Aksel Østmoe

TDT4171 Assignment 2

Task 1

a) The information given in task 1 formulated as a hidden markow model:
  - The unobservable variable for any value of t is *fish nearby*
  - The observable variable for any value of t is *birds nearby*
  - The transition model for P(Fish nearby$_t$ | Fish nearby$_{t-1}$) can be formulated in the transition matrix:

0.8	0.2
0.3	0.7

    From this matrix we can see the probabilities for fish nearby given both if there were fish nearby yesterday (then the probability would be 0.8 of there being fish nearby, and 0.2 of there not being any), and if there were no fish nearby yesterday (then the probability would be 0.3 of there being fish nearby, and 0.7 of there not being any).
  - The sensor model P(Birds nearby$_t$ | Fish nearby$_t$) can also be formulated in a matrix. This matrix will be different depending on the evidence (birds nearby or not):

    O_1:

0.75	0
0	0.2

    O_2:

0.25	0
0	0.8

  - Complete probability tables for the Hidden markow model:

Fish nearby$_{t-1}$	P(Fish nearby$_t$)
true	0.8
false	0.3

Fish nearby$_t$	P(Birds nearby$_t$)
true	0.75
false	0.2

For the tasks 1 b-e, see the python-file for calculations

b) Given that we are trying to estimate up to a current value of t based on previous states to date, this is an example of filtering. The filtering-operations provides the distribution for the current state, taking all previous evidence into account.

c) Given that we are trying to estimate a future state of t based on previous states, this is an example of prediction. The prediction-operation provides probabilities for a state in the future, based on evidence up to the current state.

d) This is an example of smoothing, because we are trying to calculate the probabilities for a past state, when we have evidence up to a current state. This operation can give calculations for some probability in the past, based both on the states before and after the state being calculated.

e) As task is to find the maximum of the probabilities, this is an example of most likely sequence. This operation provides information about what sequence of states beforehand is most likely provide a certain result.

Python-output for task 1 b-e:

```
Task 1 B:
P(X1 | e1:1) = [0.82089552 0.17910448]
P(X2 | e1:2) = [0.90197069 0.09802931]
P(X3 | e1:3) = [0.48518523 0.51481477]
P(X4 | e1:4) = [0.81645924 0.18354076]
P(X5 | e1:5) = [0.43134895 0.56865105]
P(X6 | e1:6) = [0.79970863 0.20029137]
Task 1 C:
P(X1 | e1:1) = [0.82089552 0.17910448]
P(X2 | e1:2) = [0.90197069 0.09802931]
P(X3 | e1:3) = [0.48518523 0.51481477]
P(X4 | e1:4) = [0.81645924 0.18354076]
P(X5 | e1:5) = [0.43134895 0.56865105]
P(X6 | e1:6) = [0.79970863 0.20029137]
P(X7 | e1:6) = [0.69985432 0.30014568]
P(X8 | e1:6) = [0.64992716 0.35007284]
P(X9 | e1:6) = [0.62496358 0.37503642]
P(X10 | e1:6) = [0.61248179 0.38751821]
P(X11 | e1:6) = [0.60624089 0.39375911]
P(X12 | e1:6) = [0.60312045 0.39687955]
P(X13 | e1:6) = [0.60156022 0.39843978]
P(X14 | e1:6) = [0.60078011 0.39921989]
P(X15 | e1:6) = [0.60039006 0.39960994]
P(X16 | e1:6) = [0.60019503 0.39980497]
P(X17 | e1:6) = [0.60009751 0.39990249]
P(X18 | e1:6) = [0.60004876 0.39995124]
P(X19 | e1:6) = [0.60002438 0.39997562]
P(X20 | e1:6) = [0.60001219 0.39998781]
P(X21 | e1:6) = [0.60000609 0.39999391]
P(X22 | e1:6) = [0.60000305 0.39999695]
P(X23 | e1:6) = [0.60000152 0.39999848]
P(X24 | e1:6) = [0.60000076 0.39999924]
P(X25 | e1:6) = [0.60000038 0.39999962]
P(X26 | e1:6) = [0.60000019 0.39999981]
P(X27 | e1:6) = [0.6000001 0.3999999]
P(X28 | e1:6) = [0.60000005 0.39999995]
P(X29 | e1:6) = [0.60000002 0.39999998]
P(X30 | e1:6) = [0.60000001 0.39999999]
Task 1 D:
```

