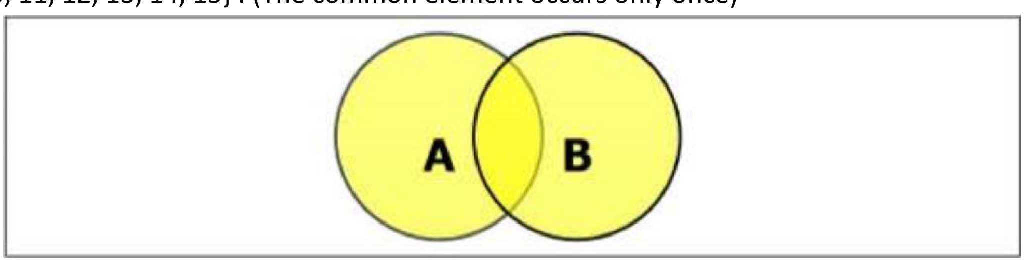
Set Operations include Set Union, Set Intersection, Set Difference, Complement of Set, and Cartesian Product.

Set Union

The union of sets A and B (denoted by A UB) is the set of elements which are in A, in B, or in both A and B. Hence, A UB = {x I e A OR x e B}.

Example If A = {10, 11, 12, 13} and {13, 14, 15} then

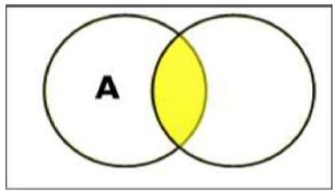
AUB {10, 11, 12, 13, 14, 15} . (The common element occurs only once)



SetIntersection

The intersection of sets A and B (denoted by A n B) is the set of elements which are in both A and B. Hence, An B = {x $x \in A\ AND\ x$ e B}.

Example - if A = {11, 12, 13} and B = {13, 14, 15} then An E$= \{13\}$

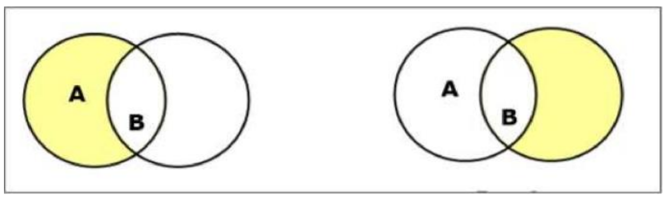


Set Difference/ Relative Complement

The set difference of sets A and B (denoted by A—B) is the set of elements which are only in A but not in B. Hence, A — B = {c I e A AND B} .

Example - If A = {10, 11, 12, 13} and B = {13, 14, 15} then (A B) {10, 11, 12}

and (B A) = {14, 15} . Here, we can see (A — B) (B$- A)$



Complement of a Set

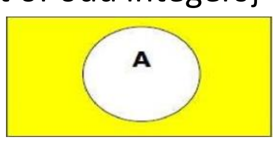
The complement of a set A (denoted by A') is the set of elements which are not in set A. Hence,

A/ = {cla A} .

More specifically, A'$= (U -$A) where U is a universal set which contains all objects.

Example A = {x I belongs to set of odd integers} then

A' = {y y does not belong to set of odd integers}



Cartesian Product / Cross Product

The Cartesian product of A and B is written as —$\times B = \{(a,1),(a,2),(b,1),(b,2)\}$The

Cartesian product of B and A is written as —$B \times A = \{(1,a),(1,b),(2,a),(2,b)\}$

Definition of Minsets

Let Bl and B2 be subsets of a set A. Notice that the Venn diagram of

Figure 4.3.1 is naturally partitioned into the subsets Al, A2, A3, and A4. Further we observe that Al, A2, A3, and A4 can be described in terms of Bl and B2 as follows:

