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# Assignment 2

## Double-Pipe Heat Exchanger - special case Analysis

To compute:- $\lim \Delta T_{LMTD}$ ~~for~~ for a counter-flow heat exchanger

soln.- $\Delta T_{LMTD}$ → Log mean temperature difference

given:- $\dot{C}_c = \dot{C}_h$ (equal heat-capacity rates)

→ The ~~xxx~~ rate of heat lost by the hot fluid must equal the rate of heat gained by the cold fluid:

$$\dot{Q} = \dot{C}_h (T_h^{in} - T_h^{out}) = \dot{C}_c (T_c^{out} - T_c^{in})$$

since, $\dot{C}_h = \dot{C}_c$ $\Rightarrow$ $T_h^{in} - T_h^{out} = T_c^{out} - T_c^{in}$

$$\Rightarrow T_h^{in} - T_c^{out} = T_h^{out} - T_c^{in} \quad \text{— (1)}$$

now, For a counter-Flow heat exchanger, the temperature difference at the two ends are defined as:

$$\Delta T_I = T_h^{in} - T_c^{out}$$

$$\Delta T_{II} = T_h^{out} - T_c^{in}$$

⇒ from (1) → $\Delta T_I = \Delta T_{II}$ (The temperature difference ~~at the two ends are defined a~~ between the hot and cold streams is constant along the entire length of the heat exchanger)

let, $\Delta T_0 = \Delta T_I = \Delta T_{II}$

$$\Rightarrow \Delta T_{LMTD} = \frac{\Delta T_I - \Delta T_{II}}{\ln\left(\frac{\Delta T_I}{\Delta T_{II}}\right)}$$

→ putting $\Delta T_I = \Delta T_{II} = \Delta T_0$

$$\Delta T_{LMTD} = \frac{\Delta T_o - \Delta T_o}{\ln\left(\frac{\Delta T_o}{\Delta T_o}\right)} = \frac{0}{\ln(1)} = \frac{0}{0} \text{ (indeterminate form)}$$

$\Rightarrow$ let, $\gamma = \frac{\Delta T_I}{\Delta T_{II}}$

now, $$\lim_{\Delta T_I \to \Delta T_{II}} \Delta T_{LMTD} = \lim_{\gamma \to 1} \left[\Delta T_{II} \cdot \frac{\gamma - 1}{\ln \gamma}\right]$$

$\rightarrow$ L'Hospital rule, $$\lim_{\gamma \to 1} \Delta T_{II} \frac{\frac{\partial}{\partial \gamma}(\gamma - 1)}{\frac{\partial}{\partial \gamma} \ln \gamma} = \lim_{\gamma \to 1} \Delta T_{II} \frac{1}{1/\gamma} = \lim_{\gamma \to 1} \Delta T_{II} \gamma = \Delta T_{II}$$

$\therefore$ The limit, $$\lim_{\dot{C}_c \to \dot{C}_h} \Delta T_{LMTD} = \Delta T_{II} \cdot (1) = \Delta T_o$$

$$\therefore \boxed{\lim_{\dot{C}_c \to \dot{C}_h} \Delta T_{LMTD} = \Delta T_o = T_h^{in} - T_c^{out} = T_h^{out} - T_c^{in}}$$

$\rightarrow$ We know that for a counter-flow heat exchanger:

$$\frac{\Delta(T_h - T_c)}{T_h - T_c} = -UA\left(\frac{1}{\dot{C}_c} - \frac{1}{\dot{C}_h}\right) \quad \text{where } A = 2\pi r \Delta x$$

$\Rightarrow$ in differential form

$$\frac{d(\Delta T)}{dx} = -UA'\left(\frac{1}{\dot{C}_c} - \frac{1}{\dot{C}_h}\right)\Delta T$$

$\rightarrow$ applying the condition $\dot{C}_c = \dot{C}_h$ we get,

$$\frac{d(\Delta T)}{dx} = -UA'(0)\Delta T \Rightarrow \boxed{\frac{d(\Delta T)}{dx} = 0}$$

$\rightarrow$ The exponential behavior fails because the term that governs the exponential change becoms zero for this special counter-flow case, leading to linear temperature profile instead of an exponential one.