HW1_nb

October 2, 2025

[1]:
```
%reload_ext autoreload
%autoreload 2
# %autoreload 1
# %aimport from kret_studies import *
# %aimport from kret_studies.notebook_imports import *
# %load_ext fireducks.pandas # linux only for now
```

[2]:
```
from kret_studies import *
from kret_studies.notebook import *
from kret_studies.complex import *

logger = get_notebook_logger()
```

```
Loaded environment variables from
/Users/Akseldkw/Desktop/Columbia/ORCS4529/.env.
/Users/Akseldkw/coding/kretsinger/data/nb_log.log
```

[12]:
```
gamma = 0.5
actions = (0.0, 0.5, 1.0)
states = (1, 2)


def reward(s: t.Literal[1, 2], action: float) -> float:
    if s == 1:
        return -action

    return (action / 12) - 1


def transition(s: t.Literal[1, 2], action: float) -> t.Tuple[float, float]:
    """
    Return the probability of staying in the same state or flipping to the next␣
 ↪state
    """
    if s == 1:
        p_1 = action**2 / 2
        return p_1, 1 - p_1

```

```
    p_2 = action**2 / 4
    return 1 - p_2, p_2
```

0.1 Quick Heuristic Visualization

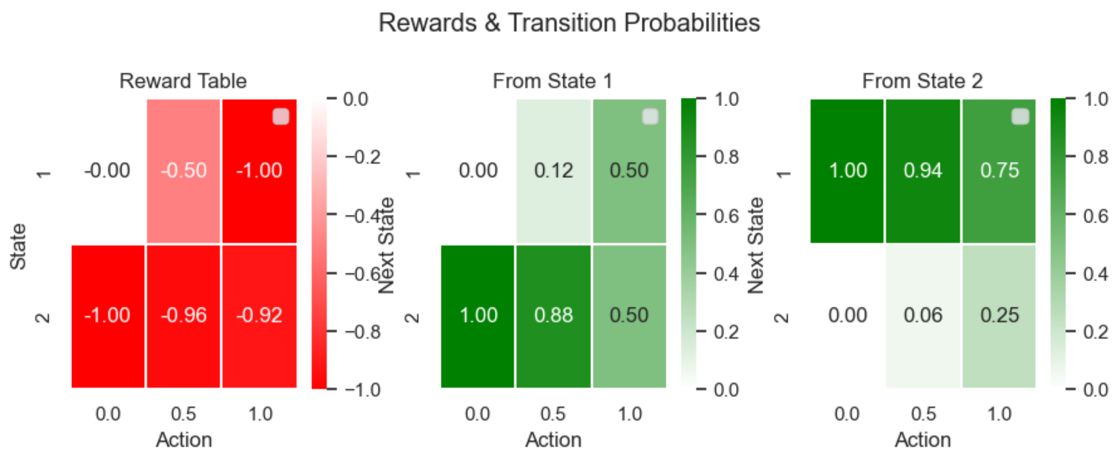
[19]:
```
rewards = pd.DataFrame(
    index=states,
    columns=actions,
    data=[[reward(s, a) for a in actions] for s in states],
).round(3)
transitions = {
    s: pd.DataFrame(
        index=states,
        columns=actions,
        data=[[transition(s, a)[i] for a in actions] for i in␣
 ↪range(len(states))],
    )
    for s in states
}
rewards.index.name = "State"
rewards.columns.name = "Action"
# rewards.title = "Reward Table"

for s in transitions:
    transitions[s].index.name = "Next State"
    transitions[s].columns.name = "Action"
    # transitions[s].title = f"Transition Probabilities (from state {s})"
```

[20]:
```
fig, ax = uks_mpl.subplots(3, 1, 3, 3)
fig.subplots_adjust(wspace=0.3)
fig.suptitle("Rewards & Transition Probabilities", y=1.15)

uks_mpl.heatmap_df(rewards, ax=ax[0])
ax[0].set_title("Reward Table")
uks_mpl.heatmap_df(transitions[1], ax=ax[1])
ax[1].set_title("From State 1")
uks_mpl.heatmap_df(transitions[2], ax=ax[2])
ax[2].set_title("From State 2")
fig
```

