# Report - Assignment 5

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**Abstract**

In this week's assignment, different plots were drawn to show the potential gradient and the current density variation across a 1cm by 1cm copper plate, having a circular wire of radius 0.35cm soldered to the center of the plate. The bottom side of the plate was grounded. The formula used to find the potential inside the plate is $\nabla^2\phi = 0$ ($\phi$ is the potential), which is the Laplace equation for a uniform region with no net charge accumulation inside the region. The boundary conditions used were that $\phi$ should not vary in the normal direction at the left, right and top boundaries (i.e $\frac{\partial\phi}{\partial n} = 0$) and $\phi$ is 0 at the bottom boundary (as it is grounded). This loop to find $\phi$ was iterated 1500 times and error between the each iteration was calculated and plotted in a semilog axis. The best fit line for error was also calculated and plotted using least squares approximation. Then, the current density ($J$) was found using the formula $j_x = -\frac{\partial\phi}{\partial x}$ (and the corresponding one for $y$) and plotted with a quiver plot. After this, the temperature variation due to this current density was also found and plotted to know which area would be the hottest.

## Libraries Used

```
from pylab import *
import mpl_toolkits.mplot3d.axes3d as p3
```

## Parameters declared

```
Nx = 25         # No. of steps along the x direction
Ny = 25         # No. of steps along the y direction
radius = 0.35   # Radius of the wire loop
Niter = 1500    # No. of iterations to find potential
errors = np.zeros(Niter)   # Error array is declared
```

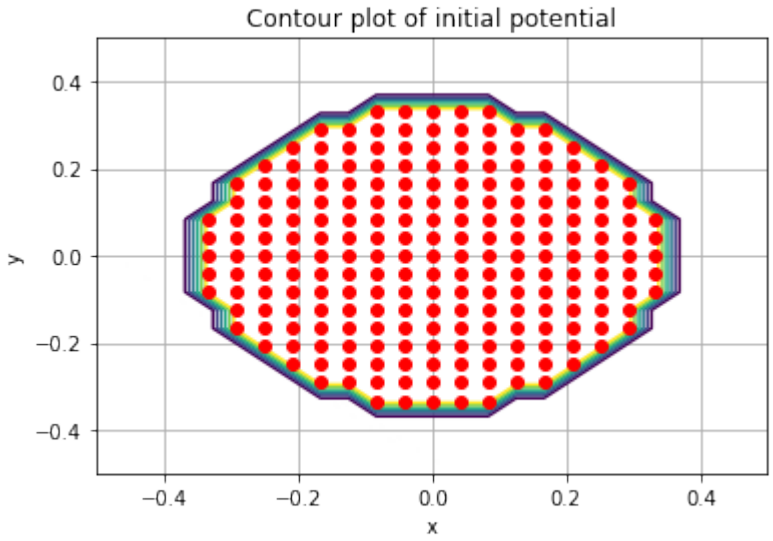
## 1 The potential is intialised

The array denoting the x and y coordinates of the plate is initialised and using that, the 2D potential array is also initialised with zeros. Then, the area covered by the ring is found (using where) and that part is given the value 1 V.

```
x = np.linspace(-0.5,0.5,25)    # x coordinate array
y = np.linspace(0.5,-0.5,25)    # y coordinate array
X,Y = meshgrid(x,y)             # The 2D grid of x and y coordinates
phi = np.zeros((Nx,Ny))         # Potential is initialised with zeros
ii = where(X*X + Y*Y <= radius*radius)  # Area covered by ring is found
phi[ii] = 1.0                   # Area covered by ring is initialised with 1 V
```

Now, the contout plot of the potential is plotted with red dots marking the region covered by the ring.

```
contour(X,Y,phi)
plot(x[ii[0]],y[ii[1]],'ro')
grid()
title('Contour plot of initial potential')
xlabel('x')
ylabel('y')
show()
```




## 2 Calculation of potential

Now, the actual iteration is performed $Niter$ (1500) times and the potential is calculated at every point using the Laplace equation and appropriate boundary conditions. The laplace equation modified for use in a 2D array of potential values ( in a computer program with discrete positions ) is:

$$\phi_{i,j} = \frac{\phi_{i+1,j} + \phi_{i-1,j} + \phi_{i,j+1} + \phi_{i,j-1}}{4}$$

```
newphi = np.zeros((Nx,Ny)) # This is to temporarily store the newly calculated values so as to calculate er
for k in range(Niter):
    oldphi = phi.copy()    # Phi before iteration is stored to calculate error
    newphi[1:-1,1:-1] = 0.25*(phi[1:-1,0:-2] + phi[1:-1,2:] + phi[0:-2,1:-1] + phi[2:,1:-1])  # Laplace equ

    newphi[1:-1,0] = newphi[1:-1,1]         # Boundary conditions applied
    newphi[1:-1,Nx-1] = newphi[1:-1,Nx-2]
    newphi[0,1:-1] = newphi[1,1:-1]
    newphi[ii] = 1.0

    errors[k] = max(np.absolute(np.subtract(oldphi.flatten(),newphi.flatten())))   # Error calculated as ma
    phi = newphi.copy()
```

## 3 Error plot

The actual error calculated from the above loop is plotted as is, by sampling every $50^{th}$ point. Then, in addition to that, two other lines are also plotted, the best fit line taking all error values obtained and that by taking all error values obtained after 500 iterations. The formula used to calculate the best fit line is:

$$y = A \exp^{Bx}$$

where $y$ is the error, $x$ is the no. of iterations and $A$ & $B$ are parameters. This is converted to its corresponding log equation:

$$\log y = \log A + Bx$$

and solved using least squares approximation to get $\log A$ & $B$, and the line thus obtained is plotted on the same plot.

```
# Best fit for all iterations
xError = np.linspace(1,Niter,1500)   # x Values for the equation
```

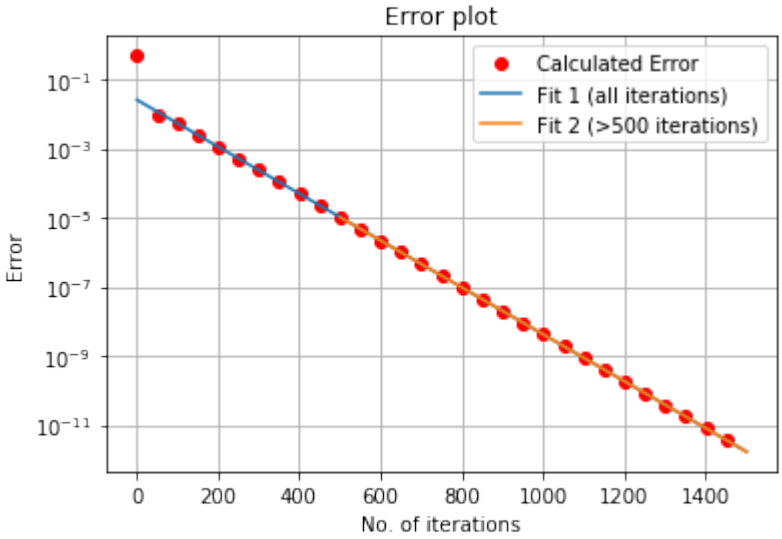
```
yError = np.log(errors)              # y values for equation
A=np.zeros((Niter,2))                # 2D matrix initialised
A[:,0] = 1
A[:,1] = xError
const = lstsq(A,yError)[0]           # parameters log(A) and B are found
yError = const[0] + const[1]*xError  # Above mentioned equation applied to find best fit line
yError = np.exp(yError)



# Best fit for greater than 500 iterations
xError2 = np.linspace(501,Niter,1000)
yError2 = np.log(errors[500:])
B=np.zeros((Niter-500,2))
B[:,0] = 1
B[:,1] = xError2
const = lstsq(B,yError2)[0]
yError2 = const[0] + const[1]*xError2
yError2 = np.exp(yError2)



# Graph plotted
semilogy(np.arange(1,1501,50),errors[0::50],'ro')
plot(xError,yError)
plot(xError2, yError2)
grid()
title('Error plot')
xlabel('No. of iterations')
ylabel('Error')
legend(('Calculated Error','Fit 1 (all iterations)','Fit 2 (>500 iterations)'))
show()
```