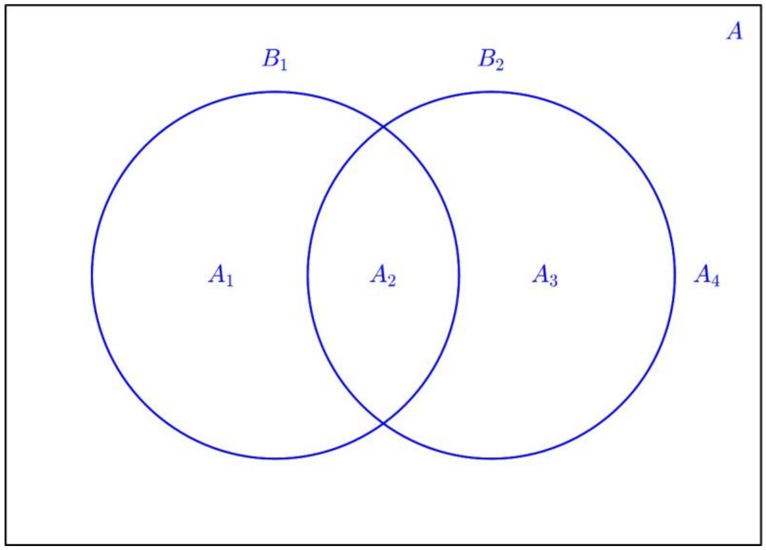


Figure 4.3.1. Venn Diagram of Minsets

Table 4.3.2. Minsets generated by two sets

$$
\begin{aligned}
A_1 &= B_1 \cap B_2^c \\
A_2 &= B_1 \cap B_2 \\
A_3 &= B_1^c \cap B_2 \\
A_4 &= B_1^c \cap B_2^c
\end{aligned}
$$

Each Ai is called a minset generated by Bl and B2. We note that each minset is formed by taking the intersection of two sets where each may be either Bk or its complement, B^Ck. Note also, given two sets, there are 22 4 minsets.

Minsets are occasionally called minterms.

The reader should note that if we apply all possible combinations of the operations intersection, union, and complementation to the sets Bl and B2 of Figure 1 , the smallest sets generated will be exactly the minsets, the minimum sets. Hence the derivation of the term minset.

Next, consider the Venn diagram containing three sets, Bl, B2, and B3. Draw it right now and count the regions! What are the minsets generated by Bl, B2, and B3? How many are there? Following the procedures outlined above, we note that the following are three of the 2 ^{3} 8 minsets. What are the others?

Table 4.3.3. Three of the minsets generated by Bl, B2, and B3

$$B_1 \cap B_2 \cap B_3^c$$
$$B_1 \cap B_2^c \cap B_3$$
$$B_1 \cap B_2^c \cap B_3^c$$

Definition 4.3.4. Minset. Let {Bl, B2, . . . , Bn} be a set of subsets of set A. Sets of the form DI n D2 n • • • n Dn, where each Di may be either Bi or Bf, is called a minset generated by Bl, B2,... and Bn.

[Example 4.3.5. A concrete example of some minsets..

4.3.2 Properties of Minsets

Theorem 4.3.8. Minset Partition Theorem. Let A be a set and let Bl, B2 B be subsets of A. The set of nonempty minsets generated by Bl, B2 • • • ,Bn is a partition of A. Proof.

One of the most significant facts about minsets is that any subset of A that can be obtained from Bl, B2 . . . , Bn, using the standard set operations can be obtained in a standard form by taking the union of selected minsets.

Definition 4.3.9. Minset Normal Form. A set is said to be in minset normal form when it is expressed as the union of zero or more distinct nonempty minsets.

Notes:

- The union of zero sets is the empty set, Ø.
- Minset normal form is also called canonical form.

Partitioning of a Set

Partition of a set, say S, is a collection of n disjoint subsets, say $P_1, P_2, \cdots$ that satisfies the following three conditions —

- Pi does not contain the empty set.

