

Assignment 1(Report)

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1. Derivation of the Overall Heat Transfer Coefficient (U_o)

This analysis considers a straight double-pipe heat exchanger. The derivation is based on a thermal resistance network for heat transfer from the hot fluid (inside) to the cold fluid (outside).

Assumptions:

- i) Steady-state heat transfer.
- ii) One-dimensional radial conduction through the cylindrical wall.
- iii) Constant properties for fluids and wall (i.e., h_i , h_o , k_{wall} are constant).
- iv) Negligible axial conduction and heat loss to the surroundings.

The total thermal resistance (R_{total}) for the heat path (based on an arbitrary length L) is the sum of four resistances in series: inner convection ($R_{\text{conv},i}$), wall conduction (R_{wall}), outer convection ($R_{\text{conv},o}$), and outer fouling ($R_{\text{fouling},o}$).

$$R_{\text{total}} = R_{\text{conv},i} + R_{\text{wall}} + R_{\text{conv},o} + R_{\text{fouling},o}$$

The individual resistances for a cylindrical system of length L are:

$$R_{\text{conv},i} = 1/(h_i \cdot A_i) = 1/(h_i \cdot \pi \cdot D_i \cdot L)$$

$$R_{\text{wall}} = \ln(D_o/D_i)/2\pi k_{\text{wall}} L$$

$$R_{\text{conv},o} = 1/(h_o \cdot A_o) = 1/(h_o \cdot \pi \cdot D_o \cdot L)$$

$$R_{\text{fouling}} = R_f/A_o = R_f/(\pi \cdot D_o \cdot L)$$

Expression for $1/U_o$

The overall heat transfer coefficient based on the outer surface area (A_o) is related to the total resistance by:

$$1/U_o = A_o \cdot R_{\text{total}}$$

Substituting the individual resistances and $A_o = \pi \cdot D_o \cdot L$

$$\frac{1}{U_o} = (\pi D_o L) \left(\frac{1}{h_i \pi D_i L} + \frac{\ln(D_o/D_i)}{2\pi k_{\text{wall}} L} + \frac{1}{h_o \pi D_o L} + \frac{R_f}{\pi D_o L} \right)$$

$$\frac{1}{U_o} = \frac{D_o}{D_i h_i} + \frac{D_o \ln(D_o/D_i)}{2k_{\text{wall}}} + \frac{1}{h_o} + R_f$$

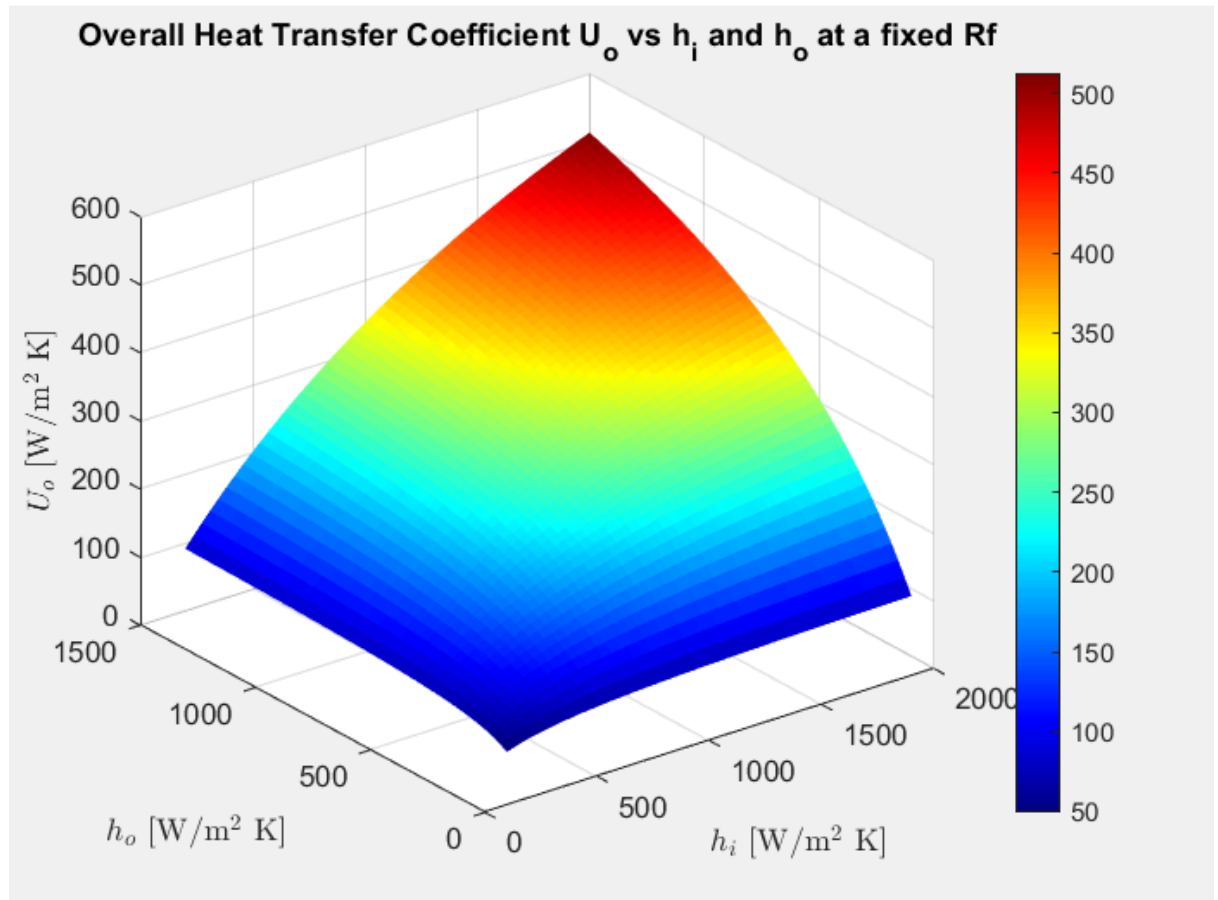
This is the final general expression for $1/U_o$

