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On the long-term response of marine structures

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ABSTRACT

This paper addresses some important issues related to the estimation of long-term extreme responses of marine structures. Several convolution models to establish the long-term distribution of a marine structure response parameter are available in the literature. These methods are typically based either on all short-term peaks, all extreme short-term peaks or all short-term upcrossing rates. The main assumptions and simplifications of the five models most usually found in the literature are discussed in this paper. A linear single-degree-of-freedom (SDOF) system along with a bi-lognormal probability model for significant wave heights and zero-crossing wave periods have been used for numerical tests. An improved approach to efficiently evaluate the long-term convolution integrals is also proposed in this paper. It is shown that a combination of the Inverse First Order Reliability Method (IFORM) and an Importance Sampling Monte Carlo Simulation (ISMCS) approach can be used to obtain a very good result for the exact solution of long-term integrals.

1. Introduction

Long-term response analysis is recognized as the most appropriate approach to obtain the N-year design response (or load-effect) of a dynamic marine structure subjected to the random environmental parameters of waves, wind and current, as stated by Naess and Moan [1]. There has been a great deal of effort towards using this approach in the analysis of marine structures [2,3]. Reliability-based design approaches for dynamically excited marine structures also make use of long-term response analyses [4,5]. Nowadays some design standards explicitly recognize this methodology as a possible alternative to evaluate the feasibility of some offshore marine systems, e.g., the DnV standard for the design of marine metallic risers [6].

For the long-term response analysis the long-term behavior of the environmental parameters is usually modeled as a series of short-term stationary conditions. The random variability of the short-term environmental parameters of waves, wind and current is represented by a joint probability distribution. In practical terms, a short-term period is usually considered as long as 3–6 h. An estimate of the N-year response is obtained by the convolution of short-term response over all short-term environmental conditions [1].

Considering that the joint probability model of the environmental parameters is available [7,8], nowadays various convolution models can be used to obtain the N-year response. They basically vary in the way they consider the short-term response. Some models are based on the probability distribution of the short-term peaks response [9–11] others are based on the probability distribution of the short-term extreme peak response [1,12] and another on the upcrossing rate of the short-term response process [13]. All these models are found separately in the literature but a comparison of their predictions is not available.

Another important issue regarding the practical use of long-term response is associated with the numerical evaluation of the convolution integrals. Standard numerical integration techniques can not be employed directly because of the huge number of short-term dynamic structural analyses that are required. However, some approximate techniques have been developed. It is recognized that the major contribution to the extreme response comes from a narrow region of the environmental parameters joint distribution [4,14] and these approximate methods try to find this region in an efficient way. The most common technique is based on the N-year environmental contour, discussed in [15], where the long-term statistics are approximately established on the basis of the critical short-term condition on the environmental contour associated with the N-yr return period. Baarholm and Moan [3] present another alternative for nonlinear responses, they locate the critical region using simplified linear models and then they perform full nonlinear analysis only for the points belonging to this region.

The first objective of this paper is to present a comparison between five convolution models commonly available in the literature for the long-term response analysis. The second one

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is to present a method to perform the numerical evaluation of the convolution integral, combining the Inverse First Order Reliability Method (IFORM) and Importance Sampling Monte Carlo Simulation (ISMCS). It is shown that accurate results can be obtained with just a small number of short-term structural analyses. Numerical results are presented for a single degree of freedom (SDOF) model using a joint probability distribution involving only the wave environmental parameters, i.e., significant wave height H_s and zero upcrossing wave period T_z. In the present paper only the all sea-states methodology [16] for long-term analysis is considered, i.e., the random storm approach is out of the scope of the paper.

2. Long-term response models

In the analysis of marine structures the weather conditions can be described by a set of parameters related to the main environmental loadings, such as, significant wave height, zero crossing wave period and wind velocity, etc. During a long time period, e.g. the design life of the structure, these parameters present significant variations. However, it is possible to define shorter time periods, of approximately 3–6 h, where the environmental parameters can be considered roughly constant and their related load effects can be represented by stationary random processes. A joint PDF (probability density function) describes the random variation of the environmental parameters during the so called long-term period. Upon assuming that the long-term variation of the environmental parameters and the short-term load process are also ergodic, it is possible to evaluate the long-term response based on some short-term response statistics, such as distribution of peaks or crossing rates. The short-term structural response, conditioned on the short-term environmental parameters, is evaluated by means of frequency or time domain dynamic analysis depending on the case. The methods discussed herein to establish the long-term response are based, in essence, on one of the following three short-term response parameters:

- short-term peaks distribution;
- short-term extreme peak distribution;
- short-term upcrossing rate.

2.1. Models based on all short-term peaks

Assuming that the short-term environmental parameters are statistically independent and also that the response peaks are statistically independent within a given short-term environmental condition, the probability of occurrence of a response peak R below a given level r within a long-term period can be defined by

$$P(R \leq r) = F_R(r) = P(R \leq r|\mathbf{S} = \mathbf{s}_1)P(\mathbf{s}_1) + P(R \leq r|\mathbf{S} = \mathbf{s}_2) \times P(\mathbf{s}_2) + \cdots + P(R \leq r|\mathbf{S} = \mathbf{s}_{N_s})P(\mathbf{s}_{N_s}) \quad (1)$$

where $P(R \leq r|\mathbf{S} = \mathbf{s}_i) = F_{R|\mathbf{S}}(r|\mathbf{s}_i)$ is the cumulative conditional distribution of the response global peaks given the short-term condition $\mathbf{S} = \mathbf{s}_i$, $P(\mathbf{s}_i)$ is the probability of occurrence associated with the ith short-term condition, $P(R \leq r) = F_R(r)$ is the long-term response peaks distribution and N_S stands for the expected number of short-term conditions in a long-term period of, say, N years. A global peak corresponds to the largest peak between two successive zero upcrossings. As the short-term conditions are mutually exclusive, a first solution for Eq. (1) can be written as

$$F_R(r) = \sum_{i=1}^{N_s} F_{R|\mathbf{S}}(r \mid \mathbf{s}_i)P(\mathbf{s}_i). \quad (2)$$

The discrete representation in Eq. (2) can straightforwardly be transformed into an integral form, given by

$$F_R(r) = \int_{\mathbf{S}} F_{R|\mathbf{S}}(r \mid \mathbf{s})f_{\mathbf{S}}(\mathbf{s})\mathrm{d}\mathbf{s} \quad (3)$$

where $f_{\mathbf{S}}(\mathbf{s})$ is the joint probability density function of the environmental parameters. For instance, considering only the main environmental parameters associated with waves $\mathbf{S} = (H_s, T_z)$, where H_s is significant wave height and T_z is the mean zero upcrossing wave period.

Eq. (3) defines the probability that the response process is below a given level r for an arbitrary occurrence of a response peak. Practical structural design verifications are often expressed in terms of some probability level of the extreme peak distribution associated with a given N-yr return period. Using order statistics [17] the N-yr extreme distribution of the response peaks can be defined as

$$F_{R_N}(r) = [F_R(r)]^{\overline{\nu}_o T_{ST} N_S} \quad (4)$$

where T_{ST} is the short-term period, usually 10,800 s (3 h), $\overline{\nu}_o$ is the long-term average zero upcrossing rate, given by

$$\overline{\nu}_o = \int_{\mathbf{S}} \nu_o(\mathbf{s})f_{\mathbf{S}}(\mathbf{s})\mathrm{d}\mathbf{s} \quad (5)$$

and $\nu_o(\mathbf{s})$ is the average of zero upcrossing rate for an arbitrary short-term environmental condition $\mathbf{S} = \mathbf{s}$.

The model presented in Eq. (3) was proposed by Nordenström [9] and can be found in other references, see, e.g., [18,19]. Hereafter it will be identified as AP1. However, this model does not take into account that the number of response peaks can be different for each individual short-term condition. Considering the relative frequency of occurrence of an individual response peak, the probability of occurrence of a peak below or equal to the level r can be defined as

$$P(R \leq r) = F_R(r) = \frac{\sum_{i=1}^{N_s} n_{r,i}}{N_p} \quad (6)$$

where $n_{r,i}$ is the number of peaks below or equal to r in the ith short-term condition and N_p is the total number of peaks considering all N_S short-term conditions. The number of peaks $n_{r,i}$ can also be expressed as the product $N_{p,i}F_{R|\mathbf{S}}(r \mid \mathbf{s}_i)$, where $N_{p,i}$ is the total number of peaks in the ith short-term condition. Considering only the global peaks, the frequency of peaks is equal to the zero upcrossing rate and then, after some manipulation, Eq. (6) can be re-written as

$$F_R(r) = \frac{\sum_{i=1}^{N_s} \nu_o(\mathbf{s}_i)F_{R|\mathbf{S}}(r \mid \mathbf{S} = \mathbf{s}_i)}{\overline{\nu}_o N_s}. \quad (7)$$

This discrete expression represents the expectation of $\nu_o(\mathbf{s})F_{R|\mathbf{S}}(r \mid \mathbf{S} = \mathbf{s})$ divided by $\overline{\nu}_o$. Then, the continuous expression for the long-term peaks distribution can be expressed as

$$F_R(r) = \int_{\mathbf{S}} \frac{\nu_o(\mathbf{s})}{\overline{\nu}_o} F_{R|\mathbf{S}}(r \mid \mathbf{s})f_{\mathbf{S}}(\mathbf{s})\mathrm{d}\mathbf{s}. \quad (8)$$

This result has been presented by Battjes [10]. The N-year extreme peak distribution $F_{R_N}(r)$ is also expressed by Eq. (4).

Eq. (8) is the most common approach used to define long-term response statistics and has been widely recommended for offshore structures [1,6]. Hereafter it will be identified as AP2.

It is worth mentioning that, although it does not take into account the number of peaks in each short-term condition, the approach AP1 is suitable for solution by means of structural reliability analysis methods, such as First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) [20,21].

2.2. Models based on all short-term extreme peaks

The two previous approaches consider the probability distribution of all global peaks during each short-term condition. Another possibility is to use the extreme peak within each short-term condition in order to obtain the long-term extreme distribution of a given response parameter R. The extreme long-term response corresponds to the largest peak among all individual short-term extreme peaks in a sequence of N_s short-term conditions expected to occur in a long-term period of N years. Then, the probability that the extreme long-term response R_N is below a certain level r can be defined as

$$P(R_N \le r) = P((R_e^1 \le r) \cap (R_e^2 \le r) \cap (R_e^3 \le r) \cap \cdots \cap (R_e^{N_s} \le r)) \quad (9)$$

where $P(R_e^i \le r) = F_{R_e|\mathbf{s}_i}(r|\mathbf{s}_i)$ corresponds to the largest peak distribution associated with the ith short-term condition. Assuming independence between peaks, this short-term distribution is given by

$$F_{R_e|\mathbf{S}}(r|\mathbf{s}_i) = [F_{R|\mathbf{S}}(r|\mathbf{s}_i)]^{\nu_o(\mathbf{s}_i)T_{ST}}. \quad (10)$$

Assuming also that all short-term extreme peaks are statistically independent, Eq. (9) can be re-written as

$$F_{R_N}(r) = \prod_{i=1}^{N_s} F_{R_e|\mathbf{S}}(r \mid \mathbf{s}_i). \quad (11)$$

It is important to notice that although the extreme peaks are considered as statistically independent they are not identically distributed in all short-term conditions. Then, a general solution for Eq. (11) is obtained by finding an equivalent short-term extreme peak distribution $\tilde{F}_{R_e}(r)$ that satisfies the following condition

$$F_{R_N}(r) = \prod_{i=1}^{N_s} \tilde{F}_{R_e}(r) = [\tilde{F}_{R_e}(r)]^{N_s} = \prod_{i=1}^{N_s} F_{R_e|\mathbf{S}}(r \mid \mathbf{s}_i). \quad (12)$$

Taking the natural logarithm on the last two terms of Eq. (12) one gets

$$\ln[\tilde{F}_{R_e}(r)] = \frac{1}{N_s} \sum_{i=1}^{Ns} \ln[F_{R_e|\mathbf{S}}(r \mid \mathbf{s}_i)] \quad (13)$$

where the right side corresponds to the expectation of $\ln[F_{R_e|\mathbf{S}}(r \mid \mathbf{s})]$. Then, the equivalent short-term extreme peak distribution $\tilde{F}_{R_e}(r)$ can be written as

$$\tilde{F}_{R_e}(r) = \exp\left(\int_{\mathbf{S}} \ln(F_{R_e|\mathbf{S}}(r \mid \mathbf{s})) f_{\mathbf{S}}(\mathbf{s}) d\mathbf{s}\right). \quad (14)$$

The long-term extreme peak distribution can finally be defined as

$$F_{R_N}(r) = \left(\exp\left(\int_{\mathbf{S}} \ln(F_{R_e|\mathbf{S}}(r \mid \mathbf{s})) f_{\mathbf{S}}(\mathbf{s}) \mathrm{d}\mathbf{s}\right)\right)^{N_s} = \left(\exp\left(T_{ST} \int_{\mathbf{S}} \nu_o(\mathbf{s}) \ln(F_{R|\mathbf{S}}(r \mid \mathbf{s})) f_{\mathbf{S}}(\mathbf{s}) \mathrm{d}\mathbf{s}\right)\right)^{N_s}. \quad (15)$$

This expression has been presented by Borgman [22] and was also used by Krogstad [23] and Ochi [24] in the context of extreme wave height analysis. Hereafter it will be identified as EP1.

An approximate solution, which is very common in the field of offshore engineering (see, e.g., [12]) and wind turbine analysis (see, e.g., [25]) is to represent $\tilde{F}_{R_e}(r)$ by the expected long-term extreme peak distribution, i.e., $\tilde{F}_{R_e}(r) = \bar{F}_{R_e}(r)$, where

$$\bar{F}_{R_e}(r) = \int_{\mathbf{S}} F_{R_e|\mathbf{S}}(r \mid \mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) \mathrm{d}\mathbf{s} = \int_{\mathbf{S}} [F_{R|\mathbf{S}}(r \mid \mathbf{s})]^{\nu_o(\mathbf{s})T_{ST}} f_{\mathbf{S}}(\mathbf{s}) \mathrm{d}\mathbf{s}. \quad (16)$$

This methodology will be denominated hereafter as EP2. It should be mentioned that Eq. (16) can also be solved by the fast integration methods FORM and SORM found in connection with structural reliability analysis [20].

2.3. Model based on the upcrossing rate of short-term response processes

Naess [13] showed, under the ergodicity assumption for the environmental parameters, which is the essential ingredient in all long-term statistics, that the long-term extreme response distribution for an N-yr return period can be expressed as

$$F_{R_N}(r) = \exp\left(-T_{LT} \int_{\mathbf{S}} \nu_R(r|\mathbf{s}) f_{\mathbf{S}}(\mathbf{s}) \mathrm{d}\mathbf{s}\right) \quad (17)$$

where $\nu_R(r|\mathbf{s})$ corresponds to the average rate of r-upcrossings during the short-term condition $\mathbf{S} = \mathbf{s}$ and $T_{LT} = N_s \cdot T_{ST}$ is the long-term period. Eq. (17) is obtained assuming also that the high level upcrossings are Poisson-distributed. This convolution model will be hereafter called the UR method. From a theoretical point of view, the model summarized by Eq. (17) involves the less restrictive assumptions [13].

2.4. Comparison of the long-term models

2.4.1. Theoretical analysis

The five long-term models mentioned above, which have been denominated as AP1, AP2, EP1, EP2 and UR, are individually found in the literature and also in different practical applications. However, a comparison among their results does not seem to be available. As mentioned before, it is well known that models AP1 and EP2 are approximations for the long-term response statistics. The next section investigates how good these approximations are by means of a numerical application. A theoretical comparison of the other three models is presented in what follows.

Under the hypothesis of independence of high level peaks of a short-term response process, the short-term peaks distribution can be approximately written as [1,24]

$$F_{R|\mathbf{S}}(r|\mathbf{s}) = 1 - \frac{\nu_R(r|\mathbf{s})}{\nu_o(\mathbf{s})}. \quad (18)$$

Then, the short-term average rate of r-upcrossings for $\mathbf{S} = \mathbf{s}$ can be written as

$$\nu_R(r|\mathbf{s}) = \nu_o(\mathbf{s})(1 - F_{R|\mathbf{S}}(r|\mathbf{s})). \quad (19)$$

By inserting Eq. (19) into Eq. (17) the same long-term model given by Eq. (8) is easily obtained. In other words, the long-term models AP2 and UR are equivalent under this approximation.

Taking the approximation $\ln(x) \cong -(1-x)$, which is valid for $0 < x < 1$, and introducing it into Eq. (15) and also using the following asymptotic relationship

$$\exp(-N.x) \approx (1-x)^N \quad (20)$$

it is not difficult to demonstrate that Eq. (15) reduces to Eq. (8), i.e., the models EP1 and AP2 are also mathematically equivalent.

In summary, by using some mathematical manipulations it has been shown in this section that in fact the models EP1, AP2 and UR are different forms of the same long-term model.

2.4.2. Numerical analysis

Focusing mainly on analyzing the accuracy of the long-term models AP1 and EP2, the long-term response of an idealized linear single-degree-of-freedom (SDOF) model under wave loading has been numerically investigated. The joint probability distribution of the environmental parameters is described in terms of the significant wave height H_s and the zero upcrossing wave period T_z. The joint probability density distribution is defined by two lognormal functions as

$$f_{Hs,Tz}(hs, tz) = f_{Hs}(hs) f_{Tz|Hs}(tz \mid hs). \quad (21)$$

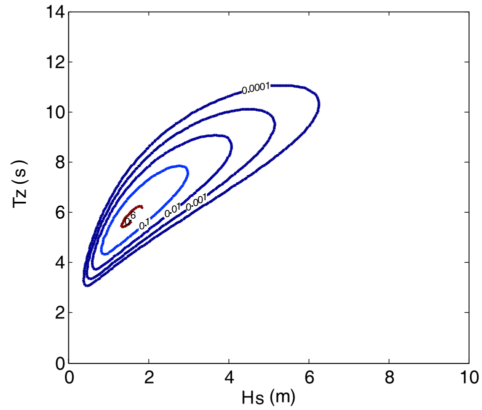

Fig. 1. Joint PDF of significant wave height and zero crossing period.

The marginal probability density function of H_s and the conditional probability density of T_z are given, respectively, by

$$f_{Hs}(hs) = \frac{1}{\xi_{Hs} hs\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(hs)-\lambda_{Hs}}{\xi_{Hs}}\right)^2\right)$$
$$f_{Tz|Hs}(tz \mid hs) = \frac{1}{\xi_{Tz} tz\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(tz)-\lambda_{Tz}(hs)}{\xi_{Tz}}\right)^2\right) \tag{22}$$

where

$$\lambda_{Tz}(hs) = \lambda_{Tz} + \rho\frac{\xi_{Tz}}{\xi_{Hs}}(\ln(hs) - \lambda_{Hs})$$
$$\xi_{Tz} = \xi_T\sqrt{1-\rho^2}. \tag{23}$$

The deterministic parameters are assumed as $\lambda_{Hs} = 0.603204$, $\xi_{Hs} = 0.329771$, $\rho = 0.90$, $\lambda_{Tz} = 1.829504$ and $\xi_{Tz} = 0.152627$. This joint distribution is presented in Fig. 1.

The response amplitude operator (RAO) of the idealized SDOF model is assumed to be simply

$$RAO(\omega) = \frac{1}{\left(\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2\right)^{1/2}} \tag{24}$$

where ω_n and ξ are the natural frequency and the damping ratio of the model, respectively. Under the linear assumption, the response spectrum for a given stationary short-term condition $\mathbf{S} = \mathbf{s} = (h_s, t_z)$ is given by [1]

$$S_{R|\mathbf{S}}(\omega|\mathbf{s}) = [RAO(\omega)]^2 S_{\eta|\mathbf{S}}(\omega|\mathbf{s}) \tag{25}$$

where $S_{\eta|\mathbf{S}}(\omega|\mathbf{s})$ is the spectral density of the sea surface elevation $\eta(t)$, in the short-term condition $\mathbf{S} = \mathbf{s}$, which in this paper has been assumed to be represented by the modified Pierson–Moskovitz spectrum [18]. Assuming that the sea surface elevation is a Gaussian stochastic process, the SDOF response is also Gaussian. Under narrow band assumption, the short-term response peaks are Rayleigh-distributed, i.e.,

$$F_{R|\mathbf{S}}(r|\mathbf{s}) = 1 - \exp\left(-\frac{r^2}{2m_0(\mathbf{s})}\right) \tag{26}$$

and their mean r-upcrossing rate is given by

$$\nu(r|\mathbf{s}) = \frac{1}{2\pi}\sqrt{\frac{m_2(\mathbf{s})}{m_0(\mathbf{s})}} \exp\left(-\frac{r^2}{2m_0(\mathbf{s})}\right) \tag{27}$$

where m_i is the ith moment of the response spectrum, i.e.,

$$m_i(\mathbf{s}) = \int_0^\infty \omega^i S_{R|\mathbf{S}}(\omega|\mathbf{s})d\omega. \tag{28}$$

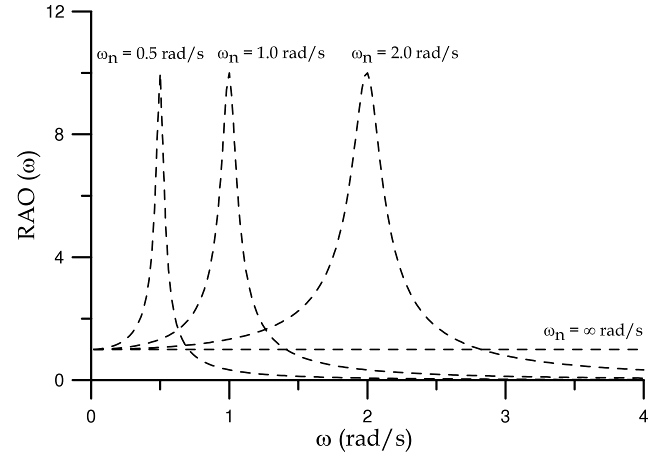

Fig. 2. Response amplitude operators.

Table 1
Most probable value of the 100-yr extreme SDOF response $r(t)$.

ω_n (rad/s)	Long-term convolution model				
	AP1	AP2	EP1	EP2	UR
0.5	35.30	34.84	34.84	34.39	34.83
1.0	17.70	17.63	17.62	17.58	17.62
2.0	10.16	9.79	9.78	9.59	9.78
∞	8.81	8.42	8.41	8.15	8.42

Table 2
Most probable value of the 100-yr extreme non-Gaussian response $s(t) = r(t)|r(t)|$.

ω_n (rad/s)	Long-term convolution model				
	AP1	AP2	EP1	EP2	UR
0.5	1238.9	1209.3	1209.3	1177.1	1209.3
1.0	311.9	311.5	311.5	311.5	311.5
2.0	101.8	95.8	95.8	91.2	95.8
∞	76.2	70.2	70.2	65.	70.2

A full numerical integration was performed to evaluate the most probable value of 100-yr long-term response according to the five long-term models presented above. Four different values for the SDOF natural frequency, i.e., $\omega_n = 0.5, 1.0, 2.0$ and ∞ (rad/s), and a damping ratio $\xi = 0.05$ were considered in the numerical analyses. The corresponding RAOs are shown in Fig. 2. In the full numerical integration a 40×40 mesh, i.e., 1600 points (h_s, t_z), was employed. The obtained results are shown in Table 1. Just for numerical checking, an additional non-Gaussian case was also considered. This case corresponds to an idealized nonlinear response parameter given by $s(t) = r(t)|r(t)|$. The corresponding peaks distribution and mean upcrossing rate of this response process are obtained by random variable transformations. Table 2 presents the results for the non-Gaussian example. As demonstrated above, the long-term models AP2, EP1 and UR predict exactly the same results. The model based on all short-peaks without considering the number of peaks within each short-term condition (AP1) overpredicts the extreme values. However, its bias is structure dependent. The results of the cases considered in the paper were overpredicted by 0.25%–8.00%. The model based on the average short-term extreme peak (see Eq. (16)) underpredicts the long-term extreme values and its bias also depends on the structure dynamic properties. The underprediction for the cases analyzed was in the range of 0.0%–8.0%.

In general terms, it can be said that a very similar behavior is observed when another $H_S - T_Z$ joint probability model is employed, for instance, using a Weibull distribution to represent H_s and a conditioned lognormal distribution for T_z. The differences

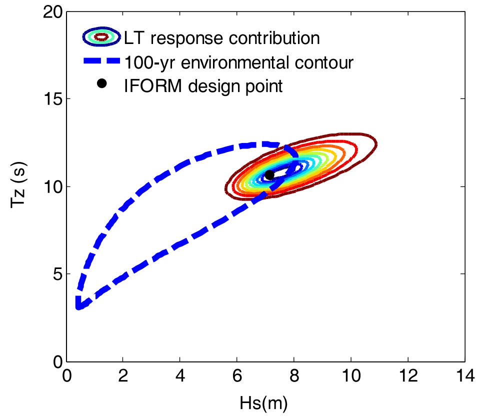

Fig. 3. Relevant short-term conditions for SDOF rsponse ($\omega_n = 0.50$ rad/s).

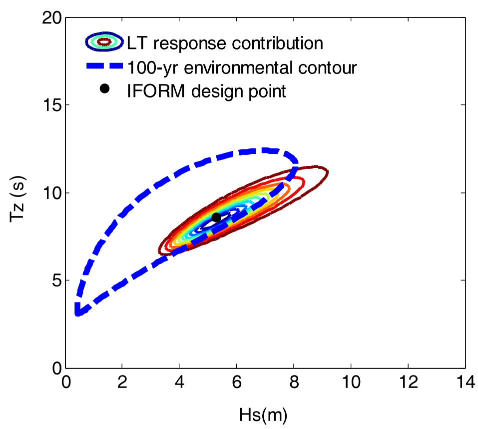

Fig. 4. Relevant short-term conditions for SDOF rsponse ($\omega_n = 1.00$ rad/s).

observed in the results are mainly related to the hypotheses behind the long-term convolution models and not to the joint probability model adopted for the environmental parameters.

3. Long-term integration methods

The numerical evaluation of any convolution integral presented above is very time-consuming for real complex structures. It is even worse when the number of environmental parameters in the vector **S** increases, e.g., when also environmental parameters associated with wind and current are considered. In order to perform the integration more efficiently, and consequently turn the utilization of long-term response approach feasible, alternative strategies must be developed. It has been shown that the contribution to the long-term extreme distribution comes from a limited region [4,14] of the joint probability distribution $f_{\mathbf{S}}(\mathbf{s})$, i.e., only some short-term conditions are relevant for the long-term response. Based on the numerical analyses presented in the previous section, Figs. 3–6 illustrate the relative contribution of the individual short-term conditions to the extreme most probable value of the SDOF linear response considering the long-term model UR. These contributions were determined according to the methodology presented in [4]. The main problem is that this region is not known in advance because it depends on the structure itself, as can be observed in the figures.

An approximate approach to overcome the tedious numerical integration problem is to use the so-called environmental contour method. Just for illustration, it is also included in Figs. 3–6 the 100-yr environmental contour. This methodology needs some sort of calibration to establish the appropriate fractile level of the short-term extreme response peak distribution to obtain an N-year

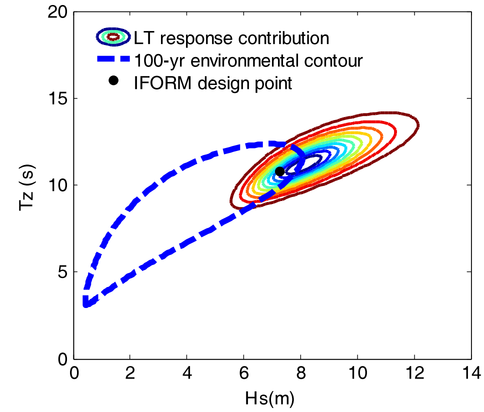

Fig. 5. Relevant short-term conditions for SDOF rsponse ($\omega_n = 2.0$ rad/s).

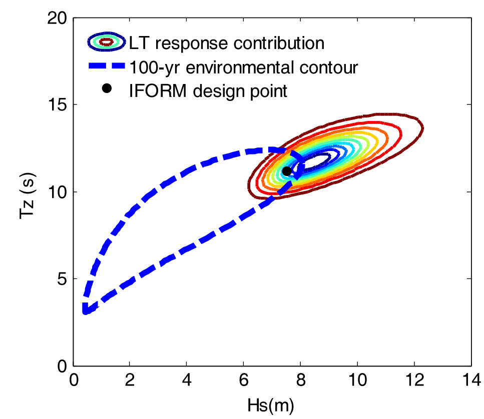

Fig. 6. Relevant short-term conditions for SDOF rsponse ($\omega_n = \infty$ rad/s).

characteristic response. This calibration can be based on the long-term models presented in this work but it was not pursued in this paper. For more details about this approach the reader is referred to Winterstein et al. [15] and Baarlholm et al. [26].

This paper proposes another approach, which is a combination between an inverse reliability method and the Importance Sampling Monte Carlo Simulation method, to efficiently obtain a very good estimate of the exact characteristic value for the long-term response.

3.1. Crude Monte Carlo Simulation

Some results using the Crude Monte Carlo Simulation (CMS) method are presented before describing the proposed approach for solving the convolution integral of the long-term models. Using the CMS method, Eq. (17), for instance, can be represented by

$$F_{R_e}(r) = \exp\left(-\frac{T_{LT}}{N_M}\sum_{i=1}^{N_M}\nu_R(r|\mathbf{s}_i)\right) \tag{29}$$

where $\mathbf{s}_i$ are random samples of the short-term environmental parameters **S** generated artificially from the joint distribution $f_{\mathbf{S}}(\mathbf{s})$ and N_M is the total number of simulated samples. The accuracy of Eq. (29) depends on the number of simulations N_M. Figs. 7 and 8 show the average and the 90% prediction interval for the 100-yr most probable SDOF responses for $\omega_n = 1.0$ rad/s and $\omega_n = \infty$ rad/s, respectively, obtained by means of 200 independent Monte Carlo Simulations for each pre-defined number of simulated samples. It can be seen that the results only converge on average to the exact value for a large number of simulations. For a small number of simulations ($<10,000$) the estimates are biased and the trend is to underpredict the theoretical values. Therefore, these

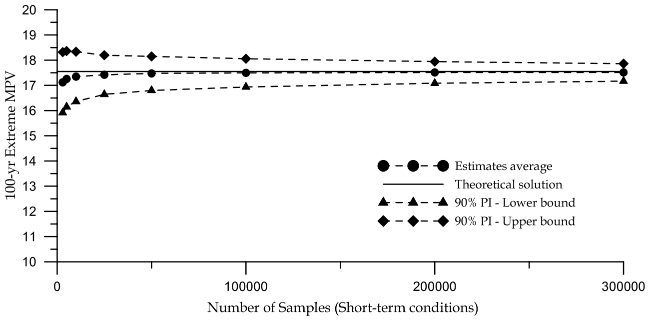

Fig. 7. Crude Monte Carlo for the SDOF long-term response ($\omega_n = 1.0$ rad/s).

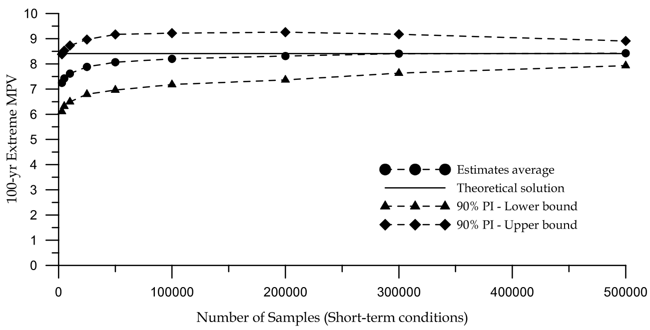

Fig. 8. Crude Monte Carlo for the SDOF long-term response ($\omega_n = \infty$ rad/s).

results indicate that much care must be taken when a small data set of measured simultaneous environmental parameters is directly used to perform any long-term prediction. In other words, as stated by Naess and Moan [1], in such a situation it is better to fit a joint probability model for the environmental parameters instead of using directly the raw data available.

3.2. IFORM and importance sampling Monte Carlo Simulation

An effective way of improving the efficiency of Monte Carlo Crude Simulation is by means of the Importance Sampling technique [27]. Using this methodology the integration of Eq. (17), for instance, is written as

$$F_{R_e}(r) = \exp\left(-T_{LT}\int_{\mathbf{S}} \nu_R(r|\mathbf{s})\frac{f_{\mathbf{S}}(\mathbf{s})}{h_{\mathbf{S}}(\mathbf{s})}h_{\mathbf{S}}(\mathbf{s})\mathrm{d}\mathbf{s}\right) \tag{30}$$

or in its discrete form as

$$F_{R_e}(r) = \exp\left(-\frac{T_{LT}}{N_M}\sum_{i=1}^{N_M} \nu_R(r|\mathbf{s}_i)\frac{f_{\mathbf{S}}(\mathbf{s}_i)}{h_{\mathbf{S}}(\mathbf{s}_i)}\right) \tag{31}$$

where $h_{\mathbf{S}}(\mathbf{s})$ is an appropriate importance sampling density function. There is no simple way to choose this joint density function; however, it is known [27] that the efficiency of the Importance Sampling technique increases significantly if it is centered in the region that contributes most to the integral.

Specifically, in the context of the problems presented in this paper there is a simple approach to obtain the center of the importance sampling density function. This point can be associated with the so-called design point obtained by an inverse reliability method based on FORM (IFORM), as proposed by Li and Foschi [28], when it is applied to solve the approximate long-term model EP2, as described in what follows.

Considering specifically the long-term model EP2, it can be noticed that a characteristic value of the long-term extreme response associated with an N-yr return period can be related to a fractile $(1 - p)$ of the distribution $\overline{F}_{R_e}(r)$, e.g., the most probable N-yr response is given by the value r whose exceedance probability p is equal to $1/Ns$. For instance, considering 3 h short-term conditions Ns is equal to 292,000 for a 100-yr return period.

As shown in [20], $\overline{F}_{R_e}(r)$ in Eq. (16) can be approximately solved in a very fast way through FORM (First Order Reliability Method) by writing a limit state function given as:

$$G(\mathbf{V}) = r - R_e(\mathbf{S}) \tag{32}$$

where r is a given value of the response R_e, $\mathbf{S}$ is the vector containing the n random short-term environmental parameters, $R_e(\mathbf{S}) \equiv R_e$ is the short-term extreme response peak for a given $\mathbf{S}$ and $\mathbf{V}$ is a vector containing all random variables considered, i.e., $\mathbf{V}^T = [S_1, S_2, \ldots, S_n, R_e]$. The FORM procedure transforms the random variables $\mathbf{V}$ to standard independent normal variables $\mathbf{U}$ by means of a suitable transformation $\mathbf{U} = T(\mathbf{V})$, such as the Rosenblatt transformation [20,27] and the so-called probability of failure p_f, i.e., $P(R_e \geq r) = P(G(\mathbf{V}) \leq 0)$, is given as $p_f \cong \Phi(-\beta)$, where $\Phi(.)$ stands for the cumulative distribution of a standard normal variable and β is the so-called reliability index, defined as the distance of the design point $\mathbf{u}^*$ to the origin in the $\mathbf{U}$-space, i.e., $\beta = |\mathbf{u}^*|$. The design point corresponds to the point in the $\mathbf{U}$-space on the failure surface ($g(\mathbf{U}) = G(\mathbf{V}) = 0$) closest to the origin. This point is the one that most contributes to p_f.

However, in its original format the FORM method must be used iteratively (trial and error) many times to find the value of r associated with a given exceedance probability p, i.e. $p_f = p$. This can be avoided by using an inverse reliability method. For a given value p and its corresponding reliability index $\beta = -\Phi^{-1}(p)$, it has been shown by Li and Foschi [28] that the design point can be obtained by the following recursive algorithm

$$\mathbf{u}^{k+1} = -\beta\frac{\nabla(g(\mathbf{u}^k))}{|\nabla(g(\mathbf{u}^k))|} \tag{33}$$

where the $\nabla(g(\mathbf{u}^k))$ denotes the gradient of $g(\mathbf{u})$ evaluated at the point $\mathbf{u}^k$. Departing from a given initial point $\mathbf{u}^o$, Eq. (33) is used recursively until the convergence is achieved, i.e., $|\mathbf{u}^{k+1} - \mathbf{u}^k|/|\mathbf{u}^{k+1}| \leq \varepsilon$, where ε is a user-specified tolerance, say, 10^{-3}. The last point $\mathbf{u}^{k+1}$ corresponds to the design point $\mathbf{u}^{k+1} = \mathbf{u}^* = T(\mathbf{v}^*)$ and, then $r = (\mathbf{v}^*)_{n+1} = (T^{-1}(\mathbf{u}^*))_{n+1}$. It must be emphasized that FORM gives an approximate result for the integral given in Eq. (16) which itself is also an approximation for the correct long-term integral. Consequently, r also corresponds to an approximation for the true long-term characteristic value. On the other hand, the coordinates of the design point associated with the environmental parameters, i.e., $s_i^* = (\mathbf{v}^*)_i$ for $i = 1, 2, \ldots, n$, are a good indication of the central part of the region that most contributes to the long-term response. It is important to notice that the number of short-term dynamic analysis, when the gradient vector is computed by a finite-difference scheme, is equal to $(n + 2)N_{IT}$, where N_{IT} is the number of iterations for convergence, which is usually small (≈5–10). It is also interesting to observe that this inverse reliability method consists of an automatic procedure to find a point close to the most relevant short-term design condition for the long-term response.

Finally, an importance sampling function $h_{\mathbf{S}}(\mathbf{s})$, centered on $\mathbf{S} = \mathbf{s}^*$, must be defined to generate the random samples $\mathbf{s}_i$ to evaluate Eq. (31). In this work, it was defined through independent normal distributions, i.e.,

$$h_{\mathbf{S}}(\mathbf{s}) = \prod_{i=1}^{n}\frac{1}{\sigma_i}\phi\left(\frac{\mathbf{s}_i - \mu_i}{\sigma_i}\right) \tag{34}$$

where $\mu_i = s_i^*$ and $\phi(.)$ stands for the standard normal probability density function. Good results were obtained for values of σ_i in the range of 1.5–2.0 times the standard deviation of the marginal distributions of the environmental parameters.

Table 3
Results for the 100-yr extreme SDOF response.

ω_n (rad/s)	Long-term approach				
	IFORM		IFORM-ISMCS (50 random samples)		Numerical integration (UR)
	100-yr MPV	No. of iterations	100-yr MPV	CoV[a] (%)	100-yr MPV
0.5	34.74	10	34.79	1.30	34.83
1.0	17.51	7	17.62	1.50	17.62
1.5	9.48	5	9.77	2.50	9.78
∞	8.09	5	8.36	2.65	8.42

[a] Obs.: (a) obtained from 100 independent ISMCS using 50 random samples (short-term conditions) in each one.

Table 3 presents the results obtained for the most probable value of the 100-yr SDOF linear extreme response. Firstly, this table shows the predictions using just IFORM to solve Eq. (16), i.e., the EP2 approach. IFORM gives a very good approximation for the results obtained by numerical integration of this equation, which are shown in Table 1. Next, the results obtained by ISMCS using the design points identified by IFORM as the center of the importance sampling density probability are presented. These design points are shown in Figs. 3–6. Only 50 random samples of short-term conditions, i.e., $N_M = 50$, have been used in ISMCS. The results predicted are in very close agreement with the true values predicted numerically by a consistent long-term model, which are repeated in the last column of the table. Also included in Table 3 are the coefficients of variation (CoVs) of the ISMCS predictions. They have been evaluated using the predictions of 100 independent simulations, each one with $N_M = 50$. The CoVs are very small indicating that just 50 random samples (or less) are enough to get a very good result for the long-term numerical integral.

An important practical aspect of the IFORM-ISMCS approach is that just a small number of structural analyses are necessary to perform an almost exact long-term response prediction. In fact, the total number of structural analyses is $(n + 2)N_{IT}$ for the IFORM method plus $\approx$50 for the ISMCS. This amount varied between 75 and 100 for all cases analyzed in this work. Since this is a general methodology, this amount of analysis should also be the same for more complex problems than those investigated in this paper.

4. Conclusions

Initially, five different approaches commonly found in the literature to deal with long-term response analysis of marine structures were investigated in this paper. Using some mathematical manipulations, it was shown that three of them correspond to the same exact model. The other two are approximate models. The accuracy of these two models is structure dependent and the errors obtained in the examples analyzed in this paper were not significant. A feature of both approximate methods is that they are suitable to be solved through the fast integration method FORM used in structural reliability analysis. In fact, an inverse FORM (IFORM) must be used to speed up the calculations. Although an approximation, the inverse reliability method IFORM has an important feature. This approach automatically gives a point very close to the center of the region that contributes the most to long-term extreme response.

Then, in the second part of the paper, it was shown that the design point obtained with the IFORM approach can be considered as the center point of an importance sampling density function to be used in connection with Importance Sampling Monte Carlo Simulation to get very accurate results for the exact long-term model. It was shown that this result can also be obtained by a relatively small number of structural analyses.

The IFORM-ISMCS methodology proposed in this paper is general and it can be useful for real complex engineering problems. For a problem where the vector **S** is composed by a larger number (n) of environmental parameters, i.e., a problem in $\Re^n$ space, IFORM can obtain automatically a point close to the most relevant short-term design condition without any previous information about its location. Additionally, the number of structural analyses that are needed to perform both steps, IFORM and ISMCS, seems not to be so high showing the feasibility of this approach even for very time-consuming computer problems. This idea can open some doors to the long-term analysis to be employed directly in everyday design practice.

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