[Pi {Ø} for all 0 < i < n]

- The union of the subsets must equal the entire original set.

$$[P_1 \cup P_2 \cup \cdots \cup P_n = S]$$

- The intersection of any two distinct sets is empty.

[Pa n Pb = {Ø}, for a b where n > a, b > 0]

Example

Let S = {a, b, c, d, e, f, g, h}

One probable partitioning is {a}, {b, c, d}, {e, f, g, h}

Another probable partitioning is {a, b}, {c, d}, {e, f, g, h}

Set Operations and the Laws of Set Theory

- The *union* of sets A and B is the set $A \cup B = \{x : x \in A \vee x \in B\}$.
- The *intersection* of sets A and B is the set $A \cap B = \{x : x \in A \wedge x \in B\}$.
- The *set difference of A and B* is the set $A \setminus B = \{x : x \in A \wedge x \in 6\ B\}$. Alternate notation: $A - B$.
- The *symmetric difference of A and B* is $A \oplus B = (A \setminus B) \cup (B \setminus A)$. Note: $A \oplus B = \{x : (x \in A \wedge x\ 6{\in}\ B) \vee (x \in B \wedge x\ 6{\in}\ A)\}$.

The *universe*, U, is the collection of all objects that can occur as elements of the sets under consideration.

- The *complement of A* is $A^c = U \setminus A = \{x : x\ 6{\in}\ A\}$.

For each Law of Logic, there is a corresponding Law of Set Theory.

- Commutative: $A \cup B = B \cup A$, $A \cap B = B \cap A$.
- Associative: $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$
- Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and also on the right: $(B \cap C) \cup A = (B \cup A) \cap (C \cup A)$, $(B \cup C) \cap A = (B \cap A) \cup (C \cap A)$
- Double Complement: $(A^c)^c = A$
- DeMorgan's Laws: $(A \cup B)^c = A^c \cap B^c, \quad (A \cap B)^c = A^c \cup B^c$

- Identity: $\emptyset \cup A = A$, $U \cap A = A$
- Idempotence: $A \cup A = A$, $A \cap A = A$
- Dominance: $A \cup U = U$, $A \cap \emptyset = \emptyset$

Arguments that prove logical equivalences can be directly translated into arguments that prove set equalities.

Set equalities of note:

- $A \setminus B = A \cap B^c$
- $A \oplus B = (A \cup B) \setminus (A \cap B)$

Principle of Duality in Discrete Mathematics

The principle of duality is a type of pervasive property of algebraic structure in which two concepts are interchangeable only if all results held in one formulation also hold in another. This concept is known as dual formulation. We will interchange unions(∪) into intersections(∩) or intersections() into the union() and also interchange universal set into the null set(∅) or null set into universal(U) to get the dual statement. If we interchange the symbol and get this statement itself, it will be known as the self-dual statement.

For example:

The dual of (X ∩ Y) ∪ Z is (X ∪ Y) ∩ Z

Duality can also be described as a property that belongs to the branch of algebra. This theory can be called lattice theory. This theory has the ability to involve order and structure, which are common to different mathematical systems. If the mathematical system has the order in a specified way, this structure will be known as lattice.

The principle of duality concept should not be avoided or underestimated. It has the ability to provide several sets of theorems, concepts, and identities. To explain the duality principle of sets, we will assume S be any identity that involves sets, and operation complement, union, intersection. Suppose we obtain the S* from S with the help of substituting ∪ → ∩ Φ. In this case, the statement S* will also be true, and S* can also be known as dual statement S.

Examples of Duality:

Examples 1:

$A \cup (B \cap A) = A$

When we perform duality, then the union will be replaced by intersection, or the intersection will be replaced by the union.

$A \cap (B \cup A) = A$

Example 2:

$A \cup ((B^C \cup A) \cap B)^C = U$

When we perform duality, then the union will be replaced by intersection, or intersection will be replaced by the union. The universal will also be replaced by null, or null will be replaced by universal.

