

Recitation 2

These problems will review key mathematical concepts that we will need throughout the course.

1. (Discrete random variable) Let X be a discrete random variable with pmf p given by:

x	-2	-1	0	1	2
p(x)	1/15	2/15	3/15	4/15	5/15

- (a) Let $Y = X^2$. Find the pmf of Y .
 - (b) Find the value the cdf of X at $-1/2, 3/4, 7/8, 1, 1.5, 5$.
 - (c) Find the value the cdf of Y at $-1/2, 3/4, 7/8, 1, 1.5, 5$.
2. (Probability mass function) Derive the probability mass function of binomial distribution and poisson distribution.
3. (Probability distribution) Given that $X \sim \text{Bin}(10, 0.7)$, $Y \sim \text{Pois}(5)$, $Z \sim \text{Geom}(0.5)$, compute the following:
- (a) The probability of $X < 3$
 - (b) The probability of $Y \geq 3$
 - (c) The probability of $Z > 2$
4. (Binomial Distribution) Suppose that X is an observation from a binomial distribution, $X \sim \text{Bin}(n, \theta)$, where n is known and θ is to be estimated. The likelihood function is

$$L(\theta) = \frac{n!}{x!(n-x)!} (\theta)^x (1-\theta)^{n-x}$$

- (a) Find the maximum likelihood estimate $\tilde{\theta}$ of θ .
 - (b) What observation can you make about the ML estimate of binomial distribution (comparing to ML estimate of Bernoulli distribution)? And why is that reasonable?
5. (Empirical Probability) Note Example 3.4
6. (Code) Free throws