1) Prediction for instance 1

a) $P(Y=low \mid X=H.S, service, <3) = P(X=H.S \mid Y=low) * P(X=service \mid Y=low) * P(X=<3 \mid Y=low) * P(Y=low)$

Before Laplace Smoothing:

$\frac{4}{6} \times \frac{4}{6} \times \frac{2}{6} \times \frac{6}{10} = 8.88\%$

After Laplace Smoothing:

$\frac{4+1}{6+2} \times \frac{4+1}{6+2} \times \frac{2+1}{6+3} \times \frac{6}{10} = \frac{5}{8} \times \frac{5}{8} \times \frac{3}{9} \times \frac{6}{10} \approx 7.813\%$

b) $P(Y=high \mid X=H.S, Service, <3) = P(X=H.S \mid Y=high) * P(X=service \mid Y=high) * P(X=<3 \mid Y=high) * P(Y=high)$

Before Laplace Smoothing:

$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{4}{10} = 0.625\%$

After Laplace smoothing:

$\frac{1+1}{4+2} \times \frac{1+1}{4+2} \times \frac{1+1}{4+3} \times \frac{4}{10} = \frac{2}{6} \times \frac{2}{6} \times \frac{2}{7} \times \frac{4}{10} \approx 1.270\%$

Prediction for instance 1 : Low

2) Prediction for instance 2

a) $P(Y=low \mid X=College, Retail, <3) = P(X=College \mid Y=low) * P(X=Retail \mid Y=low) * P(X=<3 \mid Y=low) * P(Y=low)$

Before Laplace smoothing:

$\frac{2}{6} \times \frac{0}{6} \times \frac{2}{6} \times \frac{6}{10} = 0$

After Laplace smoothing:

$\frac{2+1}{6+2} \times \frac{0+1}{6+3} \times \frac{2+1}{6+3} \times \frac{6}{10} = \frac{3}{8} \times \frac{1}{9} \times \frac{3}{9} \times \frac{6}{10} \approx 0.833\%$

b) $P(Y=high \mid X=College, Retail, <3) = P(X=College \mid Y=high) * P(X=Retail \mid Y=high) * P(X=<3 \mid Y=high) * P(Y=high)$

Before Laplace Smoothing:

$\frac{3}{4} \times \frac{0}{4} \times \frac{1}{4} \times \frac{4}{10} = 0$

After Laplace Smoothing:

$\frac{3+1}{4+2} \times \frac{0+1}{4+3} \times \frac{1+1}{4+3} \times \frac{4}{10} = \frac{4}{6} \times \frac{1}{7} \times \frac{2}{7} \times \frac{4}{10} \approx 1.088\%$

Prediction for instance 2: High

3) Prediction for Instance 3

a) $P(Y=low \mid X=Graduate, service, 3-10) = P(X=Graduate \mid Y=low) * P(X=service \mid Y=low) * P(X=3-10 \mid Y=low) * P(Y=low)$

Before Laplace Smoothing:

$\frac{0}{6} \times \frac{4}{6} \times \frac{2}{6} \times \frac{6}{10} = 0$

After Laplace smoothing:

$\frac{0+1}{6+3} \times \frac{4+1}{6+2} \times \frac{2+1}{6+3} \times \frac{6}{10} = \frac{1}{9} \times \frac{5}{8} \times \frac{3}{9} \times \frac{6}{10} \approx 1.389\%$

b) $P(Y=high \mid X=Graduate, service, 3-10) = P(X=Graduate \mid Y=high) * P(X=service \mid Y=high) * P(X=3-10 \mid Y=high) * P(Y=high)$

Before Laplace Smoothing:

$\frac{0}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{4}{10} = 0$

After Laplace smoothing:

$\frac{0+1}{4+3} \times \frac{1+1}{4+2} \times \frac{1+1}{4+3} \times \frac{4}{10} = \frac{1}{7} \times \frac{2}{6} \times \frac{2}{7} \times \frac{4}{10} \approx 0.544\%$

Prediction for instance 3: Low