

Neural Network Integration into ODE Systems

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Population Dynamics

- ▶ Key Concepts

- ▶ *Exponential Growth:*

$$\frac{dN}{dt} = rN$$

- ▶ *Logistic Growth:*

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K} \right)$$

- ▶ Key Factors

- ▶ Birth Rate and Death Rate.

- ▶ Interaction with the Environment

- ▶ Carrying Capacity (K).

- ▶ Density-Dependent and Density-Independent Factors.

- ▶ Types of Population Interactions

- ▶ Predation, Competition,

- ▶ Modeling Tools

- ▶ Mathematical and Simulation Models.

Introduction

- ▶ Overview of integrating neural networks into ODE systems.
- ▶ Objective: Visualize how neural network approximations affect ODE predictions.
- ▶ Methods: Use Julia for solving ODEs and training neural networks.

Methodology

- ▶ Define ODE system (Lotka-Volterra model).
- ▶ Generate noisy data and fit a neural network.
- ▶ Extend time span for prediction.
- ▶ Visualize network's predictions integrated into the ODE system.

Data Generation

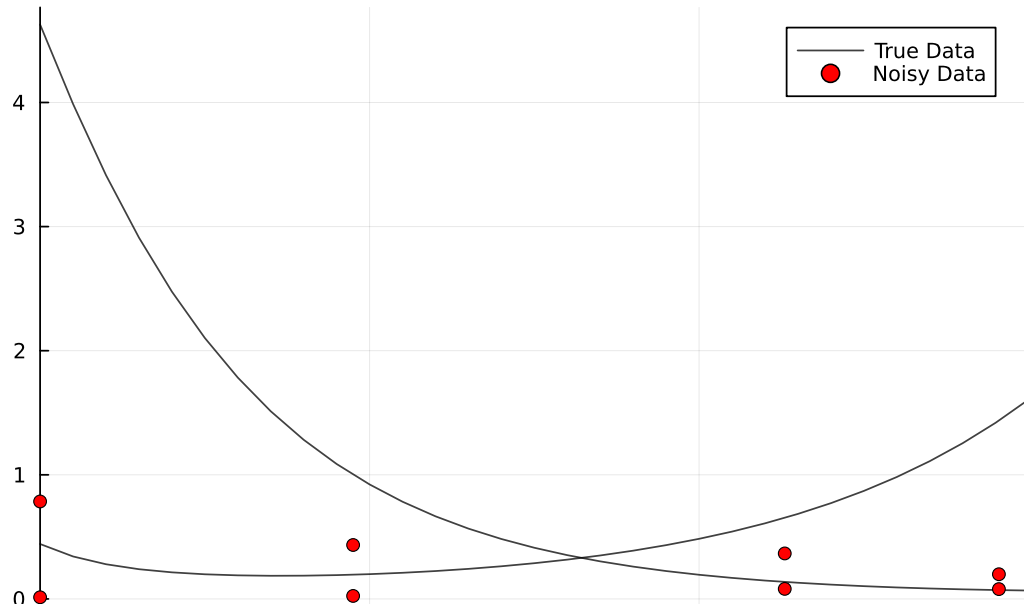
- ▶ Lotka-Volterra model equations:

$$\frac{dx}{dt} = \alpha x - \beta xy$$

$$\frac{dy}{dt} = \gamma xy - \delta y$$

- ▶ Experimental parameters and initial conditions.

Noisy Data



NODE Approach

- ▶ Network architecture: Multi-layer feedforward with RBF activation.
- ▶ Training process using ADAM and BFGS optimizers.
- ▶ Loss function and optimization.

Network Architecture

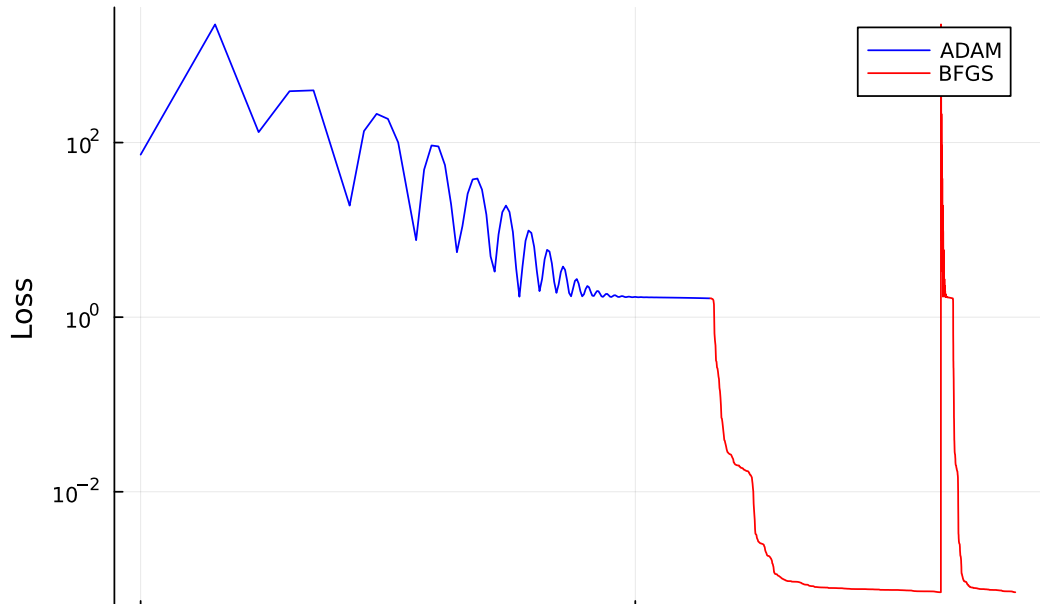
```
# Multilayer FeedForward Neural Network
U = Lux.Chain(
    Lux.Dense(2,5,rbf), Lux.Dense(5,5, rbf), Lux.Dense(5,5, rbf), Lux.Dense(5,2)
)
```

Figure: Neural Network Initialization

```
# Defines the hybrid model using the Neural Network
function ude_dynamics!(du,u, p, t, p_true)
    û = U(u, p, st)[1] # Network prediction
    du[1] = p_true[1]*u[1] + û[1]
    du[2] = -p_true[4]*u[2] + û[2]
end
```

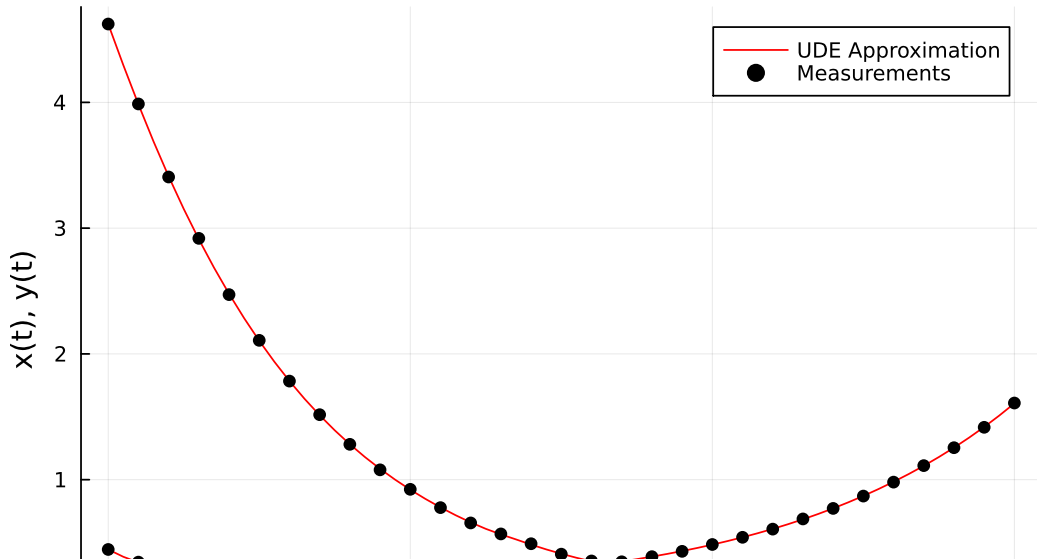
Figure: Integration into Differential Equation