$A \cap ((B^C \cap A) \cup B)^C = \Phi$

Example 3:

$(A \cup B^C)^C \cap B = A^C \cap B$

When we perform duality, then the union will be replaced by intersection, or intersection will be replaced by the union.

$(A \cap B^C)^C \cup B = A^C \cup B$

Various systems have underlying lattice structures: symbolic structure, set theory, and projective geometry. These systems also contain the principles of duality.

Relations and Functions

Relations and Functions in real life give us the link between any two entities. In our daily life, we come across many patterns and links that characterize relations such as a relation between a father and a son, brother and sister, etc. In mathematics also, we come across many relations between numbers such as a number x is less than y, line l is parallel to line m, etc. Relation and function map elements of one set (domain) to the elements of another set (codomain).

Functions are nothing but special types of relations that define the precise correspondence between one quantity with the other. In this article, we will study how to link pairs of elements from two sets and then define a relation between them, different types of relations and functions, and the difference between relation and function.

What are Relations and Functions?

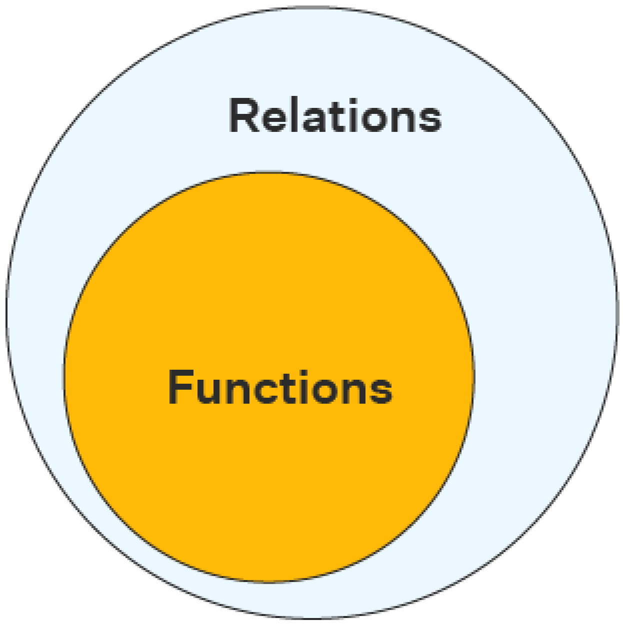
Relations and functions define a mapping between two sets (Inputs and Outputs) such that they have ordered pairs of the form (Input, Output). Relation and function are very important concepts in algebra and calculus. They are used widely in mathematics as well as in real life. Let us define each of these terms of relation and function to understand their meaning.

Relation and Function Definition

Relation and function individually are defined as:

- **Relations -** A relation R from a non-empty A to a non-empty set B is a subset of the cartesian product A × B. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in A × B.
- **Functions -** A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B. In other words, no two distinct elements of B have the same pre-image.

Relations and Functions



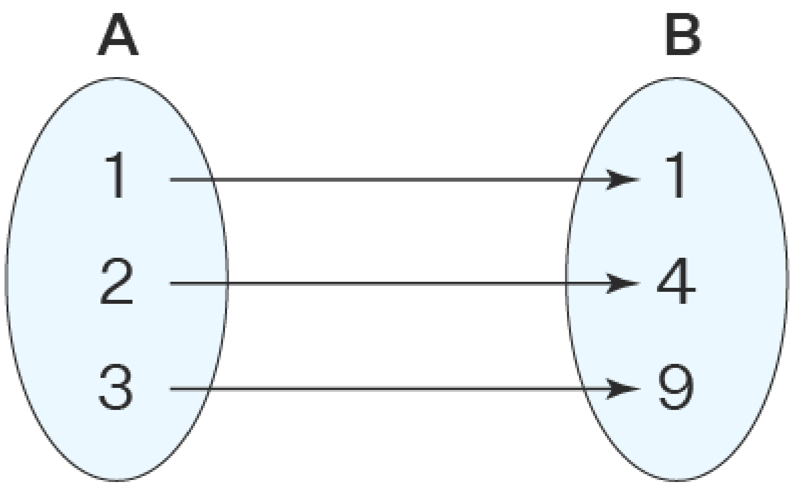
Note: Please note that all functions are relations but all relations are not functions.

