In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import os.path
import pandas as pd

# load data
npz = np.load("heart_disease_data.npz")

def ind_x_eq_val(x, val):
    return np.where(x==val)[0]

def count_x_eq_val(x, val):
    return len(ind_x_eq_val(x, val))/float(len(x))

def gaussian(x, mu, sig):
    return np.exp(-np.power(x - mu, 2.) / (2 * np.power(sig, 2.))) / sig / np.sq

data = pd.DataFrame.from_dict({item: npz[item] for item in npz.files}, orient='i
data.head()
```

Out[ ]:

	chest_pain	chest_pain_test	sex	cholesterol_test	heart_disease_test	heart_disease	choleste
0	2.0	3.0	0.0	149.0	0.0	0.0	2(
1	0.0	3.0	1.0	304.0	1.0	1.0	28
2	3.0	0.0	1.0	182.0	0.0	0.0	2(
3	0.0	1.0	0.0	204.0	0.0	0.0	24
4	3.0	2.0	1.0	360.0	0.0	1.0	24

QUESTION (b)

Non-parametric model Compute empirical pmf, derive the conditional pmf, and estimate the MAP decision by the mode of posterior distribution $p_{\tilde{h}|\tilde{s},\tilde{c}}$. (The MAP estimates should be $\tilde{h} = 0$ or 1)

In [ ]:
```
# Estimate the pmf of H, i.e. P(H=0) and P(H=1)
P_H0 = len(data[data.heart_disease == 0]) / len(data)
P_H1 = len(data[data.heart_disease == 1]) / len(data)

print('-------------------------------')
print("H0: ", P_H0, "H1: ", P_H1)
print('-------------------------------')

p_h_vals = [P_H0, P_H1]

# Estimate the conditional pmf of S given H, i.e. P(S|H=0) and P(S|H=1)
P_S_H0 = np.zeros(2)
P_S_H1 = np.zeros(2)
for ind_S in range(2):
    data_subset_0 = data[data.heart_disease == 0]
    data_subset_1 = data[data.heart_disease == 1]


    P_S_H0[ind_S] = len(data_subset_0[data_subset_0.sex == ind_S]) / len(data_su
```

```
    P_S_H1[ind_S] = len(data_subset_1[data_subset_1.sex == ind_S]) / len(data_su

print('S| H = 0 :', P_S_H0)
print('S| H = 1 :', P_S_H1)
print('------------------------------')

p_s_given_h_vals = [P_S_H0, P_S_H1]

# Estimate the conditional pmf of C given H, i.e. P(C|H=0) and P(C|H=1)
P_C_H0 = np.zeros(4)
P_C_H1 = np.zeros(4)
for ind_C in range(4):
    data_subset_0 = data[data.heart_disease == 0]
    data_subset_1 = data[data.heart_disease == 1]

    P_C_H0[ind_C] = len(data_subset_0[data_subset_0.chest_pain == ind_C]) / len(
    P_C_H1[ind_C] = len(data_subset_1[data_subset_1.chest_pain == ind_C]) / len(

print('C| H = 0 :', P_C_H0)
print('C| H = 1 :', P_C_H1)
print('------------------------------')

p_c_given_h_vals = [P_C_H0, P_C_H1]

# Calculate the MAP estimate
map_dict = {}
for s in [0,1]:
    for c in [0,1,2,3]:
        ## h = 0
        p_h0 = p_h_vals[0]
        p_s_given_h0 = p_s_given_h_vals[0][s]
        p_c_given_h0 = p_c_given_h_vals[0][c]

        h0_val = p_h0 * p_s_given_h0 * p_c_given_h0

        ## h = 1
        p_h = p_h_vals[1]
        p_s_given_h = p_s_given_h_vals[1][s]
        p_c_given_h = p_c_given_h_vals[1][c]

        h1_val = p_h * p_s_given_h * p_c_given_h

        if h0_val > h1_val:
            map_dict[s,c] = 0
        else:
            map_dict[s,c] = 1

MAP_estimate_S_C = map_dict

print(MAP_estimate_S_C)
print('------------------------------')

# Calculate the error rate - i.e. the proportion of all predictions that were in
correct = 0
for idx, row in data.iterrows():
    if idx == 50:
        break

    data['prediction'] = MAP_estimate_S_C[row['sex_test'], row['chest_pain_test'
    prediction = MAP_estimate_S_C[(row['sex_test'], row['chest_pain_test'])]
```

```
    if row['heart_disease_test'] == prediction:
        correct += 1

error_rate_S_C = (50 - correct) / 50

print("Probability of error " + str(error_rate_S_C))
```

```
-------------------------------
H0:  0.536697247706422 H1:  0.463302752293578
-------------------------------
S| H = 0 : [0.42735043 0.57264957]
S| H = 1 : [0.14851485 0.85148515]
-------------------------------
C| H = 0 : [0.09401709 0.24786325 0.4017094  0.25641026]
C| H = 1 : [0.03960396 0.06930693 0.16831683 0.72277228]
-------------------------------
{(0, 0): 0, (0, 1): 0, (0, 2): 0, (0, 3): 0, (1, 0): 0, (1, 1): 0, (1, 2): 0,
(1, 3): 1}
-------------------------------
Probability of error 0.18
```

QUESTION (d)

Maximum likelihood estimates Find the parameters of two normal distributions ($\tilde{x}|\tilde{h}=1$ and $\tilde{x}|\tilde{h}=0$) that maximize the likelihood functions.

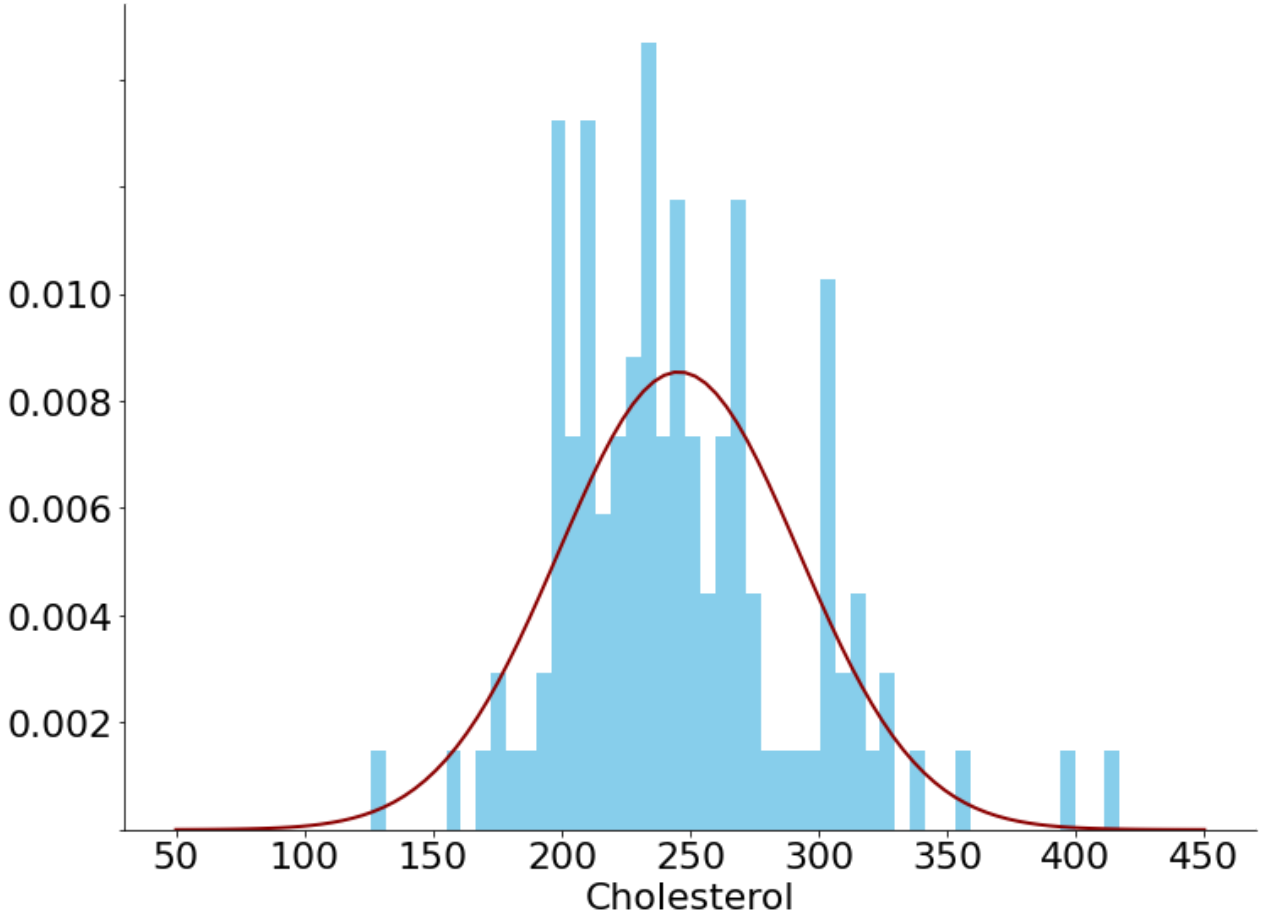
In [ ]:

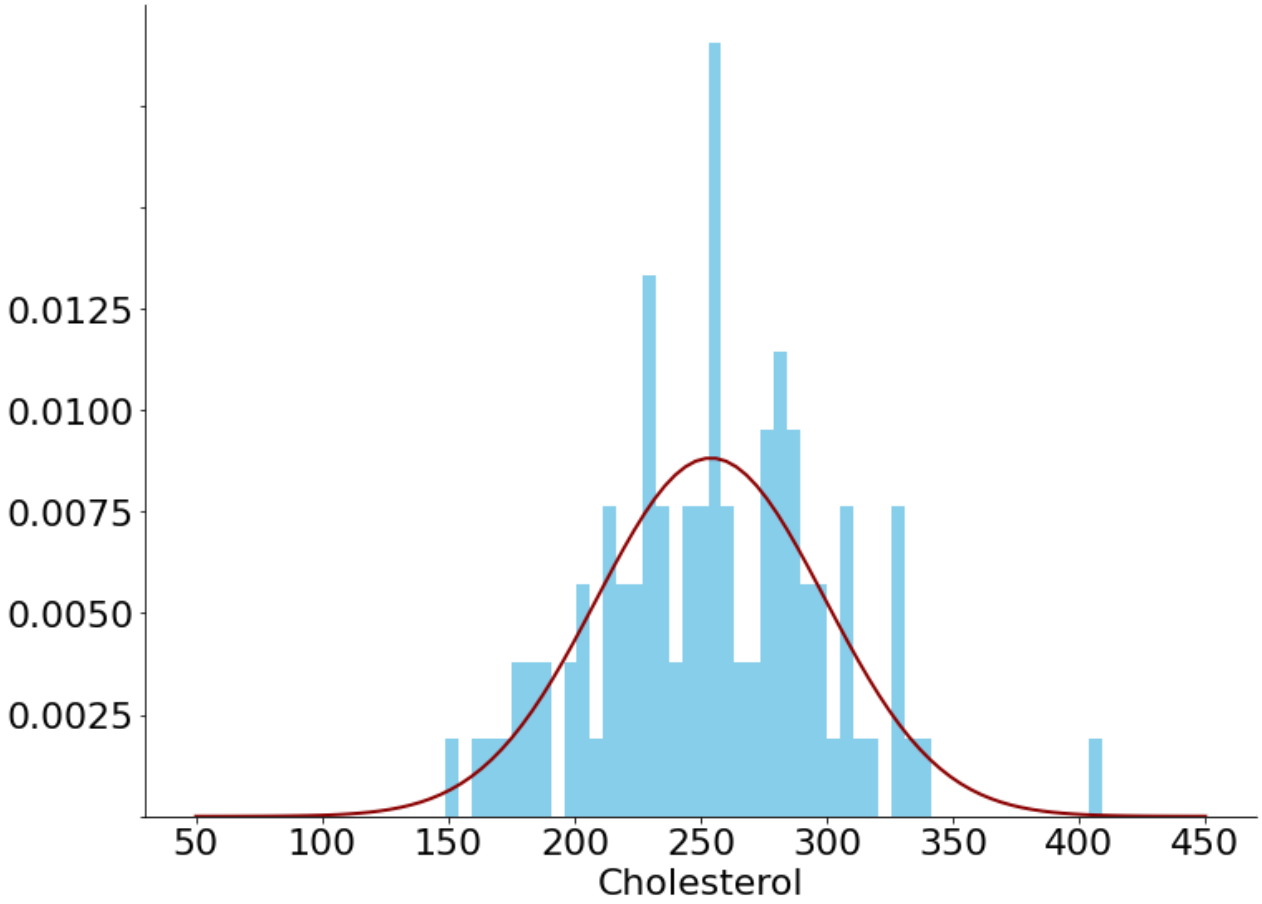
```
## Estimate MLE of X given H
mean_X_H = np.zeros(2)
std_X_H = np.zeros(2)
mean_X_H[0]= data[data.heart_disease == 0]['cholesterol'].mean()
std_X_H[0] = data[data.heart_disease == 0]['cholesterol'].std()
mean_X_H[1]= data[data.heart_disease == 1]['cholesterol'].mean()
std_X_H[1] = data[data.heart_disease == 1]['cholesterol'].std()

