Efficient Algorithm for Vector Rotation using Quaternions

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Abstract

- Comparison of two main techniques of rotation is done, namely:
  - rotation using euler angle
  - rotation using quaternions
- Rotation using Euler angle is less efficient, in terms of space complexity, than rotation using quaternion, for any general case.
- Rotation using Euler angle is less costly, in terms of time complexity, than rotation using quaternion when we do small number of rotations, for both small and general angle rotation.
- For large number of rotations, rotation using quaternion is more cost effective, in terms of time complexity, than rotation using Euler angle.
- Discussion of the reason of the change in preference of rotation technique when we go for large number of rotations and disadvantages of using Euler Rotation matrix.

Problem Statement

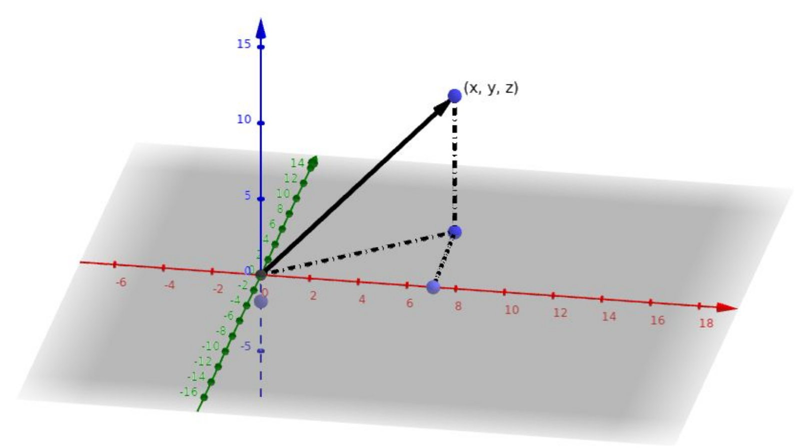
- Comparison of 3D rotations using quaternion with 3D rotation using Euler angles.

Objective

- Propose a better approach to rotate 3D objects than the conventional 3D rotation using euler angle
- Compare the 2 approaches based on:.
  - Time required for small number of rotations.
  - Time required for large number of rotations.
  - Space required for both small and large number of rotations.
- Study of Gimbal Lock

Literature Survey

✓ Paper 1



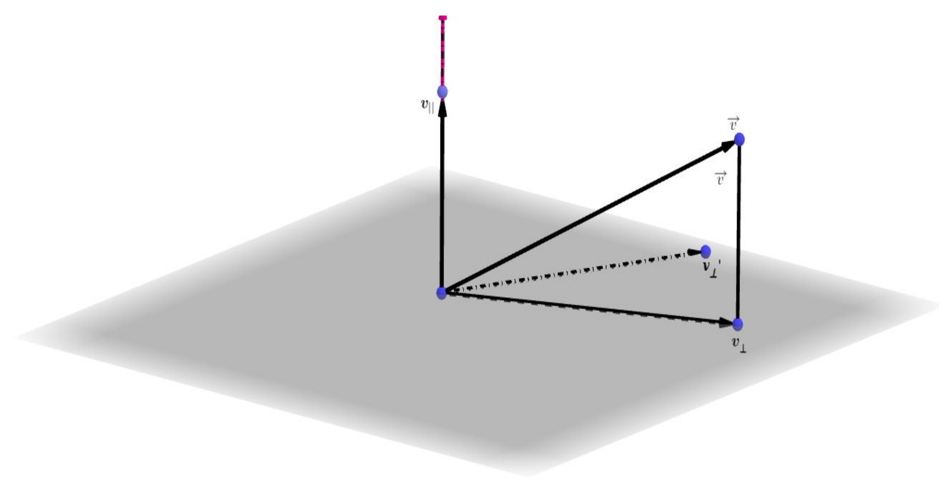
- Rotation of a point in 3D about an arbitrary axis.
  - Rotate axis of rotation in appropriate fashion to coincide it with the z-coordinate axis.
  - Now, rotate by the angle about z-axis.
  - Rotate the axis of rotation by the inverse of the transformation done in step 1.

- Chaining of rotation matrices : Series of rotation matrices can be combined to form one composite rotation matrix .

✓ Paper 2

- What are Quaternions?
- Quaternions Applications
- Quaternions Algebra
  - Quaternions Arithmetic
  - Quaternion logarithm
  - Euler Form
- Space and time complexities.
- Conjugate Normalization and Inverse

Proposed Approach



- Arbitrary axis rotation
  To rotate vector v by angle Θ using quaternions: $v' = qvq^*$, where $q = e^{n\alpha/2}$ & n is axis of rotation as shown.
- Chain of Quaternion vector rotation
  $q_2(q_1 v q^*_1)q^*_2 = (q_2 q_1)v(q_2 q_1)^*$

- Small Angle Rotation
  - Approximate of sinθ to θ and cosθ to 1
  - Chaining will result in Non-Orthogonality
  - Need to normalise the matrix / quaternion

- Rotation Matrix Normalisation
  - Replace with $M(M^T M)^{-1/2}$

- Quaternion Normalisation
  - Division by $||q||$

- Singularity and Problem of Gimbal Lock
  - About Gimbal Lock
  - When Euler Angles are used the graphic system faces the problem of gimbal lock.
  - Quaternions helps to avoid this problem.

Results & Conclusion

- Space required for chain of n quaternions vector rotations is 5n times lesser than for the rotations using euler angle.
- For single general vector rotation operations required are:
  - 45 for Euler angle vector rotation.
  - 54 for Quaternions vector rotation
- For single small angle rotation operations required are:
  - 15 for Euler angle vector rotation.
  - 52 for Quaternions vector rotation.
- For large number of rotations, quaternions vector rotation is more cost effective in terms of time.
- Problem of Gimbal Lock in case of euler angle vector rotation.

Future Work

- Fractal using self squaring procedure:
  - Quaternions can be used to model 2D, 3D & 4D fractals geometries.

References

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