TDT4171 Øving 1 Aksel Østmoe

1. $P(0) = 0{,}15$ $P(2) = 0{,}27$ $P(4) = 0{,}02$
$P(1) = 0{,}49$ $P(3) = 0{,}06$ $P(5) = 0{,}01$

a) $P(X \leq 2) = P(0) + P(1) + P(2) = \underline{\underline{0{,}91}}$

b) $P(X \geq 2 \mid X \geq 1) = \dfrac{P(X > 2 \cap X \geq 1)}{P(X \geq 1)}$

$= \dfrac{P(X > 2)}{P(X \geq 1)} = \dfrac{1 - P(X \leq 2)}{1 - P(0)} = \dfrac{1 - P(0) - P(1) - P(2)}{1 - P(0)}$

$= \underline{\underline{0{,}106}}$

c) $P(X_1 + X_2 + X_3 = 3) = (P(1))^3 + (P(2) \cdot P(1) \cdot P(0)) \cdot 3!$
$+ (P(3) \cdot (P(0))^2) \cdot 3 = 0{,}12 + 0{,}019 \cdot 3! + 0{,}00135 \cdot 3$

$= \underline{\underline{0{,}24}}$

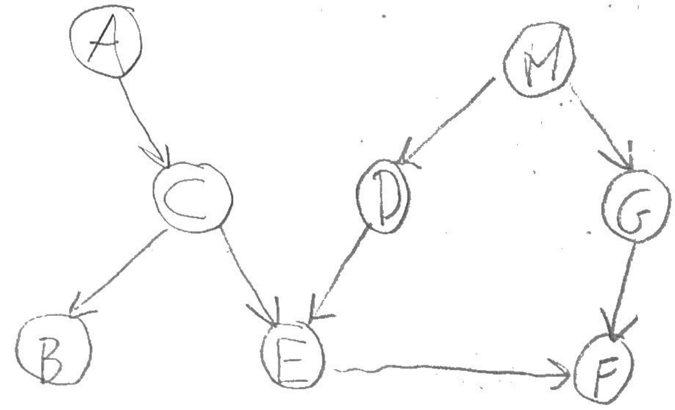
1.

d) $P(X_E = 0 \mid X_E + X_J = 3) = \frac{P(X_E + X_J = 3 \mid X_E = 0) \cdot P(X_E = 0)}{P(X_E + X_J = 3)}$

$= \frac{P(X_J = 3) \cdot P(X_E = 0)}{2(P(0) \cdot P(3)) + 2(P(1) \cdot P(2))} = \frac{0{,}06 \cdot 0{,}15}{2(0{,}06 \cdot 0{,}15) + 2(0{,}49 \cdot 0{,}27)}$

$= \underline{\underline{0{,}0318}}$

2.



a) True.

Compactness for hver variabel kan uttrykkes som 2^k, hvor k er antall foreldre.

Dette fører til følgende antall nummer:

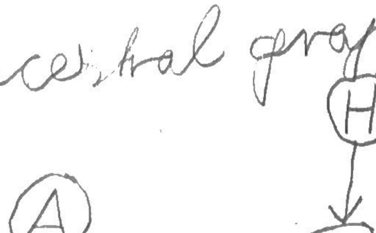
$A = 2^0 = 1$	$H = 2^0 = 1$
$C = 2^1 = 2$	$D = 2^1 = 2$
$B = 2^1 = 2$	$G = 2^1 = 2$
$E = 2^2 = 4$	$F = 2^2 = 4$

$$\Rightarrow 1+2+2+4+1+2+2+4 = \underline{\underline{18}}$$

b) $G \perp\!\!\!\perp A$ True

Bruker D-separation:

Ancestral graph for G og A:

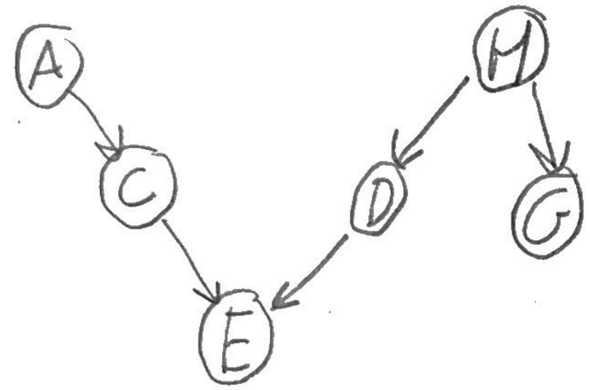


Ser allerede her at G og A ikke er forbindet, og dermed uavhengige.

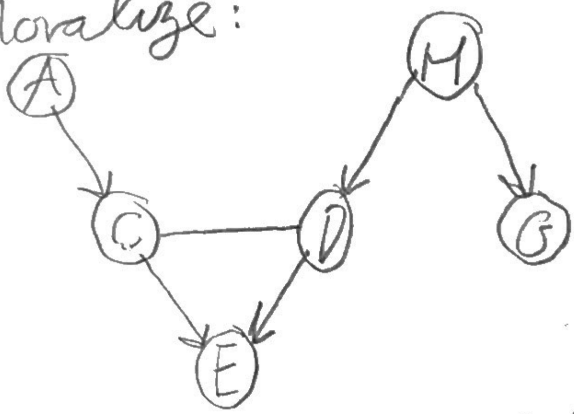
c) $E \perp\!\!\!\perp H \mid \{D, G\}$ __True__

Bruker D-separation

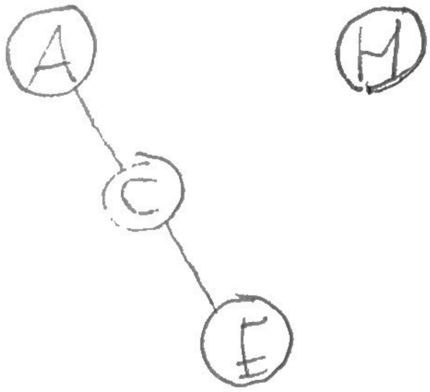
1. Ancestral graf for E, H, D, G:



2. Moralize:



3 og 4. Disorient og delete givens:

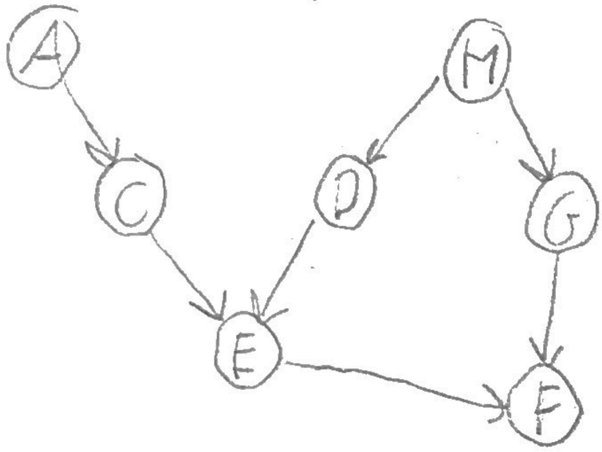


Siden E og H ikke har en sti mellom seg, er de uavhengige.

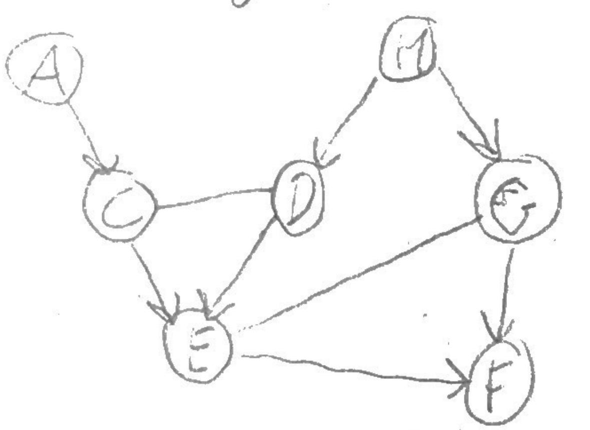
d) $E \perp\!\!\!\perp H \mid \{C, D, F\}$ <u>False</u>

Bruker D-separation

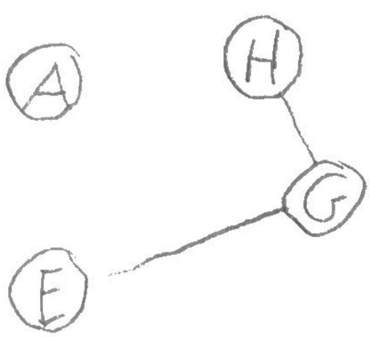
1. Ancestral graf for E, H, C, D, F:



2. Moralize:



3 og 4 Disorient og delete givens:



E og H har en sti mellom seg, og er dermed ikke uavhengige.

3. a)

$$P(b) = P(a) \cdot P(b|a) + P(\neg a) \cdot P(b|\neg a)$$
$$= 0{,}8 \cdot 0{,}5 + 0{,}2 \cdot 0{,}2 = \underline{\underline{0{,}44}}$$

b) $$P(d) = P(b) \cdot P(d|b) + P(\neg b) \cdot P(d|\neg b)$$
$$= 0{,}44 \cdot 0{,}6 + (1 - 0{,}44) \cdot 0{,}8 = \underline{\underline{0{,}71}}$$

c) $$P(c|\neg d) = \underbrace{P(c|b)}_{0{,}1} \cdot P(b|\neg d) + \underbrace{P(c|\neg b)}_{0{,}3} \cdot P(\neg b|\neg d)$$

$$= 0{,}1 \cdot \frac{\overbrace{P(\neg d|b)}^{0{,}4} \cdot \overbrace{P(b)}^{0{,}1}}{P(\neg d)} + 0{,}3 \cdot \frac{\overbrace{P(\neg d|\neg b)}^{0{,}2} \cdot \overbrace{P(\neg b)}^{0{,}3}}{P(\neg d)}$$

$$= \frac{0{,}1 \cdot 0{,}4 \cdot 0{,}44}{0{,}29} + \frac{0{,}3 \cdot 0{,}2 \cdot 0{,}56}{0{,}29} = \underline{\underline{0{,}177}}$$

