COMS 4776 Lecture 4: Distributed Representations & Neural Language Models

Richard Zemel

Neural Language Models

- Let's now see a real example of a neural net to learn feature representations of words.
  - We'll see a lot more neural net architectures later in the course.
- We'll also introduce the models used in Assignment 1.

Review: Probability and Bayes' Rule

Suppose we want to build a speech recognition system.

We'd like to be able to infer a likely sentence $\mathbf{s}$ given the observed speech signal $\mathbf{a}$. The generative approach is to build two components:

- An observation model, represented as $p(\mathbf{a} \mid \mathbf{s})$, which tells us how likely the sentence $\mathbf{s}$ is to lead to the acoustic signal $\mathbf{a}$.
- A prior, represented as $p(\mathbf{s})$, which tells us how likely a given sentence $\mathbf{s}$ is. E.g., it should know that "recognize speech" is more likely that "wreck a nice beach."

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Given these components, we can use Bayes' Rule to infer a posterior distribution over sentences given the speech signal:

$$p(\mathbf{s} \mid \mathbf{a}) = \frac{p(\mathbf{s})p(\mathbf{a} \mid \mathbf{s})}{\sum_{\mathbf{s}'} p(\mathbf{s}')p(\mathbf{a} \mid \mathbf{s}')}.$$

Language Modeling

From here on, we will focus on learning a good distribution $p(\mathbf{s})$ of sentences. This problem is known as language modeling.

Assume we have a corpus of sentences $\mathbf{s}^{(1)}, \ldots, \mathbf{s}^{(N)}$. The maximum likelihood criterion says we want our model to maximize the probability our model assigns to the observed sentences. We assume the sentences are independent, so that their probabilities multiply.

$$\max \prod_{i=1}^{N} p(\mathbf{s}^{(i)}).$$

Language Modeling

In maximum likelihood training, we want to maximize $\prod_{i=1}^{N} p(\mathbf{s}^{(i)})$.

The probability of generating the whole training corpus is vanishingly small — like monkeys typing all of Shakespeare.

- The log probability is something we can work with more easily. It also conveniently decomposes as a sum:

$$\log \prod_{i=1}^{N} p(\mathbf{s}^{(i)}) = \sum_{i=1}^{N} \log p(\mathbf{s}^{(i)}).$$

- This is equivalent to the cross-entropy loss.

Language Modeling

- Probability of a sentence? What does that even mean?

Language Modeling

- Probability of a sentence? What does that even mean?
  - A sentence is a sequence of words $w_1, w_2, \ldots, w_T$. Using the chain rule of conditional probability, we can decompose the probability as

$$p(\mathbf{s}) = p(w_1, \ldots, w_T) = p(w_1)p(w_2 \mid w_1) \cdots p(w_T \mid w_1, \ldots, w_{T-1}).$$

  - Therefore, the language modeling problem is equivalent to being able to predict the next word!
- We typically make a Markov assumption, i.e. that the distribution over the next word only depends on the preceding few words. I.e., if we use a context of length 3,

$$p(w_t \mid w_1, \ldots, w_{t-1}) = p(w_t \mid w_{t-3}, w_{t-2}, w_{t-1}).$$

  - Such a model is called memoryless.
  - Now it's basically a supervised prediction problem. We need to predict the conditional distribution of each word given the previous K.
  - When we decompose it into separate prediction problems this way, it's called an autoregressive model.

N-Gram Language Models

- One sort of Markov model we can learn uses a conditional probability table, i.e.

	cat	and	city	···
the fat	0.21	0.003	0.01	
four score	0.0001	0.55	0.0001	···
New York	0.002	0.0001	0.48	
⋮		⋮		

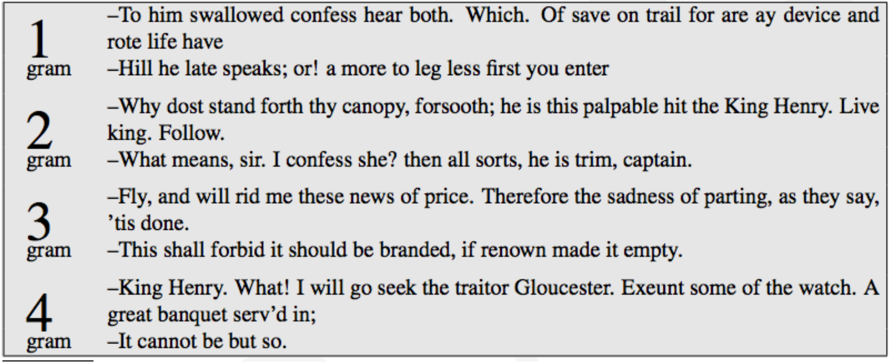
- Maybe the simplest way to estimate the probabilities is from the empirical distribution:

$$
\begin{aligned}
p(w_3 = \mathrm{cat} \,|\, w_1 = \mathrm{the}, w_2 = \mathrm{fat}) &= \frac{p(w_1 = \mathrm{the}, w_2 = \mathrm{fat}, w_3 = \mathrm{cat})}{p(w_1 = \mathrm{the}, w_2 = \mathrm{fat})} \\
&\approx \frac{\mathrm{count(the\ fat\ cat)}}{\mathrm{count(the\ fat)}}
\end{aligned}
$$

- The phrases we're counting are called n-grams (where n is the length), so this is an n-gram language model.
  - Note: the above example is considered a 3-gram model, not a 2-gram

N-Gram Language Models

Shakespeare:

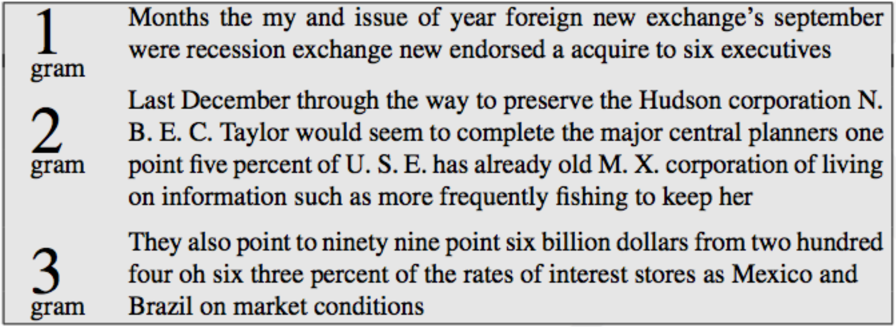


1 gram	–To him swallowed confess hear both. Which. Of save on trail for are ay device and rote life have –Hill he late speaks; or! a more to leg less first you enter
2 gram	–Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live king. Follow. –What means, sir. I confess she? then all sorts, he is trim, captain.
3 gram	–Fly, and will rid me these news of price. Therefore the sadness of parting, as they say, ’tis done. –This shall forbid it should be branded, if renown made it empty.
4 gram	–King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A great banquet serv’d in; –It cannot be but so.

Jurafsky and Martin, *Speech and Language Processing*

N-Gram Language Models

Wall Street Journal:



1 gram	Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives
2 gram	Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her
3 gram	They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Jurafsky and Martin, *Speech and Language Processing*

N-Gram Language Models

- Problems with n-gram language models

N-Gram Language Models

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  - The number of entries in the conditional probability table is exponential in the context length.
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N-Gram Language Models

- Problems with n-gram language models
    - The number of entries in the conditional probability table is exponential in the context length.
    - Data sparsity: most n-grams never appear in the corpus, even if they are possible.
- Traditional ways to deal with data sparsity
    - Use a short context (but this means the model is less powerful)
    - Smooth the probabilities, e.g. by adding imaginary counts
    - Make predictions using an ensemble of n-gram models with different n

Distributed Representations

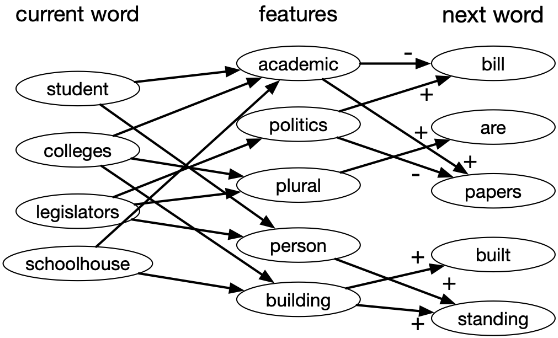
- Conditional probability tables are a kind of localist representation: all the information about a particular word is stored in one place, i.e. a column of the table.
- But different words are related, so we ought to be able to share information between them. For instance, consider this matrix of word attributes:

	academic	politics	plural	person	building
students	1	0	1	1	0
colleges	1	0	1	0	1
legislators	0	1	1	1	0
schoolhouse	1	0	0	0	1

- And this matrix of how each attribute influences the next word:

	bill	is	are	papers	built	standing
academic	−			+		
politics	+			−		
plural		−	+			
person						+
building					+	+

- Imagine these matrices are layers in an MLP. (One-hot representations of words, softmax over next word.)