Representation of Relation and Function

Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graphical form, roster form, and tabular form. Define a function f: A = {1, 2, 3} → B = {1, 4, 9} such that f(1) = 1, f(2) = 4, f(3) = 9. Now, let us represent this function in different forms.

- Set-builder form - $\{(x, y): y = x^2, x \in A, y \in B\}$
- Roster form - {(1, 1), (2, 4), (3, 9)}

- Arrow Representation -



- **Table Representation -**

x	y
1	1
2	4
3	9

Difference Between Relation and Function

The basic difference between a relation and a function is that in a relation, a single input may have multiple outputs. Whereas in a function, each input has a single output. The table given below highlights the differences between relations and functions.

Relation	Function
A relation in math is a set of ordered pairs defining the relation between two sets.	A function is a relation in math such that each element of the domain is related to a single element in the codomain.
A relation may or may not be a function.	All functions are relations.
Example: {(1, x), (1, y), (4, z)}	Example: {(1, x), (6, y), (4, z)}

Note: Look at the example of the relation above: {(1, x), (1, y), (4, z)}. This is NOT a function as a single element (1) is related to multiple elements (x and y). Hence the statement "every relation is a function" is incorrect.

Terms Related to Relations and Functions

Now that we have understood the meaning of relation and function, let us understand the meanings of a few terms related to relations and functions that will help to understand the concept in a better way:

- **Cartesian Product** - Given two non-empty sets P and Q, the cartesian product P × Q is the set of all ordered pairs of elements from P and Q, that is, $P \times Q = \{(p, q) : p \in P, q \in Q\}$
- **Domain** - The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R. It is called the set of inputs or pre-images.
- **Range** - The set of all second elements of the ordered pairs in a relation R from a set A to a set B is called the range of the relation R. It is called the set of outputs or images.
- **Codomain** - The whole set B in a relation R from a set A to a set B is called the codomain of the relation R. Note that range is a subset of codomain. i.e., Range ⊆ Codomain

Types of Relations and Functions

Different types of relations and functions have specific properties which make them different and unique. Let us go through the list of types of relations and functions given below:

Types of Relations

Given below is a list of different types of relations:

- Empty Relation - A relation is an empty relation if it has no elements, that is, no element of set A is mapped or linked to any element of A. It is denoted by R = ∅.
- Universal Relation - A relation R in a set A is a universal relation if each element of A is related to every element of A, i.e., R = A × A. It is called the full relation.
- Identity Relation - A relation R on A is said to be an identity relation if each element of A is related to itself, that is, R = {(a, a) : for all a ∈ A}
- Inverse Relation - Define R to be a relation from set P to set Q i.e., R ∈ P × Q. The relation R^{-1} is said to be an Inverse relation if R^{-1} from set Q to P is denoted by R^{-1} = {(q, p): (p, q) ∈ R}.
- Reflexive Relation - A binary relation R defined on a set A is said to be reflexive if, for every element a ∈ A, we have aRa, that is, (a, a) ∈ R.
- Symmetric Relation - A binary relation R defined on a set A is said to be symmetric if and only if, for elements a, b ∈ A, we have aRb, that is, (a, b) ∈ R, then we must have bRa, that is, (b, a) ∈ R.
- Transitive Relation - A relation R is transitive if and only if (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R for a, b, c ∈ A
- Equivalence Relation - A relation R defined on a set A is said to be an equivalence relation if and only if it is reflexive, symmetric and transitive.
- Antisymmetric Relation - A relation R on a set A is said to be antisymmetric if (a, b) ∈ R and (b , a) ∈ R ⇒ a = b.