[20]:



0.2 Define Greedy Update Function

[21]:
```python
def bellman_update(v: np.ndarray, s: t.Literal[1, 2]) -> float:
    """
    Perform a Bellman update for state s given value function v.
    """
    action_values = [
        reward(s, a) + gamma * sum(transition(s, a)[i] * v[i] for i in range(2))
        for a in actions
    ]
    best_action = actions[np.argmax(action_values)]
    assert best_action in actions
    return max(action_values)


def policy_update(v: np.ndarray, s: t.Literal[1, 2]) -> float:
    """
    Perform a policy update for state s given value function v.
    """
    action_values = [
        reward(s, a) + gamma * sum(transition(s, a)[i] * v[i] for i in range(2))
        for a in actions
    ]
    best_action = actions[np.argmax(action_values)]
    assert best_action in actions
    # pi[s - 1] = best_action
    return best_action
```

0.3 Iterative Policy Evaluation

[22]:
```python
def value_iteration(
    v_init: np.ndarray | None, max_iterations: int | None = None, print_first_k:
 ↪ int = 4
) -> np.ndarray:
    """
    Perform value iteration until convergence.
    """
    v = np.zeros(2) if v_init is None else v_init.copy()
    k = 0
    max_iter = 1000 if max_iterations is None else max_iterations
    while True:
        new_v = np.array([bellman_update(v, s) for s in states])
        if k < print_first_k:
            print(f"Iter {k+1}: V = {new_v}")
        if np.max(np.abs(new_v - v)) < 1e-6:
            break
        v = new_v
        k += 1
        if max_iter != -1 and k >= max_iter:
            print(f"Stopped after reaching max_iter={max_iter}")
            break
    print(f"Converged after {k+1} iterations.")
    return v


def policy_iteration(
    v_init: np.ndarray | None, max_iterations: int | None = None, print_first_k:
 ↪ int = 4
) -> t.Tuple[np.ndarray, np.ndarray]:
    """
    Perform policy iteration until convergence.
    """
    v = np.zeros(2) if v_init is None else v_init.copy()
    pi = np.array([0.0, 0.0])
    k = 0
    max_iter = 1000 if max_iterations is None else max_iterations
    while True:
        new_v = np.array([bellman_update(v, s) for s in states])
        new_pi = np.array([policy_update(new_v, s) for s in states])
        if k < print_first_k:
            print(f"Iter {k+1}: V = {new_v}, pi = {new_pi}")
        if np.max(np.abs(new_v - v)) < 1e-6 and np.all(new_pi == pi):
            break
        v = new_v
        pi = new_pi
```

```
        k += 1
        if max_iter != -1 and k >= max_iter:
            print(f"Stopped after reaching max_iter={max_iter}")
            break
    print(f"Converged after {k+1} iterations.")
    return v, pi
```

0.4 Results

[23]:
```
values = value_iteration(None)
print("Optimal values:", values)
```

```
Iter 1: V = [ 0.         -0.91666667]
Iter 2: V = [-0.45833333 -0.98697917]
Iter 3: V = [-0.49348958 -1.20402018]
Iter 4: V = [-0.60201009 -1.22728221]
Converged after 21 iterations.
Optimal values: [-0.65248136 -1.30496348]
```

[24]:
```
policies = policy_iteration(None)
print("Optimal policies:", policies)
```

```
Iter 1: V = [ 0.         -0.91666667], pi = [0.  0.5]
Iter 2: V = [-0.45833333 -0.98697917], pi = [0.  0.5]
Iter 3: V = [-0.49348958 -1.20402018], pi = [0.  0.5]
Iter 4: V = [-0.60201009 -1.22728221], pi = [0.  0.5]
Converged after 21 iterations.
Optimal policies: (array([-0.65248136, -1.30496348]), array([0. , 0.5]))
```

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