- Here, the information about a given word is distributed throughout the representation. We call this a distributed representation.
- In general, when we train an MLP with backprop, the hidden units won't have intuitive meanings like in this cartoon. But this is a useful intuition pump for what MLPs can represent.

Distributed Representations

- We would like to be able to share information between related words. E.g., suppose we've seen the sentence

  The cat got squashed in the garden on Friday.

- This should help us predict the words in the sentence

  The dog got flattened in the yard on Monday.

- An n-gram model can't generalize this way, but a distributed representation might let us do so.

Neural Language Model

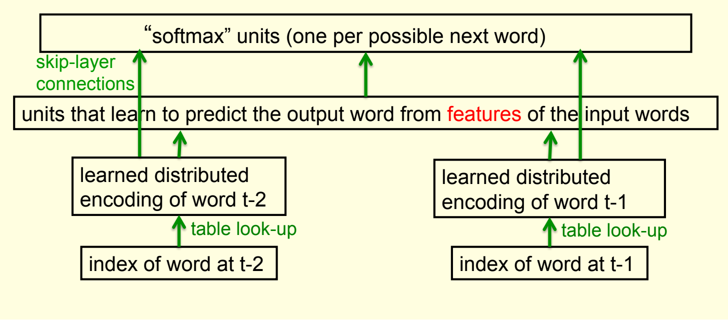
- Predicting the distribution of the next word given the previous K is just a multiway classification problem.
- **Inputs:** previous K words
- **Target:** next word
- **Loss:** cross-entropy. Recall that this is equivalent to maximum likelihood:

$$\begin{aligned} -\log p(\mathbf{s}) &= -\log \prod_{t=1}^{T} p(w_t \mid w_1, \ldots, w_{t-1}) \\ &= -\sum_{t=1}^{T} \log p(w_t \mid w_1, \ldots, w_{t-1}) \\ &= -\sum_{t=1}^{T} \sum_{v=1}^{V} t_{tv} \log y_{tv}, \end{aligned}$$

where t_{iv} is the one-hot encoding for the ith word and y_{iv} is the predicted probability for the ith word being index v.

Bengio's Neural Language Model

- Here is a classic neural probabilistic language model, or just neural language model:

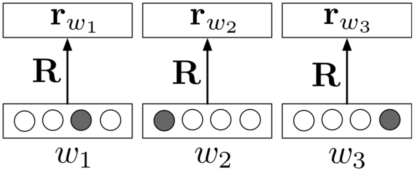


"softmax" units (one per possible next word)
skip-layer connections
units that learn to predict the output word from features of the input words
learned distributed encoding of word t-2
learned distributed encoding of word t-1
table look-up
table look-up
index of word at t-2
index of word at t-1

http://www.jmlr.org/papers/volume3/bengio03a/bengio03a.pdf

Neural Language Model

- If we use a 1-of-K encoding for the words, the first layer can be thought of as a linear layer with tied weights.



- The weight matrix basically acts like a lookup table. Each column is the representation of a word, also called an embedding, feature vector, or encoding.
  - "Embedding" emphasizes that it's a location in a high-dimensonal space; words that are closer together are more semantically similar
  - "Feature vector" emphasizes that it's a vector that can be used for making predictions, just like other feature mappigns we've looked at (e.g. polynomials)

Neural Language Model

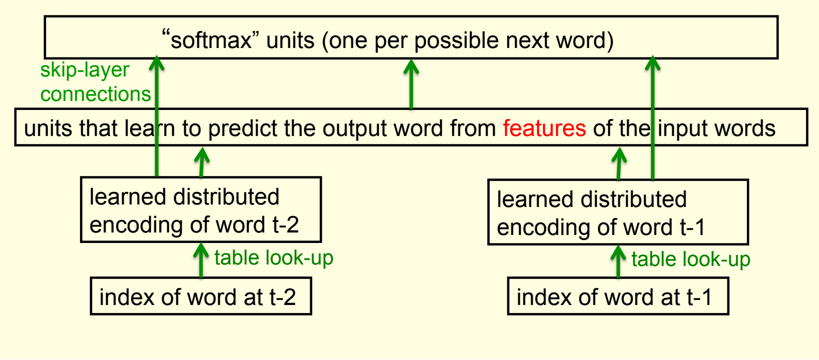
- We can measure the similarity or dissimilarity of two words using
  - the dot product $\mathbf{r}_1^\top \mathbf{r}_2$
  - Euclidean distance $\|\mathbf{r}_1 - \mathbf{r}_2\|$
- If the vectors have unit norm, the two are equivalent:

$$\begin{aligned}\|\mathbf{r}_1 - \mathbf{r}_2\|^2 &= (\mathbf{r}_1 - \mathbf{r}_2)^\top (\mathbf{r}_1 - \mathbf{r}_2) \\ &= \mathbf{r}_1^\top \mathbf{r}_1 - 2\mathbf{r}_1^\top \mathbf{r}_2 + \mathbf{r}_2^\top \mathbf{r}_2 \\ &= 2 - 2\mathbf{r}_1^\top \mathbf{r}_2\end{aligned}$$

- In this case, the dot product is called cosine similarity.

