Introduction

Indian Railways (IR) reserves tickets on a train through IR's passenger reservation system (PRS) and its website IRCTC, which reserves a specific seat per booking. A train can make many stops en route from its origin to its destination. Consequently, passengers can buy many combinations of station-to-station tickets. Tickets are booked both from a train's origin to its destination and to or from intermediate stations on its route. One of the objectives of Indian Railways is to maximize revenue by optimally utilizing the train's capacity. The goal of this study is to optimize the seating capacity of a train and reduce inefficiencies arising out of vacant seats during a train's journey.

Since, passengers deboard and board en route, multiple passengers can occupy a given seat during a train's complete journey. The seat might also be vacant for some segments. A vacant seat does not earn any revenue. Whenever a seat to or from an intermediate station is booked, a partially vacant seat is created. In the event of an abnormally high booking for intermediate station(s), many partially vacant seats are created; although these might not be filled, subsequent attempts by passengers to book end-to-end trips could be denied, thus leading to suboptimal utilization of the train's capacity. In addition, heavy cancellations of intermediate tickets near the date of a trip could cause additional losses. The optimization model in this study addressed these important issues.

Our project is based on the study by R. Gopalakrishnan and N. Rangaraj (2010), wherein the study devised a DLP model to optimize seating capacity on an IR train. Also, previously Ciancimino et al. (1999) worked on a multi-leg single-fare RM model to determine optimal booking limits for passenger train services which is similar to this project. In 1978, Hersh and Ladany worked on seat allocation for flights with intermediate stops as well as Glover et al. in 1982, Dror et al in 1988, and Rozite et al. in 2005; all studied and worked on optimizing similar problems.

Optimization Model

Figure-1: Train route schematic

The above schematic shows the train route from origin (O) to destination (D). The route has 'N' number of intermediate stations which also includes remote stations. We refer to the intermediate stations at which specific inventories of train seats are allocated as remote stations. We also define a major station as one that belongs to a set of stations comprising the origin, destination, and all remote stations; all the other stops on a train for which ticketing is permitted are minor stations. During each journey, manifests are prepared both at the origin and at each remote station. Passenger Reservation System (PRS) limits the number of remote stations per train to a maximum of six. These specific seat inventories in the form of spatial quotas are allocated for the classes available on the train.

Let,

Number of Remote Stations = n

x_i = Number of seats allocated to the i_{th} seat-split pattern

q_{jk} = Quota of seats between any pair of major stations j & k

d_{jk} = Demand of seats between any pair of major stations j & k

Minimize C = $\sum_{i=1}^{2^n} x_i$

Subject to;

1. $q_{jk} = \sum_{k=1}^{2^n} x_i \, \delta_i$ for all pattern j-k, where

   $\delta_i = 1$, if pattern j-k is present in the i_{th} seat split pattern;

   $\delta_i = 0$, otherwise;

2. $0 \leq d_{jk} \leq q_{jk}$ for all patterns j-k;
3. xi ≥ 0 for all seat splits i;

Deficiencies in Modelling

- A train cannot always accommodate all station-to-station passengers, and limitations on the number and availability of coaches that can be attached on a train impose capacity constraints. This puts a restriction on the minimum capacity of a train.
- The selection of the remote stations at which the seats have been split is intuitive and based on the station-to-station demand matrix. The choice of remote stations could be optimized.
- We have assumed that each seat demand between two major stations would require a full seat; however, we could also have used partially vacant seats between major stations.

Possible Improvements in Model

- Addition of new objective function to maximize revenue in Indian Railways.
- Adding the constraint of emergency (tatkal) ticket bookings in the seat quota between the stations.
- Including capacity constraints across different classes of trains (AC & non-AC).
- Adding the feature of priority booking on trains.

Numerical Example

Tools Used

We used Excel Solver for solving the model of our linear programming problem. The Simplex LP solving method was applied for optimizing the model.

Data Source

The base model data was taken from the research paper to compare our model's results with paper's results.