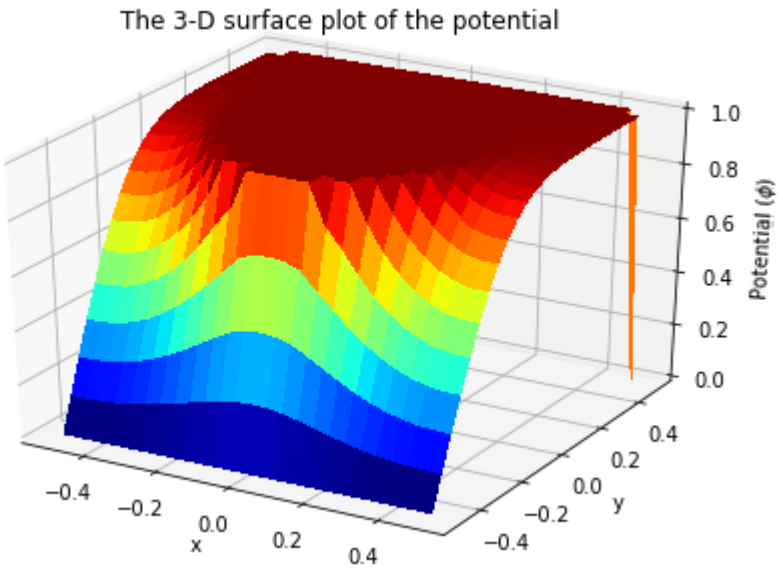



Thus, it is observed from the graph that, when all 1500 iterations are taken, although the fit is good at larger iterations, at the smaller ones, the actual calculated error vary significantly from the calculated best fit line, whereas the line calculated for errors after 500 iterations fits perfectly with the actual error values.

# 4 Potential Plots

The potential obtained after applying the Laplace equation is now plotted in a 3D surface plot so as to observe the variation of potential throughout the plate.

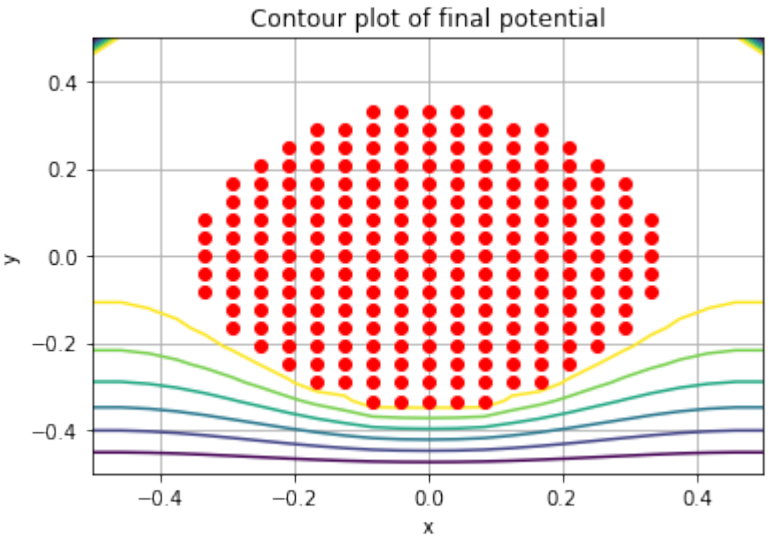
```
fig1 = figure(4)
ax = p3.Axes3D(fig1)
title('The 3-D surface plot of the potential')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('Potential $(\phi)$')
surf = ax.plot_surface(X, Y, phi, rstride=1, cstride=1, cmap=cm.jet,linewidth=0, antialiased=False)
show()
```




Thus, from the surface plot, we see that potential is almost uniform (1 V) at the top half of the plate ($y > 0.5$), whereas it decreases to 0 (ground) at the bottom half of the plate, as expected, as the bottom half of the plate is grounded.

Now, a contour plot of the potential is plotted marking the region covered by the ring with red dots.

```
contour(x,y,phi)
plot(x[ii[0]],y[ii[1]],'ro')
xlabel('x')
ylabel('y')
title('Contour plot of final potential')
grid()
show()
```




This shows that the contour lines are more dense in the region where the plate is most nearest to ground. This shows that maximum potential gradient occurs here and therefore current flow is also maximum here. Thus, in the temperature plot ( that is plotted subsequently ), this area would be the hottest.

# 5 Current Density - Calculation and Plot

From the above obtained values of $\phi$, the current density ($J$) is calculated. Since, for knowing the trend of $J$ across the plate, its actual value is not needed, the value of conductivity ($\sigma$) is taken as 1. The equations used are:

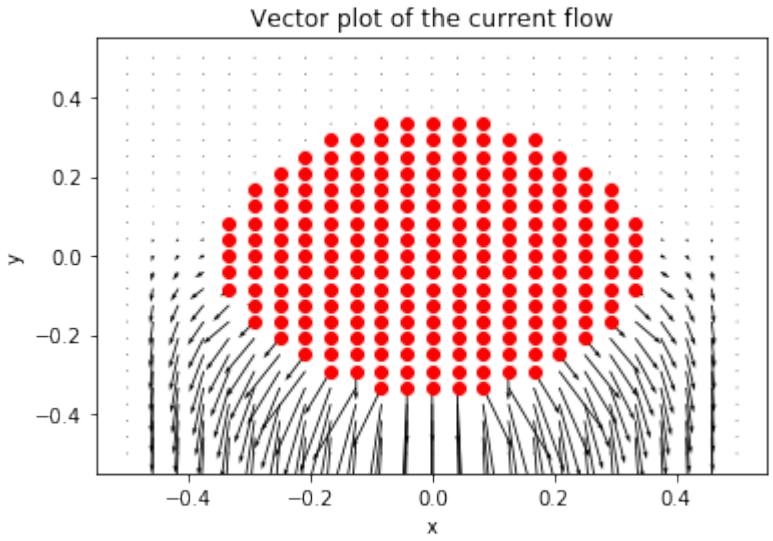
$$j_x = -\sigma \frac{\partial \phi}{\partial x}$$

$$j_y = -\sigma \frac{\partial \phi}{\partial y}$$

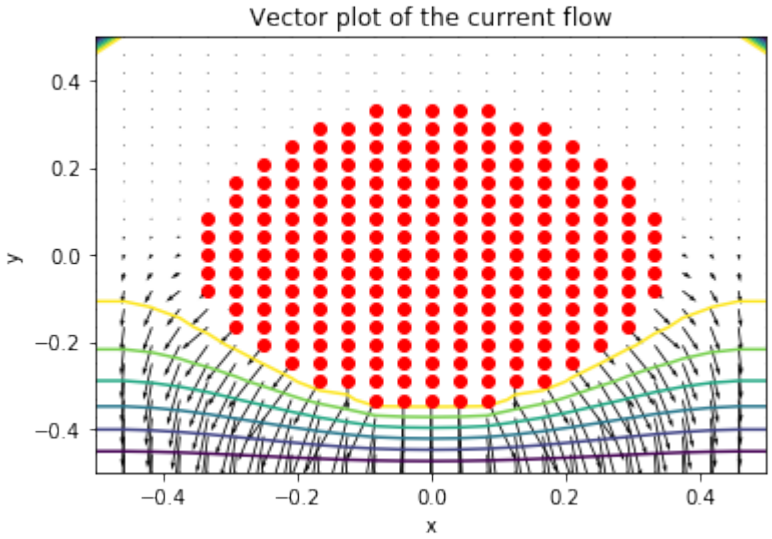
```
Jx = np.zeros((Nx,Ny))
Jy = np.zeros((Nx,Ny))
Jy[1:-1,1:-1] = 0.5*(phi[1:-1,2:] - phi[1:-1,0:-2])
Jx[1:-1,1:-1] = 0.5*(phi[2:,1:-1] - phi[0:-2,1:-1])
```

Now, a quiver plot of the above calculated current density is drawn, showing the direction and magnitude of current in the plate.

```
plot(x[ii[0]],y[ii[1]],'ro')
xlabel('x')
ylabel('y')
title('Vector plot of the current flow')
quiver(y,x,Jy[::-1,:],Jx[::-1,:])
# contour(x,y,phi)
show()
```




Thus, from the graph obtained, it is observed that, the current density has the maximum magnitude and is most dense at the region around $x = 0$ and $y <$ -0.35 (radius of the ring). This is because, this region is the one nearest to the ground, and hence has the maximum potential gradient and hence from its formula, $J$ would be maximum here. Also, when the contour plot of $\phi$ and the quiver plot of current is plotted together ( removing the comment for contour() in above code ), it is seen that the current plot is perpendicular to the potential plot which is as expected. In a static situation, there should be no component of current parallel to the conductor, and therefore the current would emerge from the conductor perpendicular to it (as shown).



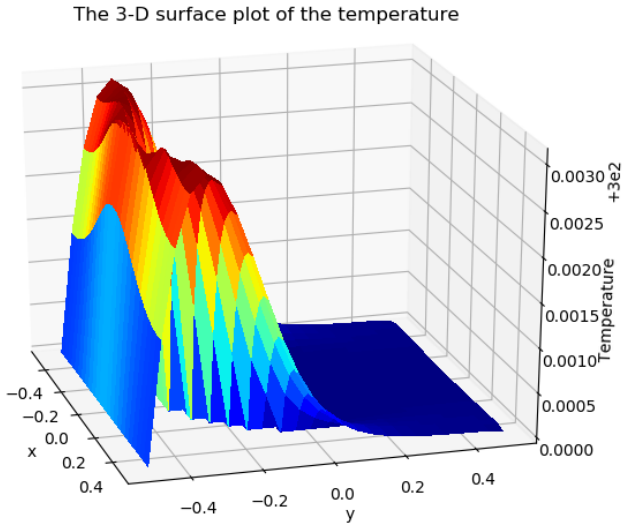

## 6 Calculation of Temperature

Now, to find which region gets heated up the most, the heat generated due to current flow (ohmic loss = $\vec{J}.\vec{E}$) is equated with the heat released corresponding to the temperature gradient of the material. Formula used is:

$$\nabla.(\kappa \nabla T) = q = \frac{1}{\sigma}|J|^2$$

```
T = np.zeros((Nx,Ny))
T[:,:] = 300
sigma = 6*(10**7)   # Electrical conductivity of copper
kappa = 385         # Thermal conductivity of copper
for i in range(Niter):
    T[1:-1,1:-1] = 0.25*(T[1:-1,0:-2] + T[1:-1,2:] + T[0:-2,1:-1] + T[2:,1:-1] + (((Jx**2)[1:-1,1:-1] + (Jy
    T[1:-1,0]=T[1:-1,1]
```

```
    T[1:-1,Nx-1]=T[1:-1,Nx-2]
    T[0,1:-1]=T[1,1:-1]
    T[ii] = 300.0
fig1=figure(4)
ax=p3.Axes3D(fig1)
title('The 3-D surface plot of the temperature')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('Temperature')
ax.plot_surface(X, Y, T, rstride=1, cstride=1, cmap=cm.jet,linewidth=0, antialiased=False)
show()
```




# 7 Inferences

From all the above plots, we see that:

1. The potential is almost uniform at the sides that are not grounded and inside the ring. This is expected, since no rigid boundary potentials are present, there would be no current flow at those sides and hence, potential would be unifrom in these areas. Also, inside the ring, the potential is maintained at 1 V due to the boundary of the ring.

2. At the region of the plate between the closest point of the ring and the ground, the potential gradient is maximum. This is because potential goes from 1 V to 0 V in a small length. Due to this, the current in this region is also maximum as current density ($J$) is directly proportional to magnitude of potential gradient.

3. Also the direction of $J$, from the quiver plot, is seen to be perpendicular to the contour plot of $\phi$. This is also as expected, as $j_i = -\sigma \frac{\partial \phi}{\partial i}$, and therefore, the direction of the current would be in the direction of decreasing potential, i.e, perpendicular to contour plot.

4. From the temperature plot, it is seen that, the temperature is highest in the region where current density is highest. This is also expected as, ohmic loss is directly related to current and as energy dissipated due to this ohmic loss increases, temperature in that region would also increase.