

# **Assignment 1(Report)**

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## **1. Derivation of the Overall Heat Transfer Coefficient (U<sub>o</sub>)**

This analysis considers a straight double-pipe heat exchanger. The derivation is based on a thermal resistance network for heat transfer from the hot fluid (inside) to the cold fluid (outside).

Assumptions:

- i) Steady-state heat transfer.
- ii) One-dimensional radial conduction through the cylindrical wall.
- iii) Constant properties for fluids and wall (i.e.,  $h_i$ ,  $h_o$ ,  $k_{wall}$  are constant).
- iv) Negligible axial conduction and heat loss to the surroundings.

The total thermal resistance ( $R_{total}$ ) for the heat path (based on an arbitrary length  $L$ ) is the sum of four resistances in series: inner convection ( $R_{conv,i}$ ), wall conduction ( $R_{wall}$ ), outer convection ( $R_{conv,o}$ ), and outer fouling ( $R_{fouling,o}$ ).

$$R_{total} = R_{conv,i} + R_{wall} + R_{conv,o} + R_{fouling,o}$$

The individual resistances for a cylindrical system of length L are:

$$R_{conv,i} = 1/(h_i * A_i) = 1/(h_i * \pi * D_i * L)$$

$$R_{wall} = \ln(D_o/D_i)/2*\pi*k_{wall}*L$$

$$R_{conv,o} = 1/(h_o * A_o) = 1/(h_o * \pi * D_o * L)$$

$$R_{fouling} = R_f/A_o = R_f/(\pi * D_o * L)$$

Expression for  $1/U_o$

The overall heat transfer coefficient based on the outer surface area ( $A_o$ ) is related to the total resistance by:

$$1/U_o = A_o * R_{total}$$

Substituting the individual resistances and  $A_o = \pi * D_o * L$

$$\frac{1}{U_o} = (\pi D_o L) \left( \frac{1}{h_i \pi D_i L} + \frac{\ln(D_o/D_i)}{2\pi k_{wall} L} + \frac{1}{h_o \pi D_o L} + \frac{R_f}{\pi D_o L} \right)$$

$$\frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{D_o \ln(D_o/D_i)}{2k_{wall}} + \frac{1}{h_o} + R_f$$

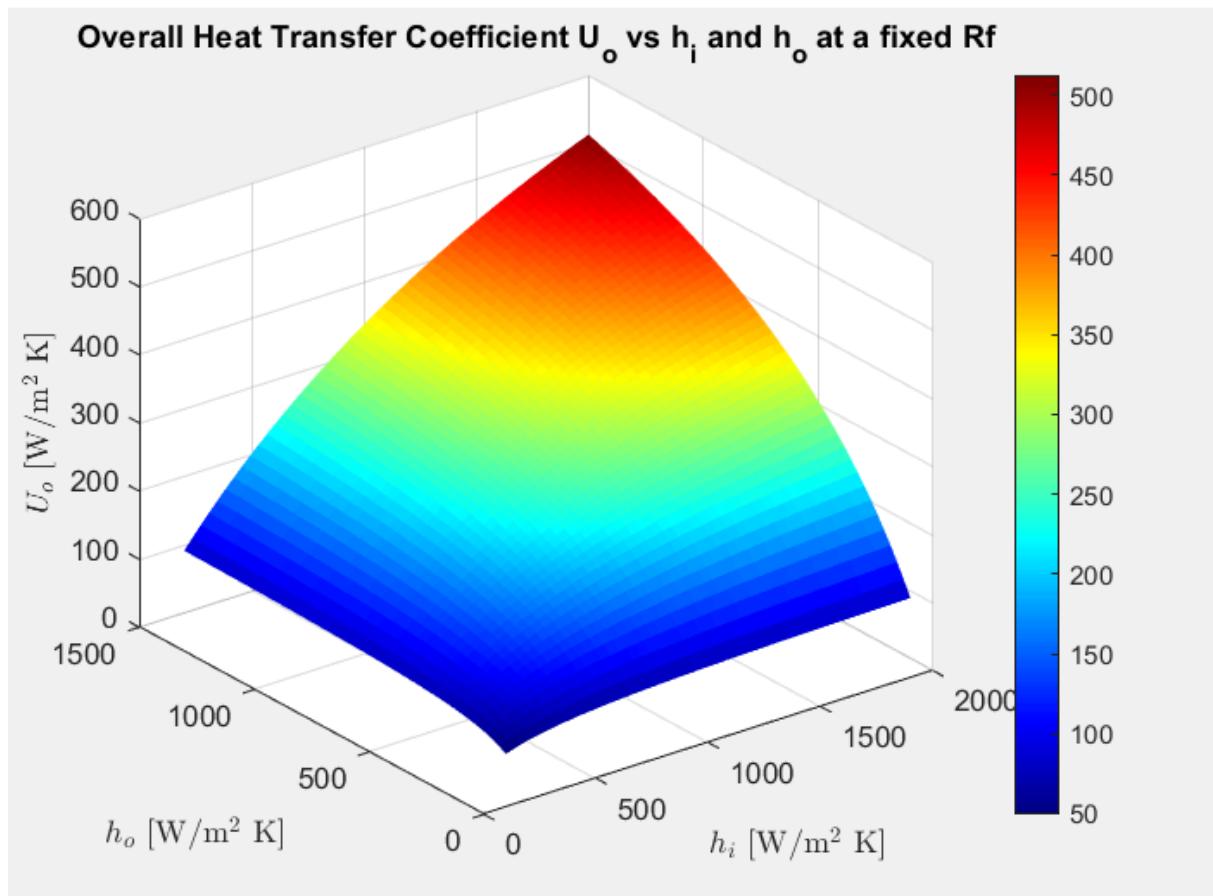
This is the final general expression for  $1/U_o$

