Neural Network Integration into ODE Systems

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Population Dynamics

- Key Concepts
  - *Exponential Growth:*
$$\frac{dN}{dt} = rN$$
  - *Logistic Growth:*
$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$
- Key Factors
  - Birth Rate and Death Rate.
- Interaction with the Environment
  - Carrying Capacity (K).
  - Density-Dependent and Density-Independent Factors.
- Types of Population Interactions
  - Predation, Competition,
- Modeling Tools
  - Mathematical and Simulation Models.

Introduction

- Overview of integrating neural networks into ODE systems.
- Objective: Visualize how neural network approximations affect ODE predictions.
- Methods: Use Julia for solving ODEs and training neural networks.

Methodology

- Define ODE system (Lotka-Volterra model).
- Generate noisy data and fit a neural network.
- Extend time span for prediction.
- Visualize network's predictions integrated into the ODE system.

Data Generation

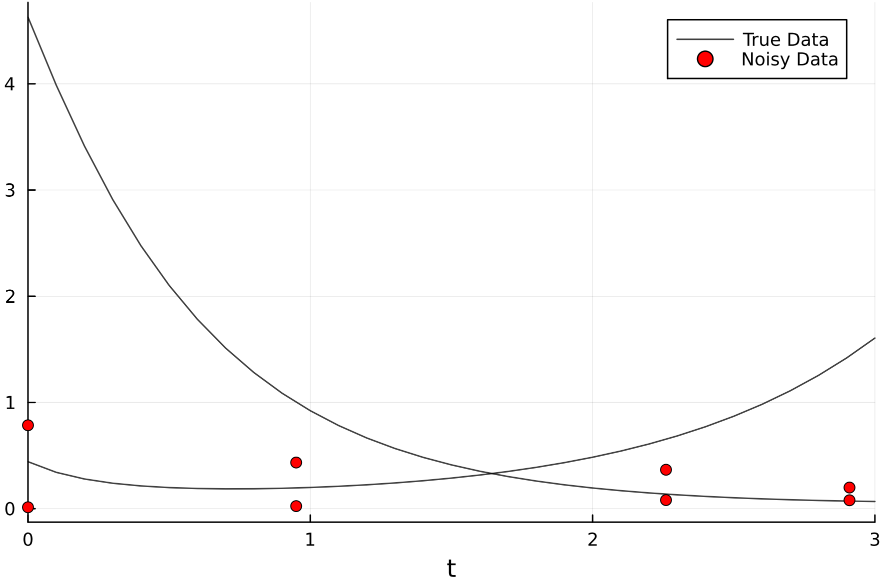
- Lotka-Volterra model equations:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \gamma xy - \delta y$$

- Experimental parameters and initial conditions.

Noisy Data



NODE Approach

- Network architecture: Multi-layer feedforward with RBF activation.
- Training process using ADAM and BFGS optimizers.
- Loss function and optimization.

Network Architecture

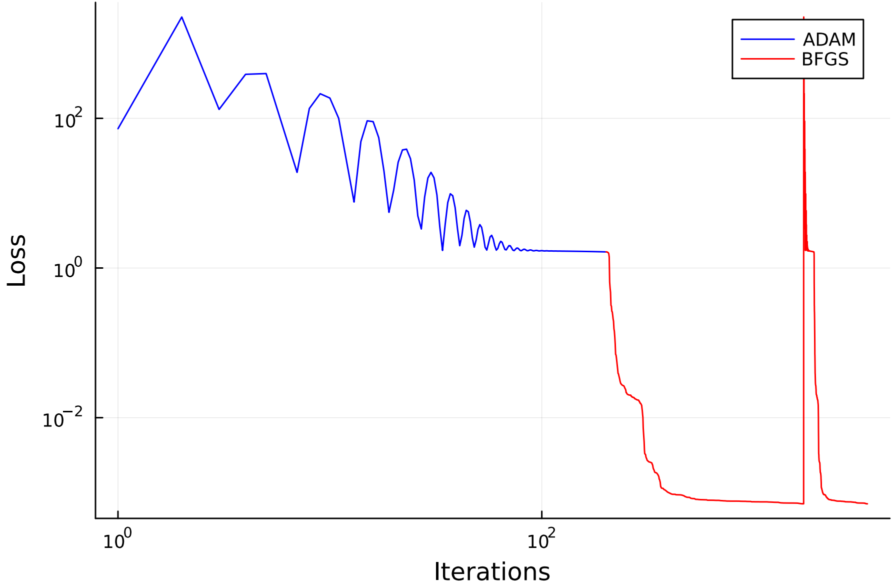
```julia
# Multilayer FeedForward Neural Network
U = Lux.Chain(
    Lux.Dense(2,5,rbf), Lux.Dense(5,5, rbf), Lux.Dense(5,5, rbf), Lux.Dense(5,2)
)
```

Figure: Neural Network Initialization

```julia
# Defines the hybrid model using the Neural Network
function ude_dynamics!(du,u, p, t, p_true)
    û = U(u, p, st)[1] # Network prediction
    du[1] = p_true[1]*u[1] + û[1]
    du[2] = -p_true[4]*u[2] + û[2]
end
```

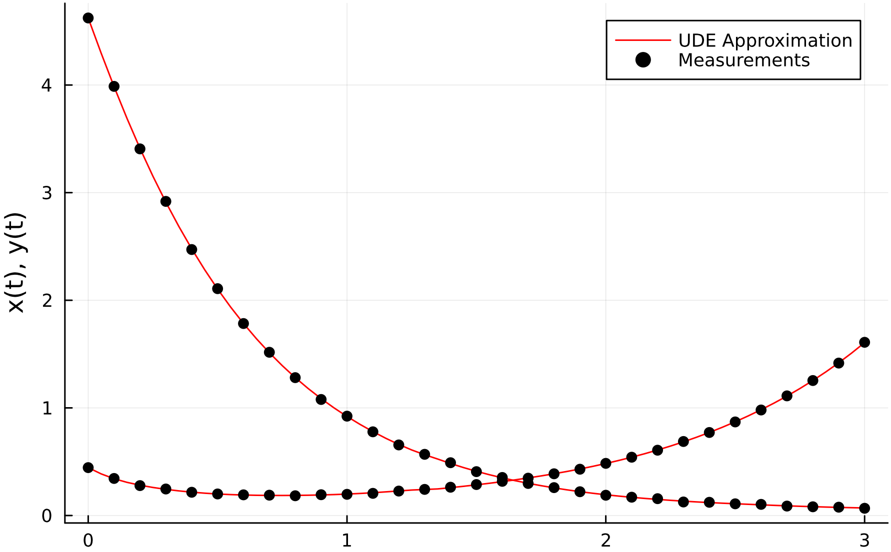
Figure: Integration into Differential Equation

Training Loss



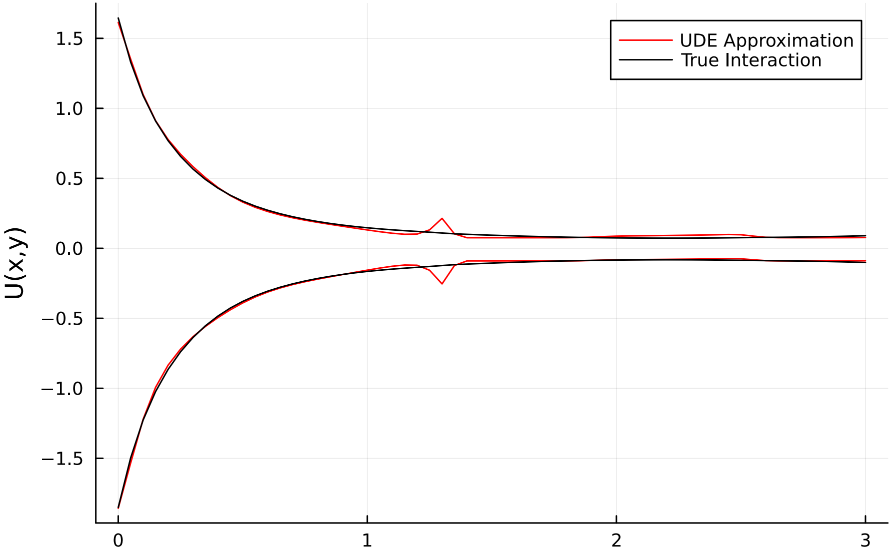
Trajectory Reconstruction

- Comparison of model predictions with true data.



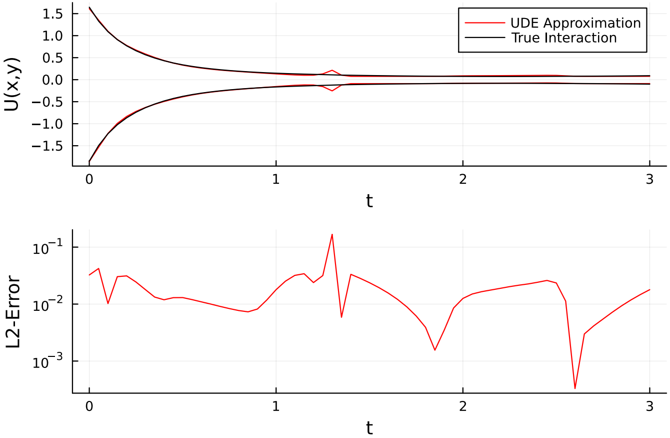
Interaction Reconstruction

- Neural network approximation vs. true interaction.



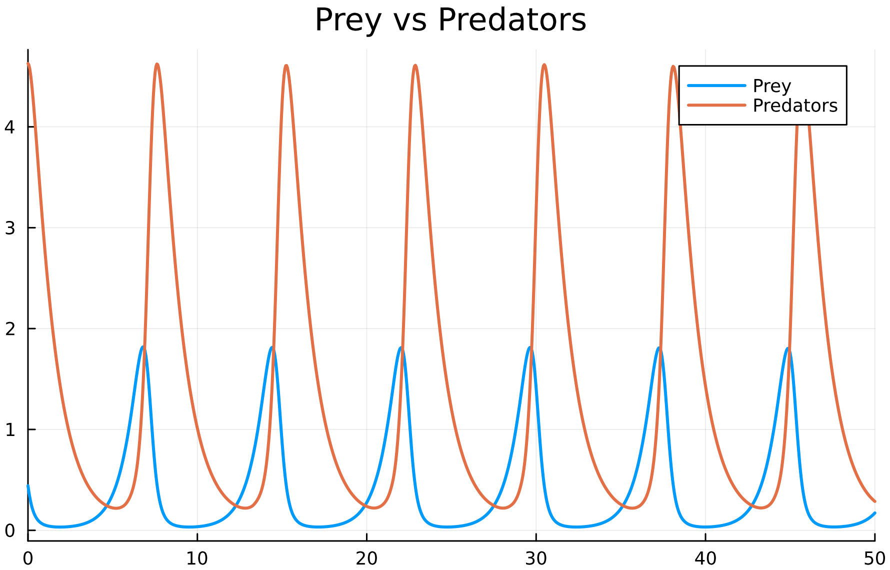
Prediction and Error

- Error analysis between true interaction and neural network guess.



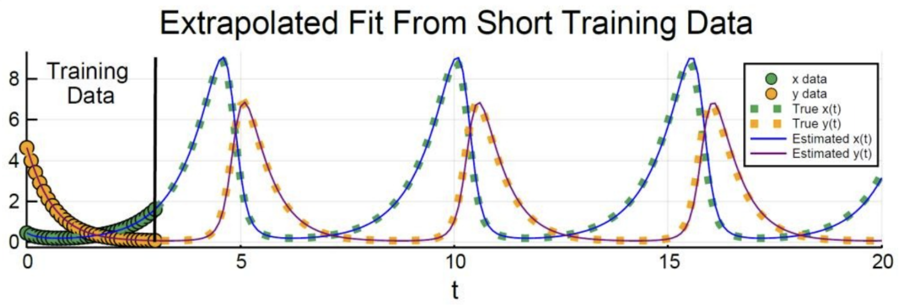
Standard Lotka-Volterra Model

- Extended predictions over 50 time periods.



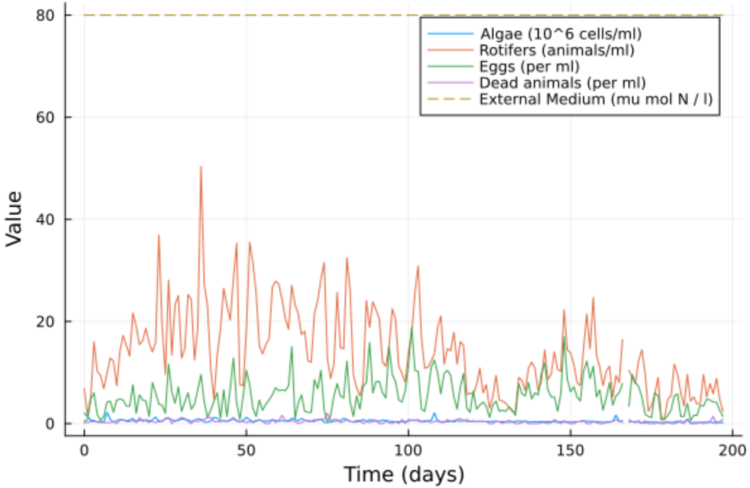
Node Model Data Extrapolation

- Extended predictions over 20 time periods.

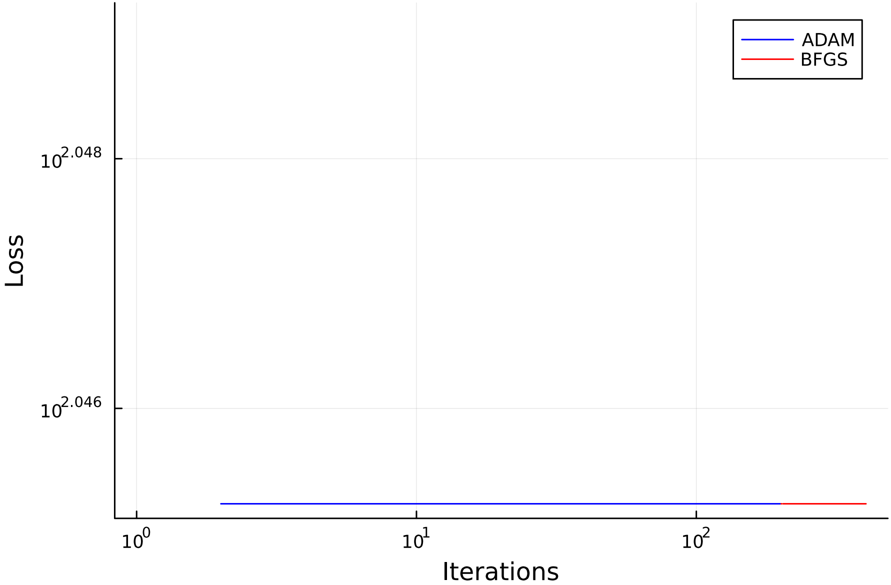


Application to Real Data

- Experimental Predator-Prey data between Planktonic Rotifers and Unicellular Algae in a Chemostat.



Application to Real Data(cont.)



Implications for Future Research

- Integration of larger neural networks.
- Application to other ODE systems.