Assumptions

The underlying assumptions in the formulation of our model are as follows:

- The remote stations are train specific and are chosen based on parameters (e.g., the number of persons boarding and detraining at the station) that are available in the PRS data warehouse. Other considerations in selecting remote stations are the importance of the city, the time of arrival at the station, and the availability of a PRS manifest-preparation facility. The model restricts to six the maximum number of stops of a train that can be selected as a remote station in the IR reservation system.
- Heuristic adjustments to the model to account for the variability that we see in practice. Informal estimates from IR regarding variability (because of seasonality and other causes) are as high as 30-40%.
- To account for variability, the slack capacity, if available on a train, is allocated to seat splits based on the judgment of IR management.
- A train cannot always accommodate all station-to-station passengers, and limitations on the number and availability of coaches that can be attached on a train impose capacity constraints.
- Other equally convenient trains on the same train route with spare capacity have not been considered.
- The selection of the remote stations at which the seats have been split is intuitive and based on the station-to-station demand matrix.
- We have assumed that each seat demand between two major stations would require a full seat and no partially vacant seats are allowed.

Model's Optimal Output

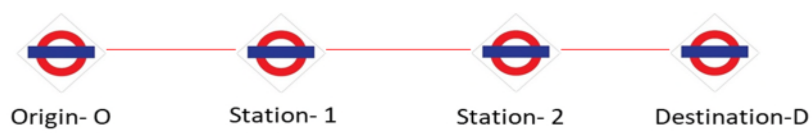

Figure-2: Base Model train route schematic

In the base model, the train route consists of two remote stations, stations 1&2. There were four possible seat split patterns in the journey against which seat quotas were optimized with specified station-to-station demands. The demand versus seat pattern quota allocation logic has been explained in Table-1:

Station-to-Station Demand	Seat Pattern	Description
O-1	O-1-D, O-1-2-D	A passenger demanding a seat from station O to station 1 can be given a seat in 'O-1-D' or 'O-1-2-D' seat pattern. This would allow a new passenger to take the seat vacated at station 1 and travel to next stations (2 and D).

O-2	O-2-D	A passenger demanding a seat from station O to station 2 can be given a seat in 'O-2-D' seat pattern. This would allow the passenger to travel his full leg from station O to station 2 and vacate a seat for a new passenger boarding from station 2.
O-D	O-D	A passenger demanding end-to-end travel will require the 'O-D' seat pattern. This would allow the passenger to travel the full leg without any booking against this seat from intermediate stations.
1-2	O-1-2-D	A passenger demanding a seat from station 1 to station 2 can be given a seat in 'O-1-2-D' seat pattern. This would allow the passenger travelling from station O to station 1 to occupy this seat and a new passenger to travel from station 2 to station D.
1-D	O-1-D	A passenger demanding a seat from station 1 to station D can be given a seat in 'O-1-D' seat pattern. This would allow the passenger travelling from station O to station 1 to occupy this seat and a new passenger to travel from station 1 to station D.
2-D	O-1-2-D, O-2-D	A passenger demanding a seat from station 2 to station D can be given a seat in 'O-1-2-D' or 'O-2-D' seat pattern. This would allow the passenger travelling from station O to station 1, station O to station 2 and station 1 to station 2 to occupy this seat and a new passenger to travel from station 2 to station D.

Table-1: Seat pattern allocation logic

Table-2 shows the passenger seat demand for each pair of major stations.

From/To	O	1	2	D	Total
O	0	70	273	487	830
1	0	0	2	27	29
2	0	0	0	59	59
D	0	0	0	0	0
Total	0	70	275	573	918

Table-2: Station-to-Station demand data

Table-3 shows the seat quota suggested by this study's model for demand data from Table-2 for each pair of major stations. For comparison, Table-4 shows the research paper's seat quota allocation against demand from Table-2 based on their optimization model.

From/To	O	1	2	D	Total
O	0	70	273	487	830
1	0	0	2	68	70
2	0	0	0	275	275
D	0	0	0	0	0
Total	0	70	275	830	1175

Table-3: Project's model suggested quota

From/To	O	1	2	D	Total
O	0	70	273	488	831
1	0	0	43	27	70
2	0	0	0	316	316
D	0	0	0	0	0
Total	0	70	316	831	1217

Table-4: Paper's model suggested quota

Table-5 shows the seat quota suggested by our model for each seat split pattern in the train.

