Common Probability Distributions

Name	Symbol / Representation	PDF and CDF	Generation and Applications	Relationship to Normal
Normal	$X \sim \mathcal{N}(\mu, \sigma^2)$	PDF: $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ CDF: $\Phi\left(\frac{x-\mu}{\sigma}\right)$	Generated via CLT or Box-Muller transform; used in modeling natural variation	Baseline distribution; many others converge to it under limiting conditions
Binomial	$X \sim \mathrm{Bin}(n, p)$	PMF: $\binom{n}{k} p^k (1-p)^{n-k}$ CDF: $\sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$	Counts successes in n Bernoulli trials; used in discrete event modeling	Approximates normal when n is large and p not near 0 or 1
Poisson	$X \sim \mathrm{Pois}(\lambda)$	PMF: $\frac{\lambda^k e^{-\lambda}}{k!}$ CDF: $\sum_{i=0}^{k} \frac{\lambda^i e^{-\lambda}}{i!}$	Models rare events over time/space; used in queuing, traffic, biology	Approaches normal as $\lambda \to \infty$
Gamma	$X \sim \Gamma(\alpha, \beta)$	PDF: $\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ CDF: $\frac{\gamma(\alpha, \beta x)}{\Gamma(\alpha)}$	Sum of α exponential variables; used in waiting times, Bayesian priors	Sum of squared normals yields chi-square, a special case of gamma
Lognormal	$X \sim \mathrm{Log}\mathcal{N}(\mu, \sigma^2)$	PDF: $\frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$ CDF: $\Phi\left(\frac{\ln x-\mu}{\sigma}\right)$	Exponentiated normal; used in finance, reliability, and multiplicative processes	Log of lognormal is normal
Exponential	$X \sim \mathrm{Exp}(\lambda)$	PDF: $\lambda e^{-\lambda x}$ CDF: $1 - e^{-\lambda x}$	Time between Poisson events; memoryless; used in survival analysis	Special case of gamma with $\alpha = 1$
Beta	$X \sim \mathrm{Beta}(\alpha, \beta)$	PDF: $\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ CDF: $I_x(\alpha, \beta)$	Models probabilities and proportions; used in Bayesian inference	No direct convergence, but appears in normalized transformations
Multinomial	$X \sim \mathrm{Mult}(n; p_1, \ldots, p_k)$	PMF: $\frac{n!}{x_1! \cdots x_k!} p_1^{x_1} \cdots p_k^{x_k}$ CDF: Not typically defined (multivariate)	Generalization of binomial for ¿2 outcomes; used in NLP, categorical modeling	Each marginal binomial can approximate normal; joint distribution approaches multivariate normal
Chi-Square	$X \sim \chi^2_k$	PDF: $\frac{1}{2^{k/2}\Gamma(k/2)} x^{k/2-1} e^{-x/2}$ CDF: $\frac{\gamma(k/2, x/2)}{\Gamma(k/2)}$	Sum of squares of k standard normals; used in hypothesis testing	Derived from squared standard normals
Student's t	$X \sim t_\nu$	PDF: $\frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{x^2}{\nu}\right)^{-\frac{\nu+1}{2}}$	Used in small-sample inference; arises from normal with unknown variance	Converges to standard normal as $\nu \to \infty$
F-distribution	$X \sim F_{d_1, d_2}$	PDF: $\frac{(d_1/d_2)^{d_1/2} x^{d_1/2-1}}{B(d_1/2, d_2/2)(1+\frac{d_1}{d_2}x)^{(d_1+d_2)/2}}$	Ratio of scaled chi-squares; used in ANOVA and variance testing	Related to normal via chi-square components
Bernoulli	$X \sim \mathrm{Bern}(p)$	PMF: $P(X=1) = p$, $P(X=0) = 1-p$	Single trial success/failure; building block for binomial	Sum of Bernoulli trials leads to binomial, then normal
Geometric	$X \sim \mathrm{Geom}(p)$	PMF: $P(X=k) = (1-p)^{k-1} p$	Counts trials until first success; memoryless	No direct convergence, but approximates exponential in continuous limit
Uniform (continuous)	$X \sim \mathcal{U}(a, b)$	PDF: $\frac{1}{b-a}$ for $x \in [a, b]$ CDF: $\frac{x-a}{b-a}$	Models equal likelihood; used in simulation and random sampling	Sum of many uniforms approximates normal (CLT)