Matrix Properties

Term	Notation	Definition	Significance
Trace	$\mathrm{tr}(A)$	Sum of diagonal elements of a square matrix A	Invariant under similarity; used in matrix calculus and spectral analysis
Gradient	∇f, $\mathrm{grad}\, f$	Vector of partial derivatives of a scalar function f	Points in the direction of steepest ascent; key in optimization
Hessian	$\nabla^2 f$, H_f	Matrix of second-order partial derivatives of f	Captures local curvature; used in Newton's method and convex analysis
Total Derivative	$Df(\vec{x})$	Linear map approximating f near $\vec{x}$	Generalizes derivative to multivariable functions; foundation of the Jacobian
Directional Derivative	$D_{\vec{v}} f(\vec{x})$	Rate of change of f at $\vec{x}$ in direction $\vec{v}$	Measures sensitivity along specified directions
Rank	$\mathrm{rank}(A)$	Dimension of the column space of A	Indicates linear independence; determines solution existence
Span	$\mathrm{span}\{\vec{v}_1, \ldots, \vec{v}_k\}$	Set of all linear combinations of given vectors	Describes the subspace generated by a set
Null Space	$\mathrm{null}(A)$ or $\ker(A)$	Set of vectors $\vec{x}$ such that $A\vec{x} = 0$	Solution space to homogeneous systems
Determinant	$\det(A)$	Scalar computed from a square matrix A	Indicates invertibility, volume scaling, and orientation
Condition Number	$\kappa(A)$	Ratio of largest to smallest singular value of A	Measures solution sensitivity to perturbations; key in numerical stability
Euclidean Norm	$\\|\vec{x}\\|_2$	$\sqrt{\sum x_i^2}$	Standard vector length; induced by the dot product
Frobenius Norm	$\\|A\\|_F$	$\sqrt{\sum_{i,j} a_{ij}^2}$	Matrix analog of the Euclidean norm; used in low-rank approximation
ℓ_1**-Norm**	$\\|\vec{x}\\|_1$	$\sum_i \|x_i\|$	Promotes sparsity; used in LASSO and compressed sensing
ℓ_2**-Norm**	$\\|\vec{x}\\|_2$	$\sqrt{\sum_i x_i^2}$	Same as Euclidean norm; minimizes energy in least-squares
Spectral Norm	$\\|A\\|_2$	Largest singular value of A	Operator norm induced by ℓ_2; bounds matrix amplification
Nuclear Norm	$\\|A\\|_*$	Sum of singular values of A	Convex surrogate for rank; promotes low-rank solutions in optimization

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Term	**Notation**	**Definition**	**Significance**
Jacobian	$J_f(\vec{x})$	Matrix of first-order partial derivatives of f	Linear approximation for multivariate functions; used in nonlinear systems
Eigenvalue	λ	Scalar satisfying $A\vec{v} = \lambda\vec{v}$	Fundamental in stability, diagonalization, and systems analysis