Do Robust Methods Identify Outliers More Reliably Than Conventional Tests for Outliers?

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Abstract

In order to identify outliers, there are two approaches: the conventional tests for outliers and robust methods. Statisticians working with robust methods argue that their results are more reliable than the conventional tests for outliers. Which one of these approaches is more reliable? This question is investigated here in view of the problems caused by masking effects, swamping effects and leverage points and discussed by simulated linear regression models. The mean success rate is used to compare the two approaches. Summarizing, the robust methods can identify outliers at a rate of 22 % more reliably than the conventional test for outliers in a simple regression.

Zusammenfassung

Um Ausreißer zu suchen, gibt es zwei Ansätze: die Tests für Ausreißer und die robusten Verfahren. Statistiker, die mit robusten Methoden arbeiten, behaupten, dass ihre Verfahren zuverlässiger als die Tests für Ausreißer sind. Welcher der beiden Ansätze ist nun zuverlässiger? Diese Frage wurde in Hinblick auf einige Probleme bei der Bestimmung von Ausreißern, wie Einfluss der Maskierung, der Überdeckung und der Hebelpunkte in einer linearen Regression durch eine Simulation untersucht. Um die beiden Ansätze miteinander zu vergleichen, wurde die mittlere Erfolgsrate benutzt. Zusammenfassend kann gesagt werden, dass die robusten Verfahren die Ausreißer um 22 % zuverlässiger bestimmen als die Tests für Ausreißer in einer einfachen Regression.

1 Introduction

If observations in a linear regression contain outliers, these bad observations disturb the estimates of the parameters in a least squares estimation (LSE) and also the decision of the linear hypothesis testing. Therefore, they have to be identified. To do that, two approaches are commonly used: the conventional tests for outliers (Baarda 1968, Pope 1976, see also Koch 1999), and robust methods (Hampel et al. 1986, Huber 1981, Koch 1996, Rousseeuw and Leroy 1987, Wilcox 1997). Many textbooks written for applied researchers still claim that conventional tests are completely satisfactory to detect outliers (Wilcox 1997). Whereas many statisticians assert that robust methods are more reliable than the other approach. Which one of these two approaches is more reliable?

There are many robust methods (Hampel et al. 1986,

Rousseeuw and Leroy 1987, Wilcox 1997). They may be divided into two broad groups:
a) M-Estimators, L_1-norm and Generalized M-Estimators (GM-Estimators),
b) Robust methods with high breakdown points such as least median of squares (LMS), or least trimmed squares (LTS).

LMS is the most successful method to detect outliers in leverage points. However, it produces artificial outliers in a simple or multiple linear regression (Hekimoğlu and Koch 1999, Hekimoğlu 2001).

To compare robust estimators with each other globally, the breakdown point ε^* is used in robust statistics (Donoho and Huber 1983, Hampel et al. 1986). It indicates the maximum proportion of gross outliers which the estimator can tolerate. For example, the breakdown point ε^* for LSE is zero. It means that LSE cannot tolerate one gross outlier. The breakdown point is defined asymptotically and does not give any information about the capability of the robust estimator to detect outliers. Hence, Hekimoğlu and Koch (1999) introduced the mean success rate (MSR) to measure the reliability of robust estimators. The success rate means that the number of successes of an estimator to identify outliers is divided by the number of the experiments.

When the observations in a linear regression contain multiple outliers, it is difficult to identify them by any robust method due to the masking or the swamping effect (Hadi and Simonoff 1993). In addition, if the bad observations lie with respect to their positions close together, their partial redundancy numbers are smaller than the other ones, and if their magnitudes lie between 3σ and 4σ, it is very difficult to identify them. To detect outliers with small partial redundancy numbers is also difficult (Hekimoğlu 1997 and 1998, Hekimoğlu and Koch 1999 and 2000). Especially problematic are the outliers in leverage points (Koch 1996). To detect them more reliably, the equiredundancy design is used (Staudte and Sheather 1990, Kampmann 1994, Koch 1999, Hekimoğlu 1998). These items are discussed in this study.

To measure the global reliability of a test procedure in robust statistics, the power and level breakdown points are introduced (He et al. 1990, Markatou and Hettmansperger 1990). The power and level breakdown points of a test are defined asymptotically. Realizing this in practice is very difficult. However, Hekimoğlu and Koch (2000) introduced also the MSR concept to measure the reliability of a test.

