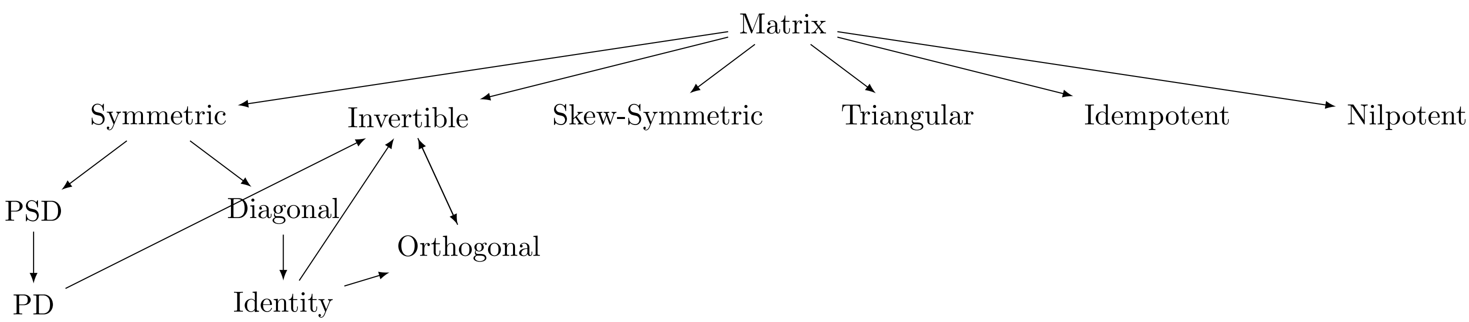

Arrows indicate subset or special case relationships. Additional arrows show overlaps.

Term	Definition	Unique Attributes / Key Properties
Diagonal	$a_{ij} = 0$ for $i \neq j$	- Determinant is product of diagonal entries - Eigenvalues are diagonal entries - A^k is diagonal with entries raised to k
Symmetric	$A = A^T$, i.e., $a_{ij} = a_{ji}$	- All eigenvalues are real - Eigenvectors for distinct eigenvalues are orthogonal - Diagonalizable via orthogonal matrices
Identity (I)	$I_{ij} = \delta_{ij}$	- $AI = IA = A$ - $I^{-1} = I$ - All eigenvalues are 1
Invertible (Non-singular)	Exists A^{-1} such that $AA^{-1} = A^{-1}A = I$	- $\det(A) \neq 0$ - Linearly independent rows and columns - No zero eigenvalues
Singular	A square matrix with $\det(A) = 0$	- Not invertible - May have linearly dependent rows or columns - At least one zero eigenvalue
Orthogonal	$A^{-1} = A^T$, so $A^TA = I$	- Rows and columns form an orthonormal set - Preserves lengths and angles - $\det(A) = \pm 1$
Pseudoinverse (Moore–Penrose)	Generalized inverse $A^\dagger$ satisfying the four Penrose conditions	- Always exists and is unique for any matrix - If $A \in \mathbb{R}^{m \times n}$ has full column rank ($m \geq n$): $A^\dagger = (A^TA)^{-1}A^T$, and $A^\dagger A = I_n$ - If A has full row rank ($n \geq m$): $A^\dagger = A^T(AA^T)^{-1}$, and $AA^\dagger = I_m$ - If A is square and invertible: $A^\dagger = A^{-1}$ - Used in least-squares solutions and underdetermined systems
Positive Definite	$\vec{x}^TA\vec{x} > 0$ for all non-zero $\vec{x}$	- All eigenvalues are positive - $\det(A) > 0$ - Invertible
Positive Semi-Definite	$A = A^T$, $\vec{x}^TA\vec{x} \geq 0$ for all $\vec{x}$	- All eigenvalues are non-negative - $\det(A) \geq 0$ - Always symmetric - Can always be written as X^TX for some X (Gram matrix) - Defines a semi-inner product
Triangular (Upper / Lower)	Upper: $a_{ij} = 0$ for $i > j$; Lower: $a_{ij} = 0$ for $i < j$	- Determinant is product of diagonal entries - Eigenvalues are diagonal entries - Solves systems efficiently via substitution

Skew-Symmetric	$A = -A^T$, so $a_{ii} = 0$	- Eigenvalues are 0 or purely imaginary - $\vec{x}^T A\vec{x} = 0$ for all $\vec{x}$
Idempotent	$A^2 = A$	- Used in projection operations - Eigenvalues are 0 or 1
Nilpotent	$A^k = 0$ for some $k \in \mathbb{N}$	- All eigenvalues are 0
Orthogonal Projection	Linear transformation P where $P^2 = P$ and $P = P^T$	- Projects onto a subspace along its orthogonal complement - Minimizes distance to the subspace (best approximation) - $\mathrm{Im}(P)$ is a subspace; $\mathrm{Ker}(P) = \mathrm{Im}(I - P)$
Linear Map (Transformation)	Function $T : V \to W$ satisfying $T(a\vec{x} + b\vec{y}) = aT(\vec{x}) + bT(\vec{y})$	- Represented by a matrix in finite dimensions - Preserves linear structure - Kernel and image define fundamental subspaces
Fundamental Subspaces	Column space, row space, null space, left null space	- Column space: span of columns (range of A) - Row space: span of rows = col space of A^T - Null space: solutions to $A\vec{x} = 0$ - Left null space: null space of A^T
Unitary	$U^\dagger = U^{-1}$, where $U^\dagger$ is the conjugate transpose of U	- $U^\dagger U = UU^\dagger = I$ - Preserves inner products: $\langle U\vec{x}, U\vec{y}\rangle = \langle \vec{x}, \vec{y}\rangle$ - All eigenvalues lie on the unit circle: $\lvert\lambda\rvert = 1$ - Generalization of orthogonal matrices to complex space