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```
t = 0
probability: [0.66485218 0.33514782]
t = 1
probability: [0.87640731 0.12359269]
t = 2
probability: [0.86578657 0.13421343]
t = 3
probability: [0.59792735 0.40207265]
t = 4
probability: [0.76663731 0.23336269]
t = 5
probability: [0.57082582 0.42917418]
Task 1 E
at t = 1 it is most likely fish nearby!
at t = 2 it is most likely fish nearby!
at t = 3 it is most likely not fish nearby!
at t = 4 it is most likely fish nearby!
at t = 5 it is most likely not fish nearby!
at t = 6 it is most likely fish nearby!
None

Process finished with exit code 0
```

TDT 4171 Assignment 2

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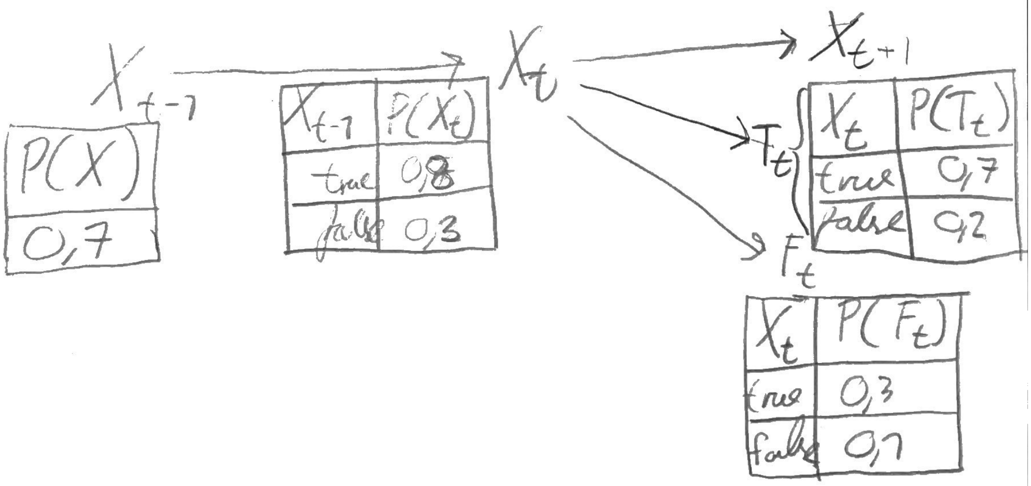
2. a) The problem formulated as a DBN:

T = Animal tracks F = food gone

N = Animals Nearby t = day

X_t = Animals Nearby on day t.



P(X)
0,7

X_{t-1}	$P(X_t)$
true	0,8
false	0,3

X_t	$P(T_t)$
true	0,7
false	0,2

X_t	$P(F_t)$
true	0,3
false	0,1

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b) Using equation 15.4 in the textbook:

$$\boxed{P(X_{t+1} \mid e_{1:t+1}) = \alpha P(e_{t+1} \mid X_{t+1}) \cdot P(X_{t+1} \mid e_{1:t})}$$

$P(X_t \mid e_{1:t})$, $t = 1,2,3,4$

$$= \alpha P(e_t \mid X_t) \cdot P(X_t \mid e_{1:t-1})$$

$$P(e_t \mid X_t) = P(T_t \wedge F_t \mid X_t) = P(T_t \mid X_t) \cdot P(F_t \mid X_t)$$

$$P(X_t \mid e_{1:t-1}) = \sum_{X_{t-1}} P(X_t \mid X_{t-1}) \cdot P(X_{t-1} \mid e_{1:t-1})$$

Solving for all values of t:

Initial value $P(X_0) = (0{,}7 \; , \; 0{,}3)$

$t=1$: $P(X_1 \mid e_1) = \alpha P(e_1 \mid X_1) P(X_1)$

$$P(X_1) = \sum_{X_0} P(X_1 \mid X_0) P(X_0) = [0{,}8, 0{,}2] \cdot 0{,}7 + [0{,}3 \; , \; 0{,}7] \cdot 0{,}3$$

matrix mul.

$$= [0{,}56 \; , \; 0{,}14] + [0{,}09 \; , \; 0{,}21]$$

$$= [0{,}65 \; , \; 0{,}35]$$

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$P(e_1|X_1) = [0{,}7,\ 0{,}2][0{,}3,\ 0{,}1] = [0{,}21,\ 0{,}02]$

$P(X_1|e_1) = \alpha[0{,}21,\ 0{,}02][0{,}65,\ 0{,}35]$

$= \alpha[0{,}1365,\ 0{,}007]$

Normalize: $0{,}1365 + 0{,}007 = 0{,}1435$

$\frac{0{,}1365}{0{,}1435} = 0{,}951 \qquad \frac{0{,}007}{0{,}1435} = 0{,}049$

$\Rightarrow P(X_1|e_1) = [0{,}951,\ 0{,}049]$

$t=2$:

$P(X_2|e_{1:2}) = \alpha P(e_2|X_2) \cdot P(X_2|e_1)$

$P(X_2|e_1) = \begin{bmatrix} 0{,}8 & 0{,}3 \\ 0{,}2 & 0{,}7 \end{bmatrix} [0{,}951,\ 0{,}049]$

$= [0{,}8,\ 0{,}2]\ 0{,}951 + [0{,}3,\ 0{,}7]\ 0{,}049 = [0{,}776,\ 0{,}224]$

Taking e_2 into account: $\alpha[0{,}3,\ 0{,}8][0{,}3,\ 0{,}7][0{,}776,\ 0{,}224]$