Types of Functions

Given below is a list of different types of functions:

- One-to-One Function - A function f: A → B is said to be one-to-one if each element of A is mapped to a distinct element of B. It is also known as Injective Function.
- Onto Function - A function f: A → B is said to be onto, if every element of B is the image of some element of A under f, i.e, for every b ∈ B, there exists an element a in A such that f(a) = b. A function is onto if and only if the range of the function = B.
- Many to One Function - A many to one function is defined by the function f: A → B, such that more than one element of the set A are connected to the same element in the set B.
- Bijective Function - A function that is both one-to-one and onto function is called a bijective function.
- Constant Function - The constant function is of the form f(x) = K, where K is a real number. For the different values of the domain(x value), the same range value of K is obtained for a constant function.
- Identity Function - An identity function is a function where each element in a set B gives the image of itself as the same element i.e., g (b) = b ∀ b ∈ B. Thus, it is of the form g(x) = x.
- Algebraic functions are based on the degree of the algebraic expression. The important algebraic functions are:
  - linear function
  - quadratic function
  - cubic function

- polynomial function
- objective functions

Important Notes on Relation and Function:

- Relations and functions define the relation between the elements of two sets and are represented as a set of ordered pairs.
- Relations and functions can be represented in different forms such as arrow representation, algebraic form, set-builder form, graph form, roster form, and tabular form.
- A relation may not be a function but every function is a relation.

Graphing Relations, Domain, and Range

A **relation** is just a relationship between sets of information. When x and y values are linked in an equation or inequality, they are related; hence, they represent a relation.

Not all relations are functions. A **function** states that given an x, we get one and only one y.

$y = 3x + 1$	$x^2 + y^2 = 5$
This is a function. For any value of x you plug in, you will get only one possible value for y. For example, if $x = 2$, y can only equal 7.	This is not a function. Any value of x can give you more than one possible y. For example, if $x = 1$, y could equal 2 or -2.

It is possible to test a graph to see if it represents a function by using the **vertical line test**. Given the graph of a relation, if you can draw a vertical line that crosses the graph in more than one place, then the relation is not a function.

This is a function.
Any vertical line will cross this graph at only one point.

This is not a function.
There is a vertical line that will cross this graph at more than one point.

The **domain** is defined as all the possible input values (usually x) which allow the formula to work. Note that values that cause a denominator to be zero, which makes the function undefined, are not allowable values.

- The function $y = 4x^2 - 9$ has a domain of all real numbers, which can be expressed using the interval . Every possible x-value will give you a legitimate y-value.
- The function has a domain of all real numbers except -5, because when $x = -5$, the denominator will be zero, and the function will be undefined. We can express this using the interval .

The **range** is the set of all possible output values (usually y), which result from using the formula.

If you graph the function $y = x^2 - 2x - 1$, you'll see that the y-values begin at -2 and increase forever. The range of this function is all real numbers from -2 onward. We can express this using the interval .

9 Important Properties Of Relations In Set Theory

1. Identity Relation: Every element is related to itself in an identity relation. It is denoted as I = {(a, a), a ∈ A}.

2. Empty relation: There will be no relation between the elements of the set in an empty relation. It is the subset ∅.

3. Reflexive relation: Every element gets mapped to itself in a reflexive relation. A relation R in a set A is reflexive if (a, a) ∈ R for all a∈R.

4. Irreflexive relation: If any element is not related to itself, then it is an irreflexive relation.

5. Inverse relation: When a set has elements which are inverse pairs of another set, then the relation is an inverse relation. For example, if A = {(p,q), (r,s)}, then R^{-1} = {(q,p), (s,r)}. Inverse relation is denoted by R^{-1} = {(b, a): (a, b) ∈ R}.