O-D	O-1-D	O-2-D	O-1-2-D	Minimum Capacity
487	68	273	2	830

Table-5: Optimal seat capacity for base model

Discussion

Scenarios

Scenario 1: The base case of our model considered a train journey with maximum demand for end-to-end journey. In unusual circumstances, the demand for travel between intermediate stations could be higher than end-to-end travel (Table-6). In this case, if the seat allocation is not optimized it could lead to a lot of vacant seats for a part of the train's journey or unconfirmed seats and higher waitlists for the intermediate travellers. We optimized one such model where the passengers requested seats for a short route leg of the train.

From/To	O	1	2	D	Total
O	0	70	100	150	320
1	0	0	200	80	280
2	0	0	0	150	150
D	0	0	0	0	0
Total	0	70	300	380	750

Table-6: Alternate Case demand data

From/To	O	1	2	D	Total
O	0	280	100	150	530
1	0	0	200	80	280
2	0	0	0	300	300
D	0	0	0	0	0
Total	0	280	300	530	1110

Table-7: Alternate case suggested seat quota

O-D	O-1-D	O-2-D	O-1-2-D	Alt Case Min Capacity
150	80	100	200	530

Table-8: Alternate Case seat capacity

When comparing the base case with alternate case we can clearly see that when the demand for travel from station O to station D was less than the demand for travel between intermediate stations 1 & 2, the maximum seat quota shifted to O-1-2-D seat pattern (Table-8). This happened because this seat pattern can cater to demands for travel between intermediate stations which were in demand for this train route.

Scenario 2: Indian Railways provisions a last-minute booking service known as 'Tatkal', where Tatkal means emergency. The tatkal tickets booking window opens 24 hours before the train's departure time. The seats under this quota are limited and gets booked instantly for busy train routes.

Tatkal seat quota is separate from the advance booking seat quota and caters to the last-minute demands. There should be a provision of transferring the advanced booked seats to tatkal quota in the case of abnormally low bookings during the advanced window. This ensures that the vacant seats are reduced in the train's journey. A constraint measures the difference between the seat's quota and the demand between particular stations. If the difference is zero, then the seats are not transferred to tatkal, otherwise if difference is greater than zero, then that difference is added to tatkal quota of seats between those stations.

In this scenario, we ran a separate seat allocation quota versus demand model, since the booking is done separately from the advanced booking that was done in the base model. However, the constraint of limited train capacity (seats) still applies to the combined quota of advanced tickets plus the tatkal tickets.

From/To	O	1	2	D	Total
O	0	10	16	35	61
1	0	0	8	12	20
2	0	0	0	7	7
D	0	0	0	0	0
Total	0	10	24	54	88

Table-9: Tatkal demand data

From/To	O	1	2	D	Total
O	0	20	16	35	71
1	0	0	8	12	20
2	0	0	0	24	24
D	0	0	0	0	0
Total	0	20	24	71	115

Table-10: Tatkal suggested seat quota

O-D	O-1-D	O-2-D	O-1-2-D	Minimum Capacity
35	12	16	8	71

Table-11: Tatkal seat capacity

Scenario 3: Indian Railways has been upgrading the railway infrastructure to accommodate pregnant ladies, disabled and physically challenged people in the trains. This has led to allocation of specific seats for the travellers with special needs. The quota for these seats is limited and requires advanced booking under special quota. The demand for such seats is generally less and most of the seats under this quota remain vacant throughout the journey. However, we can allot some of these seats under Tatkal quota considering that some seats do not get booked in advance. This could result in less vacant seats, since seats under tatkal quota usually gets booked in few hours of opening window. The overall passenger capacity gets optimized and the available seat quota is fully utilized.