Neural Language Model

- This model is very compact: the number of parameters is *linear* in the context size, compared with exponential for n-gram models.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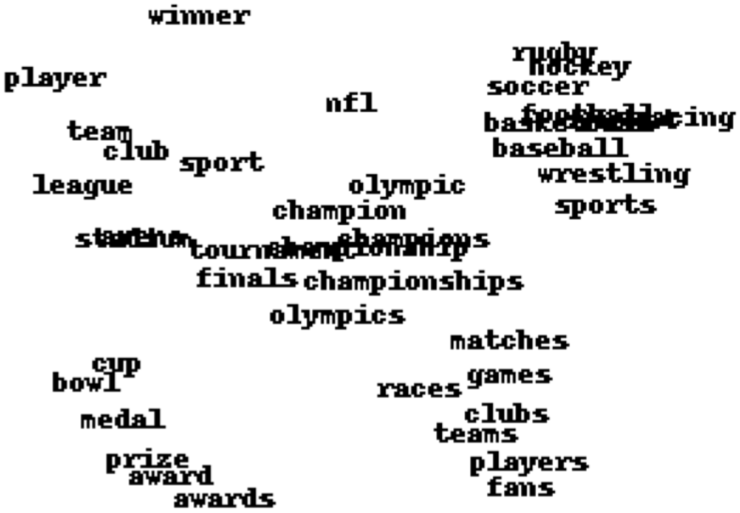
Neural Language Model

- What do these word embeddings look like?
- It's hard to visualize an *n*-dimensional space, but there are algorithms for mapping the embeddings to two dimensions.
- The following 2-D embeddings are done with an algorithm called tSNE which tries to make distnaces in the 2-D embedding match the original 30-D distances as closely as possible.
- Note: the visualizations are from a slightly different model.

Neural Language Model



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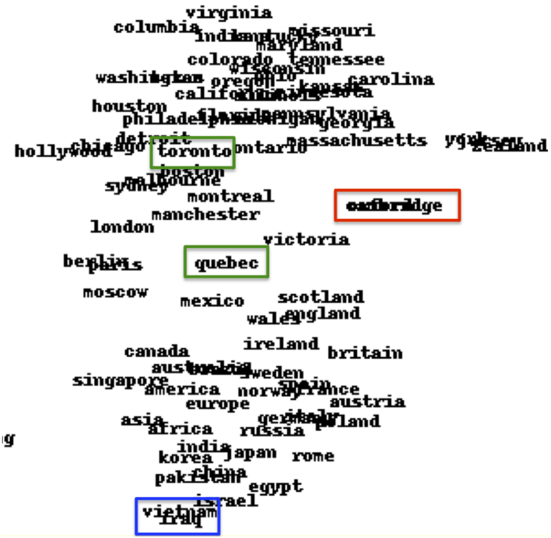

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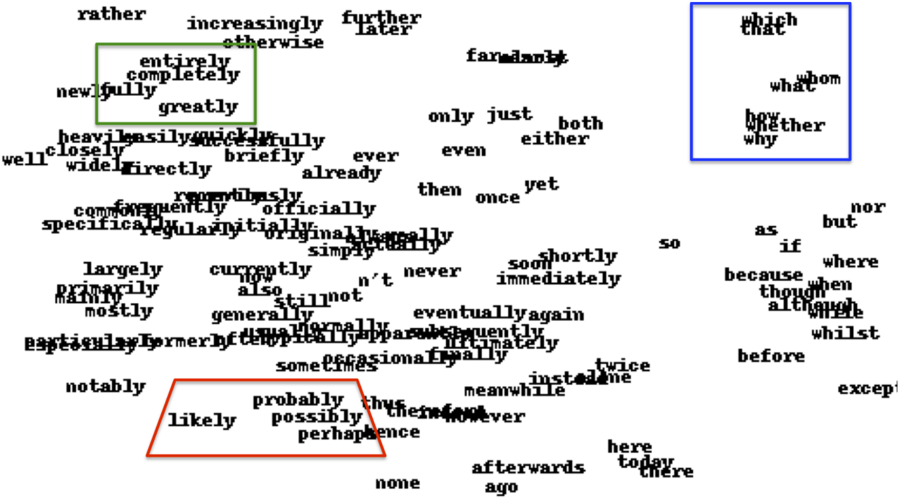