2 The tests for outliers

2.1 Gauss–Markov model

Assume that the expected values of the multi-dimensional observations l_i $(i = 1, 2, ..., n)$ can be represented by a linear combination of known coefficients and unknown parameters. Hence, a linear model, in general called Gauss-Markov model, is given (Koch 1999):

$$\mathbf{l} = \mathbf{A}\boldsymbol{\beta} + \mathbf{e} \tag{1}$$

$$\mathbf{C}_l = \sigma^2 \mathbf{P}^{-1} = \sigma^2 \mathbf{Q}_l \quad \text{and} \quad E(\mathbf{e}) = 0 , \tag{2}$$

where $\mathbf{A}$ is the $n \times u$ design matrix, $\boldsymbol{\beta}$ the $u \times 1$ unknown parameter vector, l the $n \times 1$ observation vector, $\mathbf{e}$ the $n \times 1$ random error vector which is assumed to be normally distributed, $\mathbf{C}_l$ the $n \times n$ covariance matrix, $\mathbf{P}$ the $n \times n$ weight matrix, $\mathbf{Q}_l$ the $n \times n$ cofactors matrix, σ^2 the variance factor, n the number of observations and $E(\ )$ the expected value. Let $\mathbf{A}$ have full column rank, i. e. rank $\mathbf{A} = u$, and let $\mathbf{P}$ be positive definite.

2.2 Iterative test procedures to detect multiple outliers

Outlier detection procedures are developed by Baarda (1968) and Pope (1976), see also Koch (1999, p. 302). Baarda introduced the term »data snooping«.

If it is assumed that an observation $\bar{l}_i$ has an outlier δl_i with $\bar{l}_i = l_i + \delta l_i$, the hypothesis

$$H_0 : \delta l_i = 0 \quad \text{against} \quad H_1 : \delta l_i \neq 0 \tag{3}$$

is tested. If the value of the variance σ^2 is known and the observations are uncorrelated, the residual v_i is normalized to obtain the test statistic

$$b_i = \frac{|v_i|}{\sigma \sqrt{(\mathbf{Q}_{vv})_{ii}}} , \quad i \in \{1, 2, ..., n\} , \tag{4}$$

where $(\mathbf{Q}_{vv})_{ii}$ is the ith diagonal element of the cofactors matrix $\mathbf{Q}_{vv}$ of the residuals. If $b_i > z_{(1-\alpha/2)}$ which is the upper $\alpha/2$ percentage point of the normal distribution, then the observation $\bar{l}_i$ is considered as bad observation, where the significance level α generally is chosen to be 0.001. This is Baarda's method. If there is more than one outlier among the observations, data snooping is used iteratively.

3 Robust methods

In this paper, some robust methods that are generally used in robust statistics were chosen; M-Estimation of

Huber, Danish Method, L_1-norm (Hampel et al. 1986, Huber 1981, Krarup et al. 1980, Barradole and Roberts 1974) and also Modified M-Estimation proposed by Koch (1996).

3.1 M–Estimation

The robust M-Estimation, a generalized form of the maximum likelihood estimation, was introduced by Huber (1964). The normal equations of the M-Estimation in the Gauss-Markov model are given as follows,

$$\frac{1}{\sigma^2} \sum_{i=1}^{n} w_i \ v_i \ a_{ij} = 0 , \quad j = 1, 2, ..., u , \tag{5}$$

or with matrix notation,

$$\mathbf{A}^T \mathbf{W}_k \ \mathbf{v} = \mathbf{A}^T \mathbf{W}_k \ (\mathbf{A} \ \hat{\boldsymbol{\beta}}_k - l) = 0 , \tag{6}$$

where $\mathbf{v}$ is the vector of the n residuals, $\mathbf{W}_k$ the chosen weight matrix and k the iteration number. It is readily computed by iteratively reweighting the LSE, see also Koch (1999, p. 258).

3.2 Generalized M–Estimation

GM-Estimators are introduced to bound the influence of an outlier in a leverage point by means of a weight function $\eta(x_i)$. Mallows (see: Hampel et al. 1986) proposed to replace (5) by

$$\frac{1}{\sigma} \sum_{i=1}^{n} \eta(x_i) \ w(v_i/\sigma) \ a_{ij} = 0 , \quad j = 1, 2, ..., n , \tag{7}$$

where $\eta(x_i)$ is replaced by balanced weights p_i^* in Hekimoğlu (1998), $\sqrt{r_i}$ in Huber (1981) and $r_i^{t/2}/r_d$ in Koch (1996), where $r_d = (1/n) \sum_{i=1}^{n} r_i^{t/2}$ and $t/2 = 8$ and r_i denotes the partial redundancy number and x_i is the explanatory variable in linear regression.

4 Problems with identifying outliers

4.1 Masking and swamping effects

The least squares estimates $\mathbf{v}$ may be given with the hat matrix $\mathbf{H} = \mathbf{A}(\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{P}$ by

$$\mathbf{v} = (\mathbf{H} - \mathbf{I}) \mathbf{l} . \tag{8}$$

Let the observations include two outliers and the bad observations be $\bar{l}_i$ and $\bar{l}_k$. The residual $\bar{v}_i$ can be written as follows:

$$\bar{v}_i = -(1 - h_{ii})\bar{l}_i + h_{ik}\bar{l}_k + \sum_{j=1}^{n} h_{ij}\, l_j\,,$$

$$j \neq i\,,\ k \neq i\,,\ j \neq k\,,\ i = 1, 2, ..., n\,. \tag{9}$$

If the second term on the right-hand side of (9) has the opposite sign of the first term, the two terms may cancel each other and a »bad« observation becomes a »good« observation. This is called the masking effect. Let the observation l_i be a good one. If the contributions of the second and third terms are added, the observation l_i might become a »bad« observation. This is the swamping effect.

4.2 Equiredundancy design

To detect outliers in leverage points, the equiredundancy design is used. If each observation has the same geometrical and stochastic effect, the diagonal elements h_{ii}^* of the hat matrix $\mathbf{H}^*$ must become equal. This is referred to as »equileverage design« by Staudte and Sheather (1990) and »equiredundancy design« in (Hekimoğlu 1998). In other words, the partial redundancy numbers

$$r_i^* = 1 - \frac{u}{n} \quad \text{or} \quad h_{ii}^* = \frac{u}{n}\,, \quad i \in (1, 2, ..., n)\,, \tag{10}$$

$$(\mathbf{H}^*)_{ii} = (\mathbf{A}\,(\mathbf{A}^{\mathrm{T}}\mathbf{P}^*\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}^*)_{ii} = u/n\,, \tag{11}$$

where r_i^* is the balanced redundancy number, $\mathbf{P}^*$ is the balanced weight matrix, and $\mathbf{H}^*$ is the balanced hat matrix. There are iterative methods to obtain the balanced weight p_i^* (Kampmann 1994, Koch 1996 and Hekimoğlu 1998).

4.3 Definition of an outlier

We assume that random errors e_i are normally distributed, i.e. $e_i \sim N(\mu, \sigma^2)$ with mean μ and variance σ^2. A bad error which lies outside the boundaries $\mu \pm 3\sigma$ is called *outlier*. The outliers are divided into two broad categories: random and influential outliers. Outliers that occur accidentally in the measurements are called random outliers. Their signs and magnitudes change randomly. Influential outliers have the same sign, all plus or all minus, although their magnitudes can change randomly (Hekimoğlu 1997, Hekimoğlu and Koch 1999).

A robust estimator applied to a certain corrupted sample is considered successful, if the residual $\bar{v}_i$ of each contaminated observation is greater than 3σ, i.e.,

$$|\bar{v}_i| > 3\sigma\,, \quad i = 1, 2, ..., m\,. \tag{12}$$

Let the corrupted sample have m outliers. If the robust estimator cannot identify all m outliers, we consider it not successful.

4.4 Critical outliers

Let the outliers in a linear regression lie with respect to their x-values close together and let their partial redundancy numbers r_i be smaller than the ones of the other observations, then they are here called critical outliers. For the simple regression given in Tab. 1, the possible two, three and four critical outliers are considered: $(\bar{y}_1, \bar{y}_2)$ or $(\bar{y}_9, \bar{y}_{10})$, $(\bar{y}_1, \bar{y}_2, \bar{y}_3)$ or $(\bar{y}_8, \bar{y}_9, \bar{y}_{10})$ and $(\bar{y}_1, \bar{y}_2, \bar{y}_3, \bar{y}_4)$ or $(\bar{y}_7, \bar{y}_8, \bar{y}_9, \bar{y}_{10})$, respectively.

5 Monte–Carlo Method

5.1 Observations without outliers

To investigate the problems of identifying outliers by using the robust methods and the tests for outliers, a simple straight line and a multiple linear regression model are considered as follows:

$$y_i = a + bx_i\,, \quad i = 1, 2, 3, ..., n_1 \text{ with } a = 1,\ b = 1 \text{ and} \tag{13}$$

$$z_j = a + bx1_j + cx2_j + dx3_j + ex4_j\,, \quad j = 1, 2, 3, ..., n_2 \tag{14}$$

with $a = 1$, $b = -1$, $c = 0.5$, $d = 1.2$, $e = 1.5$, $n_1 = 10$ and $n_2 = 13$.

The random errors $e1_i$, $i = 1, 2, 3, ..., 10$ and $e2_j$, $j = 1, 2, 3, ..., 13$ were generated from the normal distribution $e1_i$, $e2_j \sim N(\mu = 0, \sigma^2 = 4\,\text{cm}^2)$ by a random number generator of an International Mathematical and Statistical Library (IMSL) subroutine.

To obtain »good« observations y_i' and z_j', the random errors $e1_i$ or $e2_j$ were added to the y_i-values or the z_j-values.

Tab. 1: The (x_i, y_i)–values of a simple regression and the partial redundancy numbers r_i

Point	1	2	3	4	5	6	7	8	9	10
x_i	1	2	3	4	5	6	7	8	9	10
y_i	2	3	4	5	6	7	8	9	10	11
r_i	0.66	0.75	0.82	0.87	0.90	0.90	0.87	0.82	0.75	0.66

One thousand sets for y_i' and z_j' were generated by creating one thousand sets of the random error vectors **e1** and **e2**, respectively.

5.2 Bad Observations

To simulate a »bad« observation such as $\bar{y}_i$ or $\bar{z}_j$, the random error of a »good« observation is replaced by an outlier δy (or δz). It means that the magnitude δy (or δz) of an outlier is added to the y_i-values or the z_j-values (e. g., $\bar{y}_i = y_i + \delta y_i$ or $\bar{z}_j = z_j + \delta z_j$). Random and influential outliers are generated by the uniform distribution for a given interval in the outlier region as done in Hekimoğlu and Koch (1999 and 2000). In addition, bad observations are also considered at both ends of the interval containing the observations, i. e. where the partial redundancies are smaller than the other ones.

One hundred different contaminated samples of $\bar{y}_i$ and $\bar{z}_j$ have been simulated for each of the 1000 sets for y_i' and z_j' so that in total 100 000 contaminated samples of $\bar{y}_i$ and $\bar{z}_j$ are obtained.

The number k of iterations was taken as follows: k = 15 for Huber's method, and k = 5 for the Danish method. The constant c = 1.5 was chosen.

5.3 Results

First we applied four different robust estimators to the 1000 samples for y_i' and z_j' that have only good observations in order to verify whether an estimator produces outliers. As can be seen in the second column under heading »0« in Tab. 2, LMS produces outliers in 19 % of the cases for the simple regression. Also, Huber's estimator produces some outliers in 1 % of the cases. But, Baarda's method does not. Hence, if observations do not have any outliers, using some robust methods is a risk.

The MSRs are computed from each of the 1000 sets of contaminated samples for the random and influential outliers separately for different numbers of outliers. In addition, the MSRs are also calculated for both kinds of the outliers that lie at the ends of the interval for the

observations. Then, the minimum value of these four different MSRs is obtained for each number of outliers. They are given in Tab. 2. As can be seen in this table, the mean values of the total MSRs of the robust methods are 199 % and 336 % for the intervals of 3σ and 6σ and 6σ and 10σ, respectively. They are greater than the ones of Baarda's method with a value of 25 % $\{= (199 - 159)/159\}$ and 19 % $\{= (336 - 282)/282\}$ in total for the intervals of 3σ and 6σ and 6σ and 10σ, respectively. The mean value of 25 % and 19 % is 22 %.

However, the MSRs of all the computations applied here for the multiple regression are smaller than the ones for the simple regression. The reason is that the masking effects, the swamping effects, and the critical outliers have a stronger influence on the MSRs for the multiple regression than for the simple regression. The MSRs of the robust methods are greater than the ones of Baarda's method with a value of 46 % in total for the interval of 3σ and 6σ. But, they are smaller than the ones of Baarda's method with 20 % in total for the interval of 6σ and 10σ. They are not given here because of lack of space.

