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PROBLEM 0.1

a) $A^{n \times m} \cdot A^{n \times m} = R^{n \times m}$

$\therefore$ FALSE

b) max (multiplicity of λ) = n

TRUE

c) rank (L) + dim (ker (L)) = m

FALSE

d) FALSE

e) TRUE

PROBLEM 0.2

a) $x_1 \in S,\ x_2 \in S \rightarrow x_1 + x_2 \in S$; $\alpha x_1 \in S$

$V_1 = \begin{bmatrix} x_1 \\ 0 \\ x_1 \end{bmatrix}$, $V_2 = \begin{bmatrix} x_2 \\ 0 \\ x_2 \end{bmatrix} \longrightarrow V_1 + V_2 = \begin{bmatrix} x_1 + x_2 \\ 0 \\ x_1 + x_2 \end{bmatrix}$

$(x_1 + x_2) - (x_1 + x_2) = 0$ Addition Enclosed ✓

$V_1 = \begin{bmatrix} x_1 \\ 0 \\ x_1 \end{bmatrix} \longrightarrow \alpha V_1 = \begin{bmatrix} \alpha x_1 \\ 0 \\ \alpha x \end{bmatrix}$

$(\alpha x_1) - (\alpha x_1) = 0$ Scalar Multiplication Enclosed ✓

YES

b) $A_1 \in S,\ A_2 \in S \rightarrow A_1 + A_2 \in S$; $\alpha A_1 \in S$

$A_1 + A_2 = A_3$ st for each pair of symmetric elements in A_1 and A_2, the pair is incremented by an A_{ij} and A_{ji} st $A_{ij} = A_{ji}$. Since the pairs were equal to begin with, the remain equal. $A_3 = A_3^T$

Addition ✓ Enclosed

αA_1 : by the same logic, all elements are scaled equally. Therefore pairwise symmetric elements remain equal.

$\alpha A_1 = A_2 \in S \longrightarrow$ Scalar Mult. ✓

YES

c) Counterexample: let $x = -1$, $\alpha = -1$

$\alpha x = (-1)(-1) = 1$

$1 \leq 0$ is a contradiction

NO