n_plot = 100
for i in range(2):
    plt.figure(figsize=(12, 9))
    ax = plt.subplot(111)
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.get_xaxis().tick_bottom()
    ax.get_yaxis().tick_left()
    yticks = ax.yaxis.get_major_ticks()
    yticks[0].label1.set_visible(False)
    plt.xticks(fontsize=22)
    plt.yticks(fontsize=22)
    plt.xlabel("Cholesterol", fontsize=22)

    plt.hist(data[data.heart_disease == i]['cholesterol'],
             50, stacked=True, density= True, edgecolor = "none", color="skyblue

    plt.plot(np.linspace(50, 450, n_plot),gaussian(np.linspace(50, 450, n_plot),
                    mean_X_H[i], std_X_H[i]), color="darkred", lw=2)
```


0.010
0.008
0.006
0.004
0.002
50
100
150
200
250
300
350
400
450
Cholesterol


0.0125
0.0100
0.0075
0.0050
0.0025
50
100
150
200
250
300
350
400
450
Cholesterol

QUESTION(e)

MAP decision compute posterior $p_{\tilde{h}|\tilde{s},\tilde{c},\tilde{x}}$ and derive MAP

In [ ]:

```
# Calculate the MAP estimate
from scipy import stats
# MAP_estimate_S_C_X = ### INSERT CODE HERE ###

map_dict2 = {}
for s in [0,1]:
    for c in [0,1,2,3]:
        for x in list(set(data.cholesterol)):
            ## h = 0
            p_h0 = p_h_vals[0]
            p_s_given_h0 = p_s_given_h_vals[0][s]
            p_c_given_h0 = p_c_given_h_vals[0][c]
            p_x_given_h0 = stats.norm(mean_X_H[0], std_X_H[0]).pdf(x)

            h0_val = p_h0 * p_s_given_h0 * p_c_given_h0 * p_x_given_h0

            ## h = 1
            p_h = p_h_vals[1]
            p_s_given_h = p_s_given_h_vals[1][s]
            p_c_given_h = p_c_given_h_vals[1][c]
            p_x_given_h = stats.norm(mean_X_H[1], std_X_H[1]).pdf(x)

            h1_val = p_h * p_s_given_h * p_c_given_h * p_x_given_h

            if h0_val > h1_val:
                map_dict2[s,c,x] = 0
            else:
                map_dict2[s,c,x] = 1

print(map_dict2)


def mapEstimateX(s,c,x):
    p_h0 = p_h_vals[0]
    p_s_given_h0 = p_s_given_h_vals[0][s]
    p_c_given_h0 = p_c_given_h_vals[0][c]
    p_x_given_h0 = stats.norm(mean_X_H[0], std_X_H[0]).pdf(x)

    h0_val = p_h0 * p_s_given_h0 * p_c_given_h0 * p_x_given_h0

    ## h = 1
    p_h = p_h_vals[1]
    p_s_given_h = p_s_given_h_vals[1][s]
    p_c_given_h = p_c_given_h_vals[1][c]
    p_x_given_h = stats.norm(mean_X_H[1], std_X_H[1]).pdf(x)

    h1_val = p_h * p_s_given_h * p_c_given_h * p_x_given_h

    if h0_val > h1_val:
        h = 0
    else:
        h = 1

    return h

correct = 0
for idx, row in data.iterrows():
    if idx == 50:
```

```
        break

    s = int(row['sex_test'])
    c = int(row['chest_pain_test'])
    x = float(row['cholesterol_test'])

    h = mapEstimateX(s,c,x)

    if row['heart_disease_test'] == h:
        correct += 1


# Calculate the error rate
error_rate_S_C_X = (50 - correct) / 50
print("Probability of error using cholesterol " + str(error_rate_S_C_X))
```

```
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Probability of error using cholesterol 0.14
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In [ ]: