

Matrix Properties

Term	Notation	Definition	Significance
Trace	$\text{tr}(A)$	Sum of diagonal elements of a square matrix A	Invariant under similarity; used in matrix calculus and spectral analysis. TODO add properties
Gradient	$\nabla f, \text{grad } f$	Vector of partial derivatives of a scalar function f	Points in the direction of steepest ascent; key in optimization
Hessian	$\nabla^2 f, H_f$	Matrix of second-order partial derivatives of f	Captures local curvature; used in Newton's method and convex analysis
Total Derivative	$Df(\vec{x})$	Linear map approximating f near \vec{x}	Generalizes derivative to multivariable functions; foundation of the Jacobian
Directional Derivative	$D_{\vec{v}}f(\vec{x})$	Rate of change of f at \vec{x} in direction \vec{v}	Measures sensitivity along specified directions
Rank	$\text{rank}(A)$	Dimension of the column space of A	Indicates linear independence; determines solution existence
Span	$\text{span}\{\vec{v}_1, \dots, \vec{v}_k\}$	Set of all linear combinations of given vectors	Describes the subspace generated by a set
Null Space	$\text{null}(A)$ or $\text{ker}(A)$	Set of vectors \vec{x} such that $A\vec{x} = 0$	Solution space to homogeneous systems
Determinant	$\det(A)$	Scalar computed from a square matrix A	Indicates invertibility, volume scaling, and orientation
Condition Number	$\kappa(A)$	Ratio of largest to smallest singular value of A	Measures solution sensitivity to perturbations; key in numerical stability
Euclidean Norm	$\ \vec{x}\ _2$	$\sqrt{\sum x_i^2}$	Standard vector length; induced by the dot product
Frobenius Norm	$\ A\ _F$	$\sqrt{\sum_{i,j} a_{ij}^2}$	Matrix analog of the Euclidean norm; used in low-rank approximation
ℓ_1 -Norm	$\ \vec{x}\ _1$	$\sum_i x_i $	Promotes sparsity; used in LASSO and compressed sensing
ℓ_2 -Norm	$\ \vec{x}\ _2$	$\sqrt{\sum_i x_i^2}$	Same as Euclidean norm; minimizes energy in least-squares
Spectral Norm	$\ A\ _2$	Largest singular value of A	Operator norm induced by ℓ_2 ; bounds matrix amplification
Nuclear Norm	$\ A\ _*$	Sum of singular values of A	Convex surrogate for rank; promotes low-rank solutions in optimization

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Term	Notation	Definition	Significance
Jacobian	$J_f(\vec{x})$	Matrix of first-order partial derivatives of f	Linear approximation for multivariate functions; used in nonlinear systems
Eigenvalue	λ	Scalar satisfying $A\vec{v} = \lambda\vec{v}$	Fundamental in stability, diagonalization, and systems analysis