2/5

Aditya Srikanth
aks9136

PROBLEM 0.3

a) $\begin{pmatrix} 4 & 2 & b \\ 0 & 0 & 5 \\ 0 & a & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ c \end{pmatrix} \longrightarrow$

$4x_1 + 2x_2 + bx_3 = 0$

$5x_3 = 3$

$ax_2 = c$

$x_3 = 3/5$

$4x_1 + 2x_2 + \frac{3}{5}b = 0$

$ax_2 = c$

a	b	c	
0	1	1	
0	1	0	
0	0	1	∞
0	0	0	
1	1	1	1
1	1	0	
1	0	1	1
1	0	0	

If $a, b, c \neq 0$: x_2 is fixed → x_1 is fixed → (1 solution)

If $a = 0$: x_2 can take any value → $c = 0$ → (∞ solutions)

If $a \neq 0$, $c = 0$: $x_2 = 0$ → x_1 is a function of b → (1 solution)

If $a, c \neq 0$: x_2 is fixed → x_1 is fixed → (1 solution)

(x has ∞ solutions if $a = 0$
else, x has 1 solution)

b) $\begin{pmatrix} 4 & 2 & b \\ 0 & 0 & 5 \\ 0 & a & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 4 & b \\ 0 & 0 & 5 \\ a & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ c \end{pmatrix} \longrightarrow$

$2x_1 + 4x_2 + bx_3 = 0$

$5x_3 = 3$

$ax_1 = c$

$x_3 = 3/5$

(x has ∞ solutions if $a = 0$
else 1 solution)