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PROBLEM 0.4

a) ① $L(v+w) = L(v)+L(w)$ Show this

$$Tr(M_1+M_2) = \sum_{i=1}^{n} M1_{ii} + M2_{ii}$$
$$= \sum_{i=1}^{n} M1_{ii} + \sum_{i=1}^{n} M2_{ii}$$
$$= Tr(M_1) + Tr(M_2) \checkmark$$

② $$Tr(\alpha M_1) = \sum_{i=1}^{n} \alpha M_{ii}$$
$$= \alpha \sum_{i=1}^{n} M_{ii}$$
$$= \alpha \; Tr(M) \checkmark$$

$\therefore$ Linear Transformation

b) ① $\langle u,v \rangle = \langle v,u \rangle$

$$\langle A,B \rangle = Tr(A^T B)$$
$$= Tr((A^T B)^T)$$
$$= Tr(B^T A)$$
$$= \langle B,A \rangle \checkmark$$

by rule, $Tr(A) = Tr(A^T)$ because $A_{ii} = A^T_{ii}$

② $\langle u+v, w \rangle = \langle u,w \rangle + \langle v,w \rangle$

$$\langle A+B, C \rangle = Tr((A+B)C^T)$$
$$= Tr(AC^T + BC^T)$$
$$= Tr(AC^T) + Tr(BC^T)$$
$$= \langle A,C \rangle + \langle B,C \rangle \checkmark$$

③ $$\langle A,A \rangle = Tr(AA^T)$$
$$= \sum_{i=1}^{n} AA^T_{ii}$$
$$= \sum_{i=1}^{n} x_i x_i$$
$$= \sum_{i=1}^{n} x_i^2$$

This is positive or 0 as squares cannot be negative

This is 0 only if all terms are 0

Positive Definiteness ✓

$\therefore \langle \cdot,\cdot \rangle$ is Inner Product

c) $Tr(AB)^2 \leq Tr(A^2) + Tr(B^2)$

$Tr(AB) \cdot Tr(AB) \overset{?}{\leq} Tr(AA) + Tr(BB)$

$\sum_{i=1}^{n} A_{ii} B_{ii} \cdot \sum_{i=1}^{n} A_{ii} B_{ii} \leq \sum_{i=1}^{n} A_{ii}^2 + \sum_{i=1}^{n} B_{ii}^2$

$\|AB\|^2 \leq \|A^2\| \cdot \|B^2\|$ by Theorem

this fits the form

$\therefore Tr(AB)^2 \leq Tr(A^2) + Tr(B^2)$