$= \alpha[0{,}070,\ 0{,}018]$

Normalize: $\frac{0{,}07}{0{,}088} = 0{,}795 \qquad \frac{0{,}018}{0{,}088} = 0{,}205$

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$\Rightarrow P(X_2 | e_{1:2}) = [0{,}795 \, , \, 0{,}205]$

$t = 3$:

$$P(X_3 | e_{1:3}) = \alpha P(e_3 | X_3) \cdot P(X_3 | e_{1:2})$$

$$P(X_3 | e_{1:2}) = \sum_{x_2} P(X_3 | x_2) P(x_2 | e_{1:2})$$

$$= [0{,}8 \, , \, 0{,}2] \, 0{,}795 + [0{,}3 \;\; 0{,}7] \, 0{,}205$$

$$= [0{,}697 \, , \, 0{,}303]$$

Taking e_3 into account:

$$\alpha [0{,}3 \, , \, 0{,}8][0{,}7 \, , \, 0{,}9][0{,}697 \, , \, 0{,}303]$$

$$= \alpha [0{,}146 \, , \, 0{,}218]$$

Normalize: $\frac{0{,}146}{0{,}364} = 0{,}401$

$\frac{0{,}218}{0{,}364} = 0{,}599$

$\Rightarrow P(X_3 | e_{1:3}) = [0{,}401 \, , \, 0{,}599]$

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$t=4$:

$$P(X_4|e_{1:4}) = \alpha P(e_4|X_4)P(X_4|e_{1:3})$$

$$P(X_4|e_{1:3}) = \sum_{x_3} P(X_4|x_3)P(x_3|e_{1:3})$$

$$= [0{,}8,\ 0{,}2]\cdot 0{,}401 + [0{,}3,\ 0{,}7]\,0{,}599$$

$$= [0{,}501,\ 0{,}499]$$

Taking e_4 into account:

$$\alpha[0{,}7,\ 0{,}2][0{,}7,\ 0{,}9][0{,}501,\ 0{,}499]$$

$$= \alpha[0{,}245,\ 0{,}090]$$

Normalize: $\frac{0{,}245}{0{,}335} = 0{,}731$

$\frac{0{,}090}{0{,}335} = 0{,}269$

$$\Rightarrow P(X_4|e_{1:4}) = [0{,}731,\ 0{,}269]$$

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c)

$P(X_t | e_{1:4})$, $t = 5, 6, 7, 8$

Prediction, as we are calculating t for a future state from the evidence.

$P(X_4 | e_{1:4})$ is known as $[0{,}731\ 0{,}269]$ from task 2B.

For every value of t, we matrix-multiply the previous probability with the T-matrix.

$t = 5$:

$$P(X_5 | e_{1:4}) = \sum_{X_4} P(X_5 | X_4) P(X_4 | e_{1:4})$$

$= [0{,}8, 0{,}2]\, 0{,}731 + [0{,}3, 0{,}7]\, 0{,}269$

$= [0{,}666, 0{,}334]$

$t = 6$: $P(X_6 | e_{1:4}) = \sum_{X_5} P(X_6 | X_5) P(X_5 | e_{1:4})$

$= [0{,}8, 0{,}2]\, 0{,}666 + [0{,}3, 0{,}7]\, 0{,}334$

$= [0{,}633, 0{,}367]$

$t = 7$: $P(X_7 | e_{1:4}) = [0{,}8, 0{,}2]\, 0{,}633$

$+ [0{,}3, 0{,}7]\, 0{,}367 = [0{,}617, 0{,}383]$

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$t=8$: $[0{,}8, 0{,}2]\, 0{,}617 + [0{,}3, 0{,}7]\, 0{,}383$
$= [0{,}608, 0{,}392]$

d) Finding the stationary distribution for the transformation can be proved if the following matrix-multiplication holds:

$$[0{,}6, 0{,}4] \cdot \begin{bmatrix} 0{,}8 & 0{,}3 \\ 0{,}2 & 0{,}7 \end{bmatrix} = [0{,}6, 0{,}4]$$

$$[0{,}8, 0{,}2] \cdot 0{,}6 + [0{,}3, 0{,}7] \cdot 0{,}4$$
$$= \underline{[0{,}6, 0{,}4]} \quad \square$$

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e) $P(X_t | e_{1:4})$, $t = 0, 1, 2, 3$

As we are looking for the probability for a past state, we can use smoothing (forward-backward)

$t = 3$:

$$P(X_3 | e_{1:4}) = \alpha P(X_3 | e_{1:3}) P(e_4 | X_3)$$

$$P(e_4 | x_3) = \sum_{x_4} P(e_4 | x_4) \underbrace{P(e_5 | x_4)}_{1} P(x_4 | x_3)$$

$$P(e_4 | x_3) = 0{,}7 \cdot 0{,}7 \cdot [0{,}8 , 0{,}2] + 0{,}2 \cdot 0{,}9 \cdot [0{,}3 , 0{,}7]$$

$$= [0{,}446 , 0{,}224]$$

$$\Rightarrow \alpha \underbrace{[0{,}401 , 0{,}599]}_{\text{from task 2B}} [0{,}446 , 0{,}224]$$

$$= \alpha [0{,}179 , 0{,}134]$$

Normalize: $\frac{0{,}179}{0{,}313} = 0{,}572$ $\frac{0{,}134}{0{,}313} = 0{,}428$

$$\Rightarrow \underline{[0{,}572 , 0{,}428]}$$

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$t=2$:

$$P(X_2 \mid e_{1:4}) = \alpha P(X_2 \mid e_{1:2}) \cdot P(e_{3:4} \mid X_2)$$

$$P(e_{3:4} \mid X_2) = \sum_{x_3} P(e_3 \mid X_3) P(e_4 \mid X_3) P(X_3 \mid X_2)$$

$$= 0{,}3 \cdot 0{,}7 \cdot 0{,}446 \cdot [0{,}8, 0{,}2] + 0{,}8 \cdot 0{,}9 \cdot 0{,}224 [0{,}3, 0{,}7]$$

$$= [0{,}123, 0{,}131]$$

$$P(X_2 \mid e_{1:4}) = \alpha [0{,}795, 0{,}205][0{,}123, 0{,}131]$$

$$= \alpha [0{,}097, 0{,}027]$$

Normalize: $\frac{0{,}097}{0{,}124} = 0{,}782 \quad \frac{0{,}027}{0{,}124} = 0{,}218$

$$\Rightarrow \underline{[0{,}782, 0{,}218]}$$

$t=1$: $P(X_1 \mid e_{1:4}) = \alpha P(X_1 \mid e_1) P(e_{2:4} \mid X_1)$

$$P(e_{2:4} \mid X_1) = \sum_{x_2} P(e_2 \mid X_2) P(e_3 \mid X_2) P(X_2 \mid X_1)$$

$$= 0{,}3 \cdot 0{,}3 \cdot 0{,}123 [0{,}8, 0{,}2] + 0{,}8 \cdot 0{,}9 \cdot 0{,}131 [0{,}3, 0{,}7]$$

$$= [0{,}037, 0{,}068]$$

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$$P(x_1 | e_{1:4}) = \alpha \underbrace{[0{,}951, 0{,}049]}_{\text{from task 2B}} [0{,}037, 0{,}068]$$

$$= \alpha [0{,}035, 0{,}003]$$

Normalize: $\frac{0{,}035}{0{,}038} = 0{,}921$ $\frac{0{,}003}{0{,}038} = 0{,}079$

$$\Rightarrow \underline{[0{,}921, 0{,}079]}$$

$t = 0$:

$$P(X_0 | e_{1:4}) = \alpha P(X_0 | e_0) P(e_{1:4} | X_1)$$

$$P(e_{1:4} | X_1) = \sum_{x_1} P(e_1 | X_1) P(e_2 | X_1) P(X_1 | X_0)$$

$$= 0{,}7 \cdot 0{,}3 \cdot 0{,}037 \cdot [0{,}8, 0{,}2] + 0{,}2 \cdot 0{,}1 \cdot 0{,}068 [0{,}3, 0{,}7]$$

$$= [0{,}0066, 0{,}0025]$$

$$P(X_0 | e_{1:4}) = \alpha [0{,}7, 0{,}3] [0{,}0066, 0{,}0025]$$

$$= \alpha [0{,}0046, 0{,}00075]$$

Normalize: $\frac{0{,}0046}{0{,}00535} = 0{,}860$ $\frac{0{,}00075}{0{,}00535} = 0{,}140$

$$\Rightarrow \underline{[0{,}86, 0{,}14]}$$