## 2. MATLAB Analysis and Plots

The following plots were generated using MATLAB based on the derived expression and the given parameters ( $D_i = 0.020$  m,  $D_o = 0.040$  m,  $k_{wall} = 385$  W/m K)

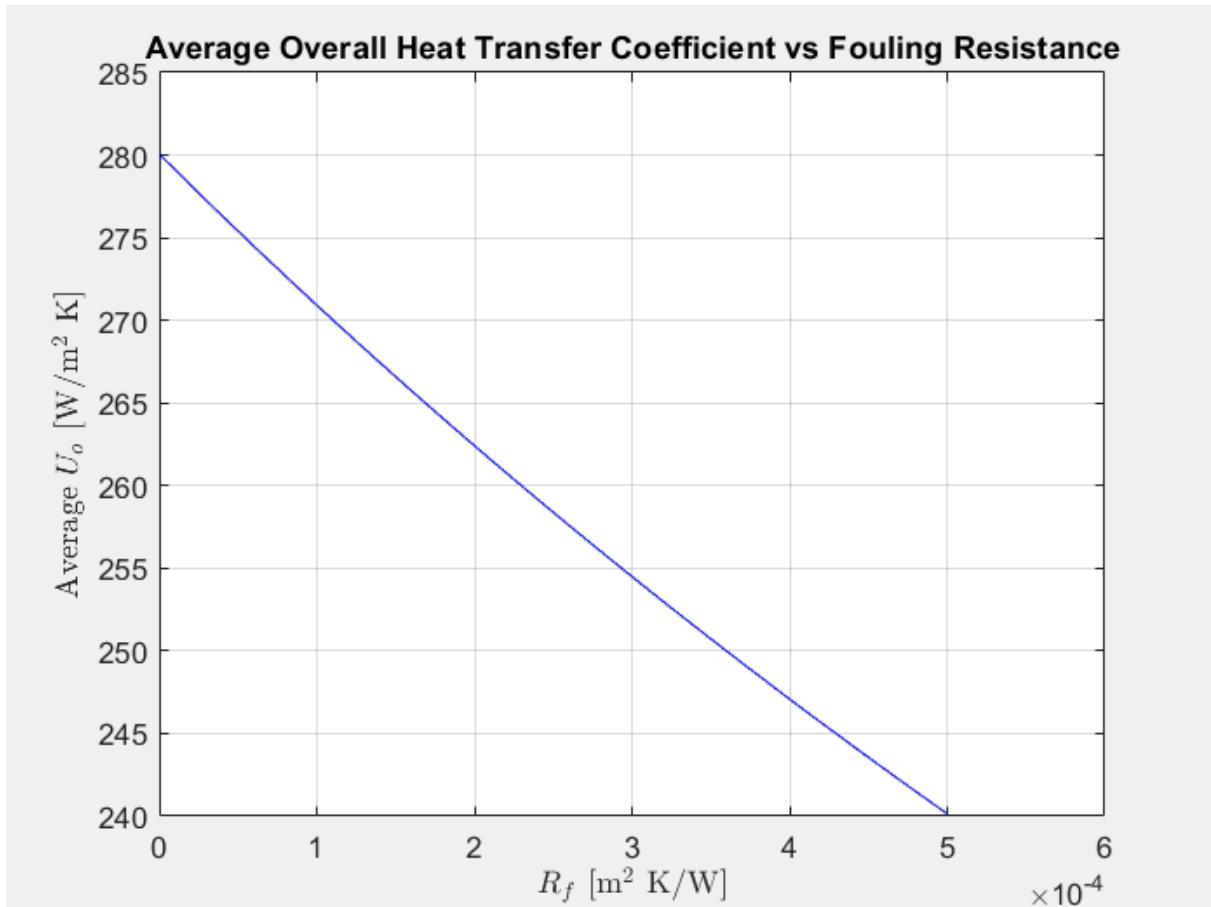
**Plot 1: 3D Surface Plot ( $U_o$  vs  $h_i$  and  $h_o$ )**

**Figure 1: Heat Transfer Coefficient  $U_o$  vs  $h_i$  and  $h_o$   
(Fixed  $R_f = 2.5e-4 \text{ m}^2 \text{ K/W}$ )**



**Plot 2: 2D Plot (Average  $U_o$  vs  $R_f$ )**

**Figure 2: Average Overall Heat Transfer Coefficient vs  
Fouling Resistance**



## Answers to Conceptual Problems —

1. For the given geometry ( $D_o/D_i = 2$ ), the **inner convective coefficient ( $h_i$ ) has a stronger influence** on the overall heat transfer coefficient ( $U_o$ ). This is because the inner convection resistance term ( $1/h_i$ ) is multiplied by the area ratio ( $D_o/D_i$ ) when based on the outer area, effectively doubling its contribution to the total resistance compared to the outer term ( $1/h_o$ ). If  $D_o$  and  $D_i$  were much more different (i.e.,  $D_o/D_i \gg 1$ ), the influence of  $h_i$  would be amplified further.
2. The wall thermal conductivity ( $k_{\text{wall}}$ ) is inversely proportional to the wall conduction resistance. A higher  $k_{\text{wall}}$  (like copper,  $k \sim 385 \text{ W/m K}$ ) results in a negligible

wall resistance and a higher  $U_o$ . Conversely, switching to a material with low  $k$  (like stainless steel,  $k \sim 16 \text{ W/m K}$ ) significantly increases the wall resistance, thereby **decreasing  $U_o$** . Since the required heat transfer area is inversely proportional to  $U_o$ , a lower  $k$  means a **larger heat exchanger** is necessary to achieve the same heat duty.

3. Heat transfer can be dominated by: 1) **Internal Convection** (low  $h_i$ , often due to viscous fluid), necessitating an increase in internal fluid velocity. 2) **Wall Conduction** (low  $k_{\text{wall}}$ ), requiring the use of a higher conductivity material. 3) **Fouling** (high  $R_f$ ), which requires strict maintenance and cleaning schedules. The regime of dominance indicates where design and operational efforts should be focused to maximize efficiency.
4. The overall heat transfer coefficient ( $U_o$ ) is **most sensitive to small increases in fouling resistance ( $R_f$ ) when the exchanger is clean** ( $R_f$  is near zero). The rate of performance drop slows down as  $R_f$  increases. A typical cleaning criterion involves determining the  $R_f$  value at which  $U_o$  drops to an unacceptable level, often **75% to 80% of its clean value**, to balance maintenance costs against the economic penalty of reduced performance.

**Key insights :-**

The analysis highlights that the **inner convective heat transfer coefficient (hi) has a stronger influence on  $U_o$**  than  $h_o$  because the inner resistance term is amplified by the area ratio ( $D_o/D_i$ ) when calculating  $U_o$  based on the outer area  $A_o$ . This is a critical design insight, suggesting that increasing flow velocity or turbulence inside the inner tube yields the most significant performance gains (Figure 1). Furthermore, while a high-conductivity wall material like copper makes the wall resistance negligible, a low-conductivity material drastically reduces  $U_o$ , necessitating a much larger required heat transfer area. Operationally, the **fouling resistance ( $R_f$ ) causes a performance decay** where  $U_o$  is most sensitive to small increases in  $R_f$  when the exchanger is clean. Therefore, an effective maintenance strategy must be based on a clear efficiency threshold (e.g., cleaning when  $U_o$  drops to 75% of its clean value) to balance maintenance costs against energy efficiency losses (Figure 2).