Neural Language Model

- Thinking about high-dimensional embeddings
  - Most vectors are nearly orthogonal (i.e. dot product is close to 0)
  - Most points are far away from each other
  - "In a 30-dimensional grocery store, anchovies can be next to fish and next to pizza toppings." – Geoff Hinton
- The 2-D embeddings might be fairly misleading, since they can't preserve the distance relationships from a higher-dimensional embedding. (I.e., unrelated words might be close together in 2-D, but far apart in 30-D.)

GloVe

- Fitting language models is really hard:
    - It's really important to make good predictions about relative probabilities of rare words.
    - Computing the predictive distribution requires a large softmax.
- Maybe this is overkill if all you want is word representations.
- Global Vector (GloVe) embeddings are a simpler and faster approach based on a matrix factorization similar to principal component analysis (PCA).
    - First fit the distributed word representations using GloVe, then plug them into a neural net that does some other task (e.g. language modeling, translation).

GloVe

- Distributional hypothesis: words with similar distributions have similar meanings ("judge a word by the company it keeps")
- Consider a co-occurrence matrix $\mathbf{X}$, which counts the number of times two words appear nearby (say, less than 5 positions apart)
- This is a $V \times V$ matrix, where V is the vocabulary size (very large)
- **Intuition pump:** suppose we fit a rank-K approximation $\mathbf{X} \approx \mathbf{R}\tilde{\mathbf{R}}^\top$, where $\mathbf{R}$ and $\tilde{\mathbf{R}}$ are $V \times K$ matrices.
  - Each row $\mathbf{r}_i$ of $\mathbf{R}$ is the K-dimensional representation of a word
  - Each entry is approximated as $x_{ij} \approx \mathbf{r}_i^\top \tilde{\mathbf{r}}_j$
  - Hence, more similar words are more likely to co-occur
  - Minimizing the squared Frobenius norm $\|\mathbf{X} - \mathbf{R}\tilde{\mathbf{R}}^\top\|_F^2 = \sum_{i,j}(x_{ij} - \mathbf{r}_i^\top \tilde{\mathbf{r}}_j)^2$ is basically PCA.

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- Global Vector (GloVe) embedding cost function:

$$\mathcal{J}(\mathbf{R}) = \sum_{i,j} f(x_{ij})(\mathbf{r}_i^\top \tilde{\mathbf{r}}_j + b_i + \tilde{b}_j - \log x_{ij})^2$$

$$f(x_{ij}) = \begin{cases} \left(\frac{x_{ij}}{100}\right)^{3/4} & \text{if } x_{ij} < 100 \\ 1 & \text{if } x_{ij} \geq 100 \end{cases}$$

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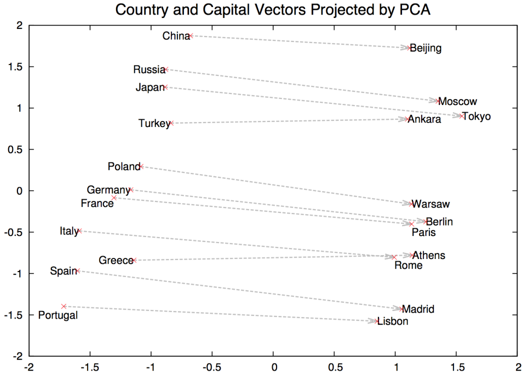
- b_i and $\tilde{b}_j$ are bias parameters.
- We can avoid computing $\log 0$ since $f(0) = 0$.
- We only need to consider the nonzero entries of **X**. This gives a big computational savings since **X** is extremely sparse!

Word Analogies

- Here's a linear projection of word representations for cities and capitals into 2 dimensions.
- The mapping city → capital corresponds roughly to a single direction in the vector space:



- Note: this figure actually comes from skip-grams, a predecessor to GloVe.

Word Analogies

- In other words,
  $\text{vector}(\text{Paris}) - \text{vector}(\text{France}) \approx \text{vector}(\text{London}) - \text{vector}(\text{England})$
- This means we can analogies by doing arithmetic on word vectors:
  - e.g. "Paris is to France as London is to _____"
  - Find the word whose vector is closest to
    $\text{vector}(\text{France}) - \text{vector}(\text{Paris}) + \text{vector}(\text{London})$
- Example analogies:

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza