Software Project 2 - Kalman Filter Model

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ECE 751 - Detection and Estimation Theory
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Problem Statement

- In MATLAB, a State Space Signal Model for a dynamic system having a straight-line trajectory with constant velocity to produce real values and measurements with noise input to the filter.
- Obtain Kalman gain that decides the measure of importance that should be given to either the measurement state or preceded state.
- Obtain Process Covariance matrix that represents the error in the estimate or process through a matrix of position or velocity.
- Compare the theoretical MSE and Instantaneous MSE where theoretical is the diagonal matrix of the Process Covariance matrix, position and velocity respectively.

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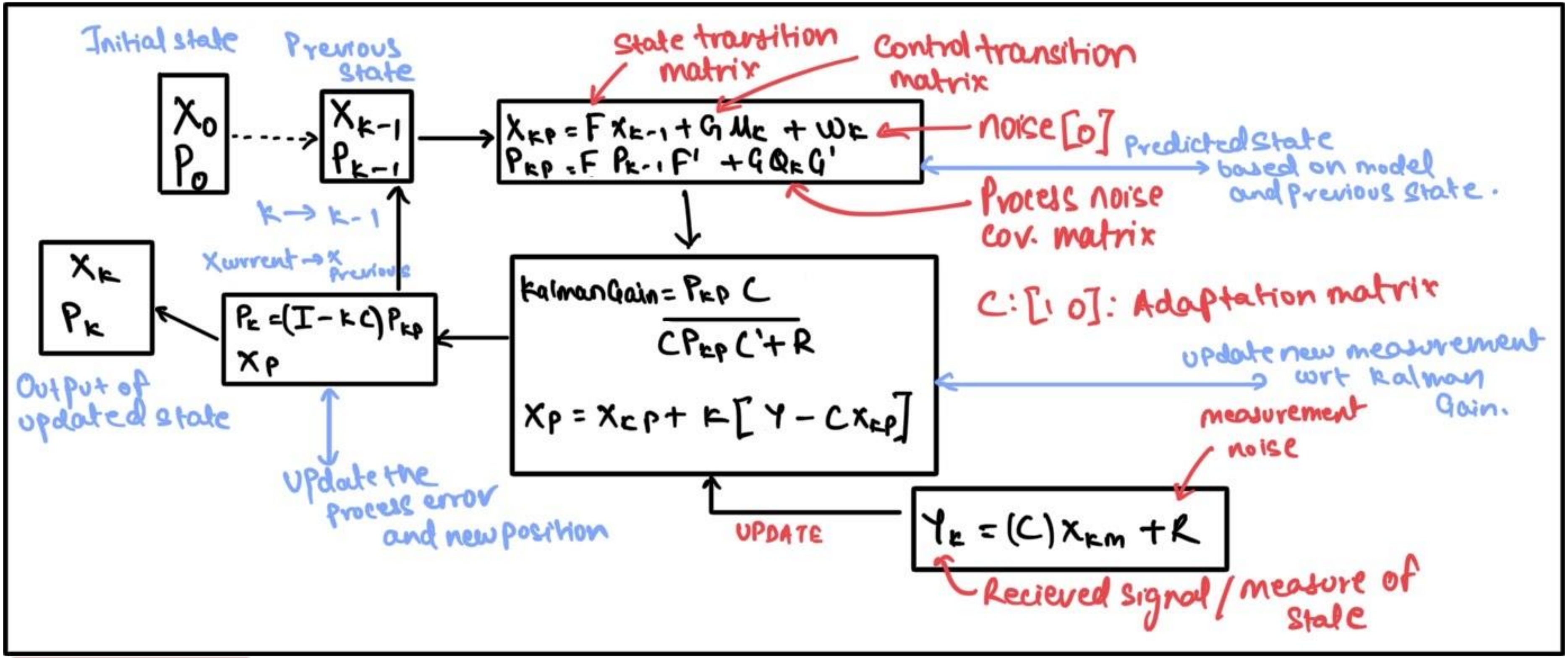
Kalman Filter Prototype