Training Loss



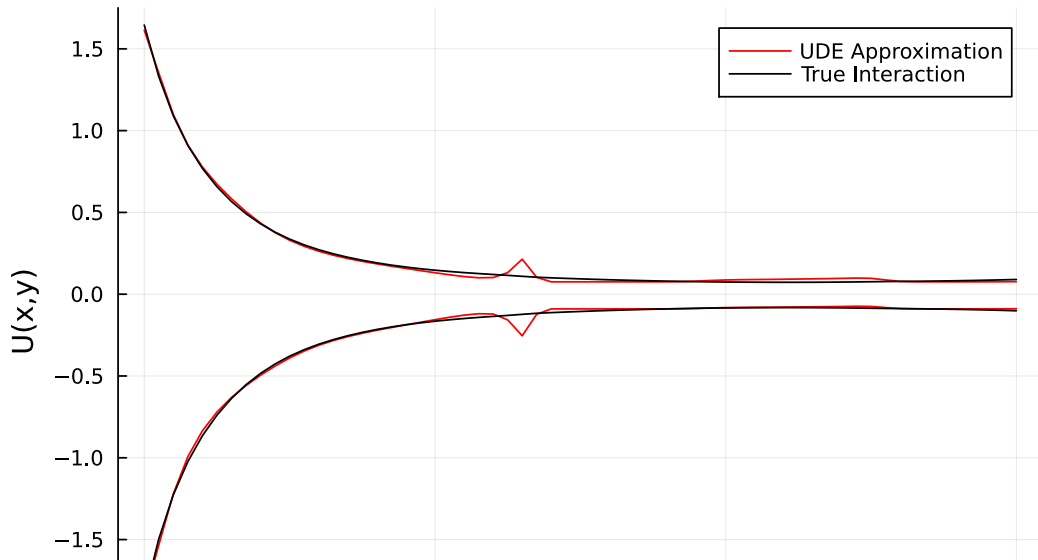
Trajectory Reconstruction

- Comparison of model predictions with true data.



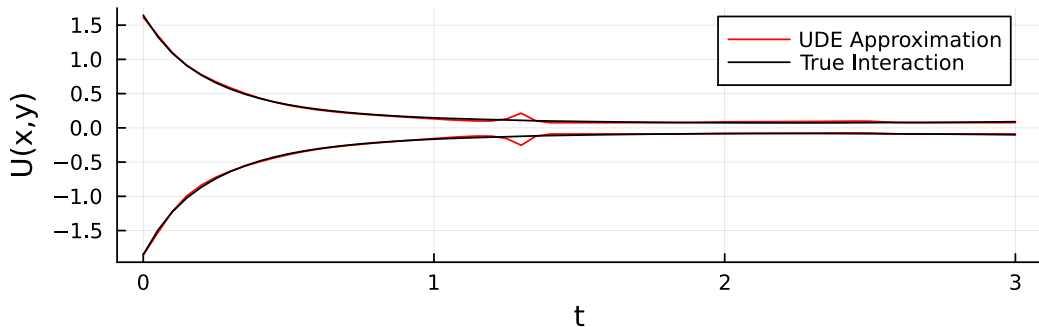
Interaction Reconstruction

- Neural network approximation vs. true interaction.



Prediction and Error

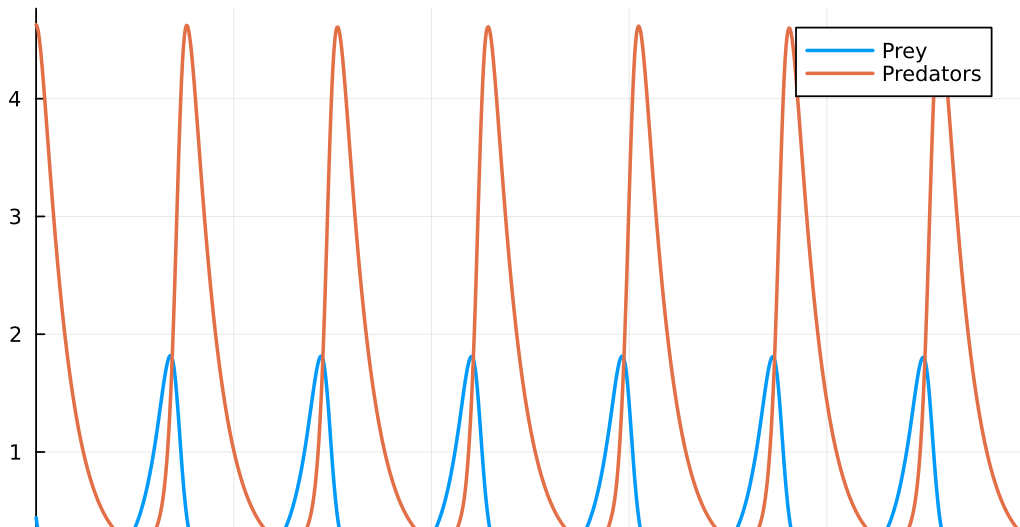
- ▶ Extended predictions over 20 time periods.
- ▶ Error analysis between true interaction and neural network guess.



Standard Lotka-Volterra Model

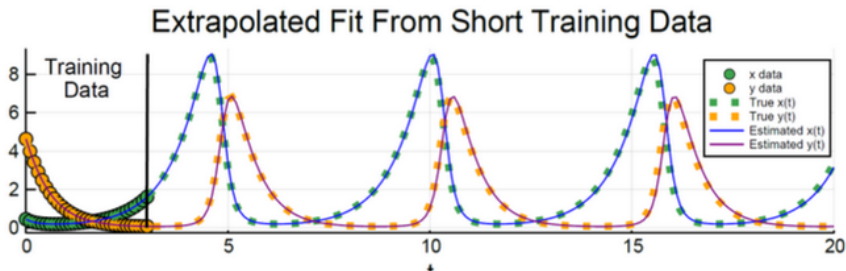
- Extended predictions over 50 time periods.

Prey vs Predators



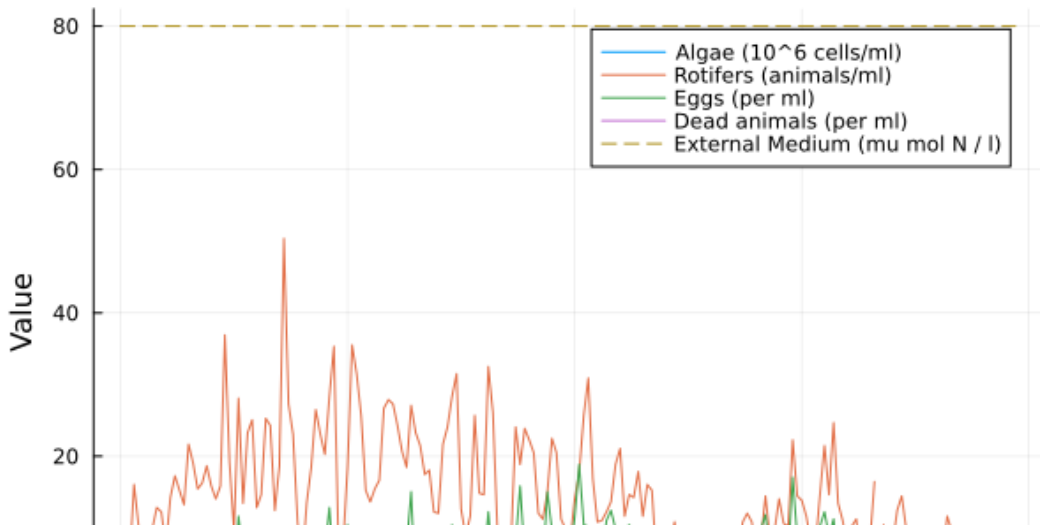
Node Model Data Extrapolation

- ▶ Extended predictions over 20 time periods.



Application to Real Data

- ▶ Experimental Predator-Prey data between Planktonic Rotifers and Unicellular Algae in a Chemostat.



Application to Real Data(cont.)



Implications for Future Research

- ▶ Integration of larger neural networks.
- ▶ Application to other ODE systems.