6 Discussion

6.1 How do the MSRs of M–Estimators change when the constant c changes?

The c-value in an M-Estimation is usually taken as constant. Xu (1993) and Wicki (1999), however, propose to keep it variable. The maximum values of the MSRs of Huber's and of the Danish method change depending on the number of outliers, the magnitudes of outliers and the c-value, and also on the number of unknowns. The changes of the c-value show that identifying multiple outliers is a nonlinear problem. We can not find a c-value which is proper for the two intervals (3σ to 6σ, and 6σ to 10σ) of outliers of a robust method. The robust methods are the most successful, but show agreement only if the c-value is taken as $3.29\sigma\sqrt{(Q_{vv})_{ii}}$ and the outliers lie between 6σ and 10σ.

Tab. 2: The minimum values of the MSRs of the methods used for the simple regression

Methods	The Magnitude of outliers between 3σ and 6σ						The Magnitude of outliers between 6σ and 10σ				
	0	1	2	3	4	Total*	1	2	3	4	Total*
Baarda	0 %	78 %	53 %	27 %	1 %	159 %	100 %	96 %	80 %	6 %	282 %
Huber	1	84	57	25	4	170	98	93	92	79	362
Danish	4	87	72	48	19	226	96	95	83	48	322
LMS	19	78	65	52	11	206	91	91	92	94	368
L_1-norm	4	84	64	38	8	194	95	90	74	32	291

* The MSRs for 0 outlier are not included. The second row denotes the number of outliers.

6.2 Equiredundancy design

Taking an equiredundancy design into account means that the balanced weights are applied instead of the weights of observations. In this case, the MSRs of the robust methods increase when bad observations lie only at the ends of the interval for the observations and decrease when outliers lie in the whole interval. However, the MSRs of Baarda's method decrease drastically for those outliers which lie only at the ends of the interval.

6.3 Leverage Points

If an x-value of an observation lies far away from the x-values of the bulk of the observations, this point is called a leverage point (Rousseeuw and Leroy 1987, p. 225). Its partial redundancy number r_i is the smallest one among the observations. We simulated a simple regression which contains one, two or more leverage points. In addition, small outliers between 3σ and 6σ are considered in these leverage points. The MSRs of these simulations are given in Tab. 3. The farther the leverage point lies from the bulk of the data, the smaller the MSRs of the robust methods and Baarda's method become except the ones of the LMS method.

As can be seen from Tab. 3, Huber's method, the Danish method and the Modified M-Estimation break down for two, three and four leverage points, respectively. But, LMS does not break down. Baarda's method breaks down for two or more leverage points.

We accept that an estimator breaks down when its MSR goes to zero. Therefore, the concept of the breakdown point is easily explained by outliers in leverage points.

6.4 Gross Outliers

Gross outliers means that their magnitudes are large with respect to the other ones, such as 1000σ and more. Only gross errors at both ends of the interval for the observations are considered, i.e. where the partial redun-

dancies are smaller than the other ones. Gross outliers whose magnitudes lie between 1000σ and 2000σ are simulated. The MSRs of LMS, L_1-norm and Baarda's method are greater than the ones of the other robust methods. The Danish method breaks down. They are not given here because of lack of space.

6.5 Equiredundancy design against outliers in leverage points

The same simulated data, i.e., the simple and the multiple regressions with the same leverage points were used as in subsection 6.3. The MSRs of the robust methods are given in Tab. 3. Huber's and the Danish method using the equiredundancy design are very successful against outliers in the leverage points. Their MSRs increase significantly compared to the ones given in Tab. 3. They are also greater than the ones for Baarda's method.

6.6 Equiredundancy design against gross outliers

The same simulated data with the same gross outliers were used as in subsection 6.4. The Danish method using the equiredundancy design breaks down. But, Huber's and L_1-norm method using the equiredundancy design are successful against the gross outliers which lie at the ends of the interval for the observations.

6.7 Critical outliers

We simulated critical outliers whose magnitudes lie between 3σ and 6σ. The Danish, LMS and Baarda's methods can identify two critical influential outliers $(\bar{y}_1, \bar{y}_2)$, or $(\bar{y}_9, \bar{y}_{10})$ given in Tab. 1 with MSRs of 27 %, 50 % and 11 % respectively for the simple regression. These rates are small as compared to the ones given in Tab. 2. If the equiredundancy design is considered, the MSRs of the Danish method increase as compared to the case when it is not considered.

Tab. 3: The MSRs of the methods for outliers in leverage points

Methods	Equiredundancy design is not taken into account					Equiredundancy design is taken into account				
	1	2	3	4	5	1	2	3	4	5
Baarda	78 %	0 %	0 %	0 %	0 %	35 %	39 %	44 %	0 %	0 %
Huber	69	0	0	0	0	81	80	78	74	66
Danish	55	55	0	0	0	58	52	52	53	0
ModME*	80	82	82	0	0					
LMS	78	81	81	85	84					

* Abbreviation: ModME – Modified M-Estimation. The second row denotes the number of the leverage points.

Tab. 4: The (x_i, y_i)–values in three contaminated simple regressions

x_i	1	2	3	4	5	6	7	8	9	10
(I) y_i	1.970	3.031	3.981	**4.932**	**6.093**	7.023	8.011	8.999	9.980	10.997
(II) y_i	1.970	3.031	3.981	4.996	6.012	7.023	8.011	**9.096**	9.980	**11.102**
(III) y_i	1.973	2.995	4.029*	4.990	6.055	7.017	7.996	8.969	**10.108**	**11.096**

Bold numbers are contaminated observations. The first row denotes the number of point.

6.8 Masking effect

The masking effect occurs if one outlier with a large magnitude lies with respect to its x-value close to another outlier with a small magnitude. As a result, the robust methods and Baarda's method can identify only the outlier with a large magnitude.

To illustrate the masking effect, we used the contaminated sample (I) given in Tab. 4, where $\bar{y}_4(\delta y_4 = -0.064)$ and $\bar{y}_5(\delta y_5 = 0.093)$ are bad observations. If a robust method is used (for example the Danish Method), only $\bar{y}_5$ can be identified as an outlier. $\bar{y}_4$ is masked by the bad observation $\bar{y}_5$ since $\left|\delta y_5\right| > \left|\delta y_4\right|$.

6.9 Swamping Effect

In the simple regression a swamping effect occurs when at least two bad observations, whose partial redundancy numbers are smaller than the other ones, enclose a good observation, for example: $(\bar{y}_1, \bar{y}_3)$, or $(\bar{y}_8, \bar{y}_{10})$. Let the bad observations be $\bar{y}_8(\delta y_8 = 0.096)$ and $\bar{y}_{10}(\delta y_{10} = 0.102)$ as given in Tab. 4 with (II). Using the Danish method, the good observation y_9 is identified as an outlier because the bad observations $\bar{y}_8$ and $\bar{y}_{10}$ are swamping. In addition, a swamping effect may also occur when at least two bad observations, whose partial redundancy numbers are smaller than the other ones, lie close together with respect to their x-value. Let the bad observations be $\bar{y}_9(\delta y_9 = 0.108)$ and $\bar{y}_{10}(\delta y_{10} = 0.096)$ as given in Tab. 4 with (III). Besides the bad observations $\bar{y}_9$ and $\bar{y}_{10}$, the good observation y_3 is identified as an outlier by the Danish method. Baarda's method is affected by the swamping effect two times more strongly than the Danish method.

7 Conclusion

Assume that a contaminated sample does not contain an outlier in a leverage point or any gross outlier. In this case, in view of the masking effects, the swamping effects and the critical outliers, the MSRs of robust methods are always greater than the ones of Baarda's

method, on the average by 22 % for the simple regression. Therefore, robust methods must be used for the identification of outliers.

If a contaminated sample has one or more outliers in leverage points, LMS or the Modified M-Estimation methods must be used. Huber's, the Danish or the L_1-norm method may be applied if the equiredundancy design is taken into account. In these cases, the MSRs of the robust methods are greater than the ones of Baarda's method.

Assume that a contaminated sample has one or more gross outliers. In this case, LMS, L_1-norm and Baarda's method may be used.

We can not be sure in advance whether the contaminated sample has outliers in leverage points or gross outliers. Considering this possibility, one starts with applying a robust method to the contaminated sample to identify outliers in the observations. Thereafter, a robust method with high breakdown point such as the Modified M-Estimation or LMS has to be applied to detect outliers in leverage points or gross outliers. The latter, however, requires excessive computations. Instead of these methods, a robust method such as Huber's or L_1-norm method can be applied to the contaminated sample by considering the equiredundancy design.

If the outliers are present in leverage points and in observations with large contributions to the redundancy, they cannot be detected reliably by any M-Estimation methods. Thus, the LMS method has to be applied.

References

Baarda, W.: A testing procedure for use in geodetic networks. Netherlands Geodetic Com., New Series, Delft, Netherlands, 2 (5), 1968.

Barrodale, I., Roberts, F. D. K.: Solution of an over determined system of equations in L_1-norm. Comm. ACM, vol. 17: 319–320, 1974.

Donoho, D. L., Huber, P. J.: The notion of breakdown point. In a Festschrift for Erich Lehmann, P. J. Bickel, K. A. Doksum and J. L. Hodges, Jr. (Eds), Wadsworth, Belmont, Cal.: 157–184, 1983.

Hadi, A. S., Siminoff, J. S.: Procedures for the identification of multiple outliers in linear models. J. Am. Stat. Assoc. 88: 1264–1272, 1993.

Hampel, F., Ronchetti, E., Rousseeuw, P., Stahel, W.: Robust statistics: The approach based on influence functions. John Wiley and Sons, New York, N. Y., 1986.

He, X., Simpson, D. G., Portnoy, S. L.: Breakdown robustness of tests. J. Am. Statistical. Assn., 85: 446–452, 1990.

Hekimoğlu, Ş.: The finite sample breakdown points of the conventional iterative outlier detection procedures. J. Surv. Engrg, ASCE, 123 (1): 15–31, 1997.

Hekimoğlu, Ş.: Equiredundancy design and its application to M-Estimation. J. Surv. Engrg. ASCE, 124 (3): 103–124, 1998.

Hekimoğlu, Ş., Koch, K. R.: How can reliability of the robust methods be measured? Third Turkish-German Joint Geodetic Days. 1–4 June 1999 Istanbul, Vol. 1: 179–196, 1999.

Hekimoğlu, Ş., Koch, K. R.: How can reliability of tests for outliers be measured?, AVN, 107 (7): 247–254, 2000.

Hekimoğlu, Ş.: Local reliability of the least median squares method. 1st International Symposium on Robust Statistics and Fuzzy Techniques in Geodesy and GIS. March 12–16. ETH, Zurich, 67–72, 2001.

Huber, P. J.: Robust estimation of a location parameter. Annals of Mathematical Statistics, 35: 73–101, 1964.

Huber, P. J.: Robust statistics. John Wiley and Sons, Inc., New York, N. Y., 1981.

Kampmann, G.: Robuste Deformationsanalyse mittels balancierter Ausgleichung. AVN, 101: 8–17, 1994.

Koch, K. R.: Robuste Parameterschaetzung. AVN, 103: 1–18, 1996.

Koch, K. R.: Parameter Estimation and Hypothesis Testing in Linear Models. 2nd Ed. Springer-Verlag, Berlin – Heidelberg – New York, 1999.

Krarup, T., Juhl, J., Kubik, K.: Götterdaemmerung over least squares adjustment. Proceedings. 14th Congress of Int. Soc. Photogr. Hamburg, 1980.

Markatou, M., Hettmansperger, T. P.: Robust bounded-influence tests in linear models. J. Am. Stat. Assoc. 85: 187–190, 1990.

Pope, A. J.: The statistics of residuals and the detection of outliers. NOAA Technical Report. NOS 65 NGS 1, U. S. Dept. of Commerce, Rockville, Md., 1976.

Rousseeuw, P. J., Leroy, A. M.: Robust regression and outlier detection. John Wiley and Sons. Inc., New York, N. Y., 1987.

Staudte, R. G., Sheather, S. J.: Robust estimation and testing. A Wiley-Interscience Publication. John Wiley and Sons. Inc., New York, 1990.

Wicki, F.: Robuste Schaetzverfahren für die Parameterschaetzung in geodaetischen Netzen. Mitteilung Nr. 67, Institut für Geodaesie und Photogrammetrie, Eidgenössische Technische Hochschule, Zürich, 1999.

Wilcox, R. R.: Introduction to robust estimation and hypothesis testing. Academic Press, New York, 1997.

Xu, P. L.: Consequences of constant parameters and confidence intervals of robust estimation. Bollettino di Geodesia e Scienze Affini, 52: 231–249, 1993.

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