Interface

- Inputs
  - T = 0.1 #delta T is considered to be 0.1
  - F = [1 T ; 0 1] #State transition matrix i.e. the Adaptation matrix, value considered from the book
  - G = [T^2/2 ; T] #Control transition matrix i.e. the Adaptation matrix, value considered from the book
  - Q = 40 #Variance of process noise, value considered from the book
  - R = 100 #Variance of measurement noise, value considered from the book
  - V = #Variance of velocity noise
  - C = [1 0] #Adaptation/Measurement matrix, value considered from the book
  - X_Estimate_Initial = [0 ; 0] #Initialize the estimate of X at time k-1
  - P_Estimate = 1000 #The error is estimated to be 1000 that is 10*(variance of R)
  - Instantaneous MSE = (1,1) diagonal element of the P estimate matrix. (Position)
  - Instantaneous MSE = (2,2) diagonal element of the P estimate matrix. (Velocity)
- Outputs
  - k_gainValue = Kalman Gain (2x1) where the the first element in the matrix is the Gain w.r.t. Position and the second is the Velocity
  - x_Estimate = Estimate state values where the the first element in the matrix is the State w.r.t. Position and the second is the Velocity
  - MSE = Instantaneous mean squared error which is a (2X2) matrix. The off diagonals are ignored and the first element of the diagonals is for Position and the other is for the Velocity

```
%State-Space Signal Model for Kalman Filter.
% Inputs:
% N: Number of time steps.
% F: State transition matrix of size Nx x Nx.
% G: Input matrix of size Nx x Nu.
% C: Measurement matrix of size Nr x Nx.
% Q: Process noise covariance matrix
% R: Measurement noise covariance matrix
% x_Initial: Initial state vector of size Nx x 1.

seed = (16+16+16+16);  %(Akshay Khanna) - a,a,a,a
rng(seed, 'twister');

i= 100;  % Number of iterations or time steps, value considered from the book 9.396
T = 0.1; %Time
max_Stepsize = 0:T:(i-1)*T; %Max Increment per second
F = [1 T ; 0 1]; %State Transition Matrix, value considered from the book 9.386
G = [T^2/2; T] ; %Control Transition Matrix, value considered from the book 9.387
sig2_Q = 40; %Variance of process noise
sig2_V = 40; %Variance of velocity, value considered from the book 9.395
sig2_R = 100; %Variance of measurement noise, value considered from the book 9.397
Q = sig2_Q; %Assigning the value to Q, u(k-1) ~ N(0,Q)
R = sig2_R; %Assigning the value to R
C = [1 0]; %Measurement matrix of size, value considered from the book 9.390
```

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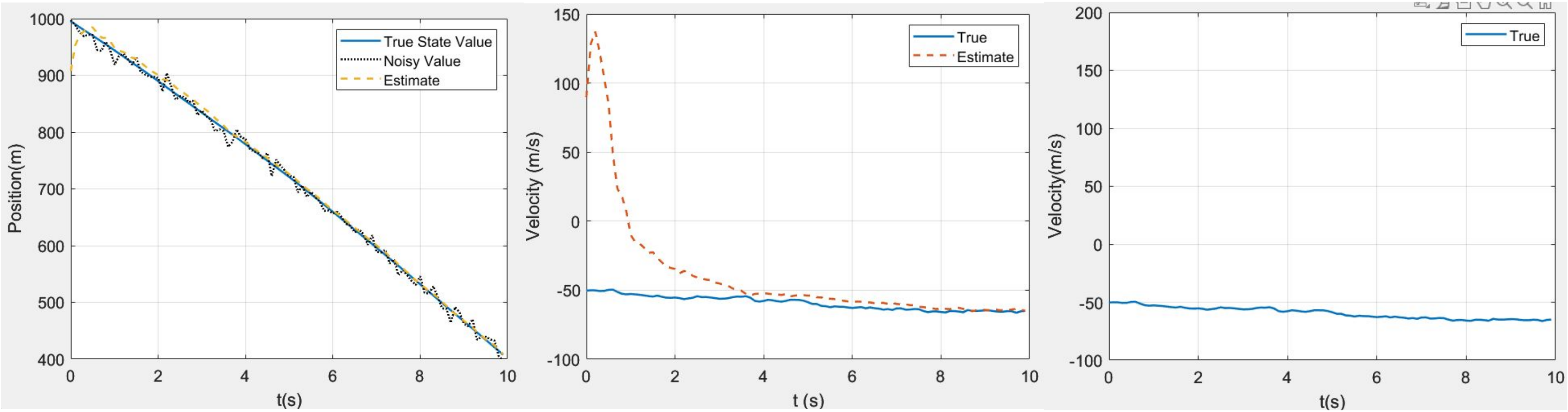
Benchmark Case

- The first figure shows the true and estimated velocity values over time.
- The true values are plotted in a solid line while the estimated values are plotted using a dashed line.
- The x-axis of the figure represents time in seconds while the y-axis represents velocity in meters per second.
- The legend in the figure distinguishes between the true and estimated values.
- The second figure displays the Kalman gain value for the position over time.
- The x-axis of the figure represents time in seconds while the y-axis represents the Kalman gain value for the position.
- The third figure displays the Kalman gain value for the velocity over time.
- The x-axis of the figure represents time in seconds while the y-axis represents the Kalman gain value for the velocity.
- The fourth figure shows the true velocity values over time. The fifth and sixth figures display the mean squared error (MSE) for position and velocity, respectively.
- The MSE is a measure of the difference between the true and estimated values. The x-axis of each figure represents time in seconds while the y-axis represents the MSE value for position and velocity, respectively.

Benchmark Case

- Calling an example function asked in the question. The values of the i/p are the same as considered in the textbook, Problem number: 9.10



A realization of Kalman Filter with range-only observation, Similar to Fig. 9.38 in the Textbook.

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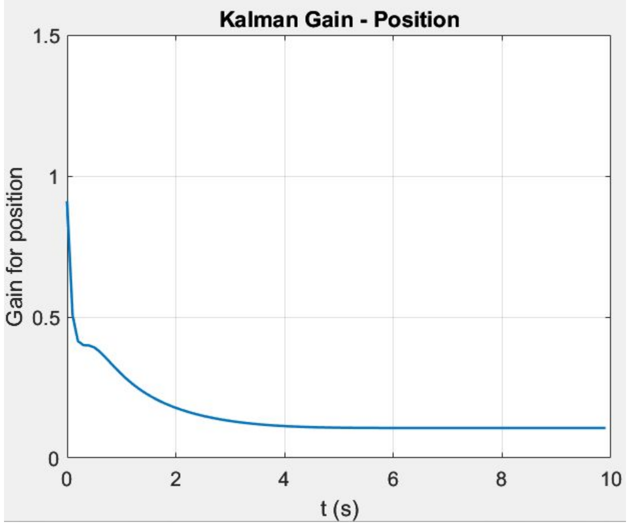
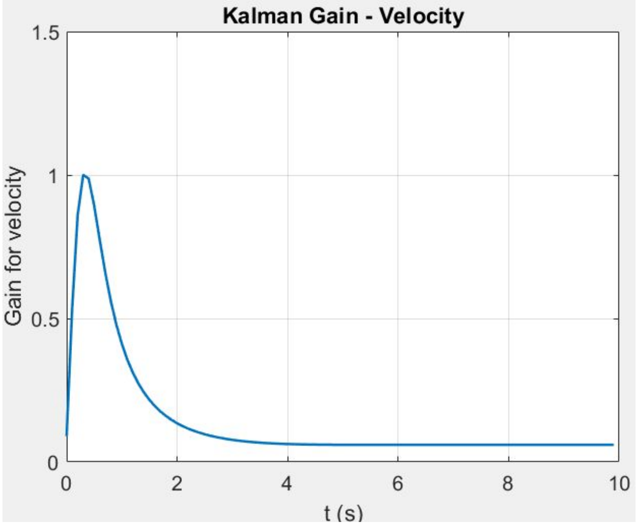
Benchmark Case