3.

d) $P(a|\neg c,d) = \dfrac{P(\neg c \wedge d|a) \cdot P(a)}{P(\neg c \wedge d)}$

$P(\neg c \wedge d|a) = P(b|a) \cdot P(\neg c|b) \cdot P(d|b)$

$+ P(\neg b|a) \cdot P(\neg c|\neg b) \cdot P(d|\neg b)$

$= 0{,}5 \cdot 0{,}9 \cdot 0{,}6 + 0{,}5 \cdot 0{,}7 \cdot 0{,}8 = 0{,}55$

$P(\neg c \wedge d|\neg a) = P(b|\neg a) \cdot P(\neg c|b) \cdot P(d|b)$

$+ P(\neg b|\neg a) \cdot P(\neg c|\neg b) \cdot P(d|\neg b)$

$= 0{,}2 \cdot 0{,}9 \cdot 0{,}6 + 0{,}8 \cdot 0{,}7 \cdot 0{,}8 = 0{,}556$

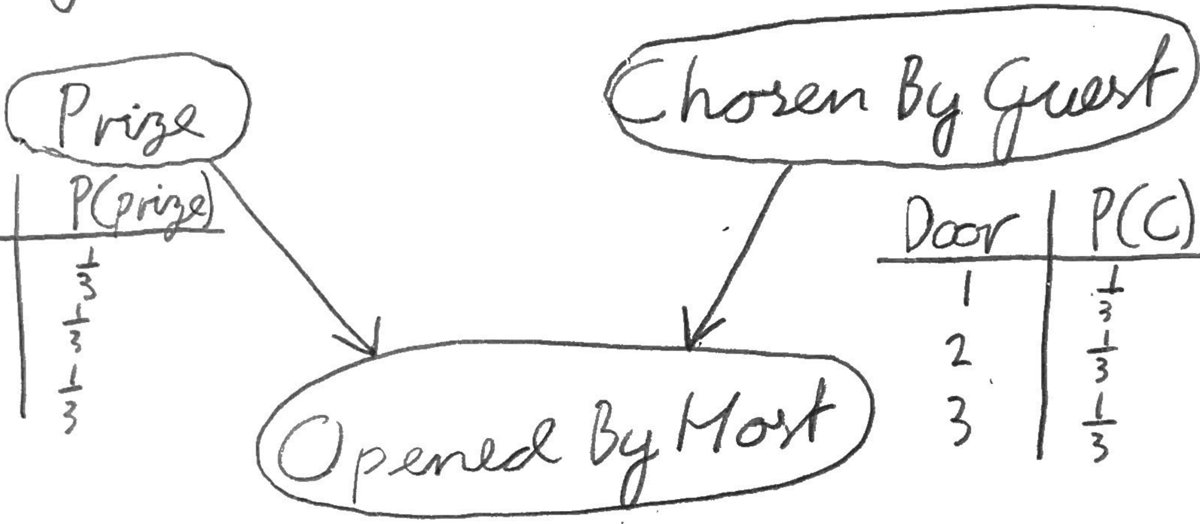
$P(\neg c \wedge d) = P(\neg c \wedge d|a) \cdot P(a) + P(\neg c \wedge d|\neg a) \cdot P(\neg a)$

$= 0{,}55 \cdot 0{,}8 + 0{,}556 \cdot 0{,}2 = 0{,}5512$

$P(a|\neg c,d) = \dfrac{0{,}55 \cdot 0{,}8}{0{,}5512} = \underline{\underline{0{,}798}}$

4, c)

Bayesian Network:



Door	P(prize)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$

Door	P(C)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$

Prize	1			2			3		
Chosen By Guest	1	2	3	1	2	3	1	2	3
Door 1	0	0	0	0	0,5	1	0	1	0,5
Door 2	0,5	0	1	0	0	0	1	0	0,5
Door 3	0,5	1	0	1	0,5	0	0	0	0

Posterior Probabilities:
(Can also be found in the python-file)
P(Prize | Chosen By Guest = 1, Opened By Host = 3)

Prize (0)	0
Prize (1)	0,6667
Prize (2)	0,3333