Sensitivity Analysis

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B3	Optimal Values x1 (O-1-D)	68	0	1	0	1
C3	Optimal Values x2 (O-1-2-D)	2	0	1	1E+30	0
D3	Optimal Values x3 (O-D)	487	0	1	1E+30	1
E3	Optimal Values x4 (O-2-D)	273	0	1	1E+30	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
F10	Station 1-D Quota ≥ Station 1-D Demand LHS	68	0	27	41	1E+30
F11	Station 2-D Quota ≥ Station 2-D Demand LHS	275	0	59	216	1E+30
F6	Station O-1 Quota ≥ Station O-1 Demand LHS	70	1	70	1E+30	41
F7	Station O-2 Quota ≥ Station O-2 Demand LHS	273	1	273	1E+30	216
F8	Station O-D Quota ≥ Station O-D Demand LHS	487	1	487	1E+30	487
F9	Station 1-2 Quota ≥ Station 1-2 Demand LHS	2	0	2	41	2

Figure-3: Sensitivity Analysis

1. Reduced Cost always equals zero for basic variables. All the variables in the objective function are basic variables according to our sensitivity analysis. Any change in co-efficient of any of the variables would cause change in the objective function value.
2. Binding constraints are Origin to Remote Station1, Origin to Remote Station2, Origin to Remote Station3. Change in the right-hand side of these constraints would change the value of objective function value by one unit.
   New C = Old C – (Shadow Price) *(Change in Value)
   For Example: If the right-hand side of the constraint of the Origin to Remote Station 1 case is reduced from 70 to 29, then what is the new objective function value?
   Solution: 70-41=29, it is in the allowable decrease range.
   Therefore, the solution still remains optimal.
   New C = 830 – (1)(41) = 789.
3. Non-binding constraints are Remote Station1 to Remote Station2, Remote Station1 to Destination, Remote Station2 to Destination. Change in the right-hand side of these constraints within the allowable increase or decrease would not change the value of objective function value. Any changes above or below the allowable margin (range of optimality) would disturb the optimality and the problem has to be resolved.

Recommendations considering the base-case and the alternative solution

The following recommendations may apply when we compare the base case with the alternative cases:

1. The seat allocation model in Indian Railways should be flexible to allow significant changes in the demand between stations be accommodated against seat quota available in the train. This would ensure robustness of the model under various situations of abnormally high or abnormally low demand for different combinations of stations.
2. Tatkal quota should be part of train's quota and its seat allocation should be optimized to increase Indian Railway's flexibility while maintaining efficient occupancy in the trains. Efficient operation and flexibility would also improve Indian Railways revenue.
3. Seats for physically challenged, disabled and pregnant ladies shall be allocated a separate quota to ensure availability of seats for people with challenges. However, in case of low demands for these seats, the same should be transferred to tatkal quota. This would ensure fairness and equal chances to different classes of passengers. Also, the occupancy efficiency will increase and keep Indian Railways more profitable.

Summary & Future work

Summary

The study's model can help Indian Railways to reduce its seat requirements and increase the availability of confirmed seats for the various station-to-station demands on several trains. The model performs well under different conditions of varying demands. Despite the limitations mentioned in model deficiencies; the model provides a good foundation for improving the capacity utilization of long-distance passenger trains. It has provided a systematic basis for one aspect of capacity management as part of overall revenue management on long distance passenger rail services.

Future Work

1. Allocate seats with the goal to maximize revenue (Limitation: Short distance travel)
2. Considering all trains running between the different stations (Line capacity estimation)
3. IR does practice revenue and peak-load management by pricing through tatkal bookings: it charges higher fares for late-decision travel and for peak periods (Dynamic pricing)
4. Splitting a train's overall capacity across various classes of travel. This topic could be considered as a part of the overall revenue management function.
5. Including pre-booking and cancellation guide constraints.
6. Adding constraints for reservations of seats for handicapped and elderly people.
7. Train service network design problem: The objective of the model is to optimize the transportation of all the shipments with minimal costs. The costs consist of accumulation costs, classification costs, train operation costs, and train travel costs (Cost Minimization).
8. Train operating problem as a mixed integer programming model to determine which pairs of terminals are to be provided with direct train connections and the frequencies of service (Scheduling).

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