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PROBLEM 0.5

a) The Projection of x is given by $VV^T x$ by rule

If $M = VV^T$, then the $Im(M) = Mx = VV^T x$

this can be written as $\langle v_1, x\rangle v_1 + \cdots + \langle v_k, x\rangle v_k$

$\alpha_1 v_1 + \cdots + \alpha_k v_k$

Because, for any x, Mx is some combination of $v_1 \cdots v_k$, $Im(M) = Mx$ is the $Span(v_1 \cdots v_k)$

$Rank(M) = K$

b) $= \langle v_1, x \rangle v_1$

$= (1/\sqrt{2} \cdot 3 + 1/\sqrt{2} \cdot 6)\, v_1$

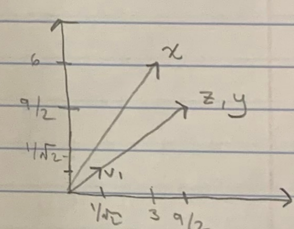
$= 9/\sqrt{2} \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 9/2 \\ 9/2 \end{bmatrix} = y$

$= \langle v_1, y \rangle v_1$

$= \langle 1/\sqrt{2} \cdot 9/2 + 1/\sqrt{2} \cdot 9/2 \rangle v_1$

$(9/2\sqrt{2} + 9/2\sqrt{2})\, v_1$

$= (9/\sqrt{2}) \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 9/2 \\ 9/2 \end{bmatrix} = z$



c) $ker(M) := VV^T x = 0$, or, for when the projection of x onto S is the 0 vector

For this to happen x itself must be 0

So $dim(ker(M)) = 0$

$Im(A) = VV^T x$, which is x projected onto S. Since $Im(A) \in S$, we can use the given orthonormal basis of S, $v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$ $dim(Im(A)) = 1$

d) $M = VV^T$

$M^2 = VV^T VV^T$

$M^2 = VV^T$

$M^2 = M$

→ We can say this because V is specifically given as an orthogonal matrix

This intuitively follows as a projection of a projector of something onto S onto S is itself.