Recitation 2 Solution

These problems will review key mathematical concepts that we will need throughout the course.

1. (Discrete random variable)

   (a) Note: Y = 1 when X = 1 or X = -1, so P(Y = 1) = P(X = 1) + P(X = -1).

y	0	1	4
$p_Y(y)$	3/15	6/15	6/15

   (b) for (b) and (c), using the tables in part (a) and the definition $F_X(a) = P(X \leq a)$, we get

a	-1/2	3/4	7/8	1	1.5	5
$F_X(a)$	3/15	6/15	6/15	10/15	10/15	1
$F_Y(a)$	0	3/15	3/15	9/15	9/15	1

2. (Probability mass function) Derive the probability mass function of binomial distribution and poisson distribution.

   Note Example 3.2 and Example 3.7

3. (Probability distribution)

   (a)

$$\begin{aligned}
P(X < 3) &= P(X = 0) + P(X = 1) + P(X = 2) && (1)\\
&= (0.3)^{10} + 10 \times 0.7 \times (0.3)^9 + 45 \times (0.7)^2 \times (0.3)^8 && (2)\\
&\approx 0.0016 && (3)\\
& && (4)
\end{aligned}$$

   (b)

$$\begin{aligned}
P(Y \geq 3) &= 1 - P(Y < 3) && (5)\\
&== 1 - P(Y = 0) - P(Y = 1) - P(Y = 2) && (6)\\
&= 1 - e^{-5} - 5e^{-5} - \frac{25}{2}e^{-5} && (7)\\
&= 1 - \frac{37}{2}e^{-5} && (8)\\
& && (9)
\end{aligned}$$

   (c)

$$P(Z > 2) = 1 - P(P \leq 2) = 1 - P(Z = 1) - P(Z = 2) = 1 - 0.5 - 0.25 = 0.25$$

4. (Binomial Distribution)

   (a) The log-likelihood function is

$$log(L(\theta)) = log(\frac{n!}{x!(n-x)!}(\theta)^x(1-\theta)^{n-x})$$

To find the maximum, we ignore the term $\frac{n!}{x!(n-x)!}$, since it's a constant and doesn't affect the maximum. Take derivative of the log-likelihood and set it to 0:

$$\frac{x}{\theta} - \frac{n-x}{1-\theta} = 0$$

$$\theta = \frac{x}{n}$$

(b) The ML estimate of a bernoulli distribution with probability of sucess=θ on each trial is $\frac{k}{n}$ where k is the number of total success and n is the number of total trials. Since binomial is the result of n independent Bernoulli trials, it's not surprising that the MLE based on n independent Bernoulli random variables and the MLE based on a single binomial random variable are the same.