2. MATLAB Analysis and Plots

The following plots were generated using MATLAB based on the derived expression and the given parameters ($D_i = 0.020$ m, $D_o = 0.040$ m, $k_{\text{wall}} = 385$ W/m K)

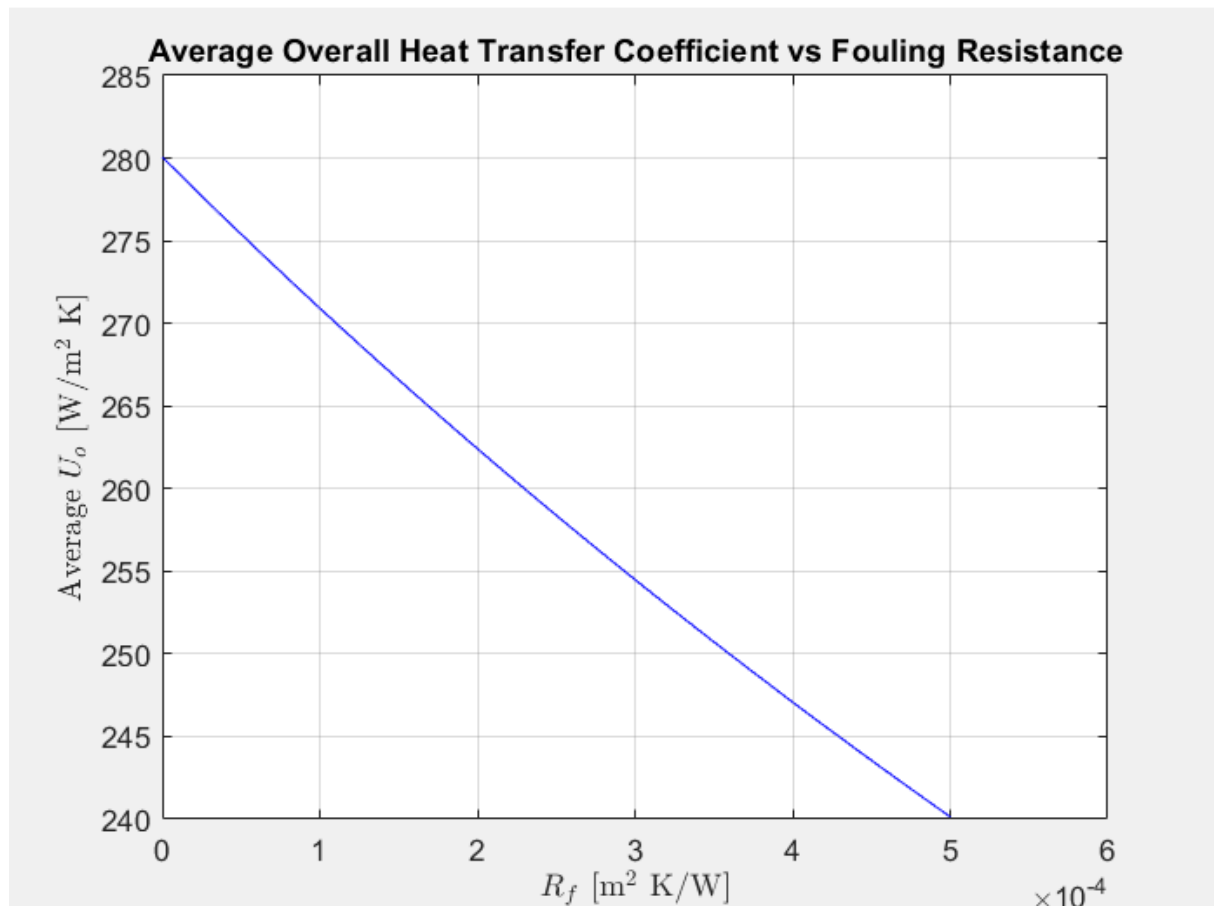
Plot 1: 3D Surface Plot (U_o vs h_i and h_o)

**Figure 1: Heat Transfer Coefficient U_o vs h_i and h_o
(Fixed $R_f = 2.5 \times 10^{-4} \text{ m}^2 \text{ K/W}$)**



Plot 2: 2D Plot (Average U_o vs R_f)

**Figure 2: Average Overall Heat Transfer Coefficient vs
Fouling Resistance**



Answers to Conceptual Problems —

1. For the given geometry ($D_o/D_i = 2$), the **inner convective coefficient (h_i) has a stronger influence** on the overall heat transfer coefficient (U_o). This is because the inner convection resistance term ($1/h_i$) is multiplied by the area ratio (D_o/D_i) when based on the outer area, effectively doubling its contribution to the total resistance compared to the outer term ($1/h_o$). If D_o and D_i were much more different (i.e., $D_o/D_i \gg 1$), the influence of h_i would be amplified further.
2. The wall thermal conductivity (k_{wall}) is inversely proportional to the wall conduction resistance. A higher k_{wall} (like copper, $k \sim 385 \text{ W/m K}$) results in a negligible

wall resistance and a higher U_o . Conversely, switching to a material with low k (like stainless steel, $k \sim 16 \text{ W/m K}$) significantly increases the wall resistance, thereby **decreasing U_o** . Since the required heat transfer area is inversely proportional to U_o , a lower k means a **larger heat exchanger** is necessary to achieve the same heat duty.

3. Heat transfer can be dominated by: 1) **Internal Convection** (low h_i , often due to viscous fluid), necessitating an increase in internal fluid velocity. 2) **Wall Conduction** (low k_{wall}), requiring the use of a higher conductivity material. 3) **Fouling** (high R_f), which requires strict maintenance and cleaning schedules. The regime of dominance indicates where design and operational efforts should be focused to maximize efficiency.
4. The overall heat transfer coefficient (U_o) is **most sensitive to small increases in fouling resistance (R_f) when the exchanger is clean** (R_f is near zero). The rate of performance drop slows down as R_f increases. A typical cleaning criterion involves determining the R_f value at which U_o drops to an unacceptable level, often **75% to 80% of its clean value**, to balance maintenance costs against the economic penalty of reduced performance.

Keys insights :-

The analysis highlights that the **inner convective heat transfer coefficient (h_i) has a stronger influence on U_o** than h_o because the inner resistance term is amplified by the area ratio (D_o/D_i) when calculating U_o based on the outer area A_o . This is a critical design insight, suggesting that increasing flow velocity or turbulence inside the inner tube yields the most significant performance gains (Figure 1). Furthermore, while a high-conductivity wall material like copper makes the wall resistance negligible, a low-conductivity material drastically reduces U_o , necessitating a much larger required heat transfer area. Operationally, the **fouling resistance (R_f) causes a performance decay** where U_o is most sensitive to small increases in R_f when the exchanger is clean. Therefore, an effective maintenance strategy must be based on a clear efficiency threshold (e.g., cleaning when U_o drops to 75% of its clean value) to balance maintenance costs against energy efficiency losses (Figure 2).