Term	Notation	Definition	Significance
Trace	$\operatorname{tr}(A)$	Sum of diagonal elements of a square matrix $\cal A$	Invariant under similar- ity; used in matrix calcu- lus and spectral analysis
Gradient	∇f , grad f	Vector of partial derivatives of a scalar function \boldsymbol{f}	Points in the direction of steepest ascent; key in optimization
Hessian	$\nabla^2 f$, H_f	Matrix of second-order partial derivatives of \boldsymbol{f}	Captures local curva- ture; used in Newton's method and convex analysis
Total Deriva- tive	$Df(\vec{x})$	Linear map approximating f near \vec{x}	Generalizes derivative to multivariable func- tions; foundation of the Jacobian
Directional Derivative	$D_{ec{v}}f(ec{x})$	Rate of change of f at \vec{x} in direction \vec{v}	Measures sensitivity along specified directions
Rank	$\operatorname{rank}(A)$	Dimension of the column space of \boldsymbol{A}	Indicates linear indepen- dence; determines solu- tion existence
Span	$\operatorname{span}\{\vec{v}_1,\ldots,\vec{v}_k\}$	Set of all linear combinations of given vectors	Describes the subspace generated by a set
Null Space	$\operatorname{null}(A)$ or $\ker(A)$	Set of vectors \vec{x} such that $A\vec{x} = 0$	Solution space to homogeneous systems
Determinant	$\det(A)$	Scalar computed from a square matrix \boldsymbol{A}	Indicates invertibility, volume scaling, and orientation
Condition Number	$\kappa(A)$	Ratio of largest to smallest singular value of ${\cal A}$	Measures solution sen- sitivity to perturbations; key in numerical stability
Euclidean Norm	$\ \vec{x}\ _2$	$\sqrt{\sum x_i^2}$	Standard vector length; induced by the dot product
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Term	Notation	Definition	Significance	
Frobenius Norm	$ A _F$	$\sqrt{\sum_{i,j} a_{ij}^2}$	Matrix analog of the Euclidean norm; used in low-rank approximation	
$\ell_1 ext{-Norm}$	$\ \vec{x}\ _1$	$\sum_{i} x_i $	Promotes sparsity; used in LASSO and compressed sensing	
ℓ_2 -Norm	$\ \vec{x}\ _2$	$\sqrt{\sum_i x_i^2}$	Same as Euclidean norm; minimizes energy in least-squares	
Spectral Norm	$ A _2$	Largest singular value of $\cal A$	Operator norm induced by ℓ_2 ; bounds matrix amplification	
Jacobian	$J_f(\vec{x})$	Matrix of first-order partial derivatives of f	Linear approximation for multivariate functions; used in nonlinear systems	
Eigenvalue	λ	Scalar satisfying $A\vec{v}=\lambda\vec{v}$	Fundamental in stability, diagonalization, and systems analysis	