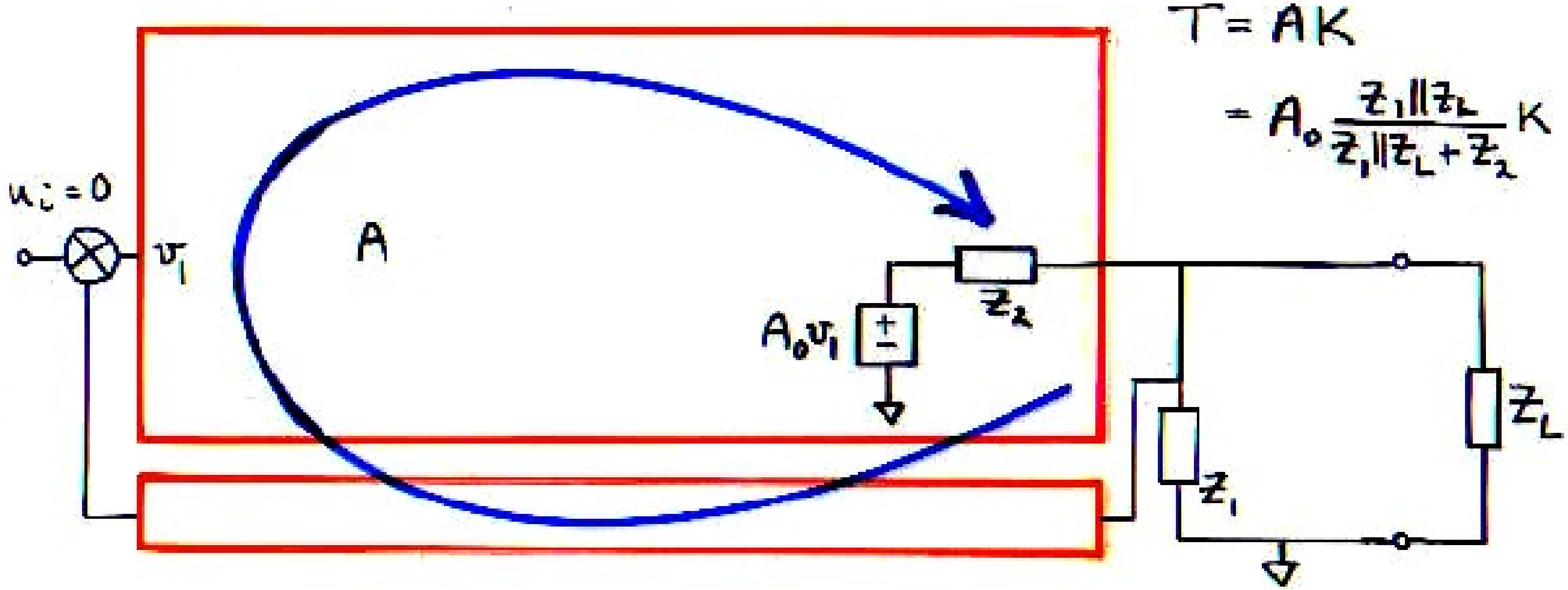
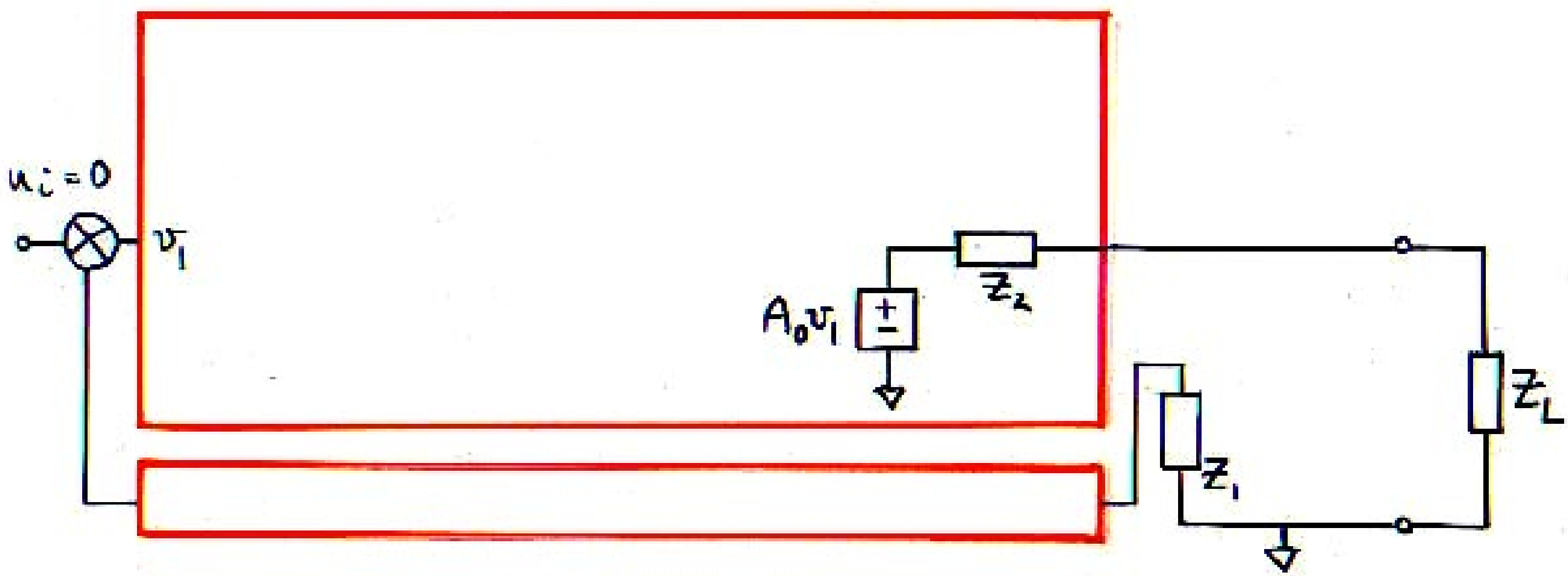
9

ALL ABOUT T

$$T = AK = A_0 \frac{Z_1 \| Z_L}{Z_1 \| Z_L + Z_2} K$$

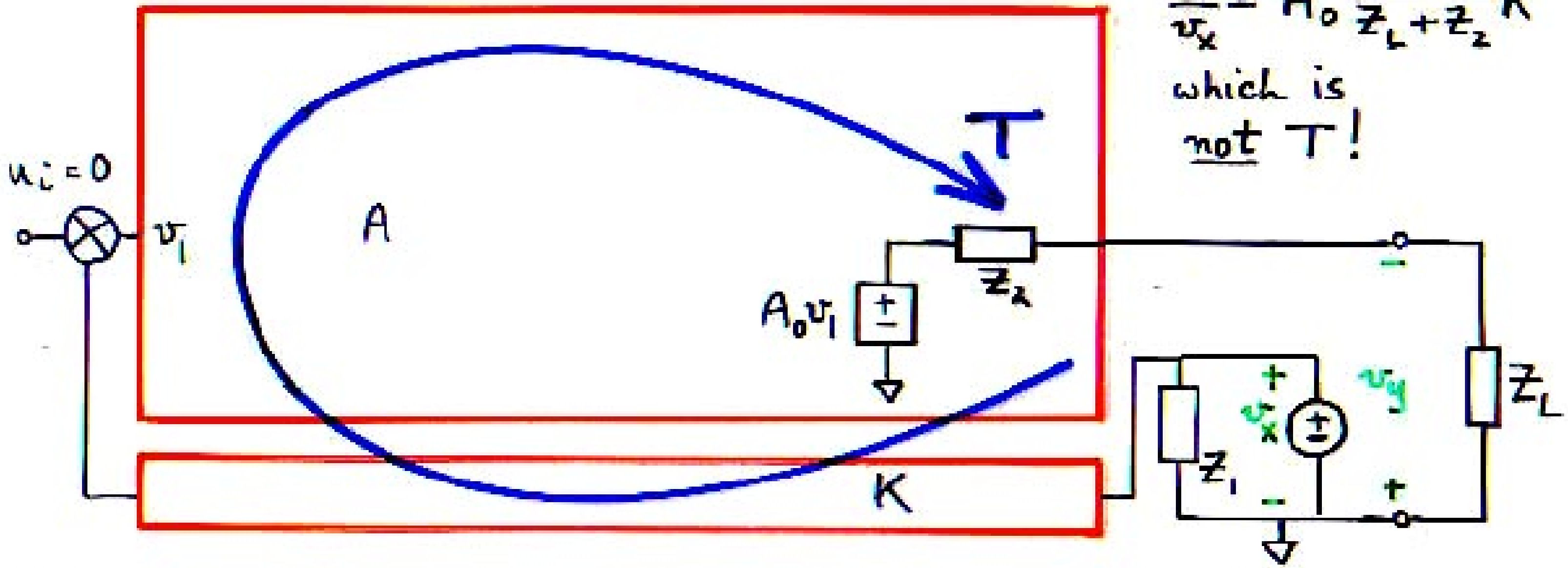




$u_i = 0$
v_1
$A_0 v_1$
Z_2
Z_1
Z_L

$$\frac{v_y}{v_x} = A_0 \frac{Z_L}{Z_L + Z_2} K$$

which is <u>not</u> T!



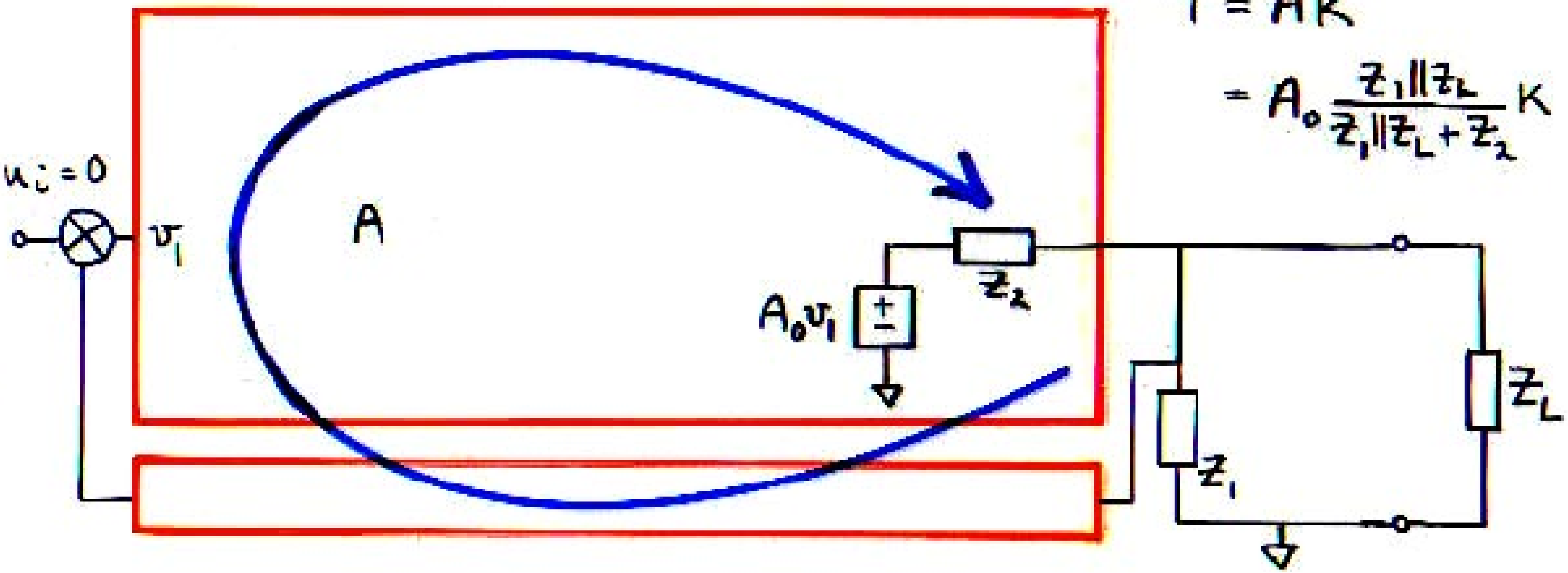
Conventional procedure: break the loop at the output (or input) of the forward path, and find $T = \frac{v_y}{v_x} = AK$.

However, often the feedback path loads the forward path at the output (or input), so the value found for A and/or K is not the same as when the loop is closed, and so the wrong answer is obtained for T.

In some cases the loading may be simulated, but involves guesswork.

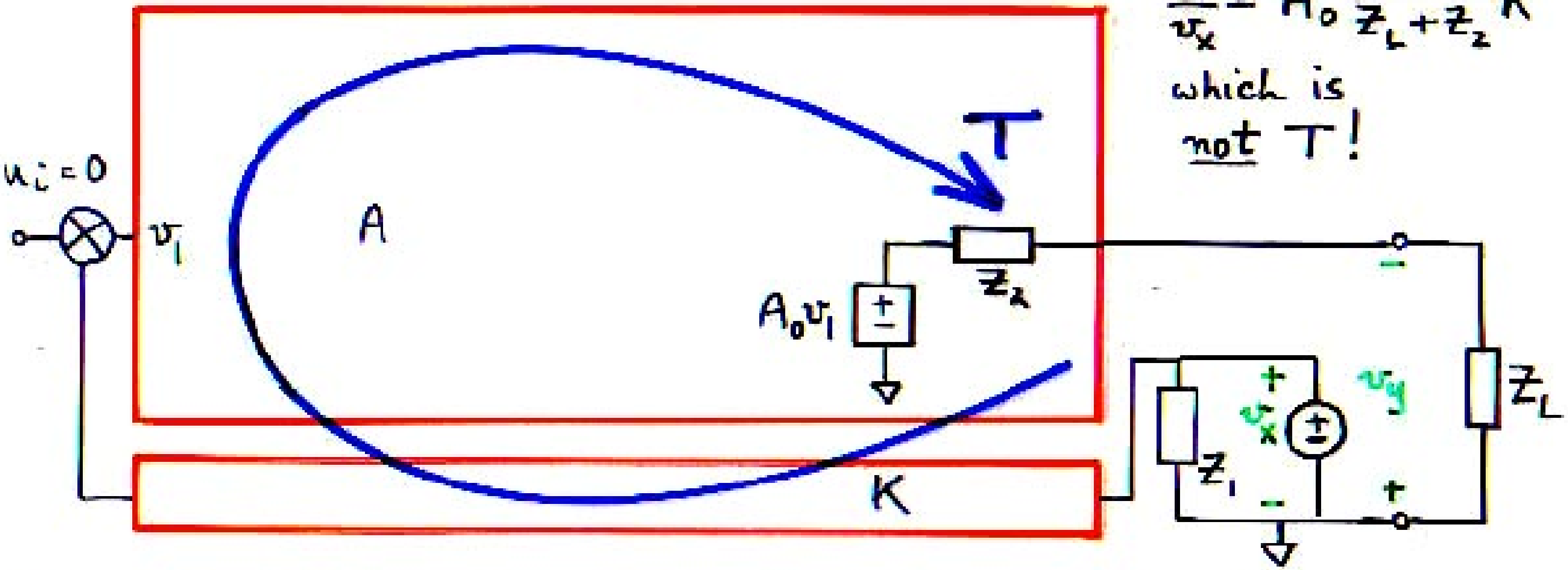
$$T = AK$$

$$= A_0 \frac{Z_1 \| Z_L}{Z_1 \| Z_L + Z_2} K$$



$$\frac{v_y}{v_x} = A_0 \frac{Z_L}{Z_L + Z_2} K$$

which is <u>not</u> T!



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However, often the feedback path loads the forward path at the output (or input), so the value found for A and/or K is not the same as when the loop is closed, and so the wrong answer is obtained for T.

In some cases the loading may be simulated, but involves guesswork.

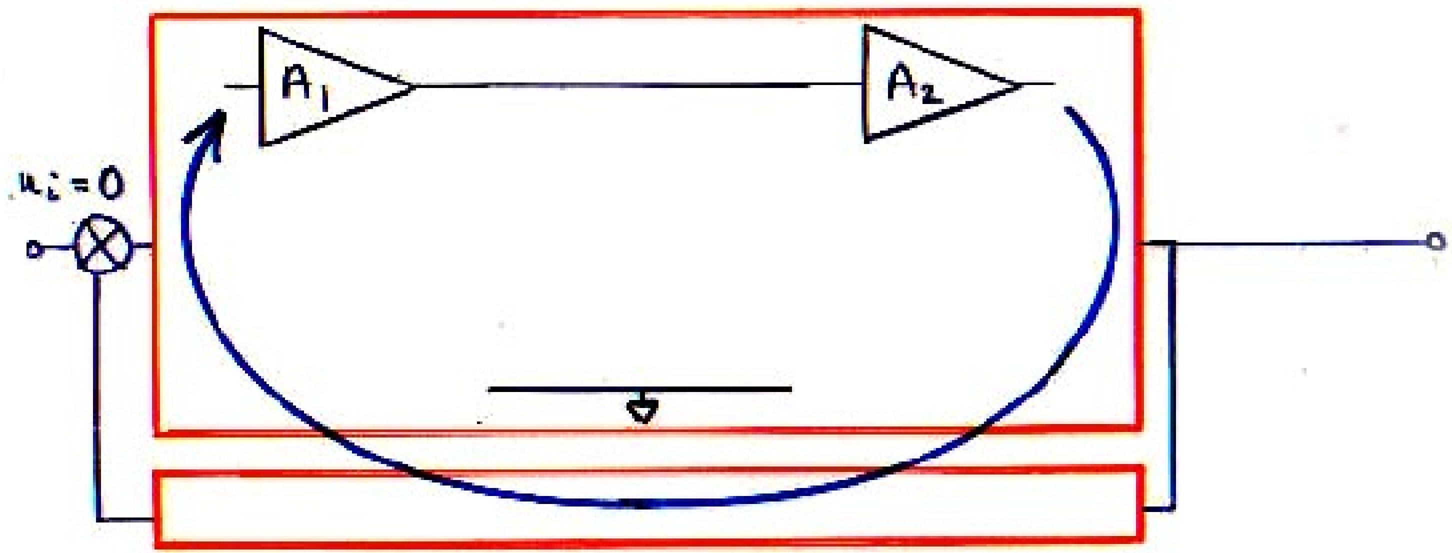
Instead, since A need not be known separately, break the loop at a point where the loading is not disturbed (usually somewhere in the forward path).

Loop Gain by Signal Injection into the Closed Loop

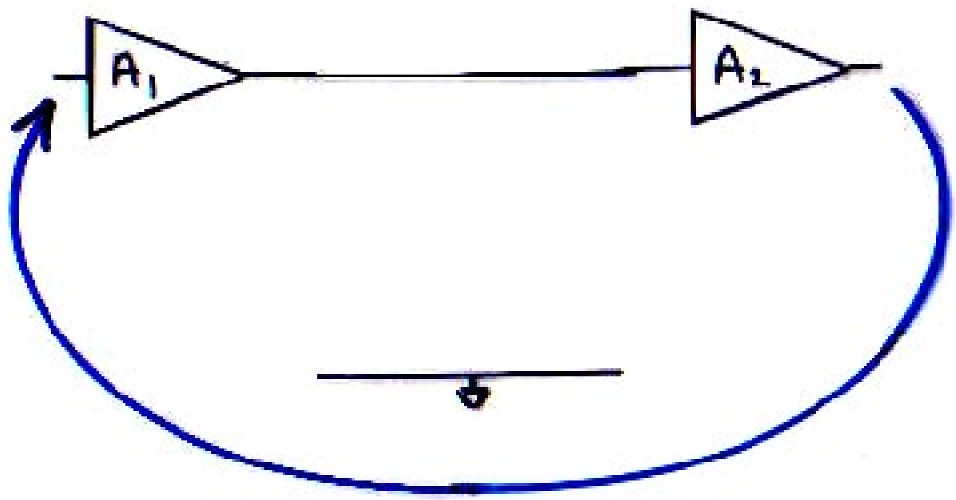
One of the most valuable benefits of the improved feedback formula is that, since A does not have to be known separately, the loop need not be broken at either the input or the output.

Instead, the break can be made anywhere in the loop, so that a test point can be chosen where the loading is not upset by the break.

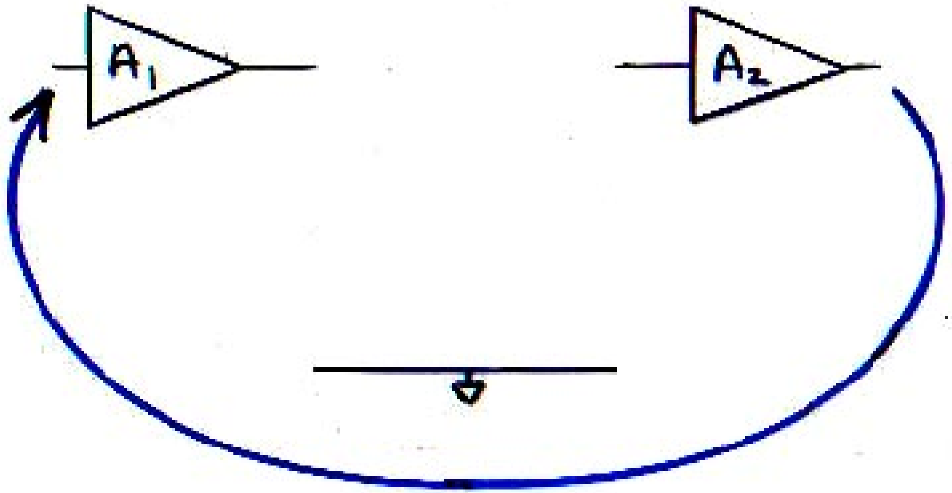
A convenient test point is at a dependent generator in the model.



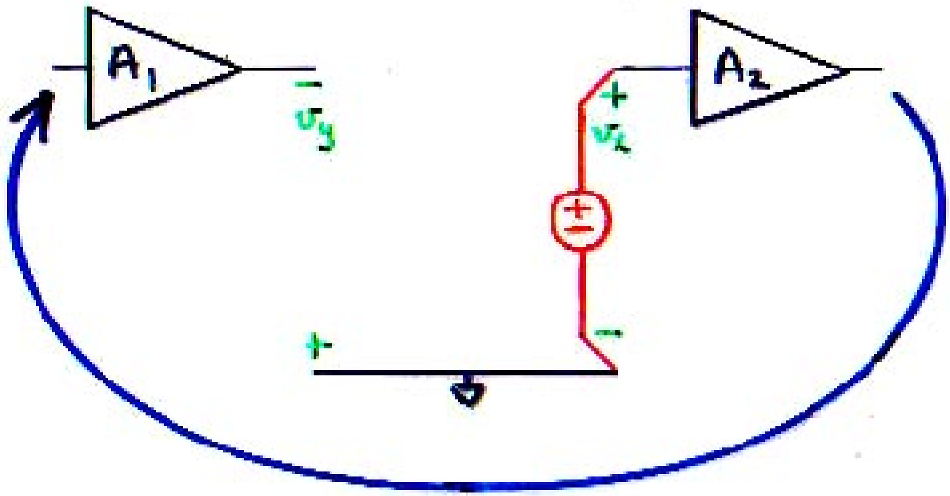
$u_i = 0$
A_1
A_2



A1
A2

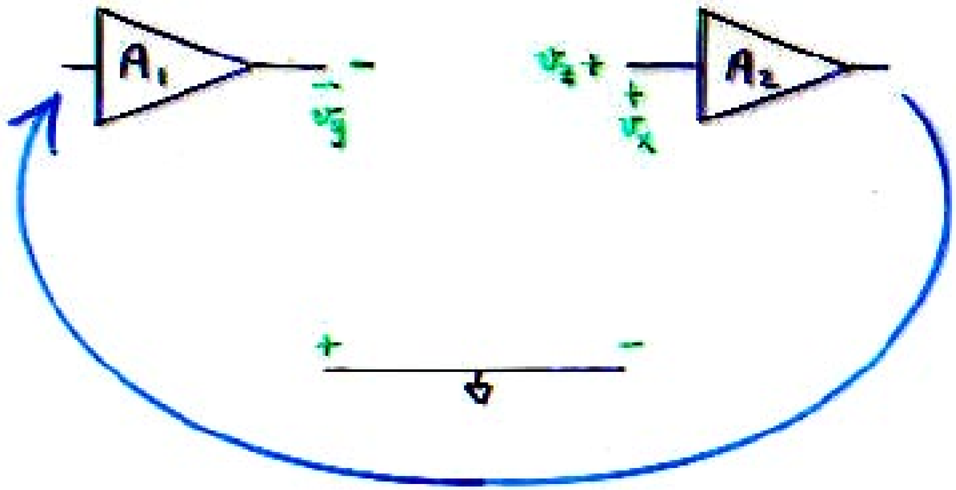


A1
A2



If A_1 and A_2 are unaffected by breaking the loop between A_1 and A_2, a test signal v_x may be applied at the input of A_2, and the resulting signal v_y at the output of A_1 gives the correct value of the loop gain T as

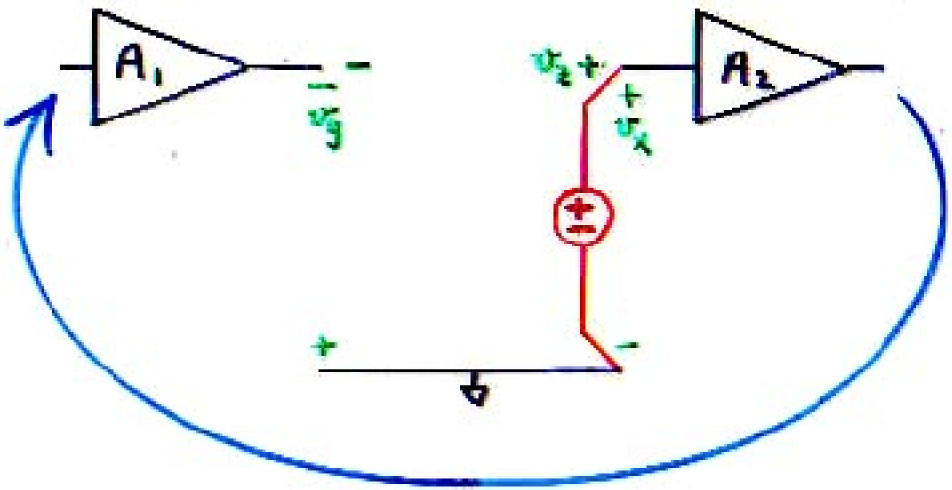
$$T = \frac{v_y}{v_x}$$



Under such conditions, a voltage $v_z = v_x + v_y$ exists across the break.

It makes no difference to the ratio v_y / v_x whether the driving signal is v_x or v_z.

Hence $T = \frac{v_y}{v_x}$, regardless of whether the system is driven by v_x or v_z.



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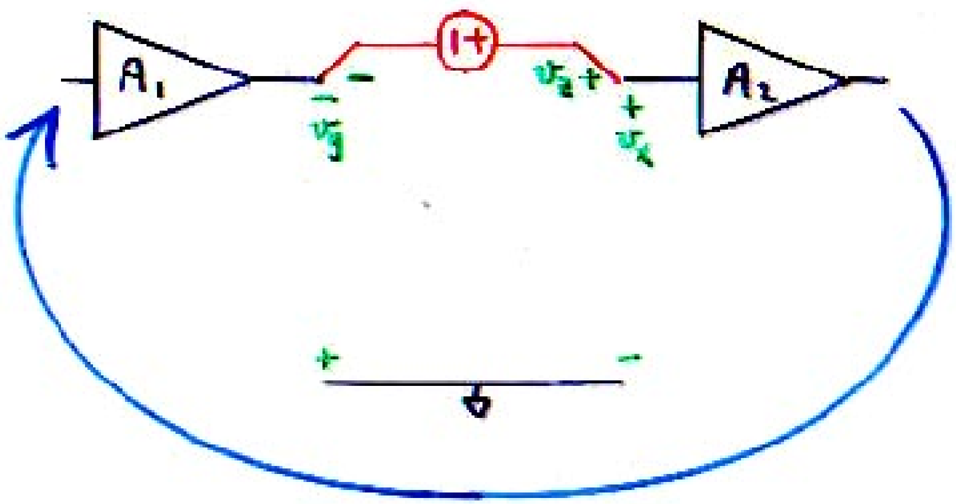

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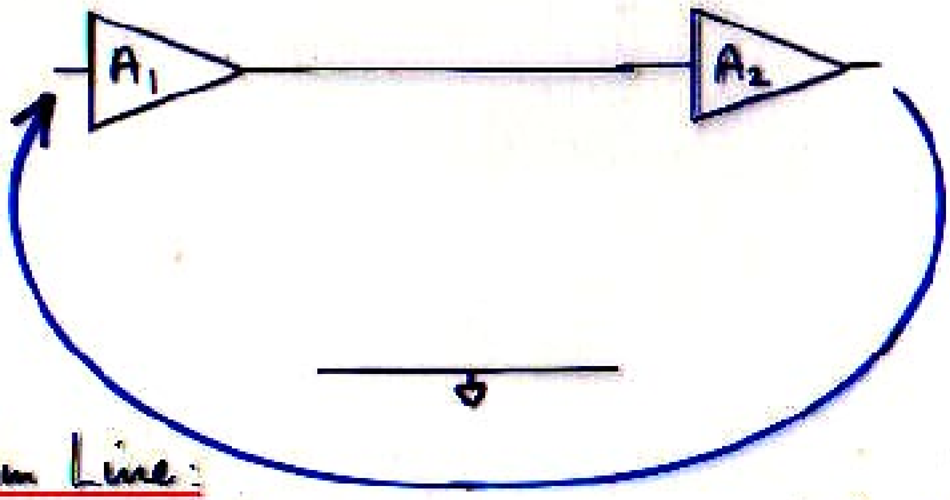
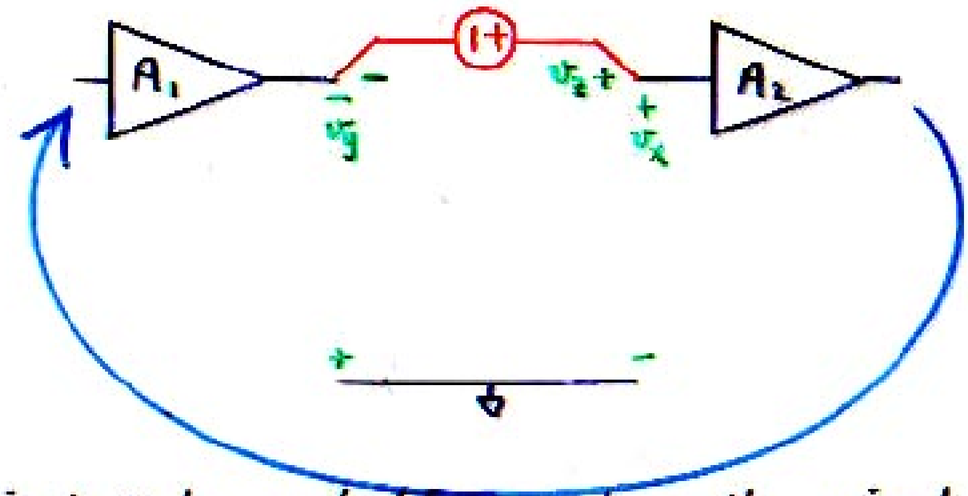


Bottom Line:

There are great advantages, especially for experimental application, in use of an injected test signal into the closed loop:

1. The loop does not have to be broken, with consequent difficulties in maintaining the proper operating point.

2. The circuit remains in the same configuration (loop remains closed).



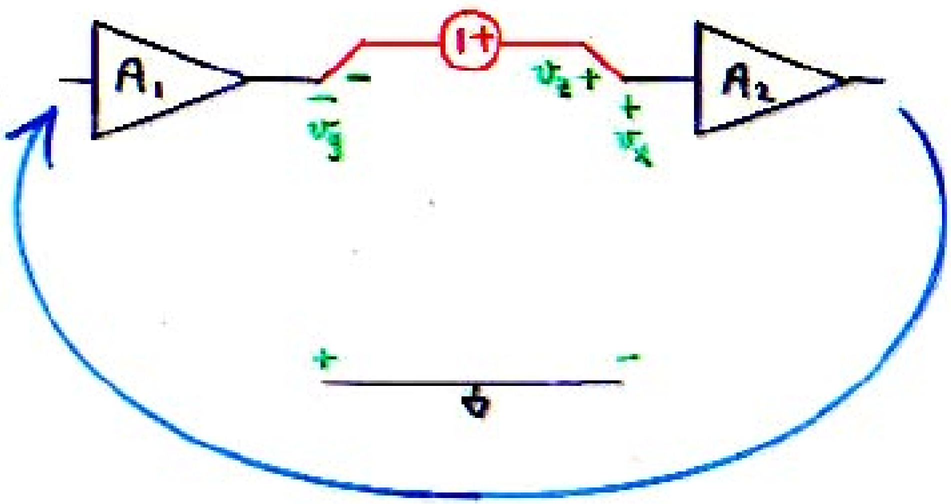
Physical interpretation of how the signals adjust to injection of a test signal into the closed loop

$v_x + v_y = v_z$ phasor sum, fixed by the drive v_z

$\frac{v_y}{v_x} = T$ phasor ratio, fixed by the circuit

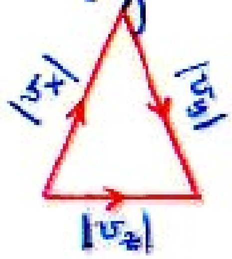
Hence, the values of v_x and v_y adjust themselves to satisfy simultaneously these two conditions:

$$v_x = \frac{1}{1+T} v_z \qquad v_y = \frac{T}{1+T} v_z$$



Phasor diagrams for given $|v_z|$ at increasing frequencies:

Below crossover frequency, $f \ll f_c$: $\left|\frac{v_y}{v_x}\right| \gg 1$



At crossover frequency, $f = f_c$: $\left|\frac{v_y}{v_x}\right| = 1$

Above crossover frequency, $f \gg f_c$: $\left|\frac{v_y}{v_x}\right| \ll 1$

Generalization: Determination of Loop Gain by Injection of a Test Signal into the Closed Loop

Preferable to application of a test signal to a broken loop, because:

1. The loop remains closed, avoiding disturbances of operating point.
2. The circuit remains in the same configuration.

However, injection must be done at a point where, if the loop were broken, there would be negligible change in the loading effects.

Determination of Feedback System Parameters — General

The method of Loop Gain T determination T by injection of a test signal into the closed loop can be generalized and, by inclusion of the Null Double Injection technique, also leads to determination of the Ideal Closed-Loop Gain G_∞.

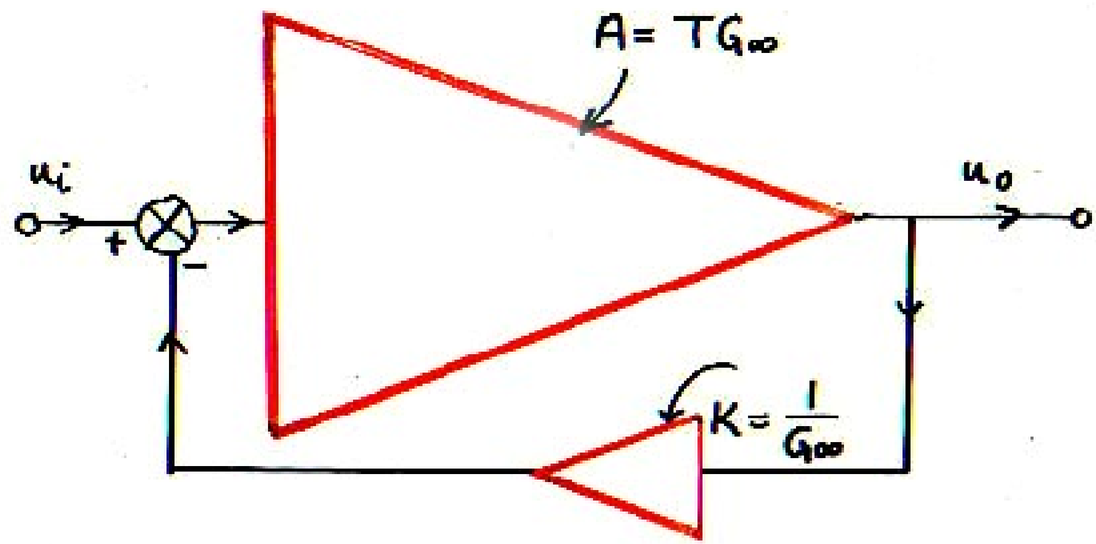
Basic relations:

$$G = \frac{A}{1+AK} = \frac{A}{1+T}$$

$$= \frac{1}{K}\frac{T}{1+T}$$

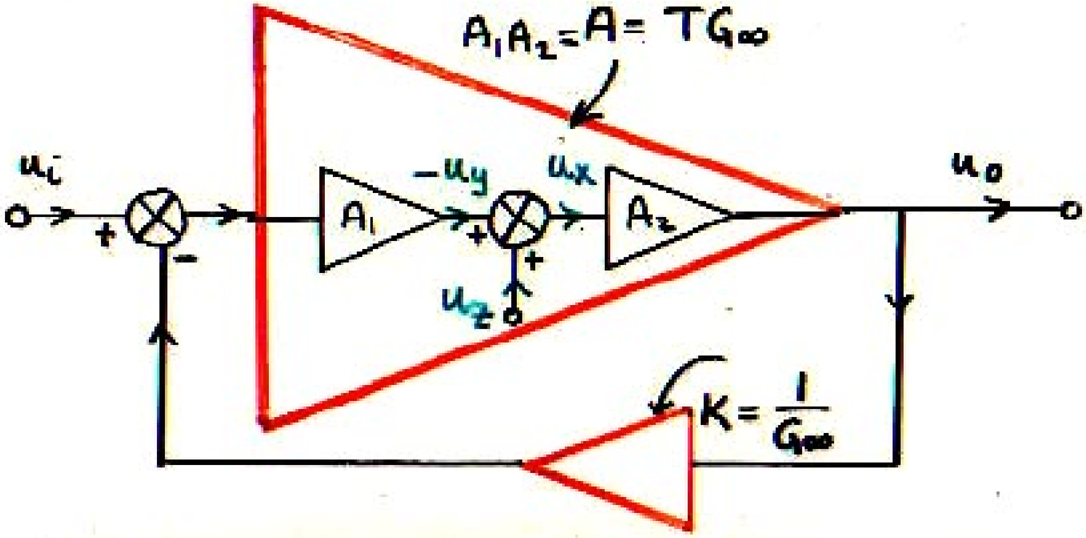
$$= G_\infty D$$

Note that $A = TG_\infty$ and $K = 1/G_\infty$:



Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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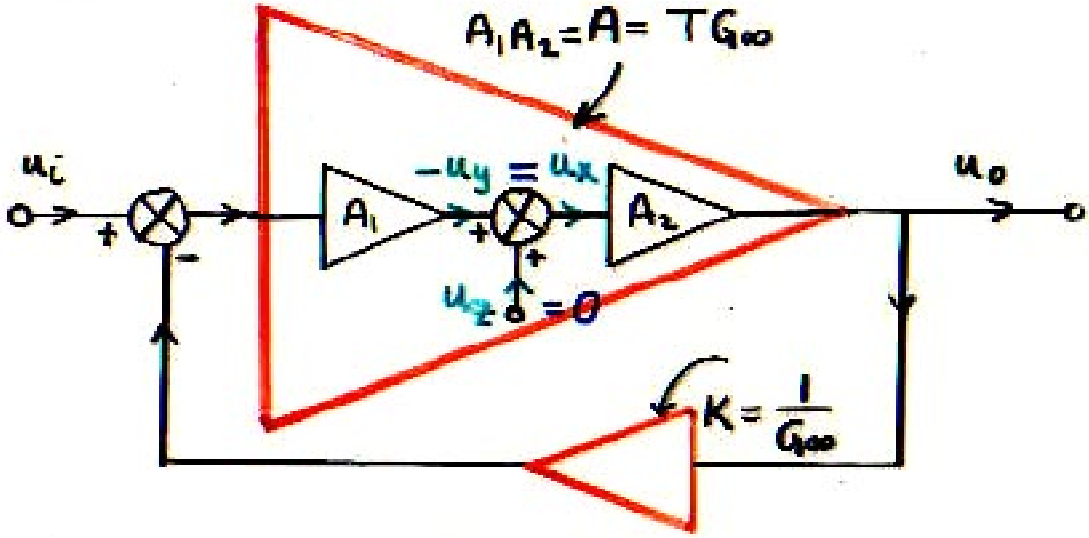
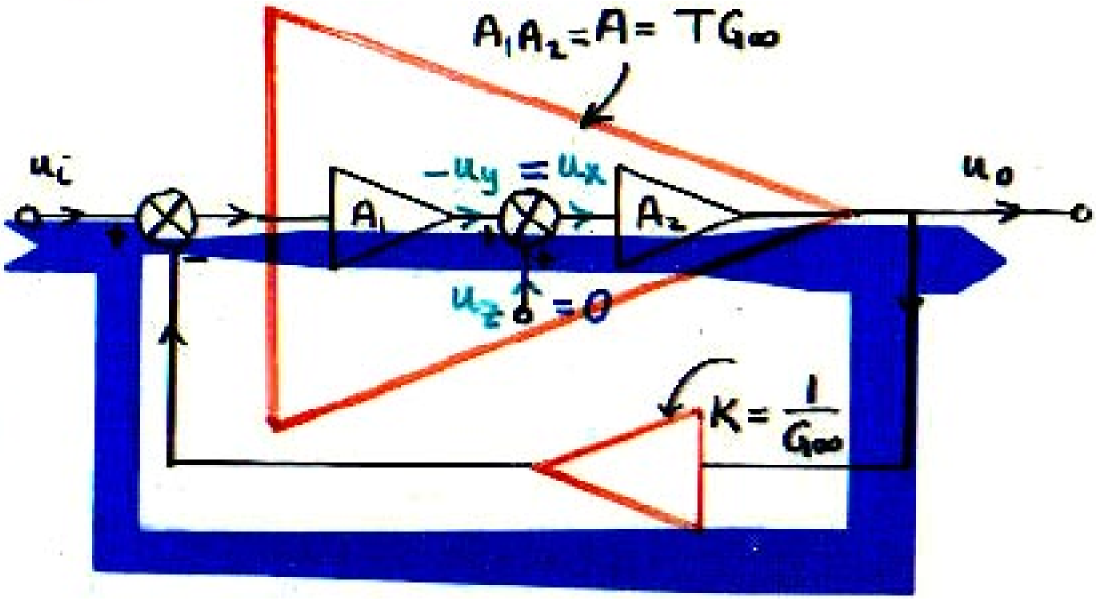


For single injection, u_i only (normal operation):

$$\left.\frac{u_o}{u_i}\right|_{u_z=0} \equiv G = G_\infty \frac{T}{1+T}$$ closed-loop gain

Note that $A = T G_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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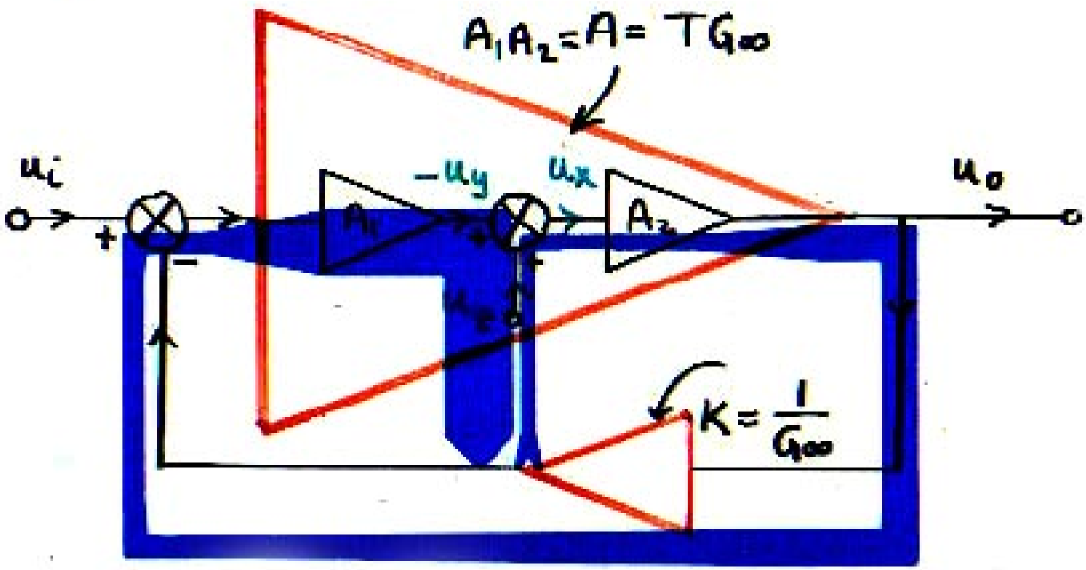

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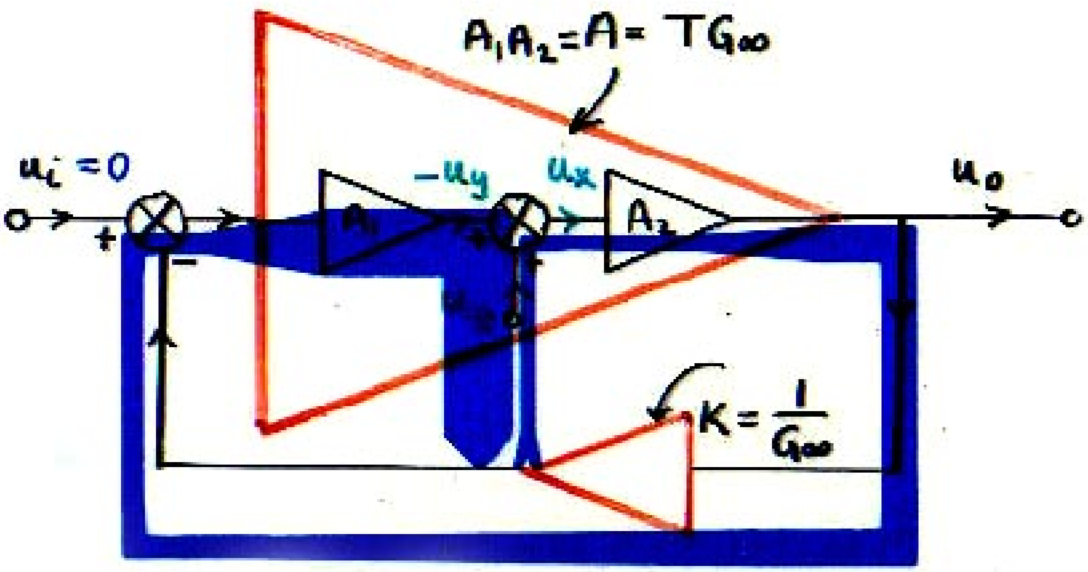
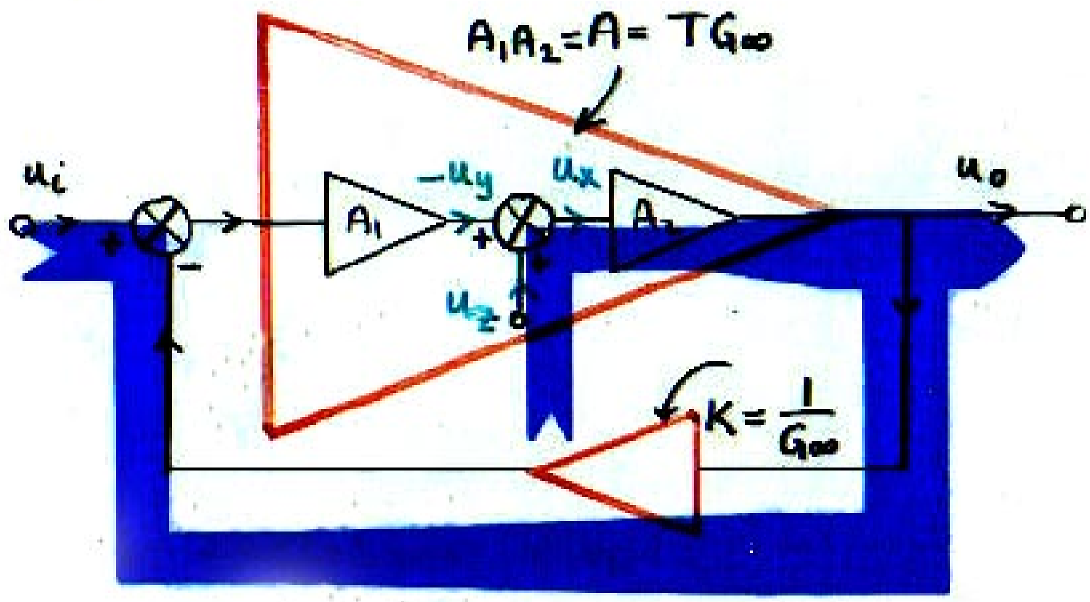


For single injection, u_z only,

$$\left.\frac{u_y}{u_x}\right|_{u_i=0} = A_2 K A_1 = AK \equiv T \quad \text{loop gain}$$

Note that $A = TG_\infty$ and $K = 1/G_\infty$:

Consider a second driving signal u_z injected into the forward path:



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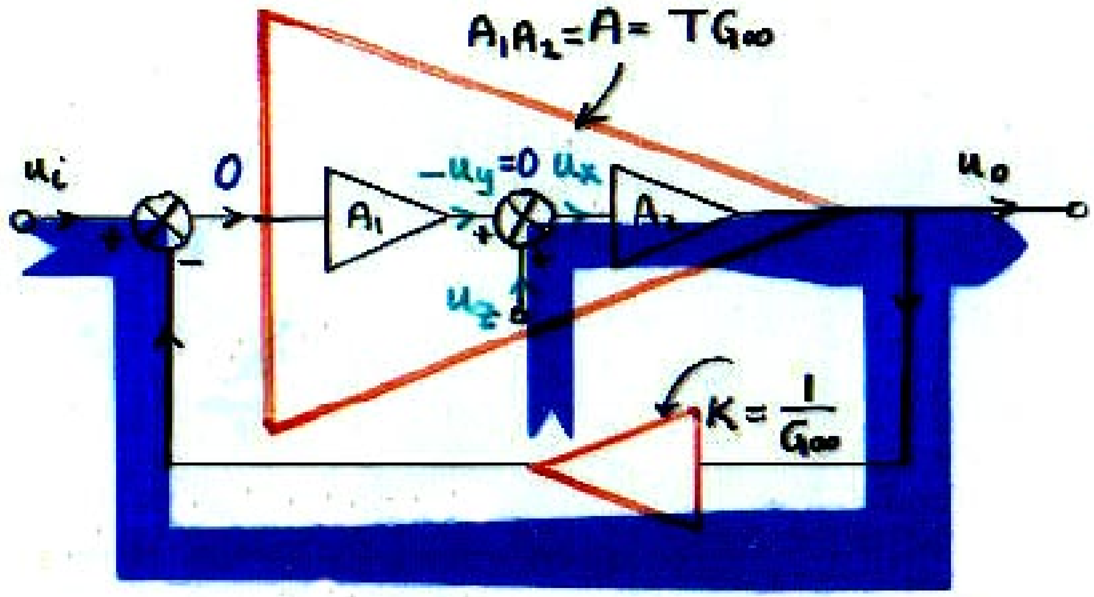


For double injection, u_i and u_z, adjusted to null u_y:

$$\left.\frac{u_o}{u_i}\right|_{u_y=0} = \frac{1}{K} \equiv G_\infty \quad \text{ideal closed-loop gain}$$

Generalization: The Feedback Theorem

The feedback path does several things:

1. Provides the feedback signal — ideal (desired)
2. Loads the output
3. Loads the input

$\left.\begin{matrix} \text{2.} \\ \text{3.} \end{matrix}\right\}$ nonideal (undesired)

Conventional form of the theorem:

$$G = \frac{A}{1+T} \quad \text{where} \quad T \equiv AK$$

Disadvantages:

Breaking the feedback path at the input or the output disturbs the loading effects.

Recommended form of the theorem:

$$G = G_\infty D \quad \text{where } G_\infty \equiv \frac{1}{k} \quad D \equiv \frac{T}{1+T}$$

Advantages:

1. G_∞ and D are directly related to the important properties of the system:

   G_∞, the Ideal Loop Gain, is the design Specification;

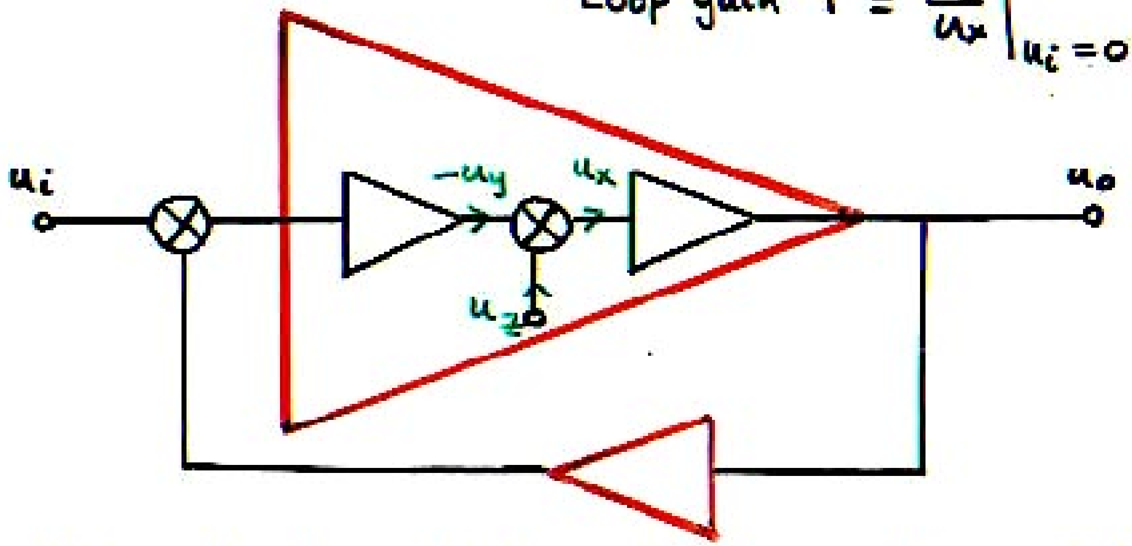
   D, the Discrepancy Factor, must be designed to be close to unity over the specified frequency range.

2. T can be found by injection of a test signal into the closed loop, without disturbance of the feedback path loading effects, and G_∞ can be found by Null Double Injection of both the input and a test signal.

NOTE: It is never necessary to know A, the open-loop forward gain: A is always embedded in T, which is why the nonideal loading effects of the feedback path are automatically accounted for.

Implementation of injection of second signal for loop gain determination

Loop gain $T \equiv \left.\frac{u_y}{u_x}\right|_{u_i=0}$



Conditions to be satisfied by injection point:

1. Must be inside the feedback loop

2. Injected signal must add to the forward signal without affecting the impedance loading

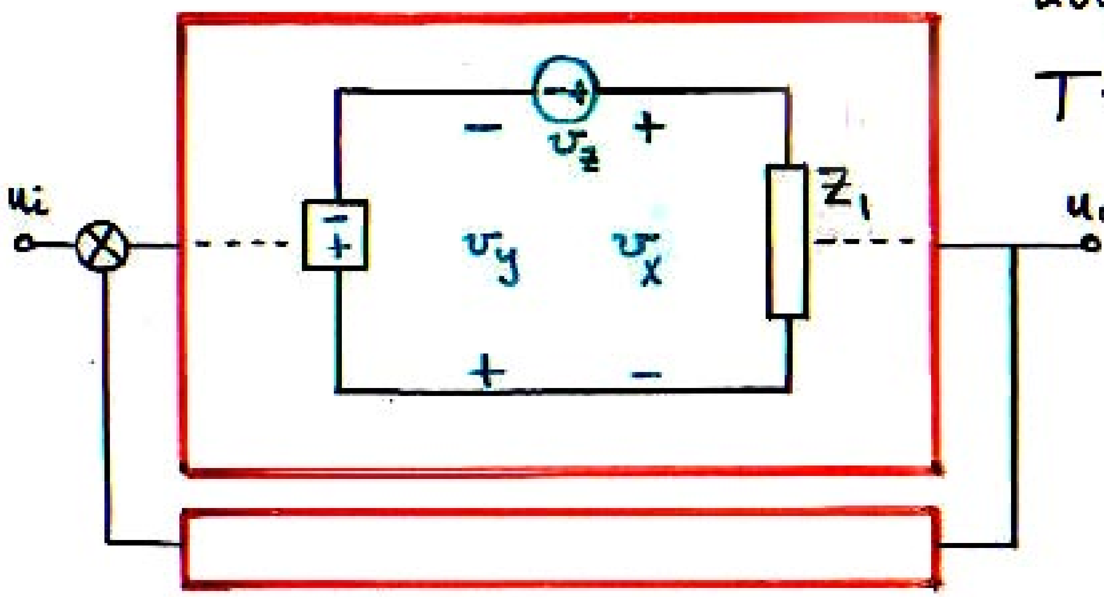
Two points satisfy these conditions

1. Inject a voltage in series with a controlled voltage generator:

$v_z = v_x + v_y$

Loop gain:

$$T = \left.\frac{v_y}{v_x}\right|_{u_i = 0}$$



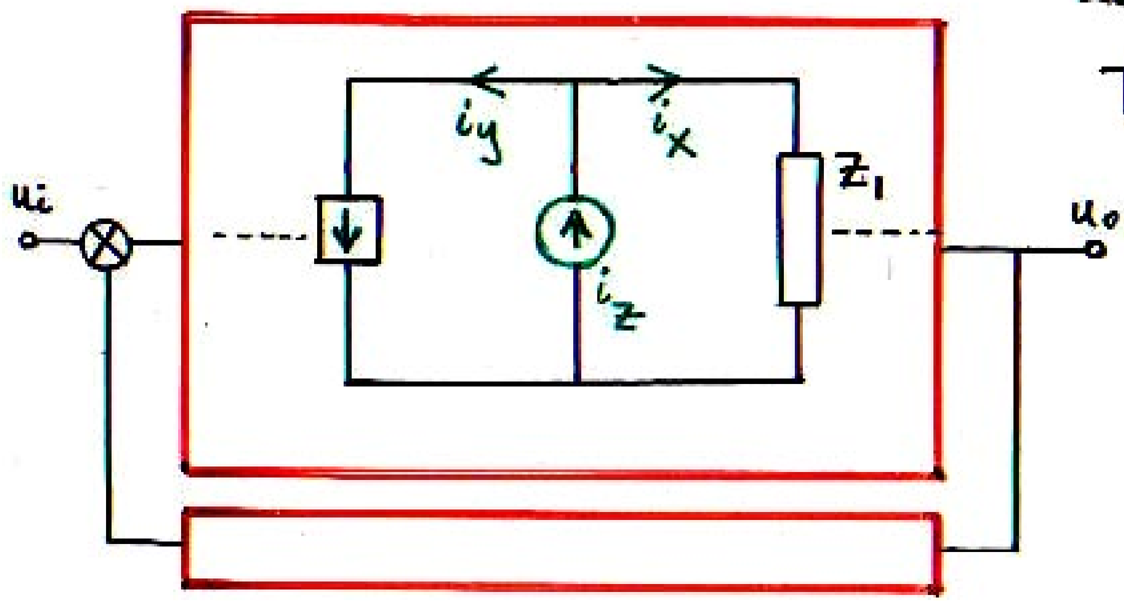
Two points satisfy these conditions

2. Inject a current in shunt with a controlled current generator:

$i_z = i_x + i_y$

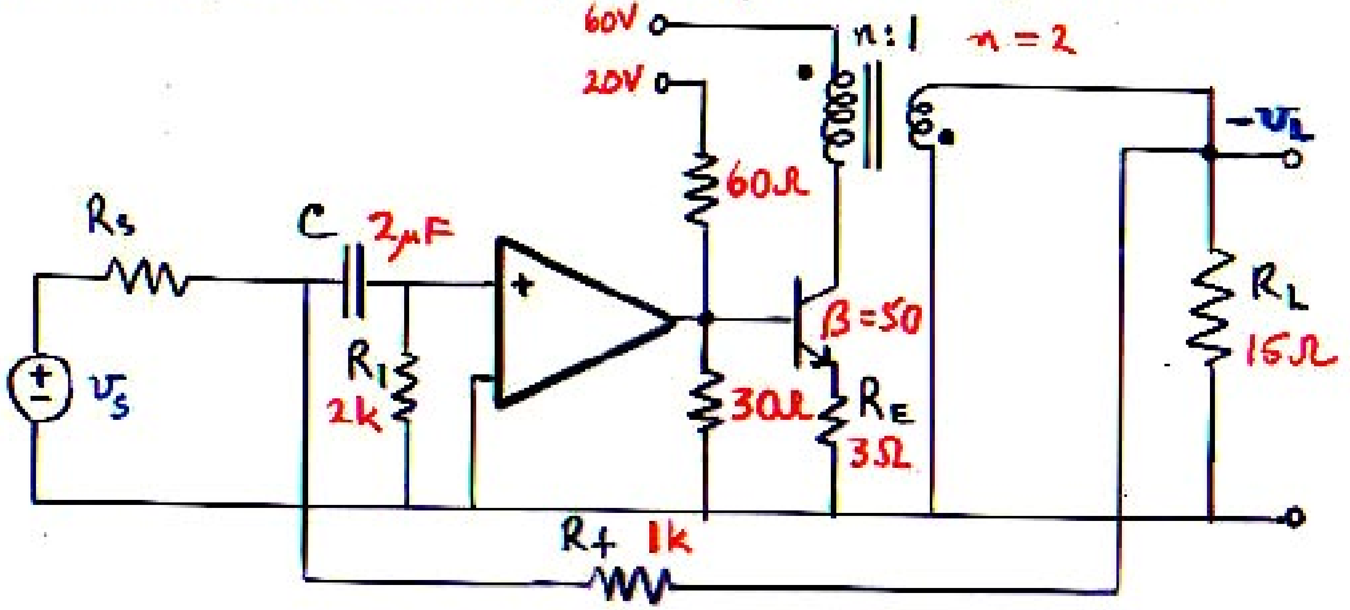
Loop gain:

$$T = \left.\frac{i_y}{i_x}\right|_{u_i=0}$$



Determination of Feedback System Parameters – Example

Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8\,dB$ and $f_A = 2\,kHz$, and an output impedance of $4.5\,\Omega$



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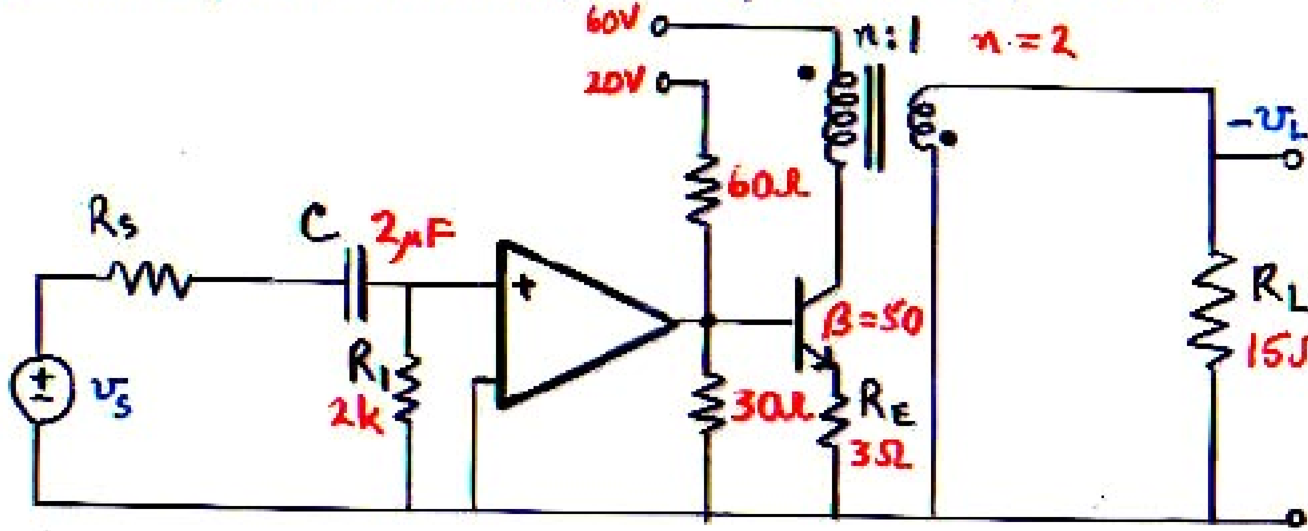
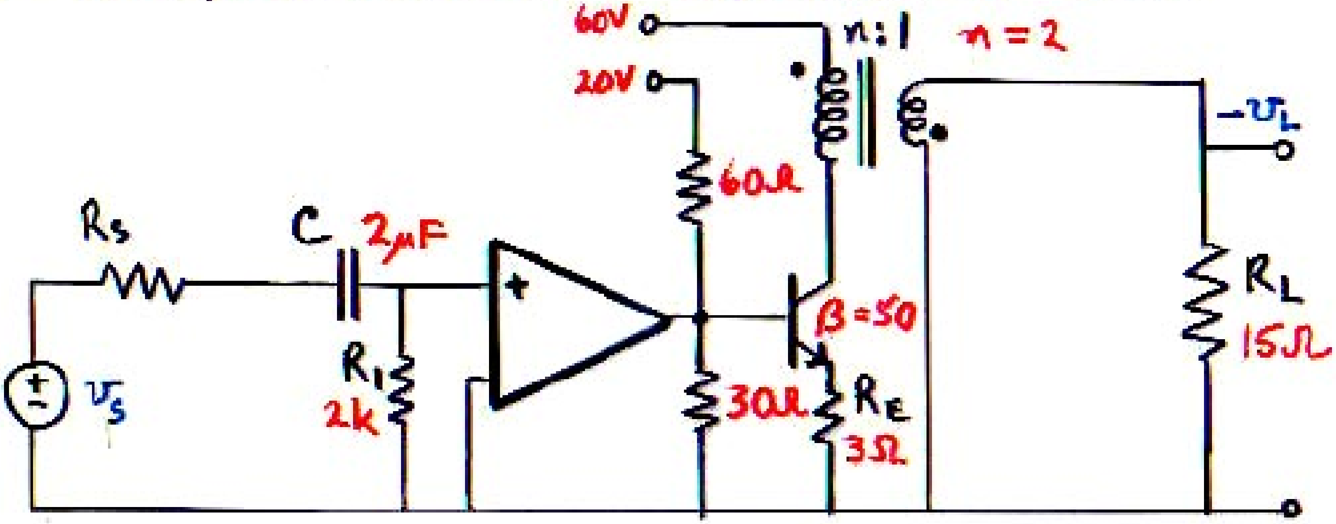


The conventional analysis method is to break the loop by opening the feedback path, then calculate A and K separately to get T = AK and G = A/(1+T).

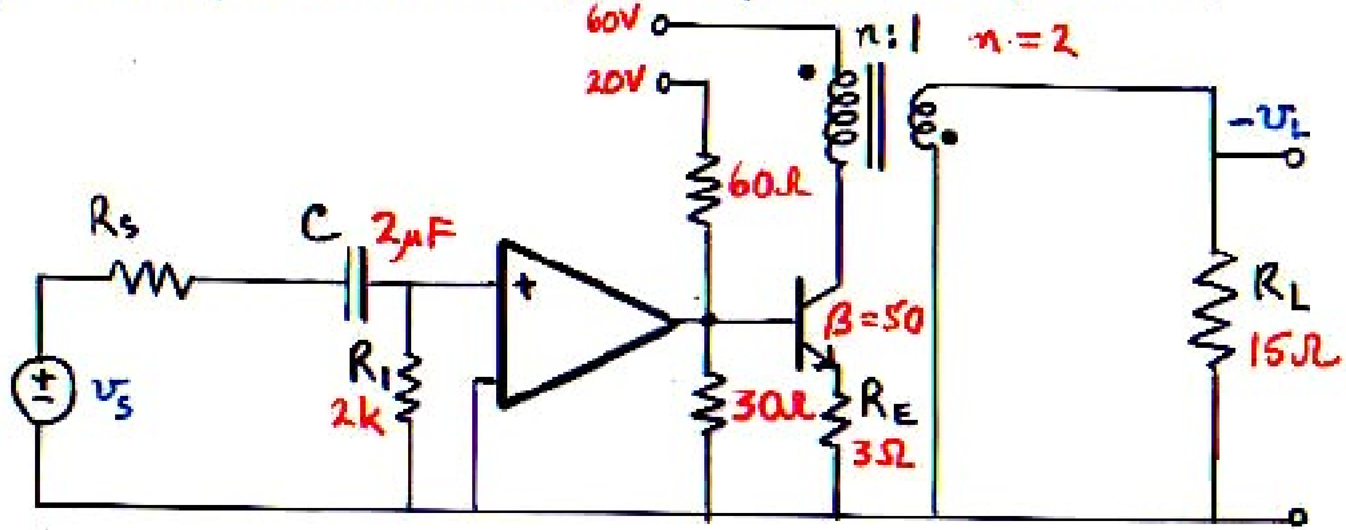
Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A))$, where $A_{10} = 8$ dB and $f_A = 2$ kHz, and an output impedance of 4.5Ω.



This is wrong, because loading by the feedback network at output and input is ignored.

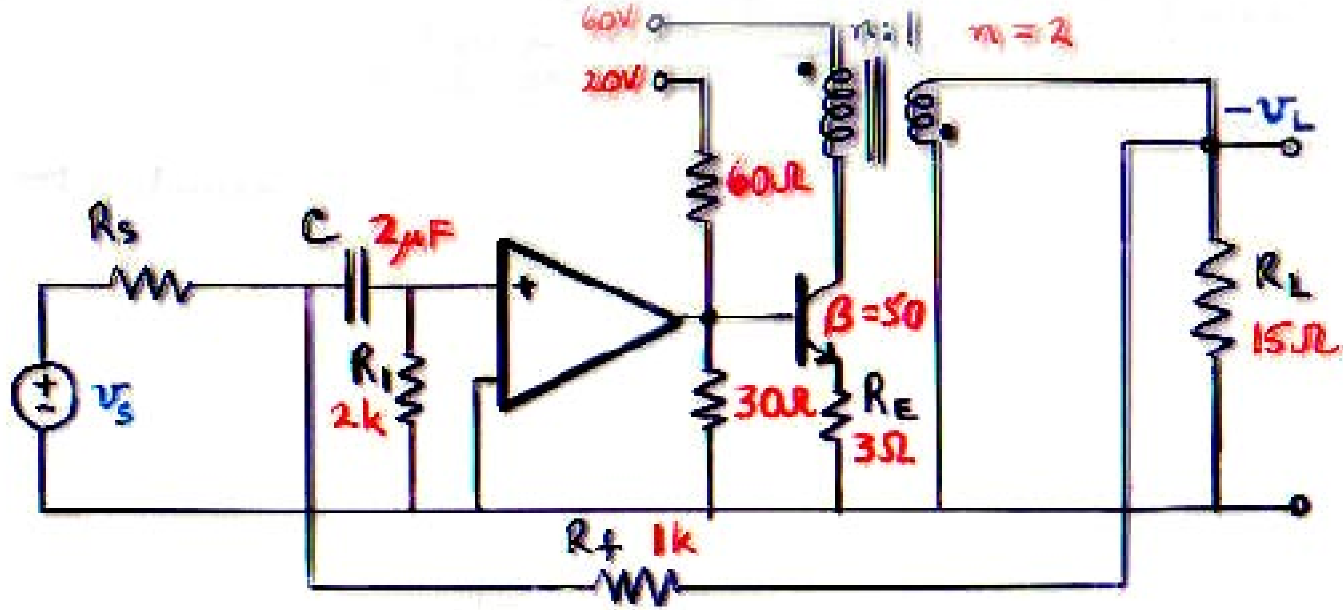
The loading could be simulated, but involves guesswork.

Single-ended Class A audio feedback amplifier. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A))$, where $A_{10} = 8\,dB$ and $f_A = 2\,kHz$, and an output impedance of $4.5\,\Omega$.

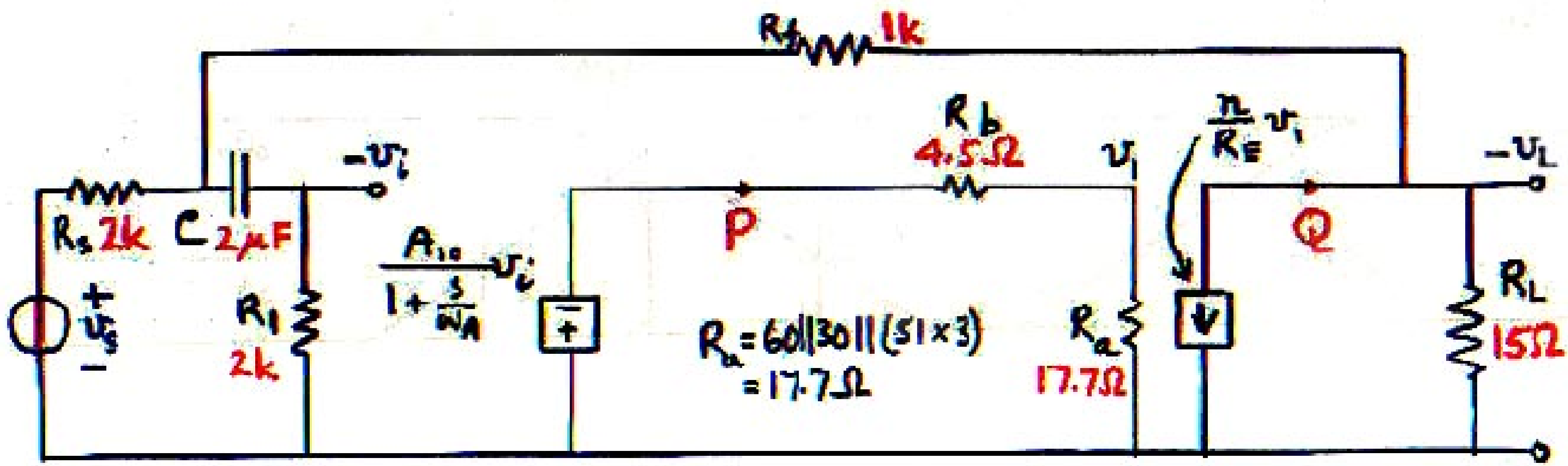


Instead, break the loop in the forward path and find T directly.

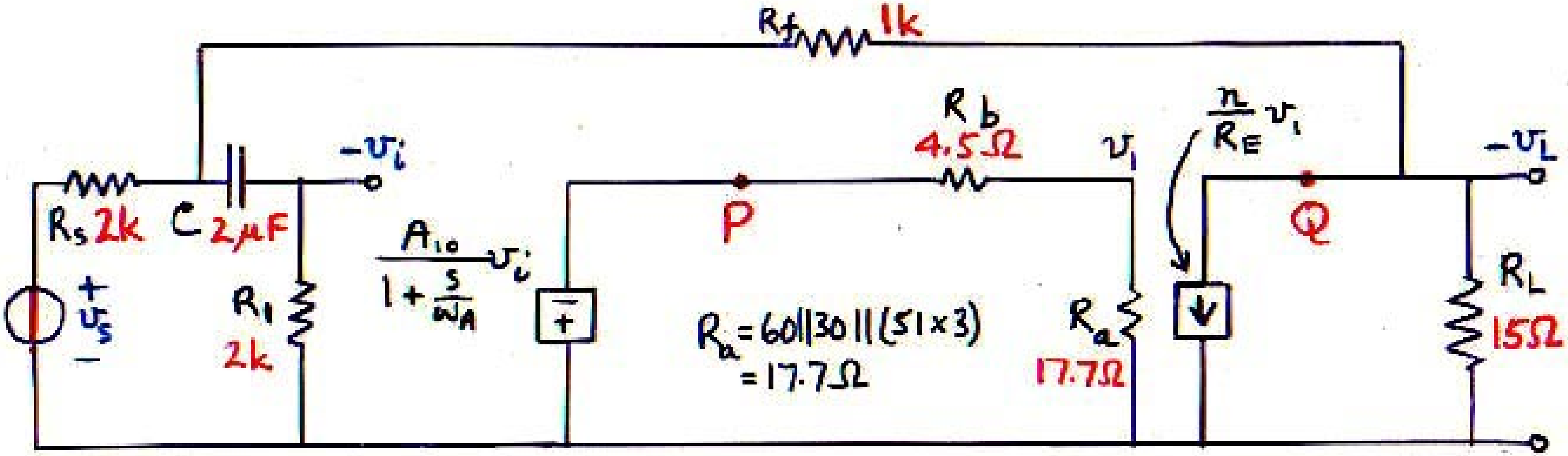
First, make an equivalent circuit model of the closed loop.



60V
20V
n:1
n = 2
$-v_L$
60Ω
R_s
C
2μF
R_L
R_1
2k
β=50
15Ω
30Ω
R_E
3Ω
v_s
R_f 1k



R_f 1k
R_b
4.5Ω
$\frac{n}{R_E} v_i$
v_i
$-v_L$
$-v_i$
P
Q
R_s 2k
C 2μF
$\frac{A_{10}}{1+\frac{s}{\omega_A}} v_i'$
R_L
R_1
2k
v_s
$R_a = 60 \| 30 \| (51\times3) = 17.7\Omega$
R_a
17.7Ω
15Ω

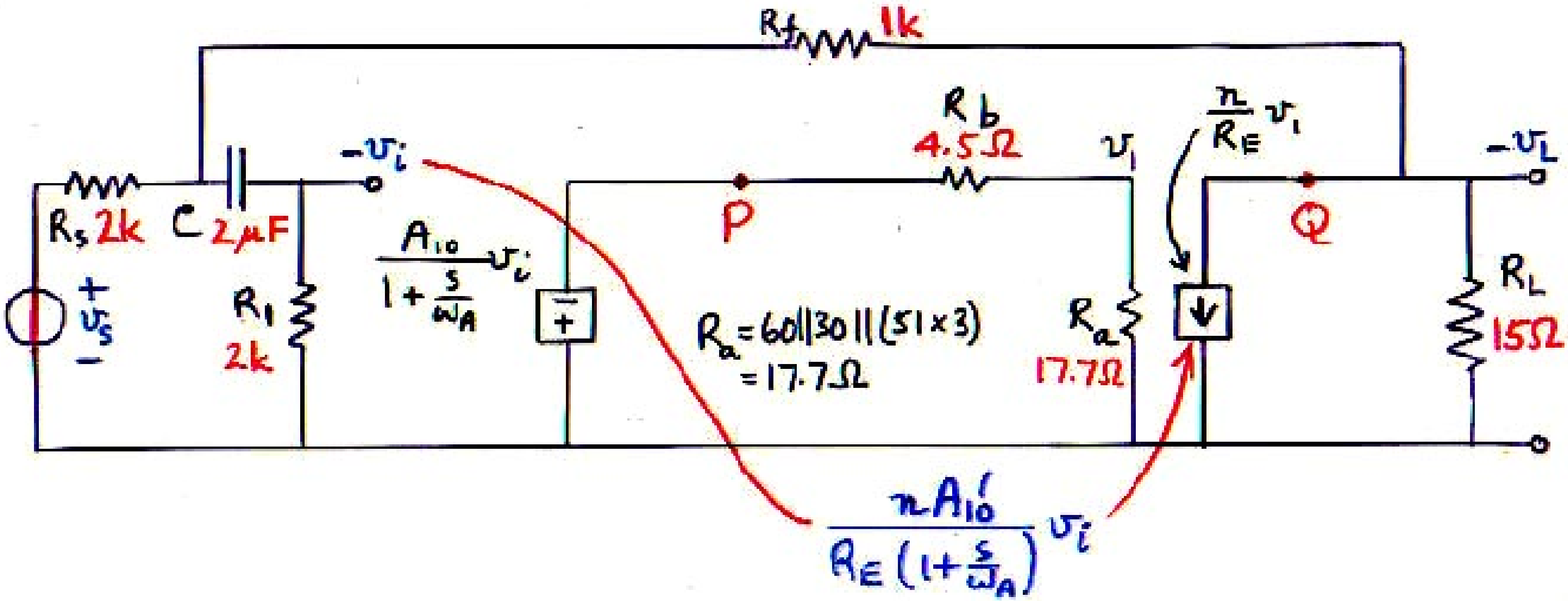


For all calculations concerning loop gain, the input voltage v_s is zero.

Suitable injection points:

Series voltage at point P

Shunt current at point Q



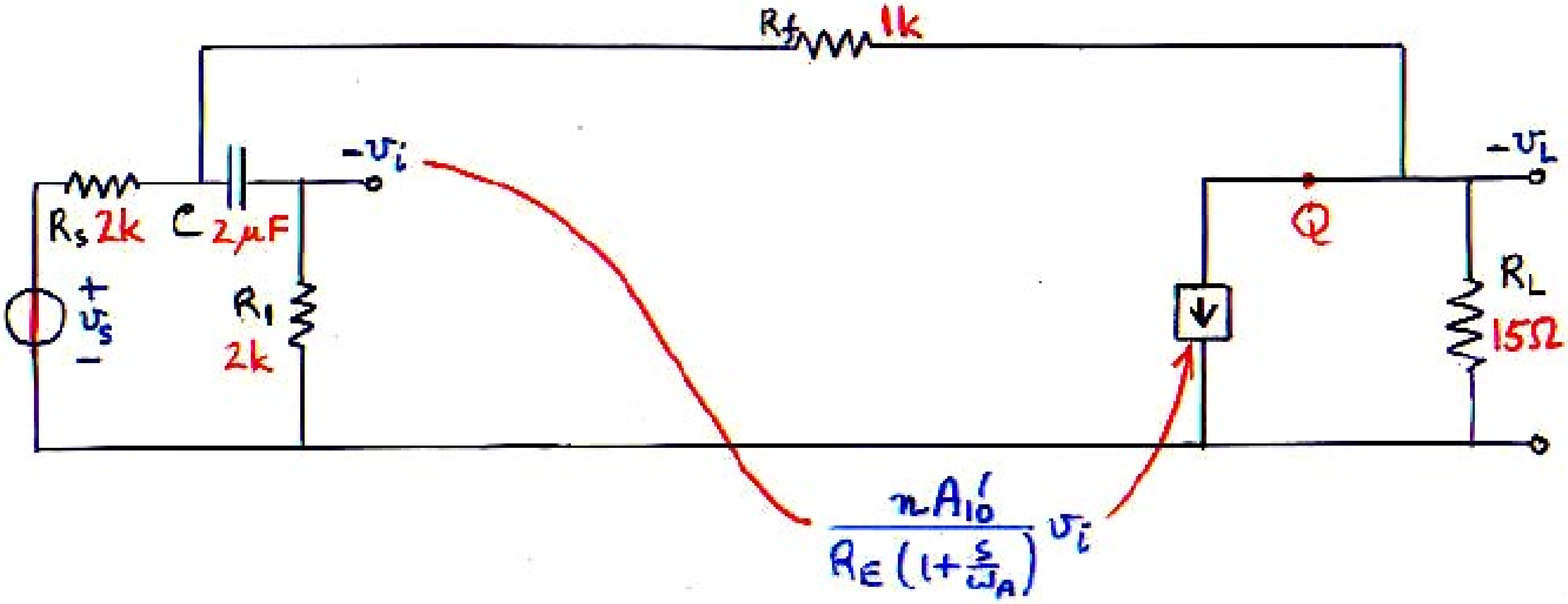
If current injection is chosen, the gain from v_i to the power stage output generator can be condensed into a single factor:

$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\,\frac{R_a}{R_a+R_b}\,\frac{n}{R_E}\,v_i = \frac{nA_{10}'}{R_E(1+\frac{s}{\omega_A})}v_i$$

where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,\mathrm{dB}\times\frac{17.7}{17.7+4.5} = 2.51\times0.80$$
$$= 2.0 \Rightarrow 6\,\mathrm{dB}$$

is the loaded gain of the opamp.



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$$\frac{A_{10}}{1+\frac{s}{\omega_A}}\,\frac{R_a}{R_a+R_b}\,\frac{n}{R_E}\,v_i = \frac{nA_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}v_i$$

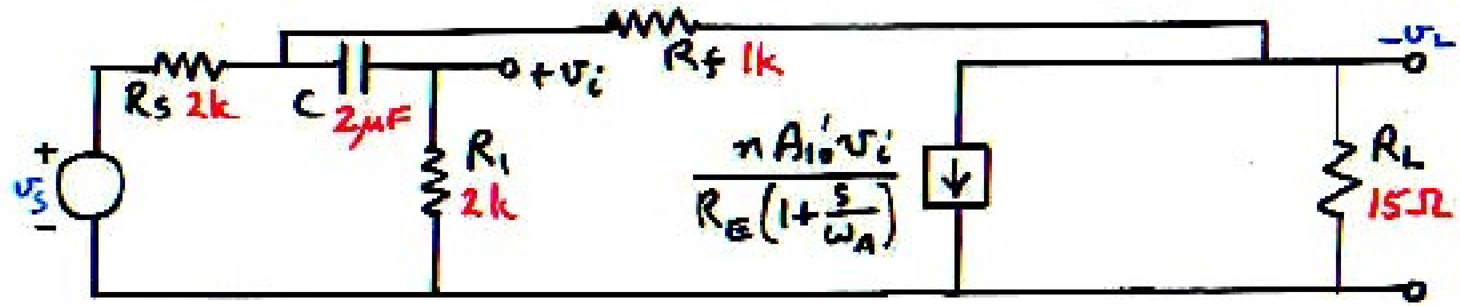
where

$$A_{10}' = A_{10}\frac{R_a}{R_a+R_b} = 8\,\mathrm{dB}\times\frac{17.7}{17.7+4.5} = 2.51\times 0.80$$

$$= 2.0 \Rightarrow 6\,\mathrm{dB}$$

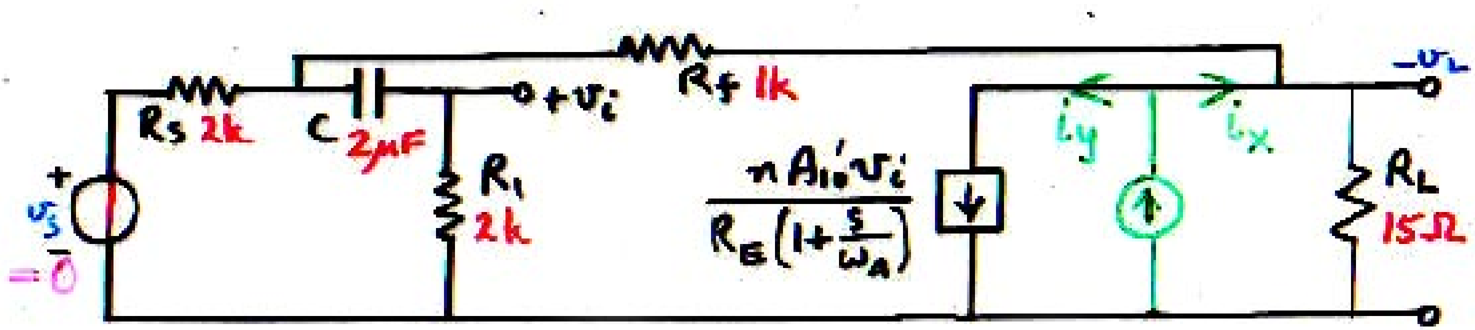
is the loaded gain of the opamp.

Ac model



R_s 2k
C 2µF
$+v_i$
R_f 1k
$-v_L$
+
v_s
−
R_1
2k
$\frac{n A_{10}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L
15Ω

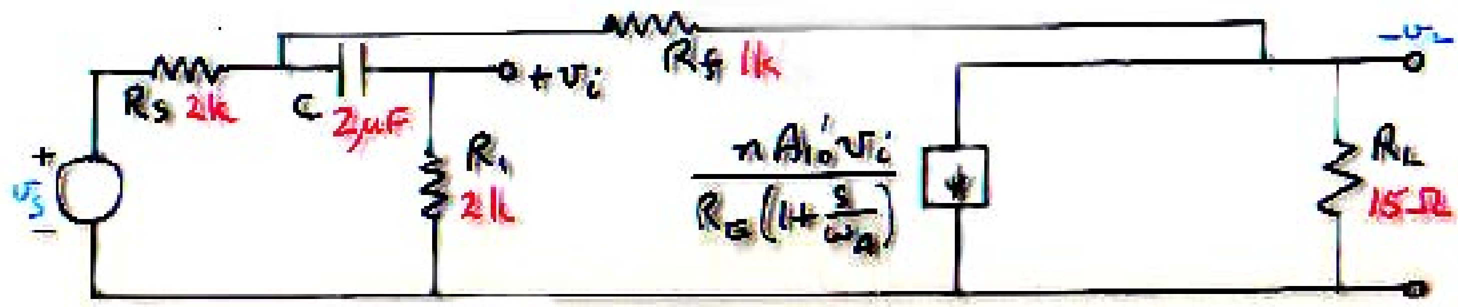
Ac model



$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_s \| R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_s}{R_s + R_1\left(1+\frac{\omega_1}{s}\right)} \; \frac{R_1 n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2\times 2} = 40\,\text{Hz}$$



This method requires additional algebraic force to find the corner frequencies. Therefore, choose

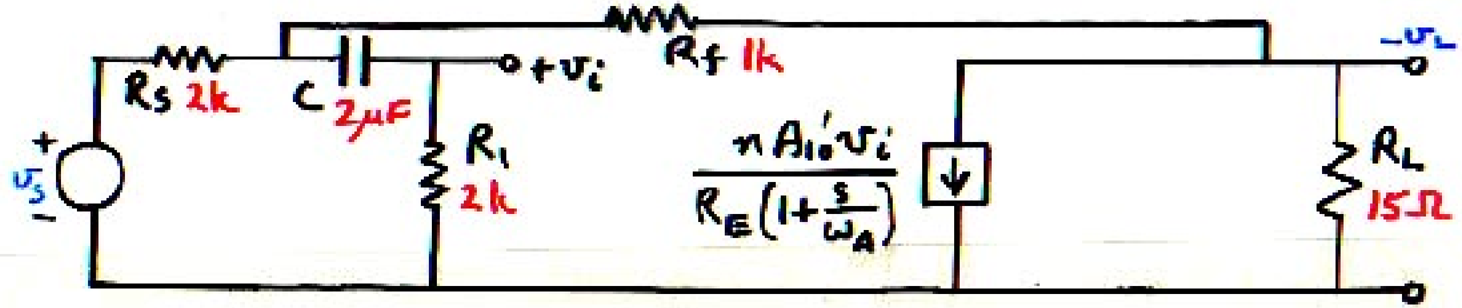
Better Method:

First, find T_m:

$$T_m = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L} + R_f + R_s \| R_1} (R_s \| R_1) \frac{nA'_{10}}{R_E}$$

$$= \frac{(R_f \| R_s \| R_1)\, nA'_{10} R_L}{R_f R_E} = \frac{0.5 \times 2 \times 2 \times 15}{1 \times 3}$$

$$= 10 \Rightarrow 20\,\text{dB}$$



Second, find corner frequencies due to C by use of Extra Element Theorem.

Apply test signal u_z in place of C.

To find $Z_n = R_n$:

"input" signal for T ($v_s = 0$)

Adjust u_z in presence of i_x to null i_y

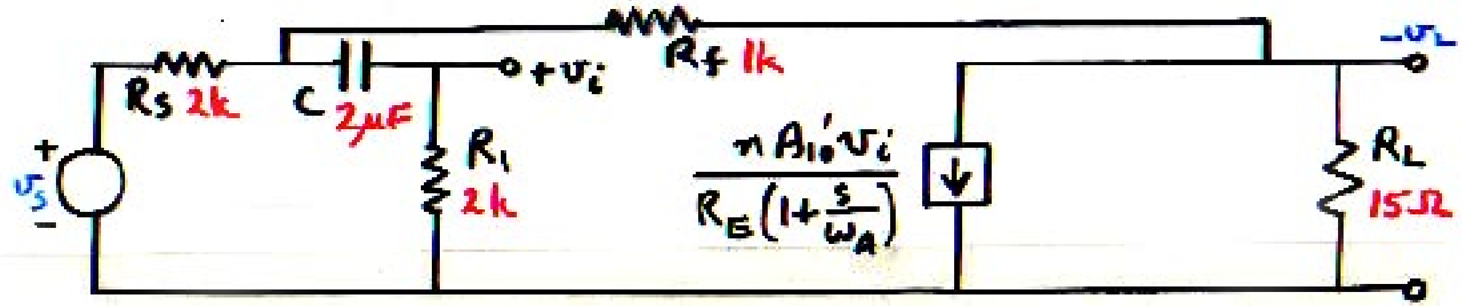
"output" signal for T

Hence: $v_i = 0$, and $R_n = \infty$

To find $Z_d = R_d$:

Apply u_z with $i_x = 0$

Hence: $R_d = (\cancel{R_L} + R_f) \| R_s + R_1$



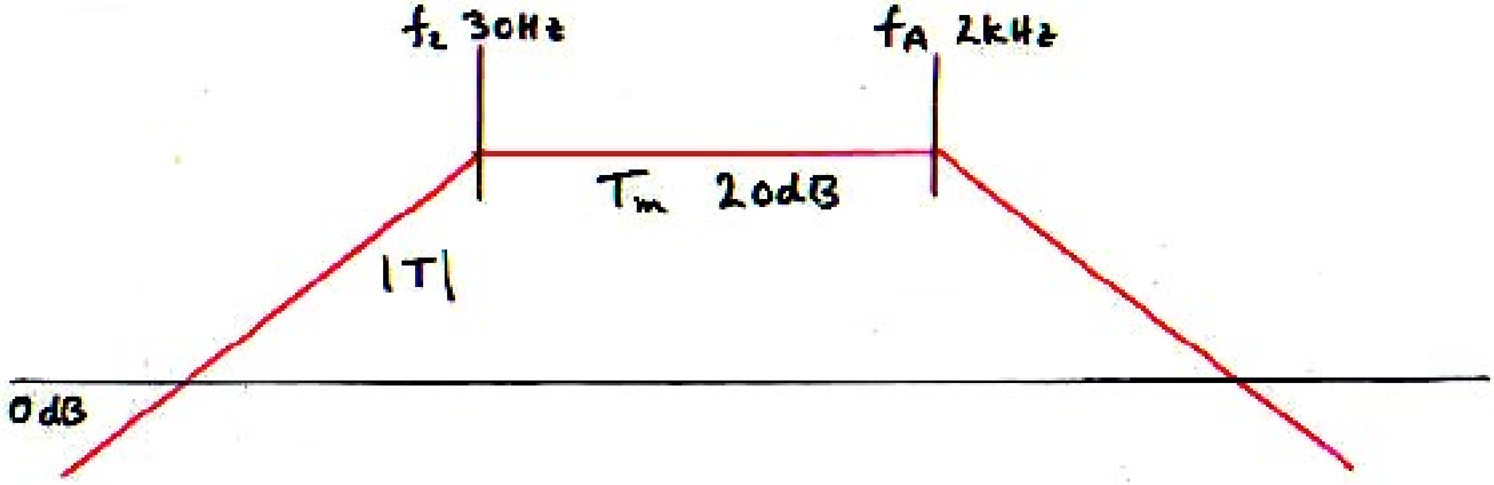
Then:

$$T = T_m \frac{1 + \frac{Z}{\overset{\infty}{Z_n}}}{1 + \frac{\underset{\frac{1}{sC}}{Z}}{\underset{R_d}{Z_d}}} \frac{1}{1 + \frac{s}{\omega_A}} \quad \text{already explicit}$$

$$= T_m \frac{1}{\left(1 + \frac{\omega_2}{s}\right)\left(1 + \frac{s}{\omega_A}\right)}$$

where

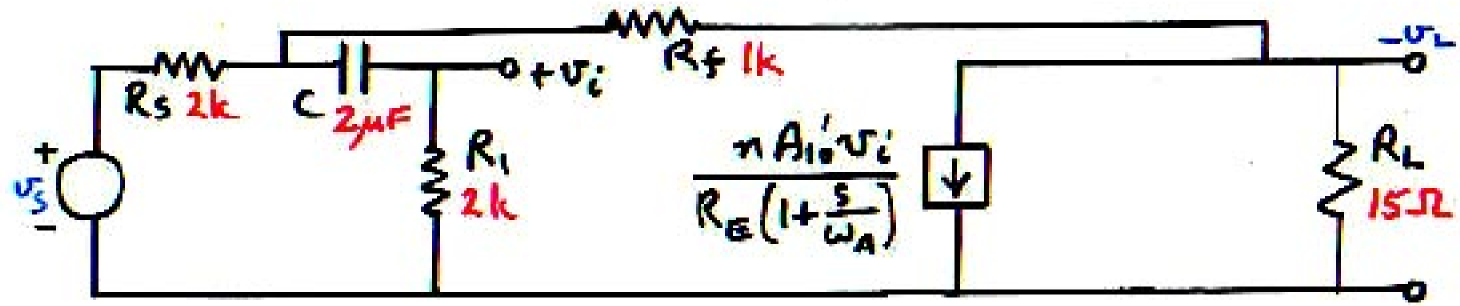
$$\omega_2 \equiv \frac{1}{CR_d} = \frac{1}{C[R_f \| R_s + R_1]} \qquad f_2 = \frac{159}{2[1\|2+2]} = 30\,\text{Hz}$$



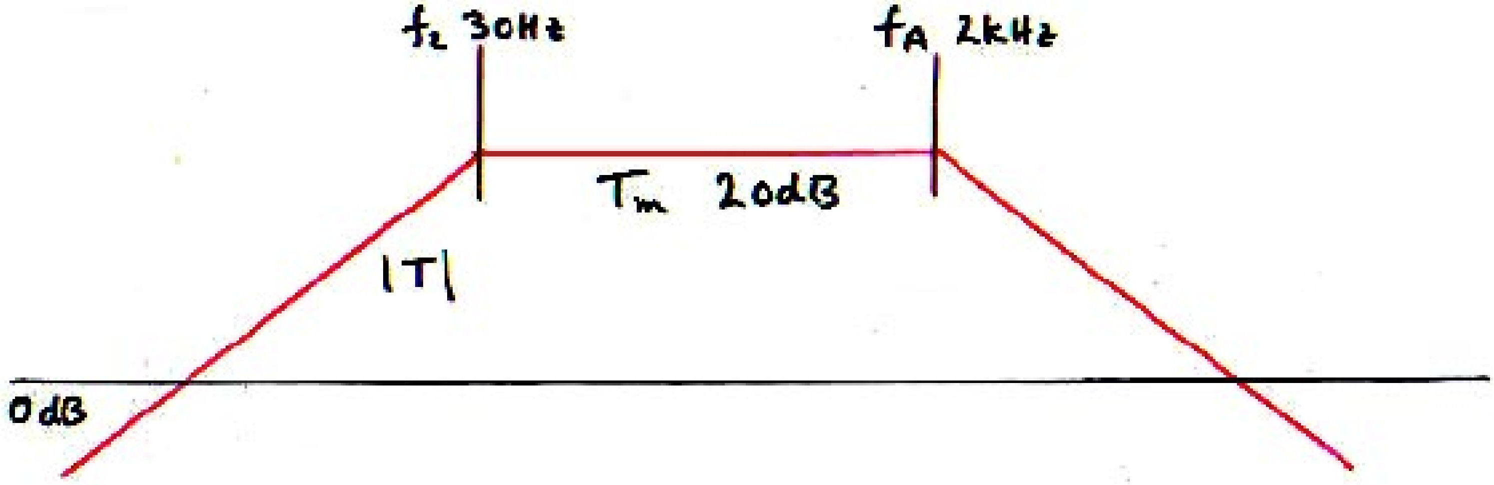
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1 + T$ and $D = T/(1+T)$

Ac model



R_S 2k
C 2μF
$+v_i$
R_f 1k
$-v_L$
v_S
+
−
R_1
2k
$\frac{n A_{10}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
R_L
15Ω



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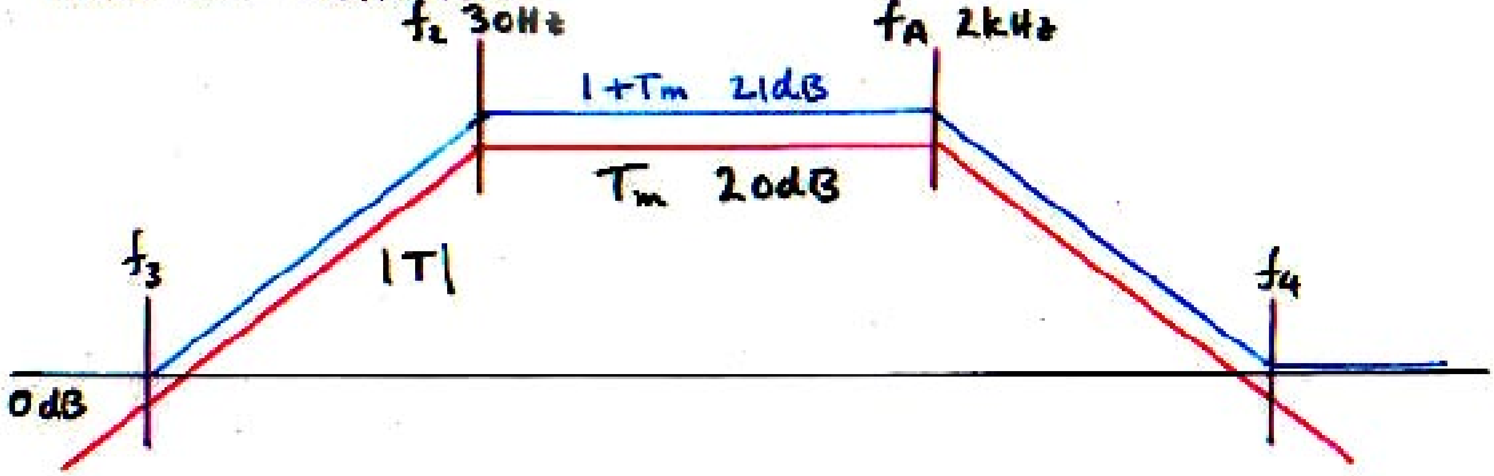
Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

where $\omega_3 \equiv \dfrac{\omega_2}{1+T_m}$ $\qquad f_3 = \dfrac{30}{1+10} = 2.7\,\text{Hz}$

$\omega_4 \equiv (1+T_m)\,\omega_A \qquad f_4 = 11 \times 2 = 22\,\text{kHz}$

Calculation of $F = 1+T$ the Hard Way (by algebra):

$$F = 1+T = 1 + \frac{T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right) + T_m \frac{s}{\omega_2}}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{1 + \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)s + \left(\frac{1}{\omega_2 \omega_A}\right)s^2}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) + \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2 \omega_A}}\right]s\right)\left(1 + \left[\frac{1}{2}\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right) - \frac{1}{2}\sqrt{\left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2 - \frac{4}{\omega_2 \omega_A}}\right]s\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \frac{\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)} = \frac{\omega_2}{\omega_{z1}} \frac{\left(1+\frac{\omega_{z1}}{s}\right)\left(1+\frac{s}{\omega_{z2}}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This result gives no insight into the interpretation of the two zeros ω_{z1} and ω_{z2}, or into the midband value $F_m \equiv \frac{\omega_2}{\omega_{z1}}$.

However, a much simpler result is obtaine if the approximate real root form is used.

Check the value of $Q^2 = ac/b^2$ for the numerator quadratic of F:

$$Q^2 = \frac{1}{\omega_2 \omega_A \left(\frac{1+T_m}{\omega_2} + \frac{1}{\omega_A}\right)^2} = \frac{\omega_2}{\omega_A (1+T_m)^2 \left(1 + \cancel{\frac{\omega_2}{\omega_A(1+T_m)}}\right)^2} \ll (0.5)^2$$

Hence, the approximate factorization for well-separated real roots can be adopted:

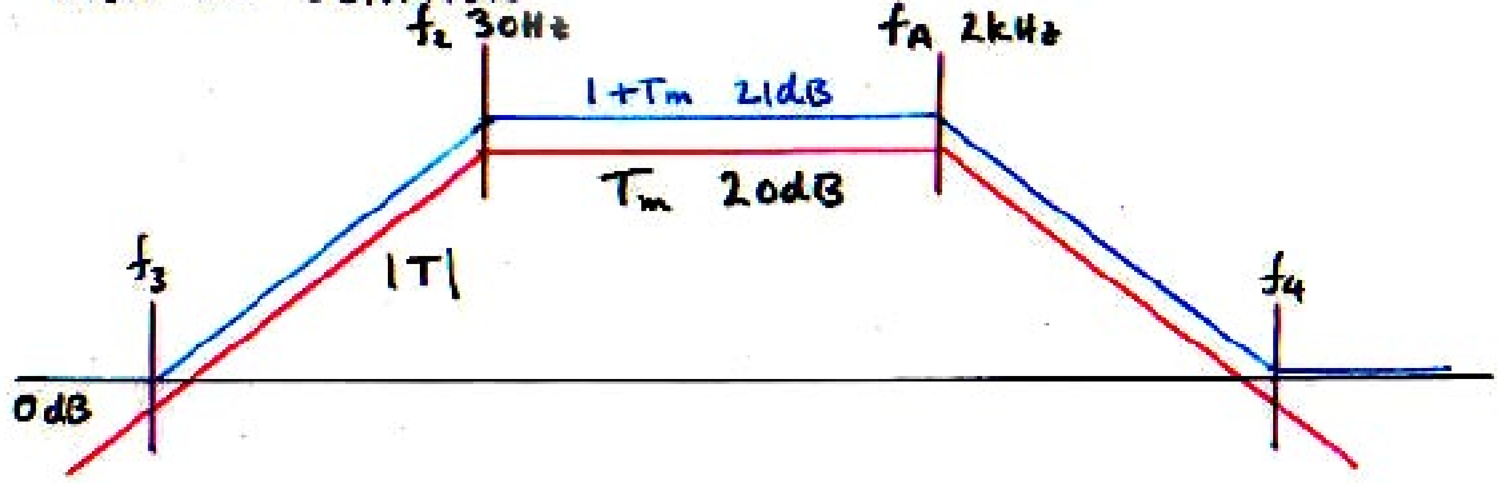
$$F \approx \frac{\left(1 + \left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}}\right]s\right)\left(1 + \frac{s}{\omega_2 \omega_A \left[\frac{1+T_m}{\omega_2} + \cancel{\frac{1}{\omega_A}}\right]}\right)}{\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$= \left(1 + T_m + \cancel{\frac{\omega_2}{\omega_A}}\right) \frac{\left(1 + \frac{\omega_2/(1+T_m)}{\left[1 + \cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right] s}\right)\left(1 + \frac{s}{\left[1 + \cancel{\frac{\omega_2}{(1+T_m)\omega_A}}\right](1+T_m)\omega_A}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

This is the same result obtained by Doing the Algebra on the Graph.

The algebraic factorization could not have been done at all if T had three or more poles.

Exercise Solution



Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$F = (1+T_m)\,\frac{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

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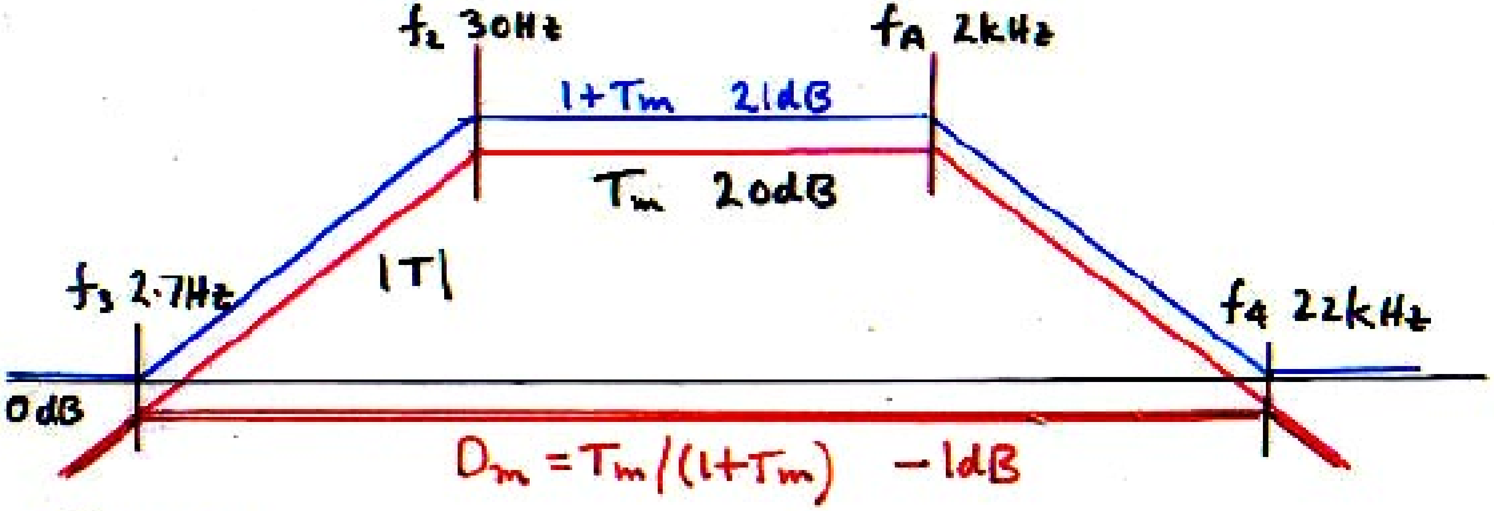


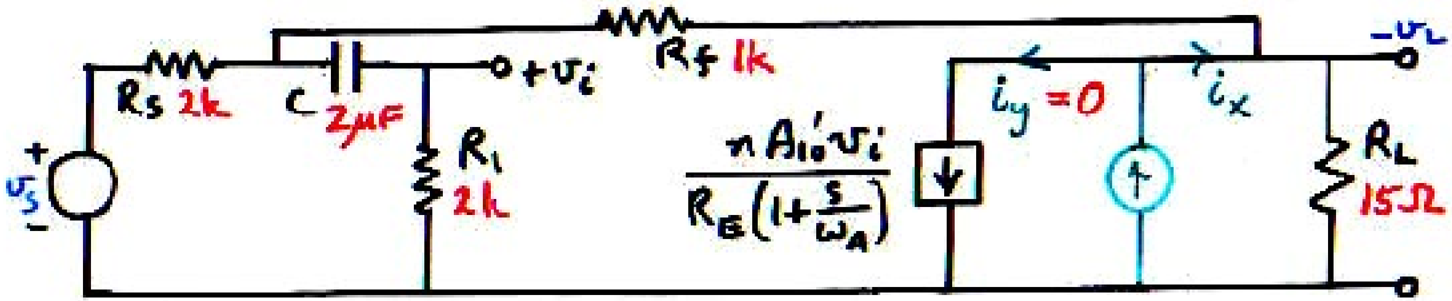
Exercise

By "doing the algebra on the picture," find the analytic pole-zero forms for $F = 1+T$ and $D = T/(1+T)$

$$D = \frac{T_m}{1+T_m}\,\frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

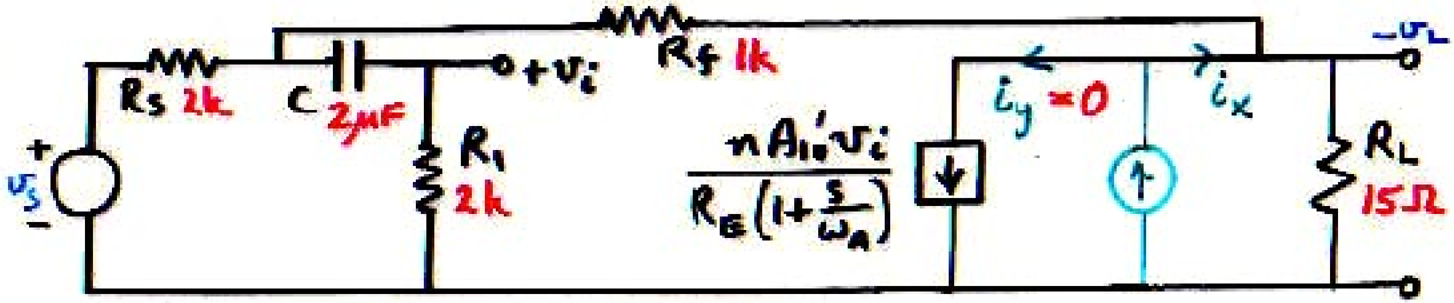
Ac model
Exercise

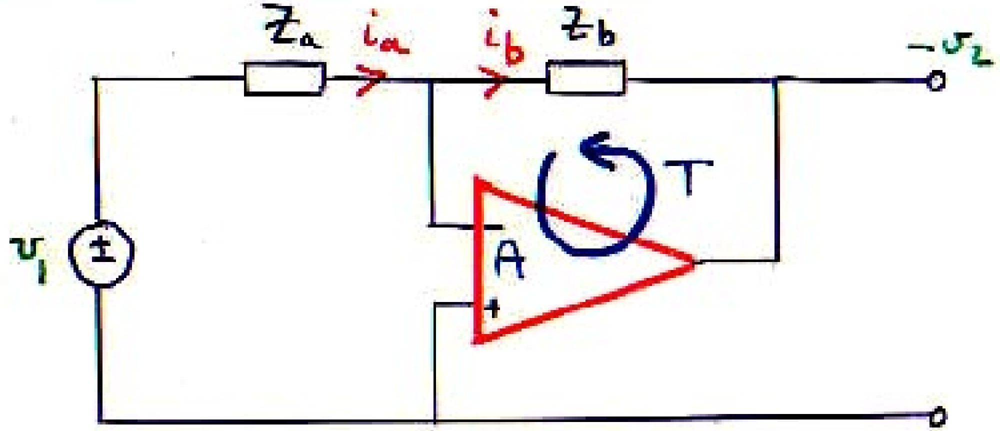
Find G_∞



Ac model
Exercise

Find G_∞

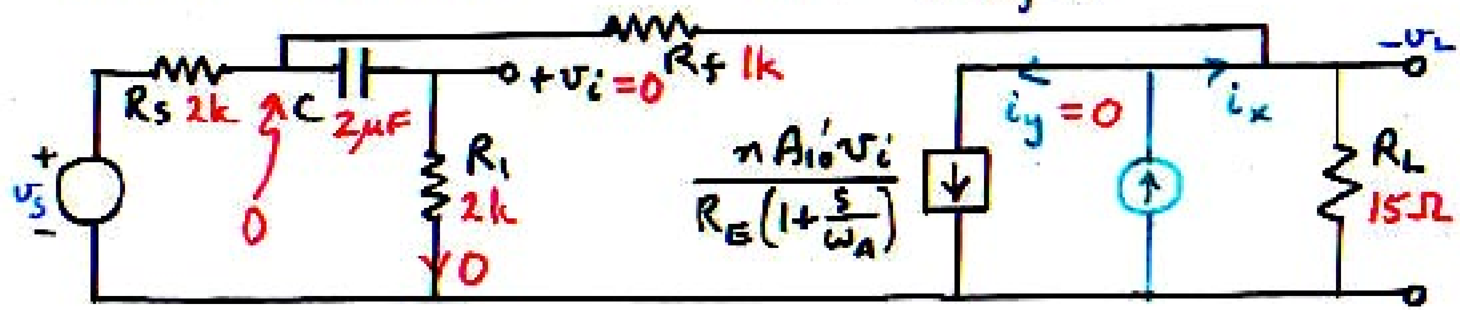




Ac model

Exercise Solution

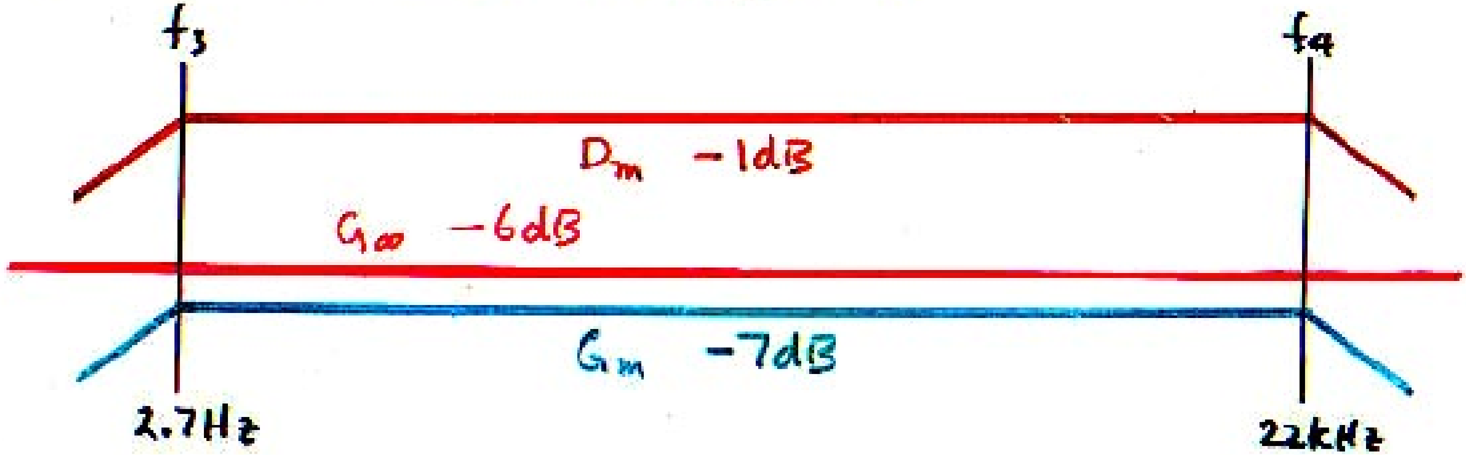
$$G_{\infty} = \left.\frac{v_L}{v_S}\right|_{i_y=0}$$



$$\frac{v_S}{R_S} = \frac{v_L}{R_f}$$

$$G_{\infty} = \frac{R_f}{R_S} = \frac{1}{2} \Rightarrow -6dB$$

Hence the closed-loop gain G is



$$G = G_m \frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}$$

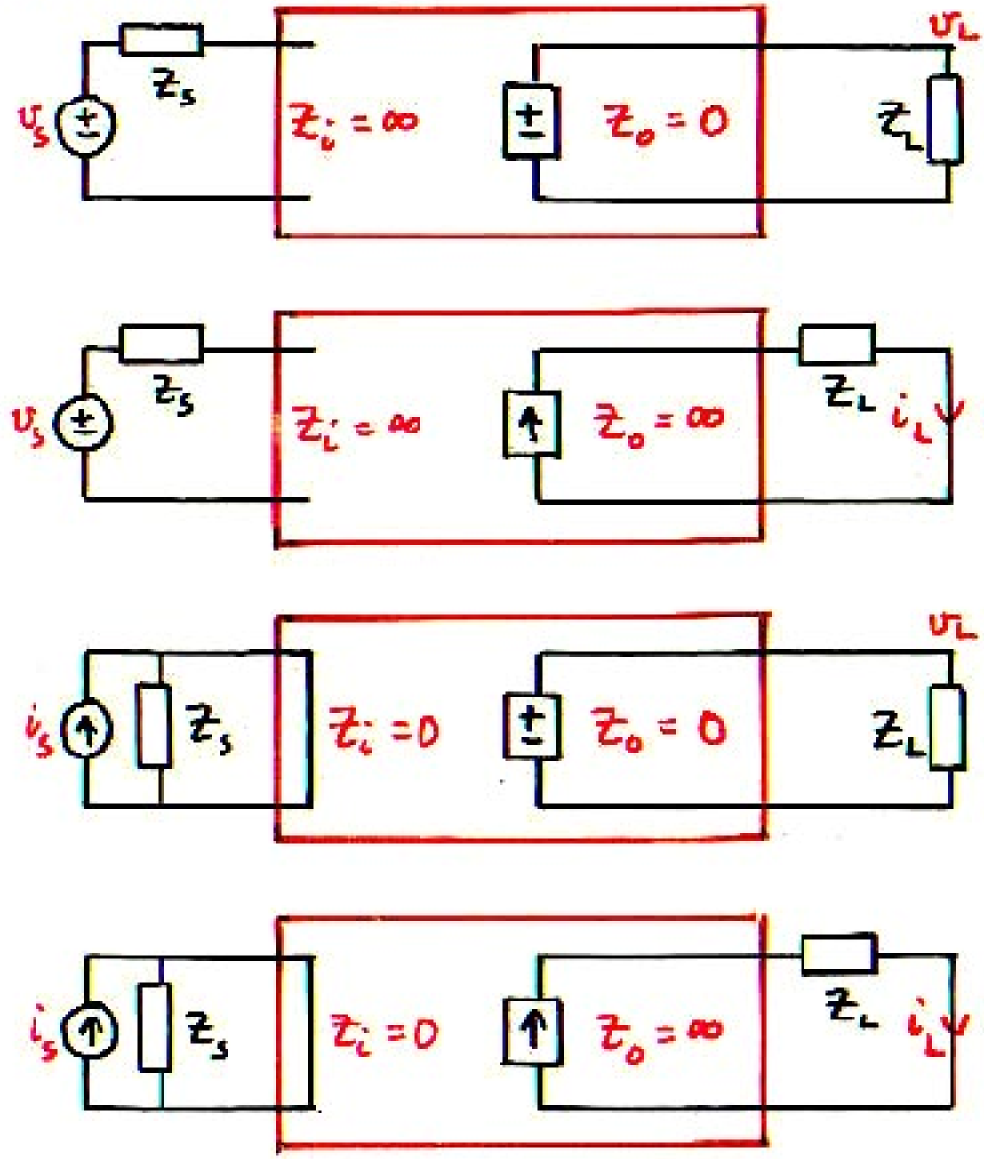
Generalization: Two Conditions for Injection of a Test Signal into a Closed Loop

1. Must be inside the feedback loop

2. Injected signal must add to the forward signal without affecting the impedance loading

Condition 2 can be met by injection of a voltage in series with a dependent voltage source, or by injection of a current in parallel with a current source.

Ideal amplifiers



$$A = \frac{v_L}{v_s} = \frac{\text{voltage}}{\text{voltage}}$$

$$A = \frac{i_L}{v_s} = \frac{\text{current}}{\text{voltage}}$$

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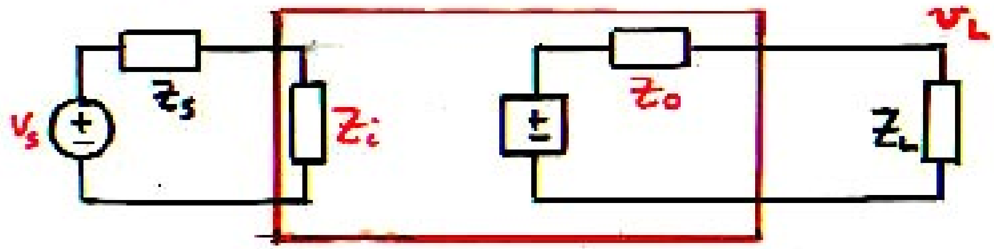
In the ideal cases, the gain is <u>independent</u> of Z_s and Z_L

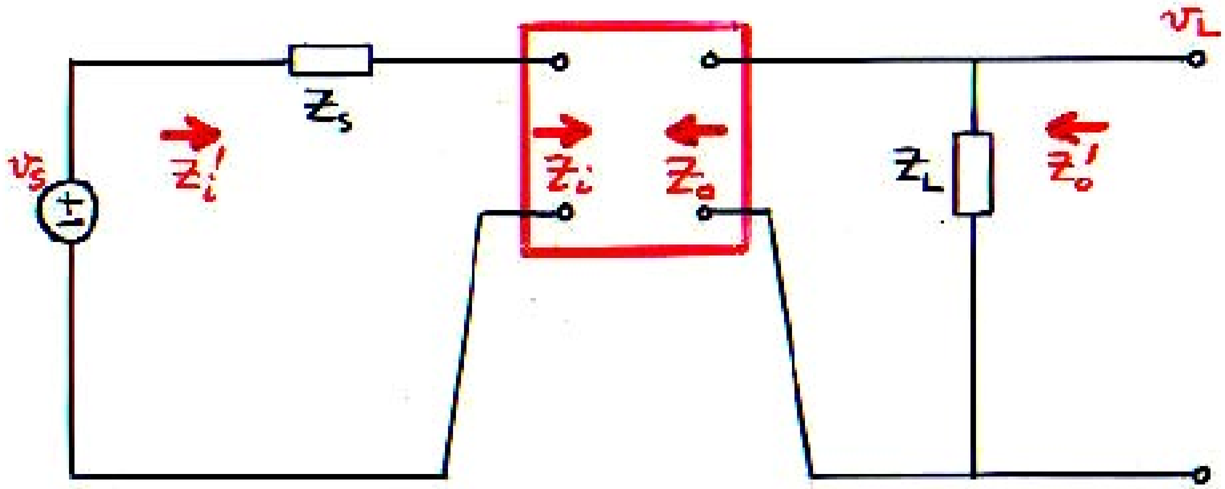
A practical amplifier is not ideal, and the gain does depend on Z_s and Z_L:



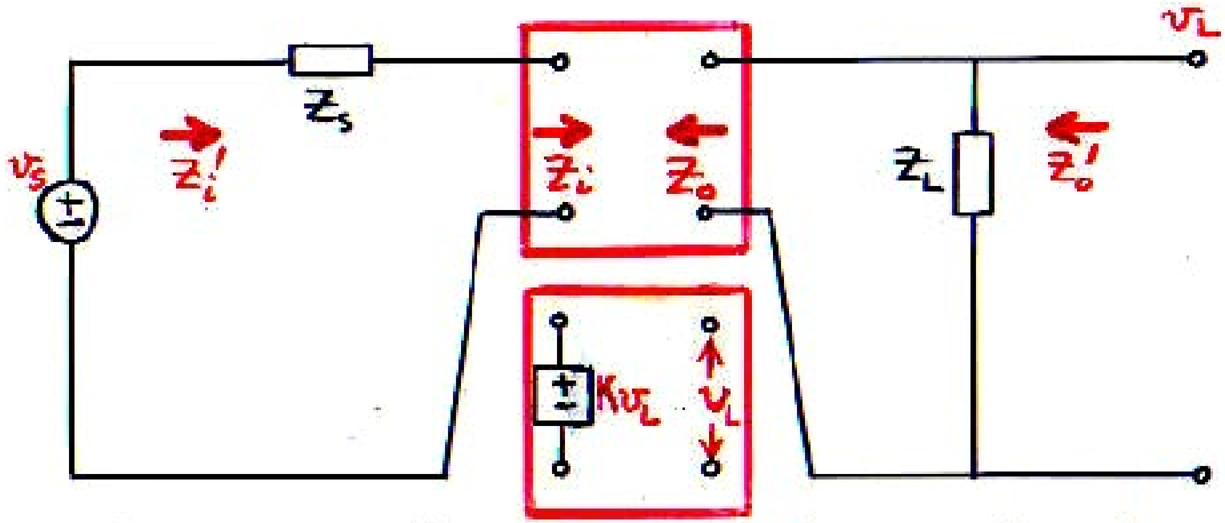
$A = A(Z_s, Z_L)$

However, an appropriate connection of feedback can make a nonideal amplifier approach more closely the properties of any one of the ideal amplifiers.

Feedback causes the closed-loop gain G to approach the reciprocal feedback ratio $1/K = G_\infty$. Thus G_∞ must be designed to have the same current or voltage transfer properties as the desired ideal amplifier.



v_S
Z_S
Z_i'
Z_i
Z_o
Z_L
Z_o'
v_L



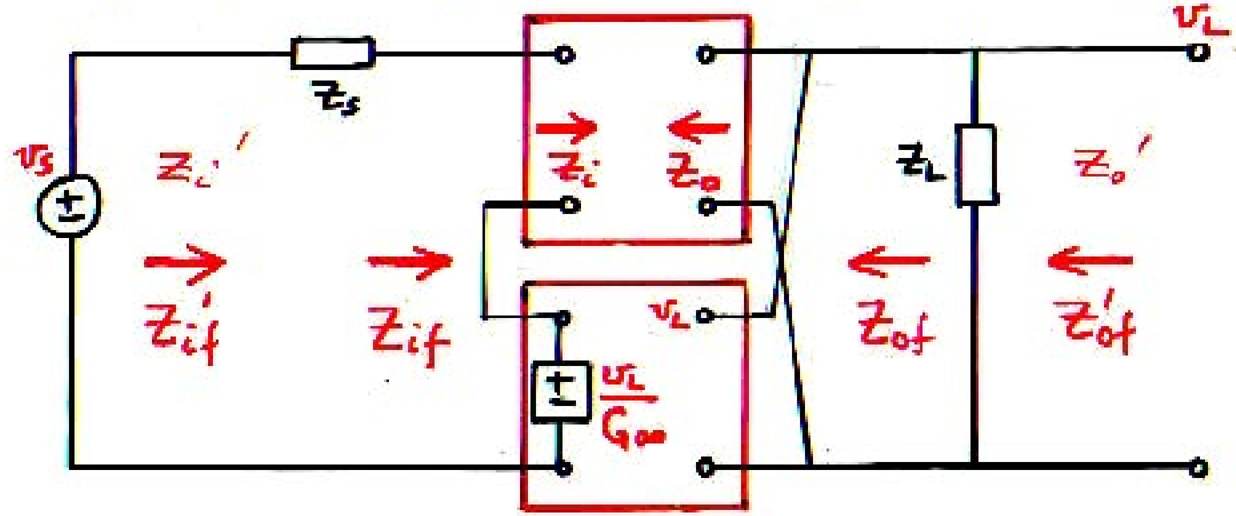
Ideal feedback path must "sense" output voltage and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{v_L}{v_S} = \frac{\text{voltage}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

where

$$A = \left. \frac{v_L}{v_S} \right|_{K = 1/G_\infty = 0}$$

1. Voltage-to-voltage (series voltage feedback)



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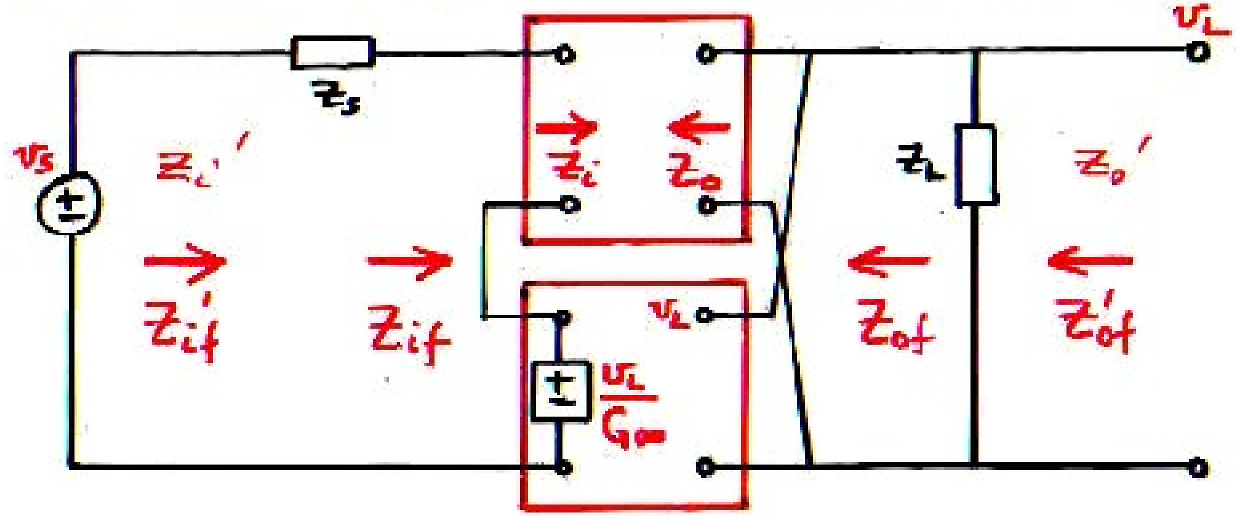

$$Z'_{of} = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0} \frac{1}{1+T|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}} \frac{1+\cancel{T|_{Z_L\to 0}}}{1+T} = \frac{Z_o'}{1+T}$$

$$Z_{of} = \frac{Z_o}{1+T|_{Z_L\to \infty}}$$

OR:

$$Z'_{of} = \frac{G_\infty \frac{T}{1+T}}{G_\infty \left.\frac{T}{Z_L}\right|_{Z_L\to 0} \frac{1}{1+\cancel{T|_{Z_L\to 0}}}} = \frac{T}{1+T}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

$$Z_{of} = \left.\frac{T}{1+T}\right|_{Z_L\to\infty}\left[\left.\frac{Z_L}{T}\right|_{Z_L\to 0}\right]$$

Outside output impedance is reduced by the factor $1+T$.

Inside output impedance is reduced by the factor $1+T|_{Z_L\to\infty}$, which is larger than $1+T$.

Outside and inside output impedances Z_{of}' and Z_{of} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much smaller than Z_L.

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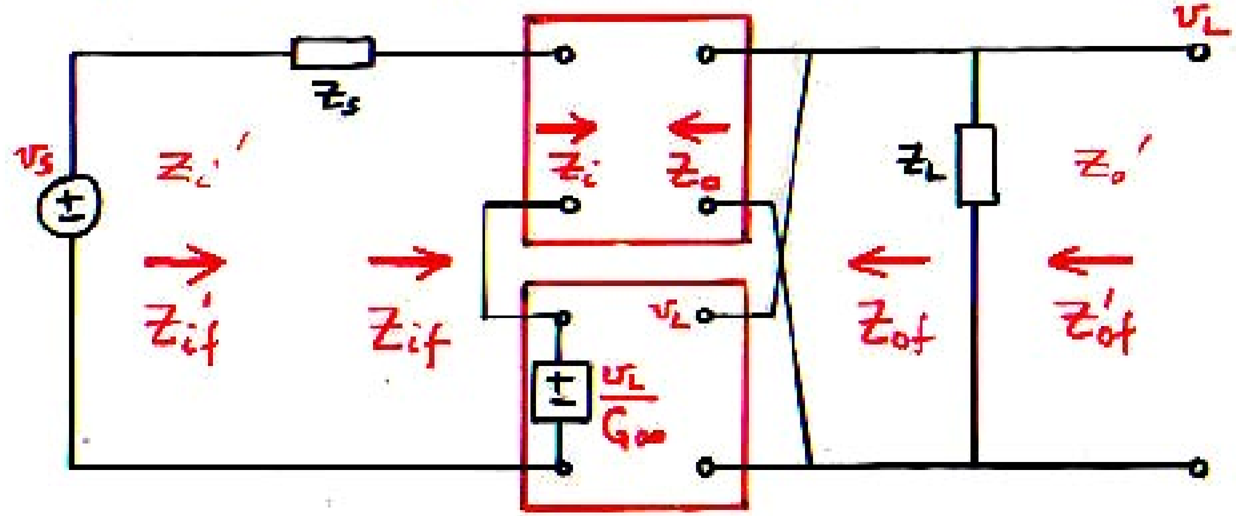

$$Z'_{if} = \frac{Z_s G|_{Z_s\to\infty}}{G} = \frac{\dfrac{Z_s A|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_s A|_{Z_s\to\infty}}{A}\,\frac{1+T}{\cancel{1+T|_{Z_s\to\infty}}} = Z_i'(1+T)$$

$$Z_{if} = Z_i(1+T|_{Z_s\to 0})$$

OR:

$$Z'_{if} = \frac{G_\infty \dfrac{Z_s T|_{Z_s\to\infty}}{\cancel{1+T|_{Z_s\to\infty}}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_s T|_{Z_s\to\infty}\right]$$

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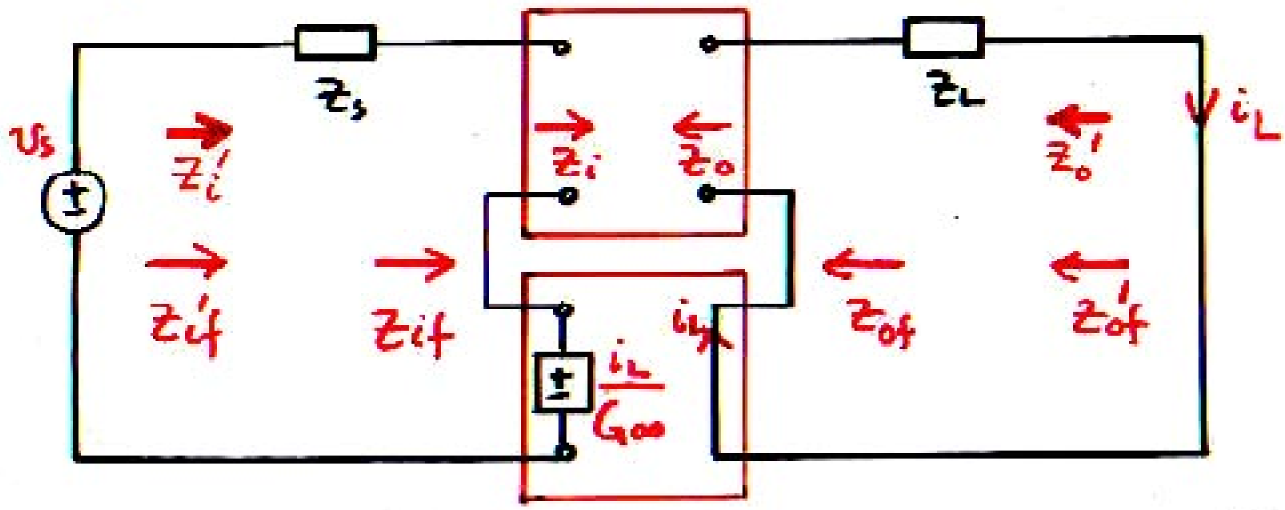
Outside and inside input impedances Z'_{if} and Z_{if} can each be found directly from the loop gain T. They are almost equal (for large T), hence both are much larger than Z_s.

Bottom Line:

The input and output impedances of a feedback amplifier can be found from a knowledge solely of the loop gain, which further emphasizes the fact that the loop gain T is the single central, important property of a feedback amplifier.

2. Voltage-to-current (Series current feedback)



Ideal feedback path must "sense" output current and convert it to a fedback voltage. Voltage summing is done in series.

$$G = \frac{i_L}{v_s} = \frac{\text{current}}{\text{voltage}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

where

$$A = \left.\frac{i_L}{v_s}\right|_{k=1/G_\infty=0}$$

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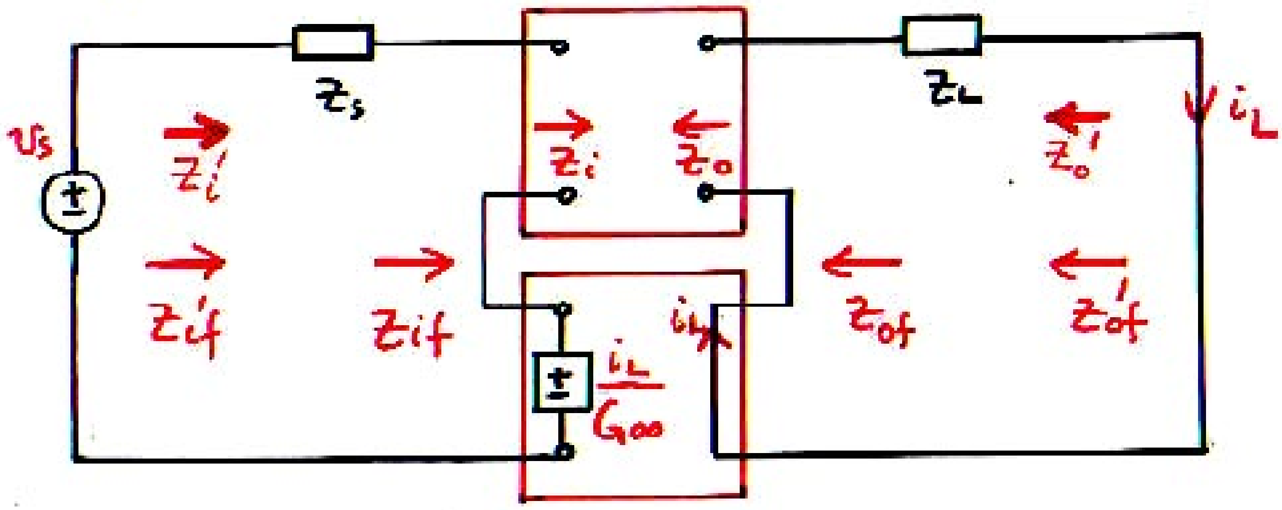

$$Z_{of}' = \frac{Z_L G|_{Z_L\to\infty}}{G} = \frac{\dfrac{Z_L A|_{Z_L\to\infty}}{1+T|_{Z_L\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_L A|_{Z_L\to\infty}}{A}\,\frac{1+T}{1+\cancel{T|}_{Z_L\to\infty}} = Z_o'(1+T)$$

$$Z_{of} = Z_o(1+T|_{Z_L\to\cancel{\infty}\,0})$$

OR:

$$Z_{of}' = \frac{G_\infty \dfrac{Z_L T|_{Z_L\to\infty}}{1+\cancel{T}|_{Z_L\to\infty}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_L T|_{Z_L\to\infty}\right]$$

$$Z_{of} = \frac{1+T}{T}\Big|_{Z_L\to 0}\left[Z_L T|_{Z_L\to\infty}\right]$$

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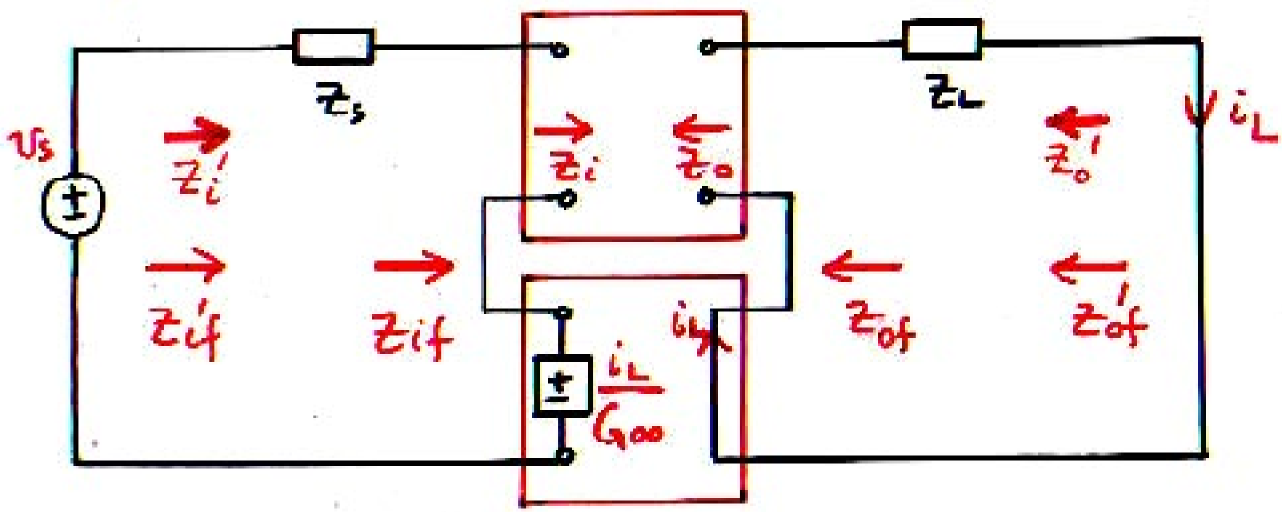

$$Z'_{if} = \frac{Z_s G|_{Z_s \to \infty}}{G} = \frac{\dfrac{Z_s A|_{Z_s\to\infty}}{1+T|_{Z_s\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_s A|_{Z_s\to\infty}}{A}\,\frac{1+T}{1+\cancel{T|_{Z_s\to\infty}}} = Z'_i(1+T)$$

$$Z_{if} = Z_i\left(1+T|_{Z_s\to 0}\right)$$

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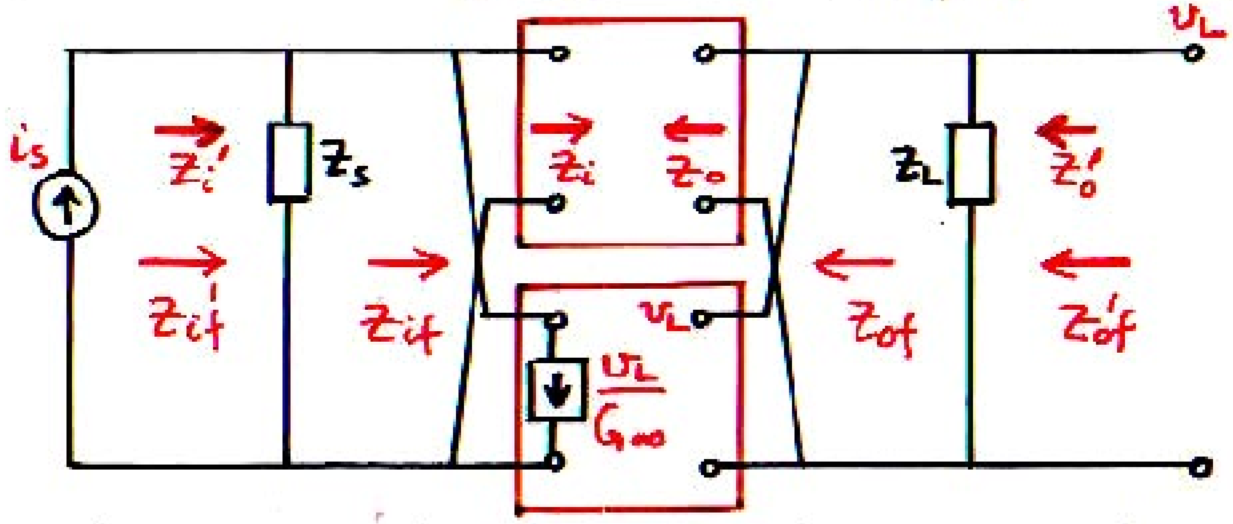
$$Z_{if} = \left.\frac{1+T}{T}\right|_{Z_s\to 0}\left[Z_s T|_{Z_s\to\infty}\right]$$

3. Current-to-voltage (shunt voltage feedback)



Ideal feedback path must "sense" output voltage and convert it to a fedback current. Current summing is done in shunt.

$$G = \frac{v_L}{i_s} = \frac{\text{voltage}}{\text{current}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

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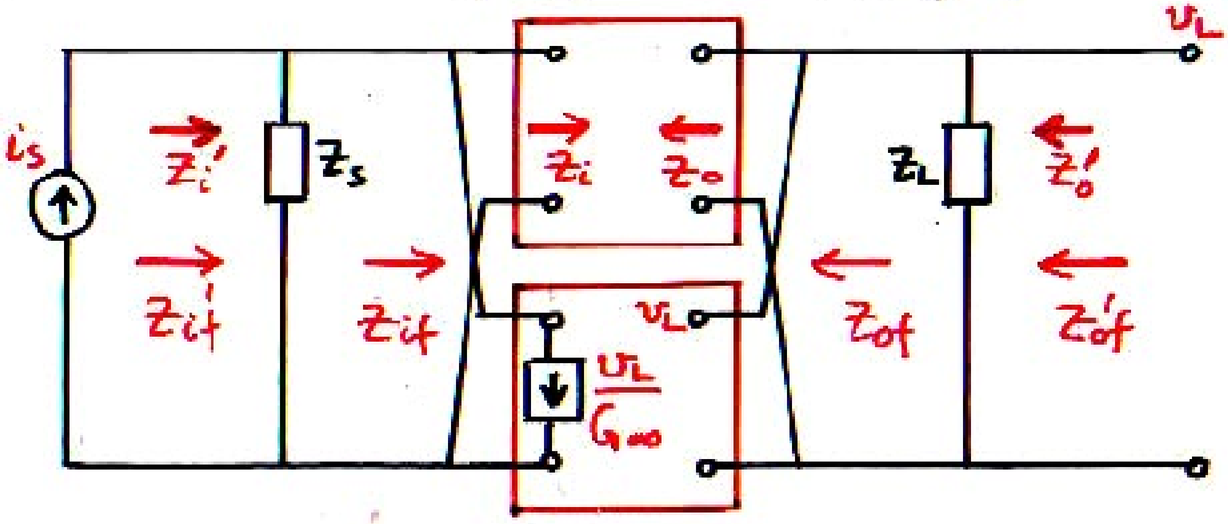

$$Z_{of}' = \frac{G}{\left.\frac{G}{Z_L}\right|_{Z_L\to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}\frac{1}{1+T|_{Z_L\to 0}}} = \frac{A}{\left.\frac{A}{Z_L}\right|_{Z_L\to 0}}\ \frac{1+\cancel{T|_{Z_L\to 0}}}{1+T} = \frac{Z_o'}{1+T}$$

$$Z_{of} = \frac{Z_o}{1+T|_{Z_L\to\infty}}$$

OR:

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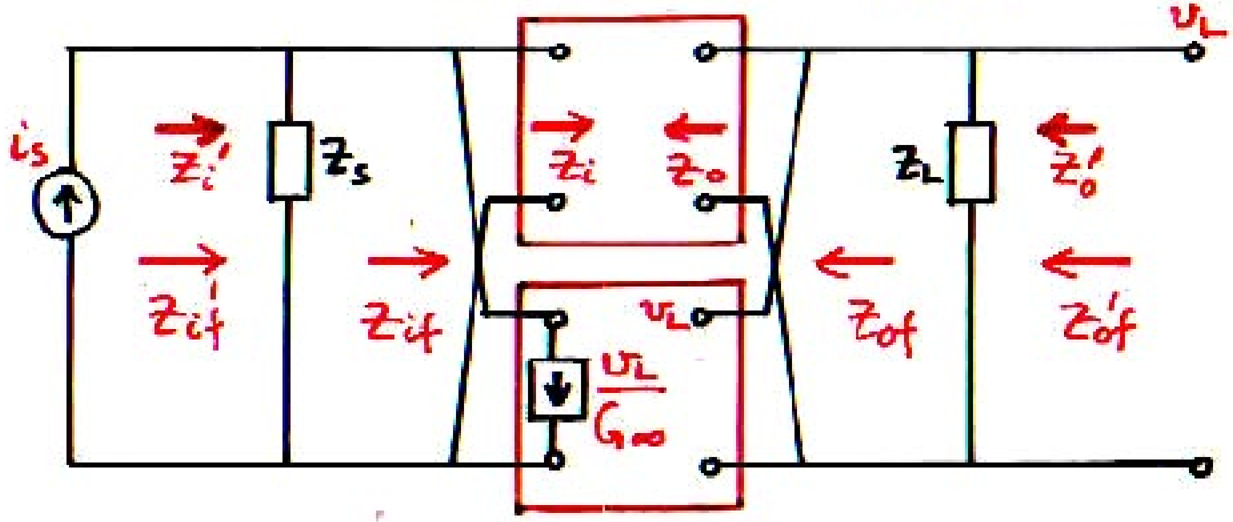

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$$Z_{if} = \frac{Z_i}{1+T|_{Z_s\to\infty}}$$

OR:

$$Z_{if}' = \frac{G_\infty\frac{T}{1+T}}{\left.G_\infty\frac{T}{Z_s}\right|_{Z_s\to 0}\frac{1}{1+\cancel{T}|_{Z_s\to 0}}} = \frac{T}{1+T}\left[\left.\frac{Z_s}{T}\right|_{Z_s\to 0}\right]$$

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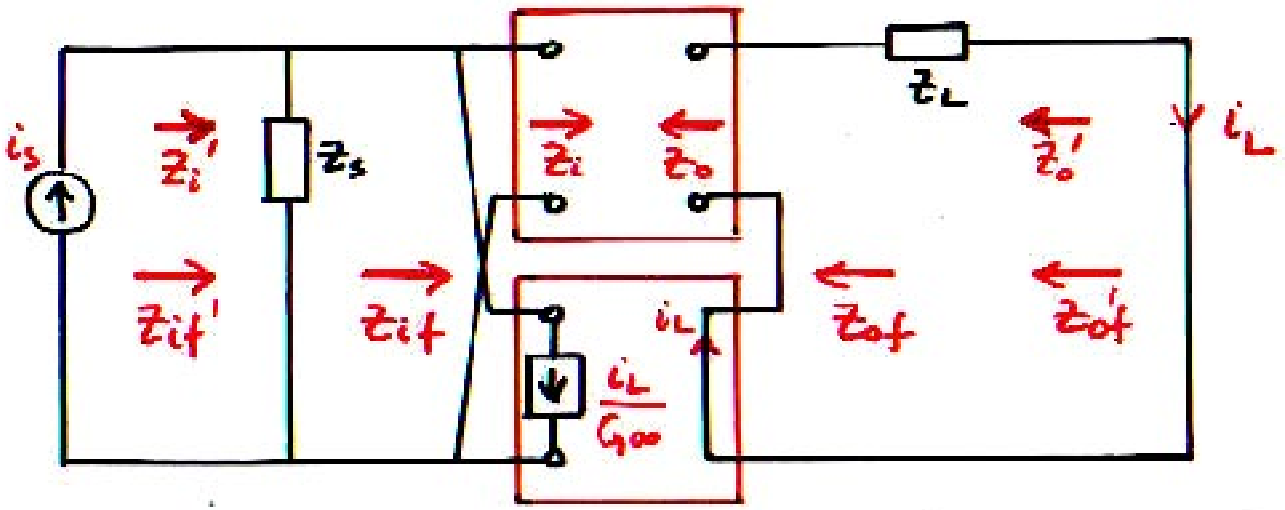
4. Current-to-current (shunt current feedback)



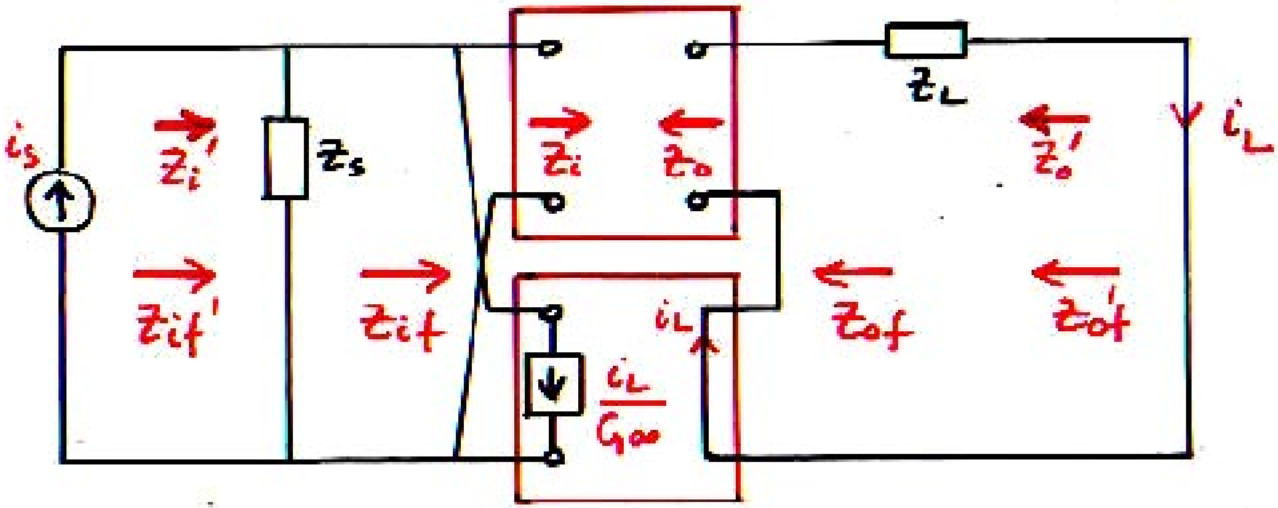
Ideal feedback path must "sense" output current and convert it to a fedback current. Current summing is done in shunt.

$$G = \frac{i_L}{i_s} = \frac{\text{current}}{\text{current}} = \frac{A}{1+T} = G_\infty \frac{T}{1+T}$$

where

$$A = \left.\frac{i_L}{i_s}\right|_{K = 1/G_\infty = 0}$$

4. Current-to-current (shunt current feedback)



$$Z_{of}' = \frac{Z_L G|_{Z_L\to\infty}}{G} = \frac{\dfrac{Z_L A|_{Z_L\to\infty}}{1+T|_{Z_L\to\infty}}}{\dfrac{A}{1+T}} = \frac{Z_L A|_{Z_L\to\infty}}{A}\;\frac{1+T}{1+\cancel{T}|_{Z_L\to\infty}} = Z_o'(1+T)$$

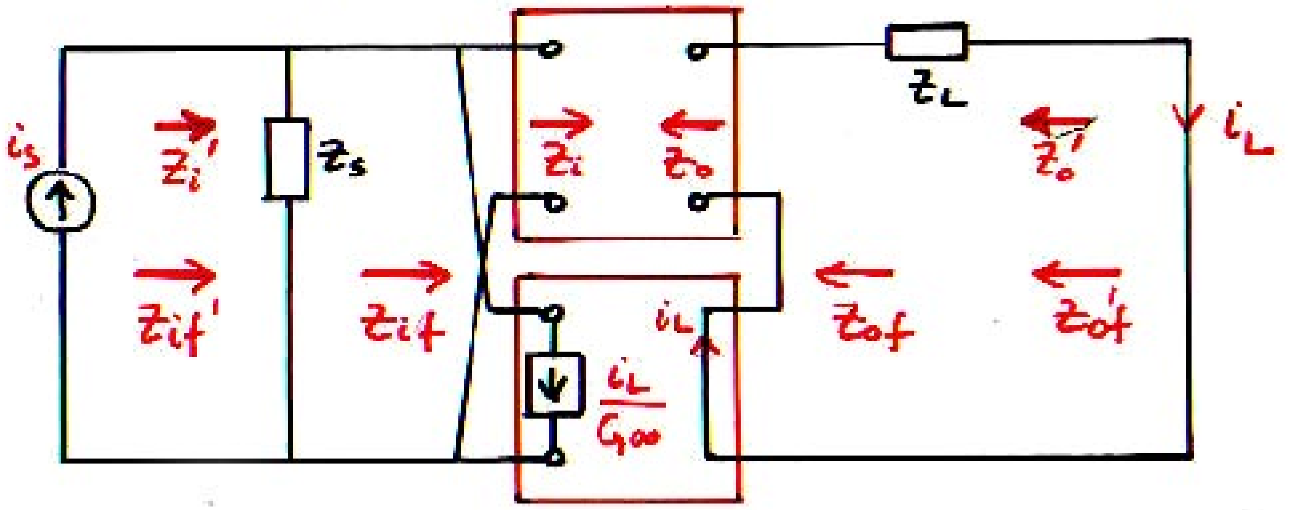
$$Z_{of} = Z_o(1+T|_{Z_L\to \overset{0}{\cancel{\infty}}})$$

OR:

$$Z_{of}' = \frac{G_\infty \dfrac{Z_L T|_{Z_L\to\infty}}{1+\cancel{T}|_{Z_L\to\infty}}}{G_\infty \dfrac{T}{1+T}} = \frac{1+T}{T}\left[Z_L T|_{Z_L\to\infty}\right]$$

$$Z_{of} = \frac{1+T}{T}\Big|_{Z_L\to 0}\left[Z_L T|_{Z_L\to\infty}\right]$$

4. Current-to-current (shunt current feedback)



$$Z_{if}' = \frac{G}{\left.\frac{G}{Z_s}\right|_{Z_s \to 0}} = \frac{\frac{A}{1+T}}{\left.\frac{A}{Z_s}\right|_{Z_s \to 0} \frac{1}{1+T|_{Z_s \to 0}}} = \frac{A}{\left.\frac{A}{Z_s}\right|_{Z_s \to 0}} \frac{1+\cancel{T|_{Z_s \to 0}}}{1+T} = \frac{Z_i'}{1+T}$$

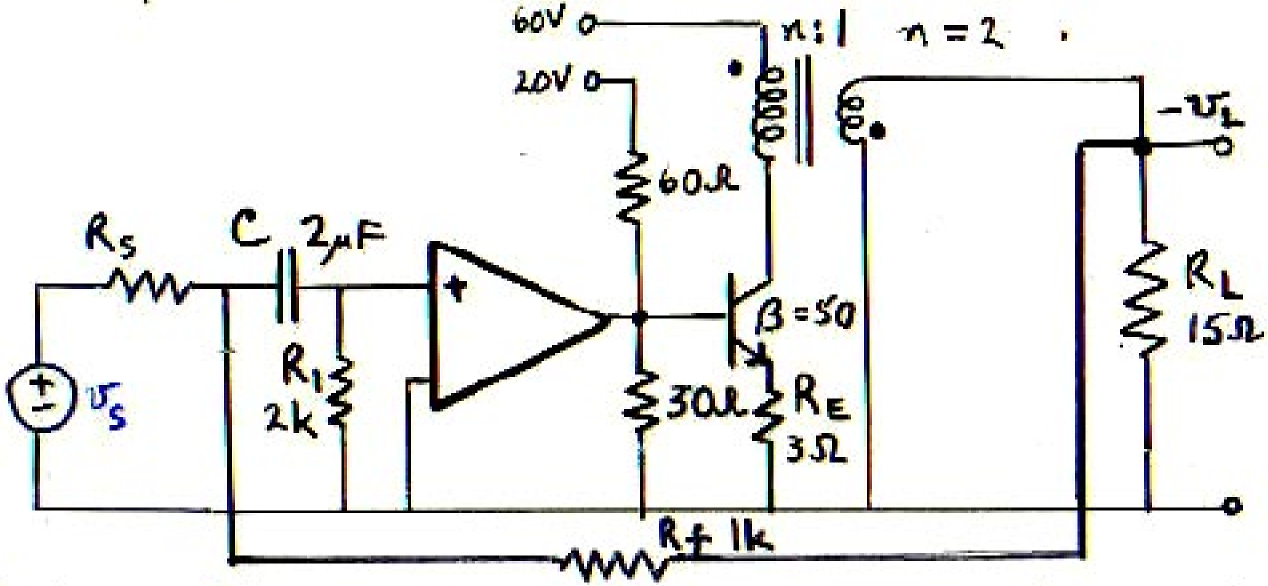
$$Z_{if} = \frac{Z_i}{1+T|_{Z_s \to \infty}}$$

OR:

$$Z_{if}' = \frac{G_\infty \frac{T}{1+T}}{G_\infty \left.\frac{T}{Z_s}\right|_{Z_s \to 0} \frac{1}{1+\cancel{T}|_{Z_s \to 0}}} = \frac{T}{1+T}\left[\left.\frac{Z_s}{T}\right|_{Z_s \to 0}\right]$$

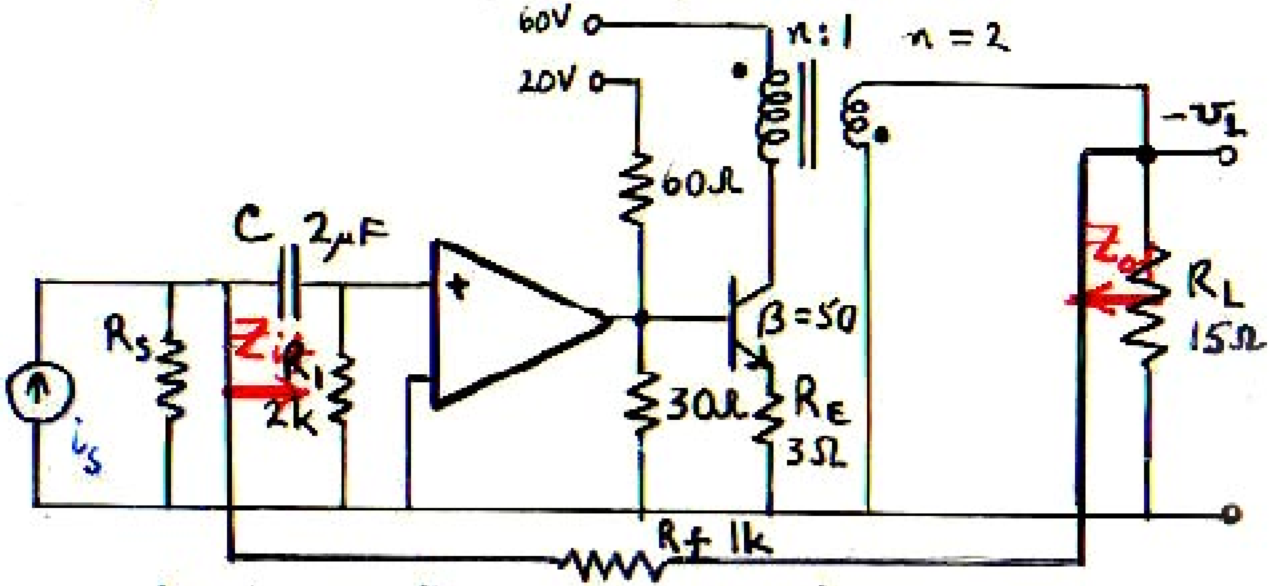
$$Z_{if} = \left.\frac{T}{1+T}\right|_{Z_s \to \infty}\left[\left.\frac{Z_s}{T}\right|_{Z_s \to 0}\right]$$

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8$ dB and $f_A = 2$ kHz, and an output impedance of 4.5Ω



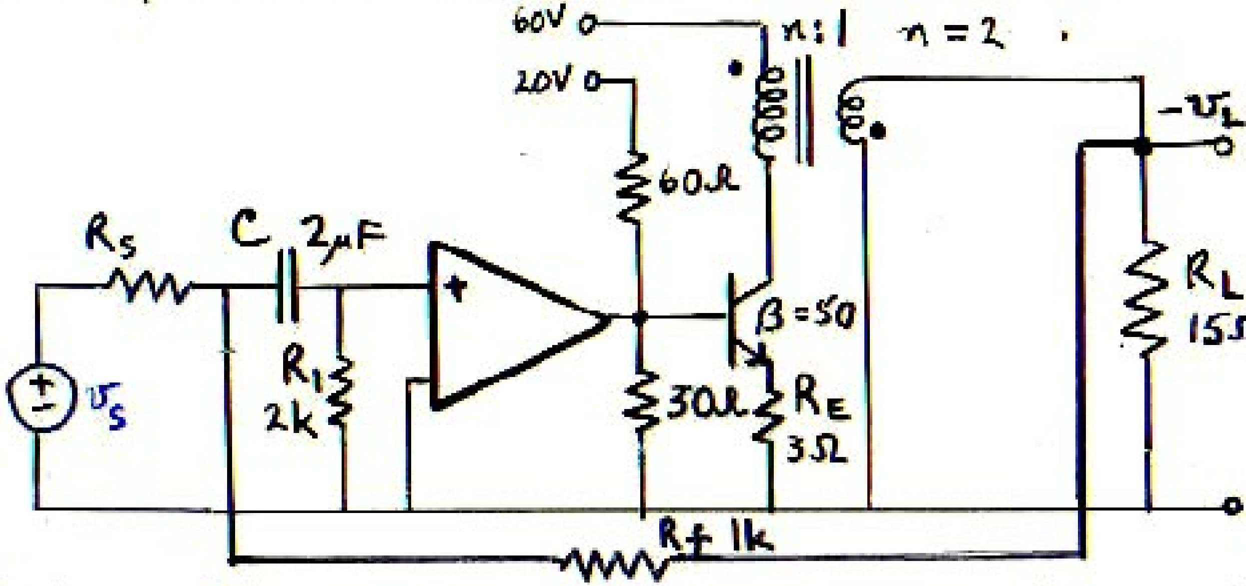
Although the gain is expressed as a voltage ratio, this is really a "Type 3" current-to-voltage feedback amplifier, because the feedback signal is connected at a shunt current-summing junction.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/((1+s/\omega_A)$, where $A_{10} = 8\,dB$ and $f_A = 2\,kHz$, and an output impedance of $4.5\,\Omega$.



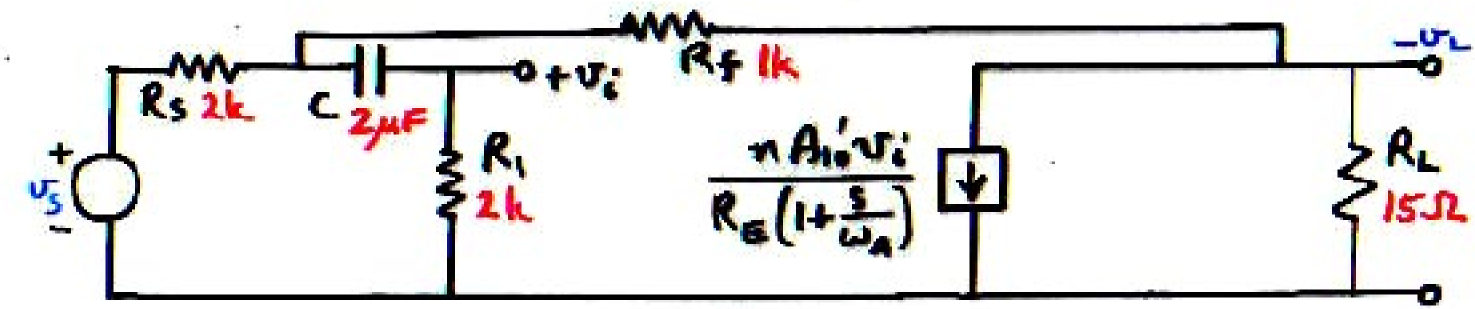
Hence, the "inside" output and input impedances Z_{of} and Z_{if} are both lowered by the feedback.

Single-ended Class A audio feedback power amplifier, based on the same power stage previously discussed. The driver opamp has a gain $A_1 = A_{10}/(1+s/\omega_A)$, where $A_{10} = 8\,dB$ and $f_A = 2\,kHz$, and an output impedance of $4.5\,\Omega$



Although the gain is expressed as a voltage ratio, this is really a "Type 3" current-to-voltage feedback amplifier, because the feedback signal is connected at a shunt current-summing junction.

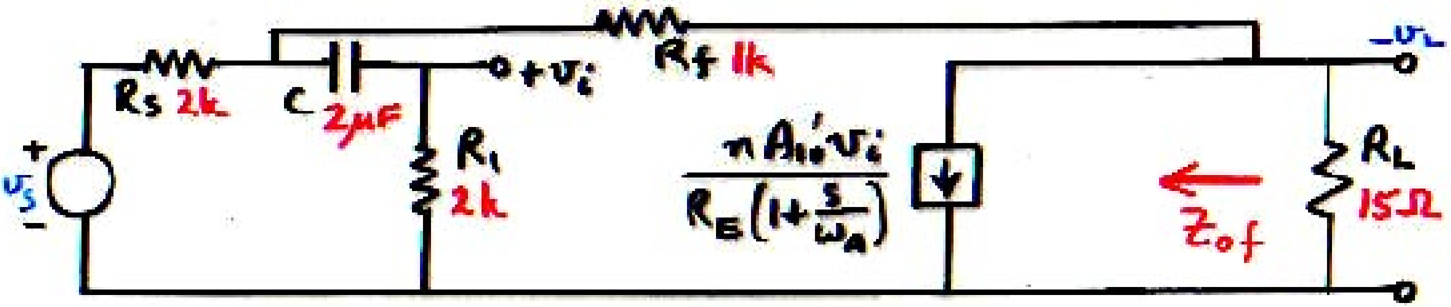
Ac model



R_S 2k
C 2μF
$+v_i$
R_f 1k
R_1 2k
v_S
+
−
$\frac{n A_{io}' v_i}{R_E\left(1+\frac{s}{\omega_A}\right)}$
$-v_L$
R_L 15Ω

Ac model

Find Z_{of}



Use $$Z_{of} = \frac{T}{1+T}\bigg|_{R_L \to \infty} \left[\frac{R_L}{T}\bigg|_{R_L \to 0} \right]$$

where

$$T = T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$T_m = \frac{(R_f \| R_s \| R_1)\, n\, A'_{10} R_L}{R_f R_E}$$

$$\omega_2 \equiv \frac{1}{C[R_f \| R_s + R_1]}$$

$$\left.\frac{R_L}{T}\right|_{R_L \to 0} = \frac{R_f R_E}{(R_f \| R_s \| R_i)\, n A_{io}'}\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)$$

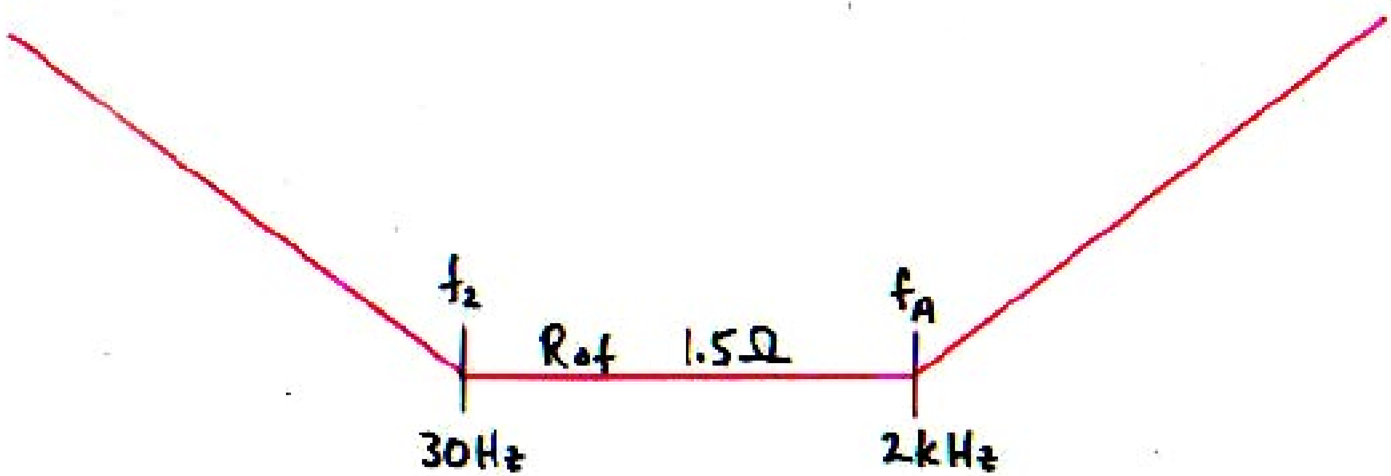
$$T|_{R_L \to \infty} = \infty, \quad \text{so} \quad \left.\frac{T}{1+T}\right|_{R_L \to \infty} = 1$$

Hence

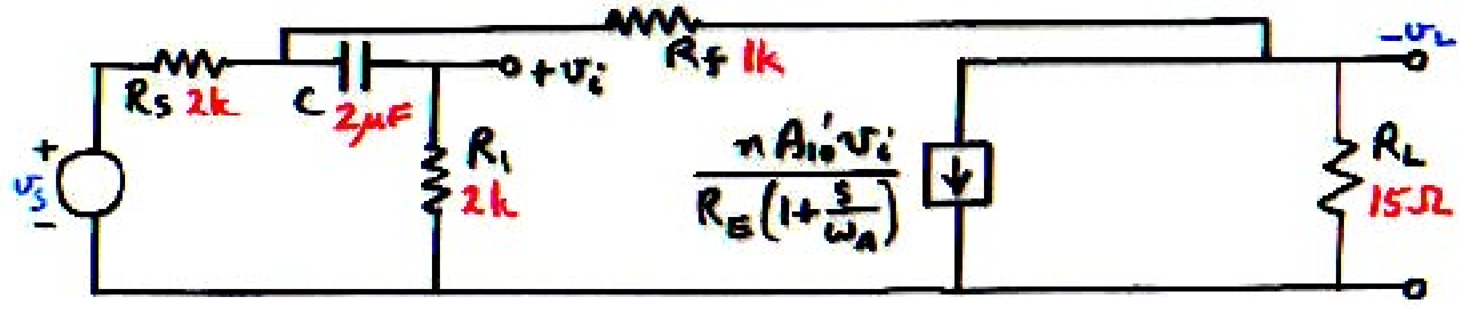
$$Z_{of} = R_{of}\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)$$

where

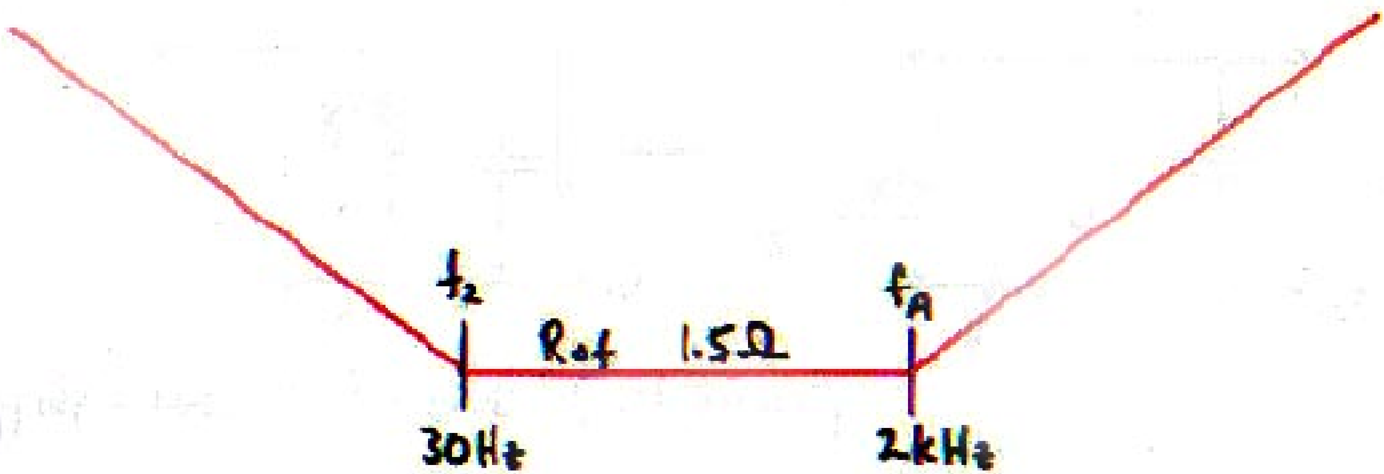
$$R_{of} \equiv \frac{R_f R_E}{(R_f \| R_s \| R_i)\, n A_{io}'} = \frac{1 \times 3}{0.5 \times 2 \times 2} = 1.5\,\Omega$$



Ac model

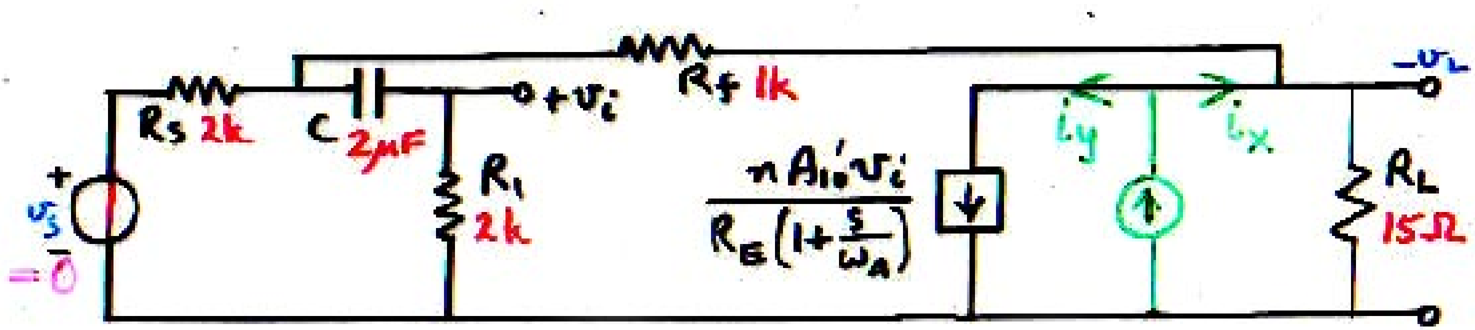


Rf 1k
Rs 2k
C 2μF
+ vi
R1 2k
−vL
$\frac{n A_{10}' v_i}{R_E \left(1+\frac{s}{\omega_A}\right)}$
vs
RL 15Ω



f2
fA
Rof 1.5Ω
30Hz
2kHz

Ac model



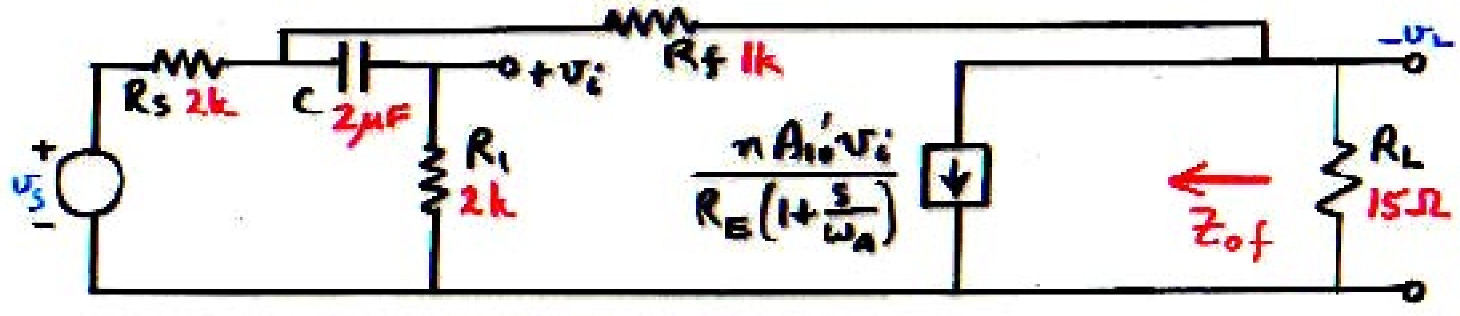
$$T = \left.\frac{i_y}{i_x}\right|_{v_s=0} = \frac{R_L}{\cancel{R_L}+R_f+R_s \| R_1\left(1+\frac{\omega_1}{s}\right)} \frac{R_s}{R_s+R_1\left(1+\frac{\omega_1}{s}\right)} \frac{R_1 n A_{10}'}{R_E\left(1+\frac{s}{\omega_A}\right)}$$

where

$$\omega_1 \equiv \frac{1}{CR_1} \qquad f_1 = \frac{159}{2\times 2} = 40\,\text{Hz}$$

Ac model

Find Z_{of}



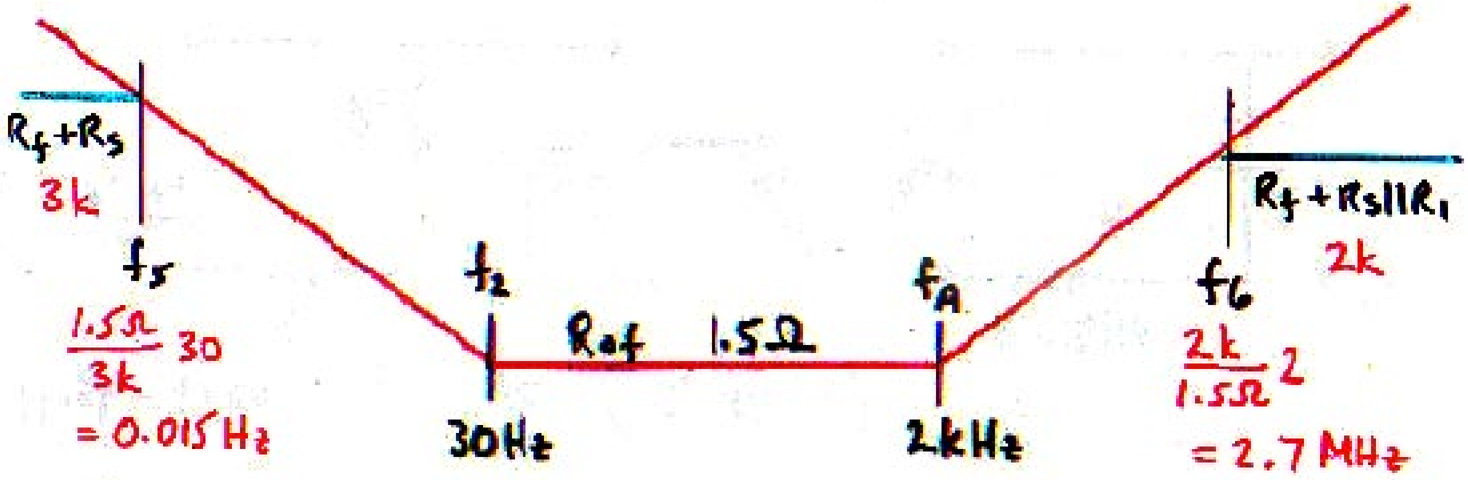
Use $$Z_{of} = \frac{T}{1+T}\bigg|_{R_L\to\infty}\left[\frac{R_L}{T}\bigg|_{R_L\to 0}\right]$$

where

$$T = T_m \frac{1}{\left(1+\frac{\omega_2}{s}\right)\left(1+\frac{s}{\omega_A}\right)}$$

$$T_m = \frac{(R_f \| R_s \| R_1)\, n A_{10}' R_L}{R_f R_E}$$

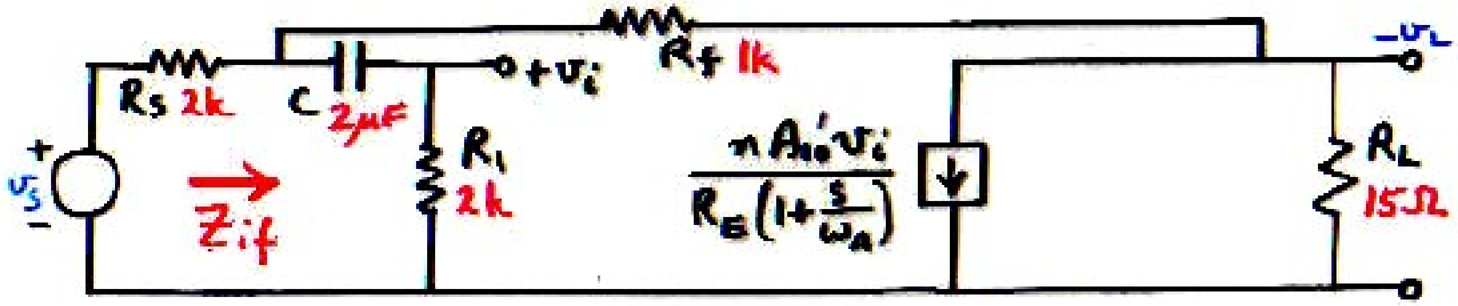
$$\omega_2 \equiv \frac{1}{C[R_f \| R_s + R_1]}$$



Note from the actual circuit that Z_{of} would limit at both low and high frequencies, because of the loading effect of the feedback path on R_L. This nonideality was ignored in the solution, but could have been included with further complication but no additional difficulty.

Ac model

Find Z_{if}



Use
$$Z_{if} = \frac{T}{1+T}\Big|_{R_s\to\infty} \cdot \frac{R_s}{T}\Big|_{R_s\to 0}$$

where
$$D = \frac{T}{1+T} = \frac{T_m}{1+T_m}\,\frac{1}{\left(1+\frac{\omega_3}{s}\right)\left(1+\frac{s}{\omega_4}\right)}$$

$$\omega_3 = \frac{\omega_2}{1+T_m} \qquad \omega_2 = \frac{1}{C\left[R_f \| R_s + R_1\right]}$$

$$\omega_4 = (1+T_m)\,\omega_A$$

$$\frac{R_s}{T}\bigg|_{R_s \to 0} = \frac{R_f R_E}{n A_{io}' R_L}\left(1 + \frac{\omega_7}{s}\right)\left(1 + \frac{s}{\omega_A}\right)$$

where

$$\omega_7 \equiv \omega_2\big|_{R_s \to 0} = \frac{1}{C R_1} = \omega_1$$

$$D\big|_{R_s \to \infty} = \frac{(R_f \| R_1)\, n A_{io}' R_L}{R_f R_E \left(1 + T_m\big|_{R_s \to \infty}\right)} \frac{1}{\left(1 + \frac{\omega_8}{s}\right)\left(1 + \frac{s}{\omega_9}\right)}$$

where

$$\omega_8 \equiv \omega_3\big|_{R_s \to \infty} = \frac{\omega_2\big|_{R_s \to \infty}}{1 + T_m\big|_{R_s \to \infty}}$$

$$= \frac{1}{C(R_f + R_1)\left(1 + T_m\big|_{R_s \to \infty}\right)}$$

$$T_m\big|_{R_s \to \infty} = \frac{(R_f \| R_1)\, n A_{io}' R_L}{R_f R_E} = \frac{(1 \| 2)\, 2 \times 2 \times 15}{1 \times 3} = 13$$

$$f_8 = \frac{159}{2(1+2)(1+13)} = 1.9\,\text{Hz}$$

$$\omega_9 \equiv \left(1 + T_m\big|_{R_s \to \infty}\right)\omega_A$$

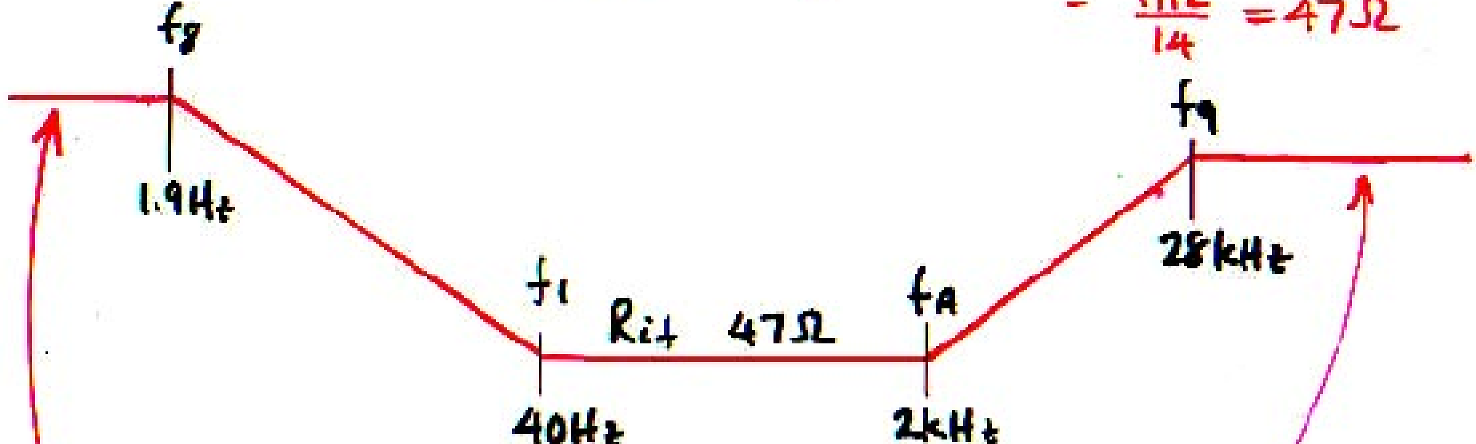
$$f_9 = 14 \times 2 = 28\,\text{kHz}$$

Hence

$$Z_{if} = \frac{(R_f \| R_1)\, n A'_{io} R_L}{R_f R_E \left(1 + T_m|_{R_s \to \infty}\right)} \; \frac{R_f R_E}{n A_{io} R_L} \; \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_A}\right)}{\left(1+\frac{\omega_8}{s}\right)\left(1+\frac{s}{\omega_9}\right)}$$

$$= R_{if} \frac{\left(1+\frac{\omega_1}{s}\right)\left(1+\frac{s}{\omega_A}\right)}{\left(1+\frac{\omega_8}{s}\right)\left(1+\frac{s}{\omega_9}\right)} \qquad R_{if} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}}$$

$$= \frac{1\|2}{14} = 47\Omega$$



$$R_{if}\frac{\omega_1}{\omega_8} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}} \; \frac{C(R_f + R_1)\left(1 + T_m|_{R_s \to \infty}\right)}{C R_1}$$

$$= R_f$$

$$= 1k$$

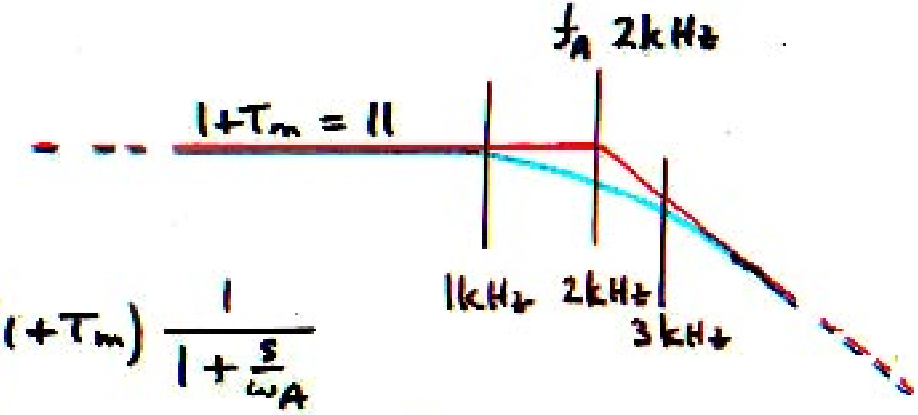
$$R_{if}\frac{\omega_9}{\omega_A} = \frac{R_f \| R_1}{1 + T_m|_{R_s \to \infty}} \left(1 + T_m|_{R_s \to \infty}\right)$$

$$= R_f \| R_1 = 0.67k$$

Example: Distortion

When the open-loop amplifier is delivering full output power at 1kHz, the output stage develops 3% 2nd harmonic and 5% 3rd harmonic distortion. Find the full output power closed-loop total harmonic distortion at 1kHz.

Percentage of each harmonic is reduced by $|1+T|$ at the harmonic frequency.



$$1+T = (1+T_m)\frac{1}{1+\frac{s}{\omega_A}}$$

$$|1+T|_{2kHz} = 11\frac{1}{\sqrt{1+\left(\frac{2}{2}\right)^2}} = \frac{11}{\sqrt{2}} = 7.8$$

$$|1+T|_{3kHz} = 11\frac{1}{\sqrt{1+\left(\frac{3}{2}\right)^2}} = \frac{11}{\sqrt{3.8}} = 6.1$$

$$THD = \sqrt{\left(\frac{3\%}{7.8}\right)^2 + \left(\frac{5\%}{6.1}\right)^2} = \sqrt{0.39^2 + 0.82^2}$$

$$= 0.91\%$$

Generalization: Effect of Feedback on Impedances

1. Output impedance is $\left\{\begin{matrix}\text{decreased}\\ \text{increased}\end{matrix}\right\}$ by $\left\{\begin{matrix}\text{voltage}\\ \text{current}\end{matrix}\right\}$ feedback from the output.

   Input impedance is $\left\{\begin{matrix}\text{increased}\\ \text{decreased}\end{matrix}\right\}$ by $\left\{\begin{matrix}\text{series}\\ \text{shunt}\end{matrix}\right\}$ feedback to the input.

2. The "outside" impedances are changed by a factor $(1+T)$

   The "inside" impedances are changed by a larger factor.

3. Alternatively, the input and output impedances can be found solely from the loop gain T.

Stability

If the open-loop gain A is stable, the closed-loop gain

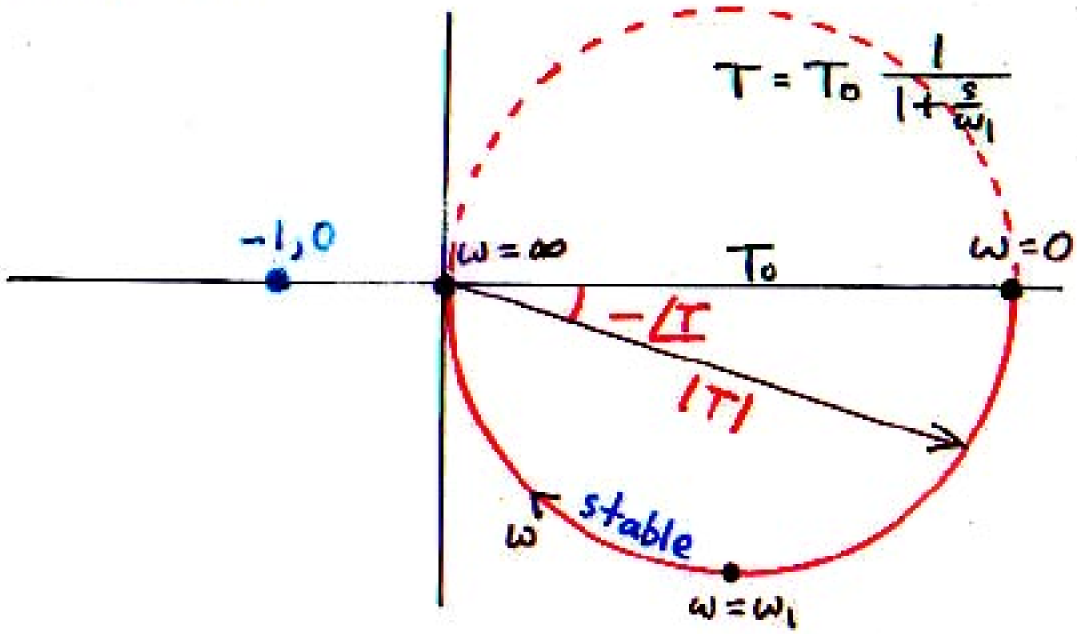
$$G = \frac{A}{1+T}$$

is stable if $1+T$ has no roots in the right half-plane (rhp).

By complex variable theory, this implies that a polar plot of $1+T$ must not encircle the origin; or, equivalently, that a polar plot of T must not encircle the $(-1,0)$ point. (Nyquist Stability Criterion).

Simple cases of the Nyquist plot of loop gain T:

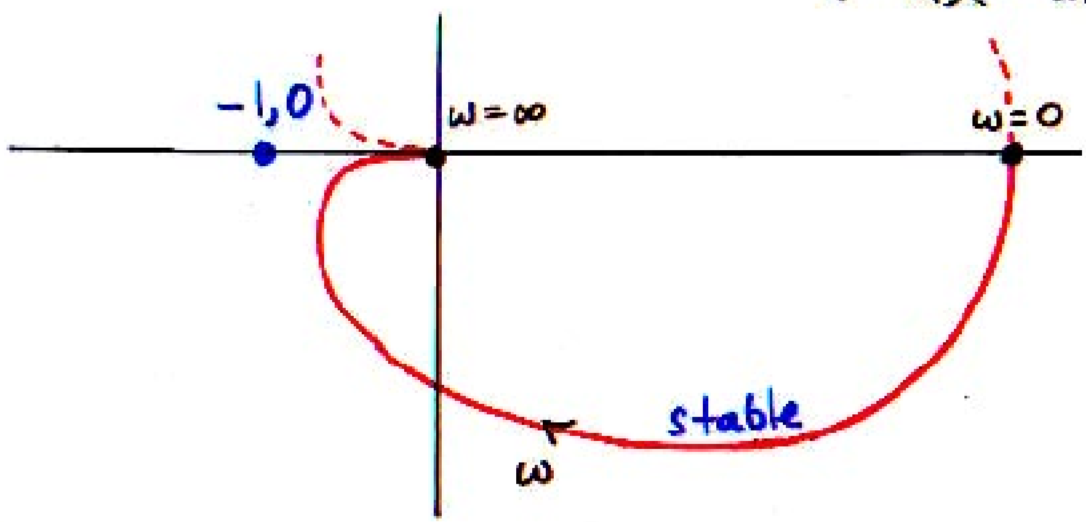
1-pole response:



Always stable

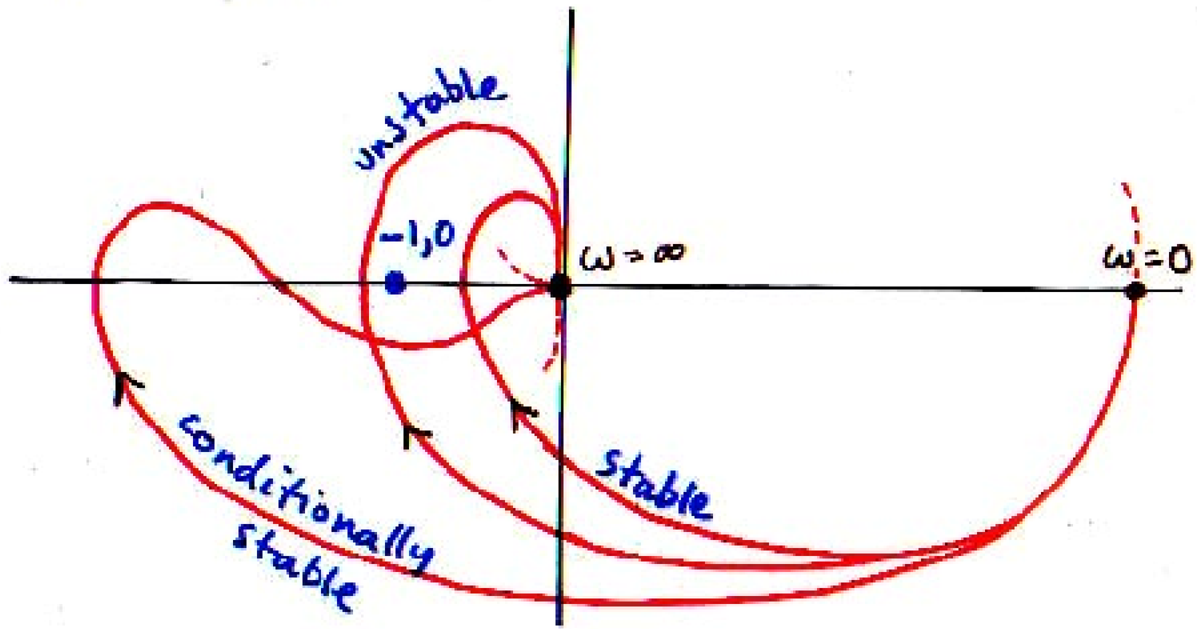
2-pole response

$$T = T_0 \frac{1}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)}$$



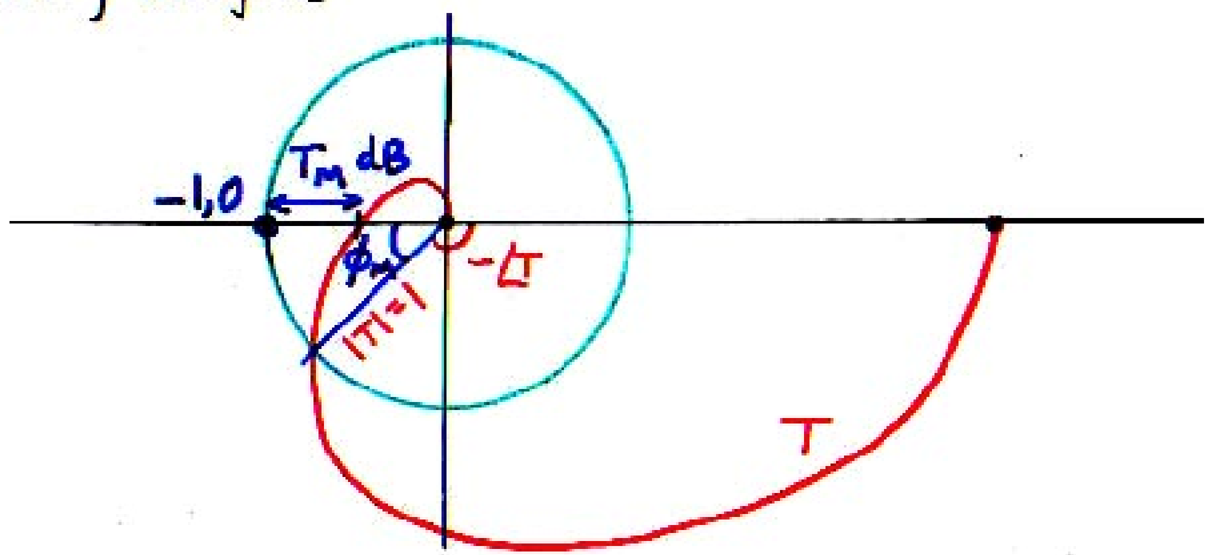
Always stable

3-pole response



Can be stable or unstable

Stability margins

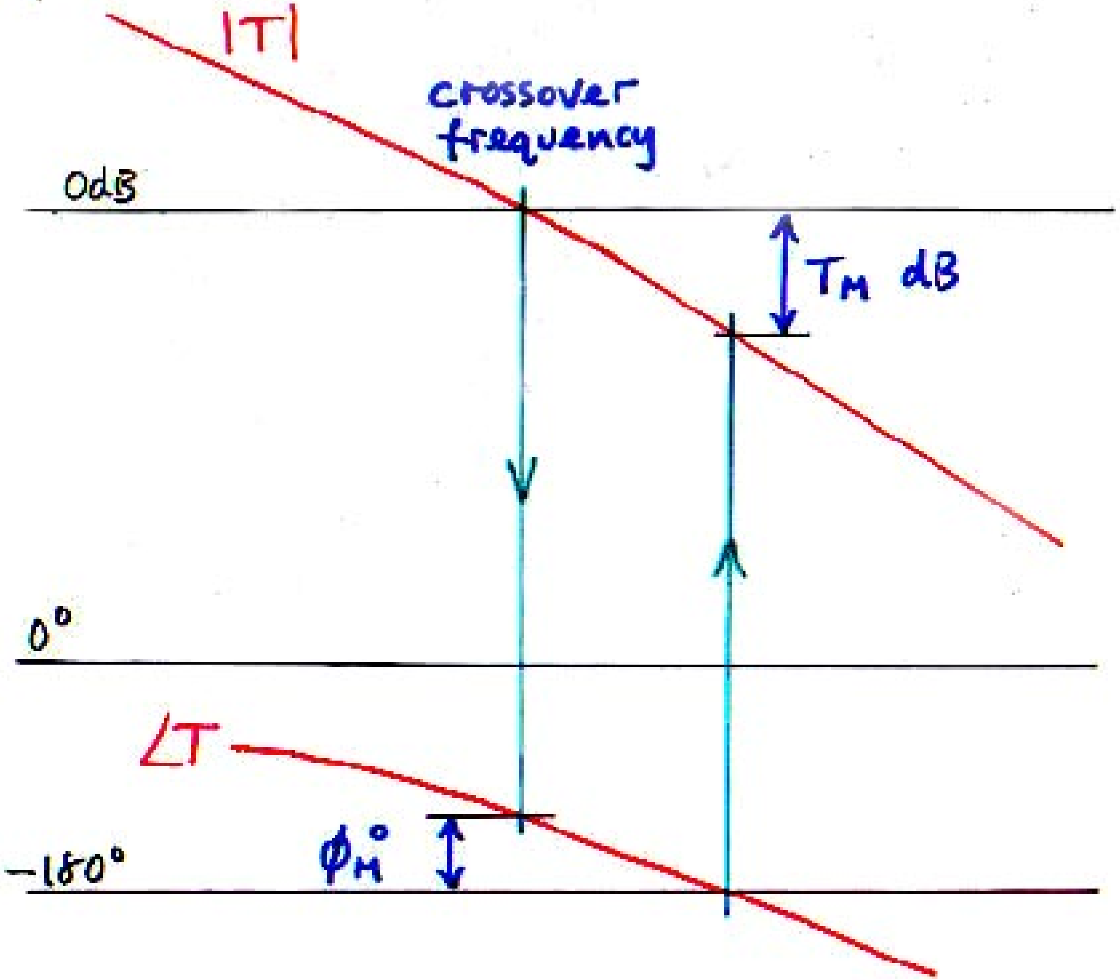


Phase margin $\phi_M \equiv 180° + \angle T$ when $|T| = 1$ (0dB)

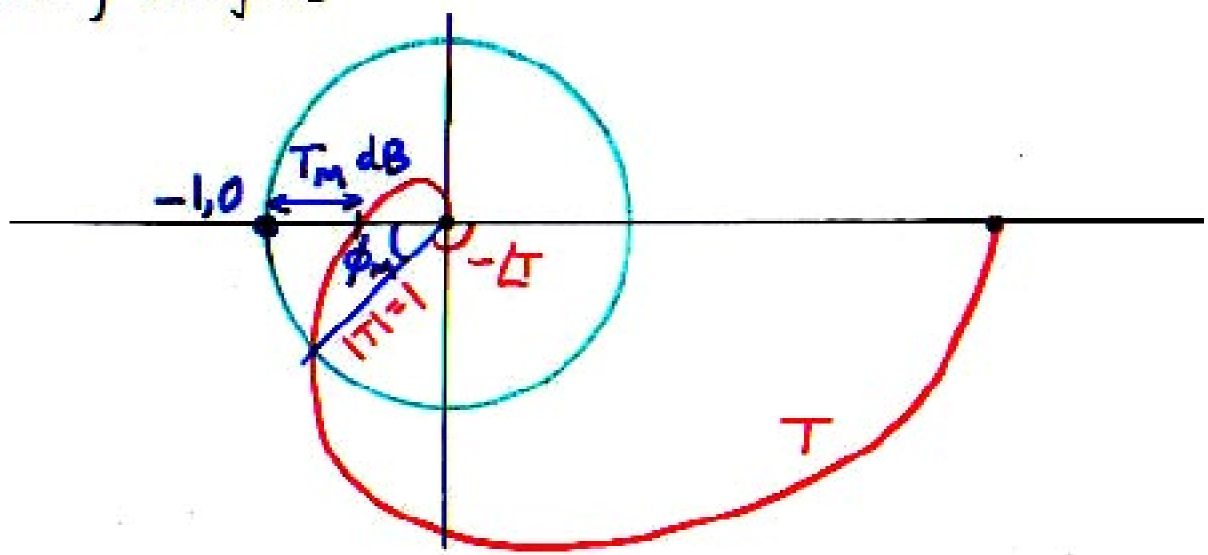
$\angle T$ negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180°$

Bode plot:



Stability margins

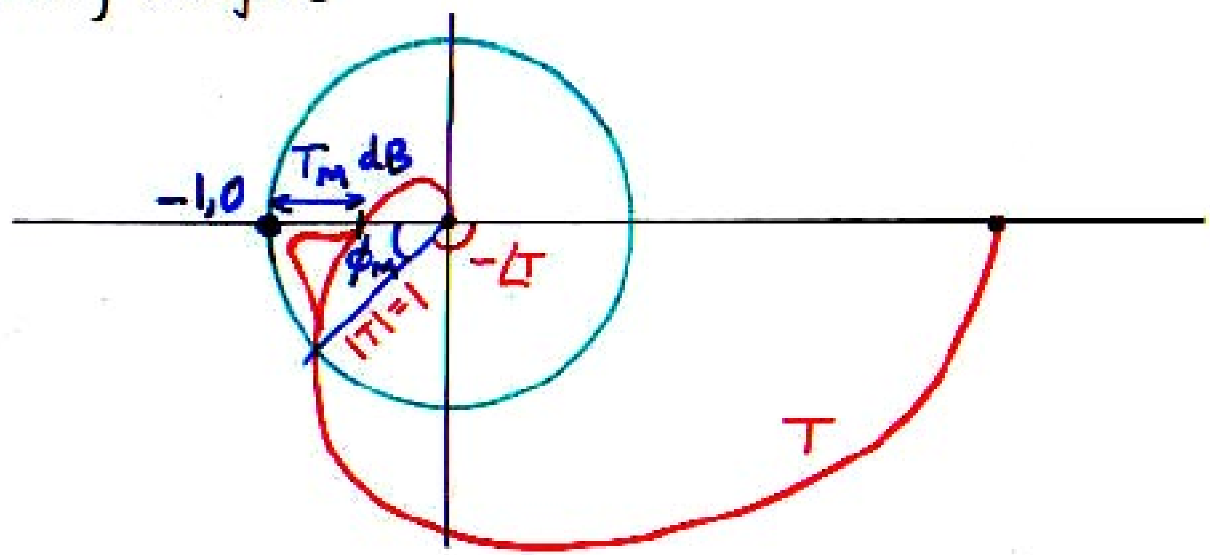


Phase margin $\phi_M = 180° + \angle T$ when $|T| = 1$ (0dB)

negative (lag)

Gain margin $T_M|_{dB} = -T|_{dB}$ when $\angle T = -180°$

Stability margins

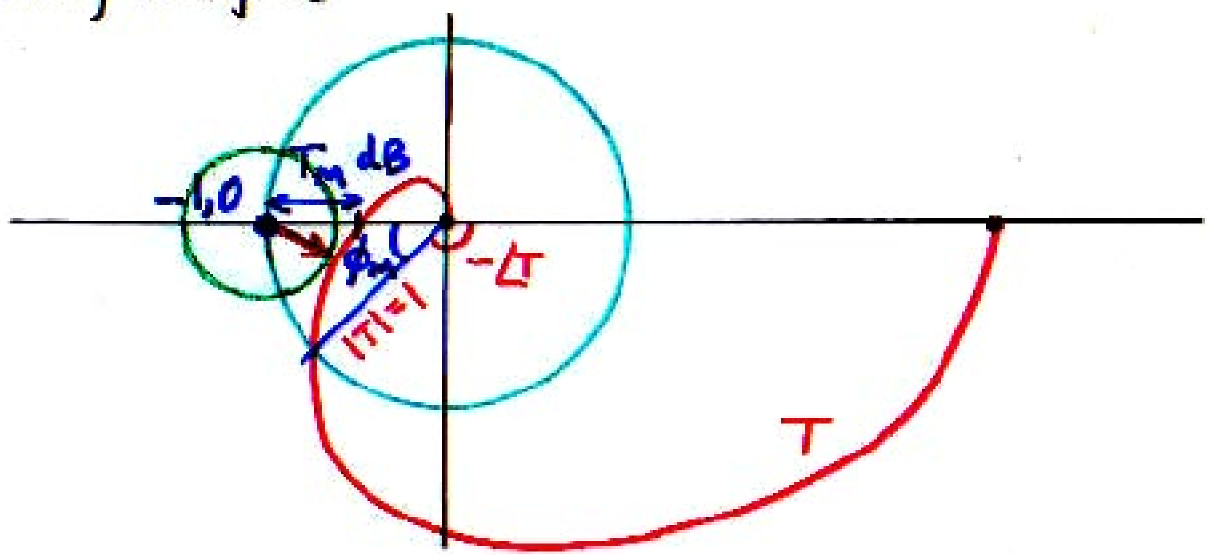


Phase margin $\phi_M \equiv 180^\circ + \angle T$ when $|T| = 1$ (0dB)

negative (lag)

Gain margin $T_M|_{dB} \equiv -T|_{dB}$ when $\angle T = -180^\circ$

Stability margins



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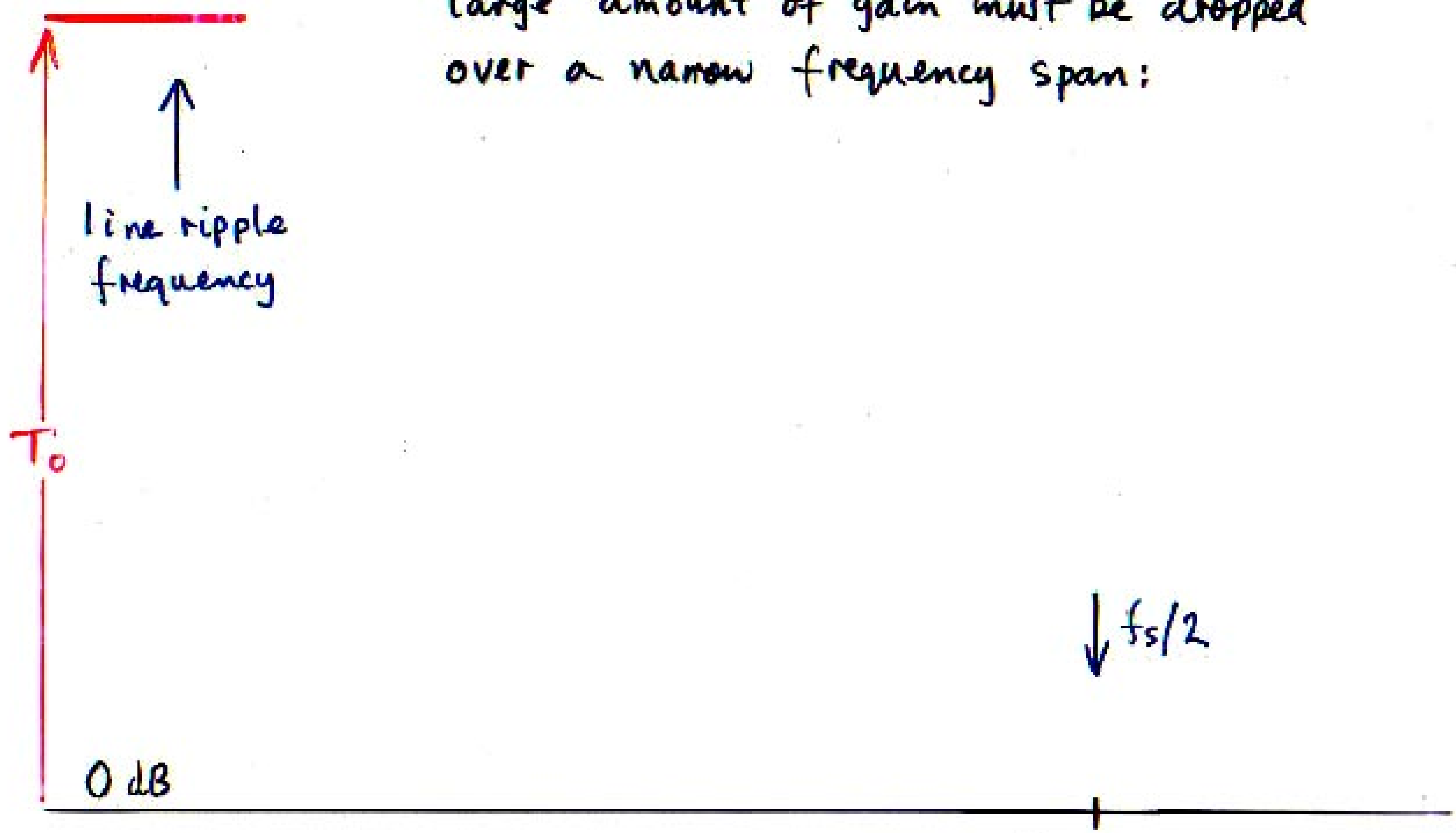
A conditionally stable system may be necessary when a large amount of gain must be dropped over a narrow frequency span:



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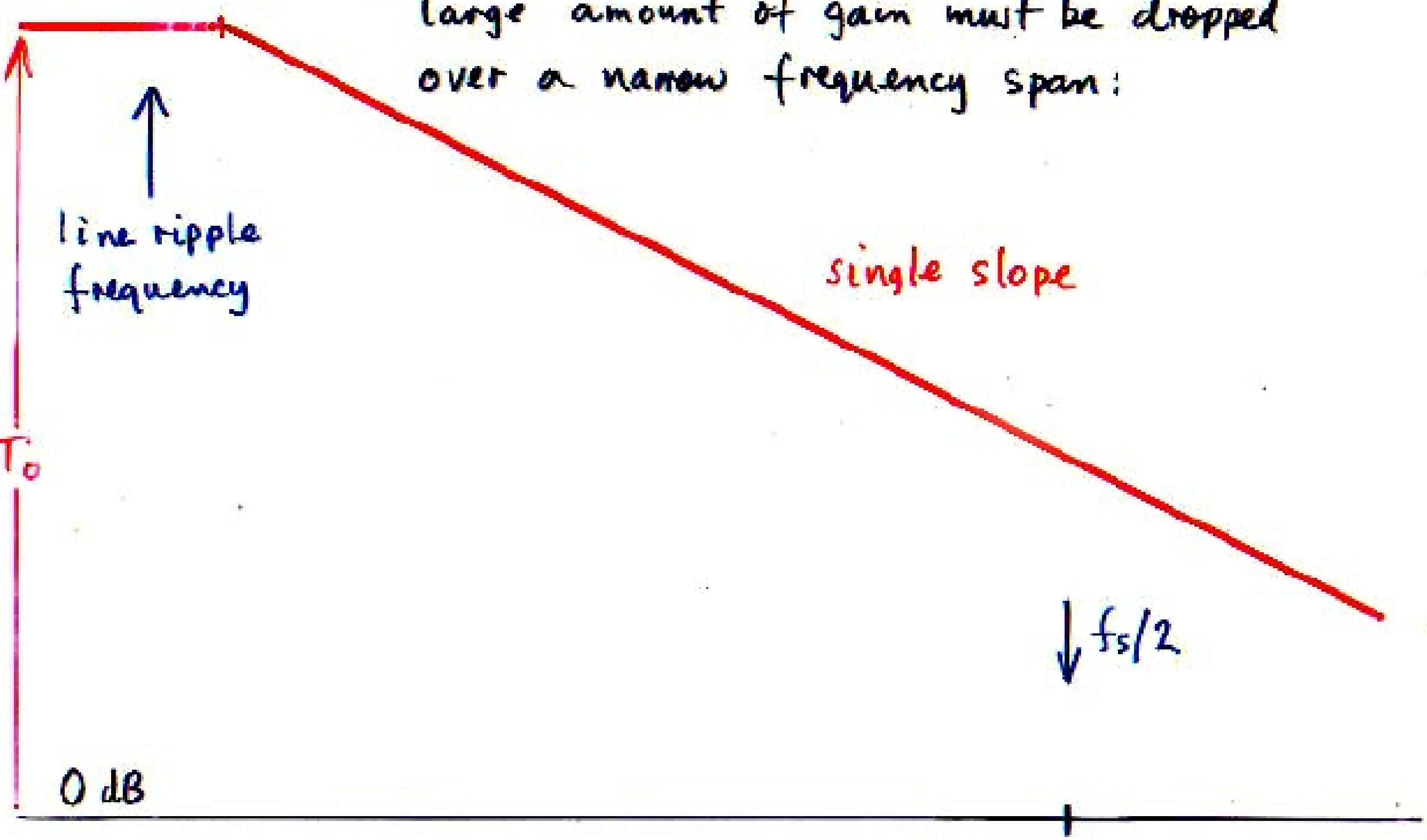

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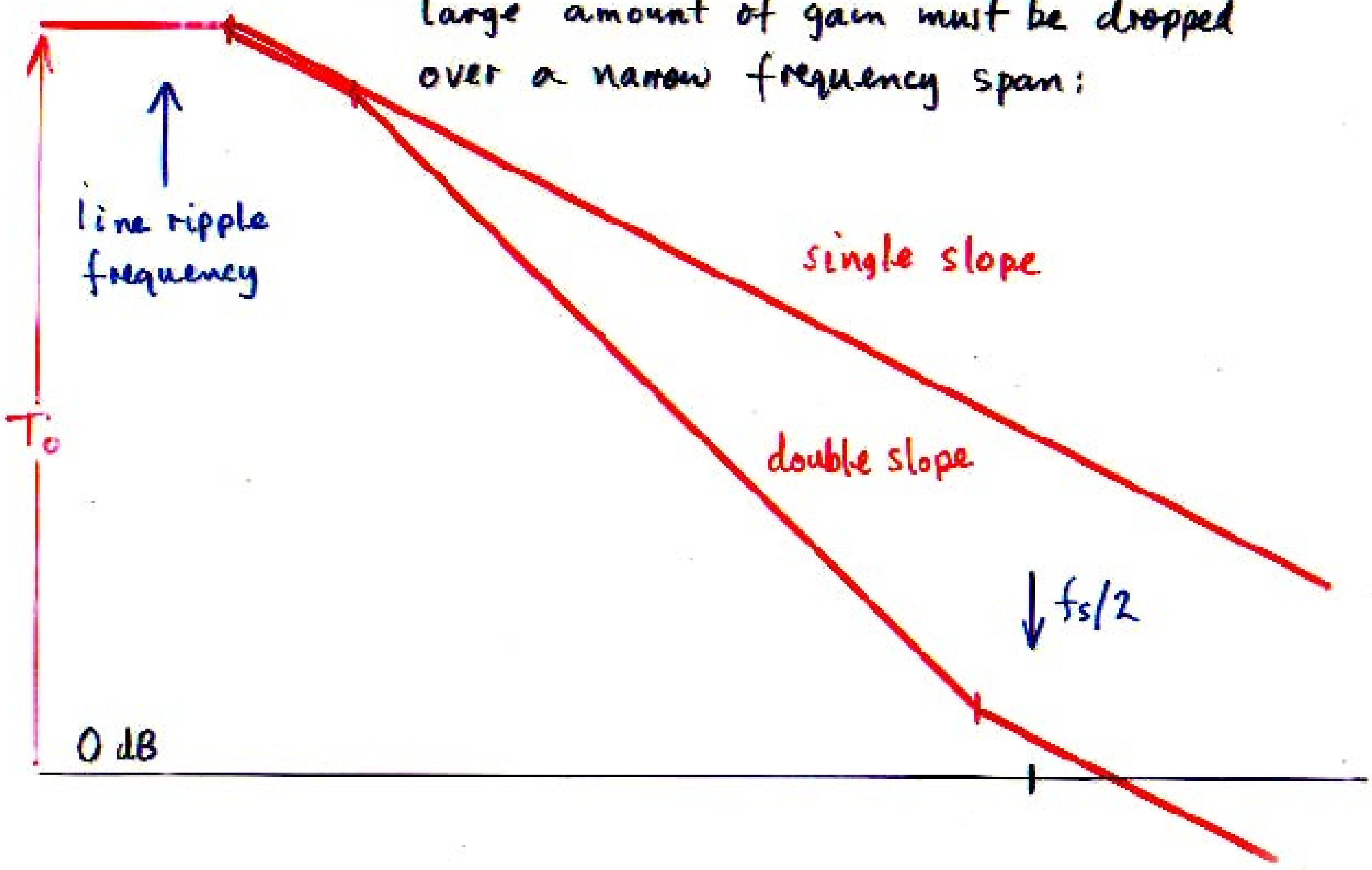

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