6. Symmetric relation: A relation R is a symmetric relation if (b, a) ∈ R is true when (a,b) ∈ R. For example R = {(3, 4), (4, 3)} for a set A = {3, 4}. Symmetric relation is denoted by

aRb ⇒ bRa, ∀ a, b ∈ A.

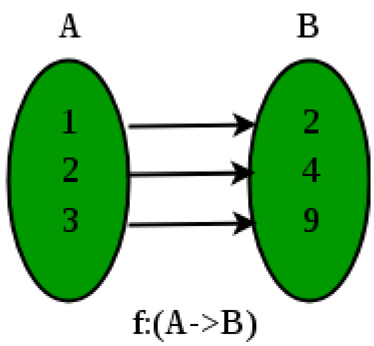
7. Transitive relation: A relation is transitive, if (a, b) ∈ R, (b, c) ∈ R, then (a, c) ∈ R. It is denoted by aRb and bRc ⇒ aRc ∀ a, b, c ∈ A

8. Equivalence relation: A relation is called equivalence relation if it is reflexive, symmetric, and transitive at the same time.

9. Universal relation: A relation is said to be universal relation, If each element of A is related to every element of A, i.e. R = A × A.

Injective, surjective, Bijective of Functions

1. **One to one function(Injective):** A function is called one to one if for all elements a and b in A, if f(a) = f(b),then it must be the case that a = b. It never maps distinct elements of its **domain** to the same element of its **co-domain**.

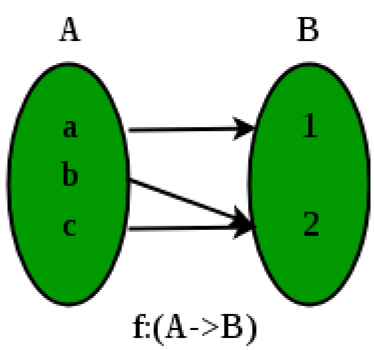
   

   We can express that f is one-to-one using quantifiers

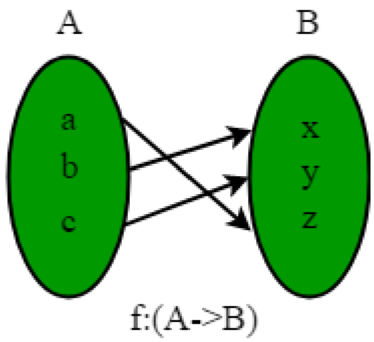
   as or

   equivalently , where the universe of discourse is the domain of the function.

2. **Onto Function (surjective):** If every element b in B has a corresponding element a in A such that f(a) = b. It is not required that a is unique; The function f may map one or more elements of A to the same element of B.

3. **One to one correspondence function(Bijective/Invertible):** A function is Bijective function if it is both one to one and onto function.



4. **Inverse Functions:**Bijection function are also known as invertible function because they have inverse function property. The inverse of bijection f is denoted as f^{-1}. It is a function which assigns to b, a unique element a such that f(a) = b. hence f^{-1} (b) = a.

Function Composition: let g be a function from B to C and f be a function from A to B, the composition of f and g, which is denoted as fog(a)= f(g(a)).

Properties of function composition:

1. fog ≠ gof
2. f^{-1} of = f^{-1} (f(a)) = f^{-1}(b) = a.
3. fof^{-1} = f(f^{-1} (b)) = f(a) = b.
4. If f and g both are one to one function, then fog is also one to one.
5. If f and g both are onto function, then fog is also onto.
6. If f and fog both are one to one function, then g is also one to one.
7. If f and fog are onto, then it is not necessary that g is also onto.
8. (fog)$^{-1}$ = g^{-1} o f^{-1}

Some Important Points:

1. A function is one to one if it is either strictly increasing or strictly decreasing.
2. one to one function never assigns the same value to two different domain elements.
3. For onto function, range and co-domain are equal.
4. If a function f is not bijective, inverse function of f cannot be defined.