- Calling an example function asked in the question. The values of the i/p are the same as considered in the textbook, Problem number: 9.10





Kalman Gain versus Time for Position and Velocity, Similar to Fig. 9.39 in the Textbook.

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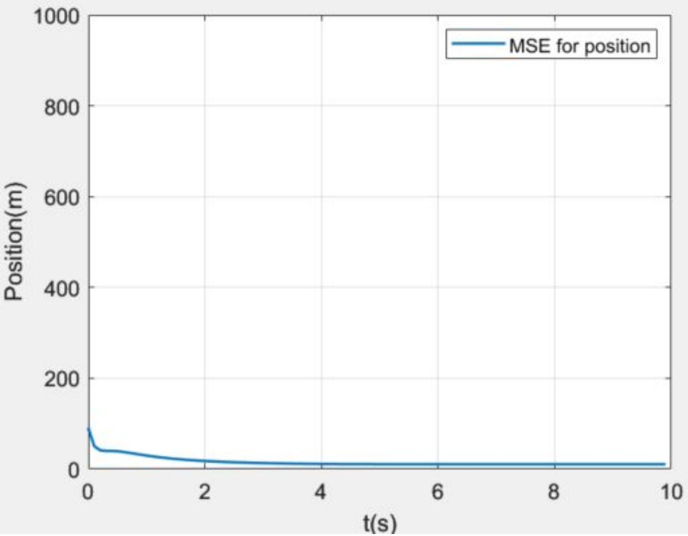
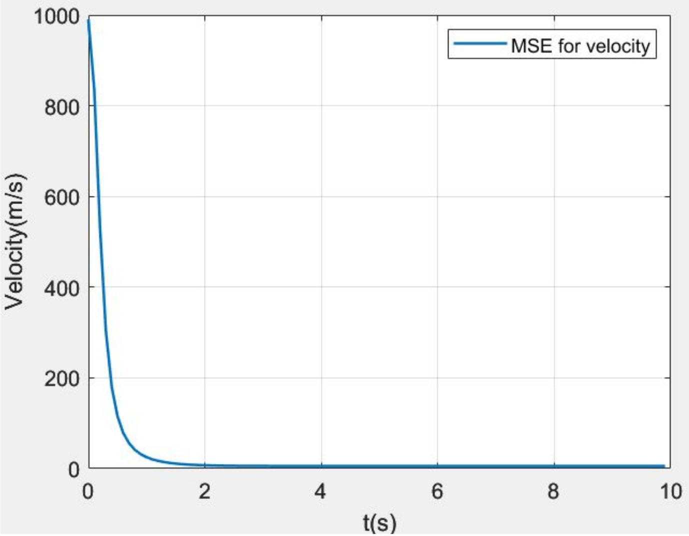
Benchmark Case

- Calling an example function asked in the question. The values of the i/p are the same as considered in the textbook, Problem number: 9.10





MSE versus Time for Position and Velocity, Similar to Fig. 9.40 in the Textbook.

- In this case, the MSE that is considered is the diagonal values of the P error matrix for position and velocity. We are not using the formula for MSE.

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Conclusion

- The code implements a state-space signal model for a Kalman filter.
- The model takes in several inputs including the number of time steps, state transition matrix, input matrix, measurement matrix, process noise covariance matrix, measurement noise covariance matrix, and initial state vector.
- The code generates random noise values based on a seed value.
- The model estimates the true state vector values of position and velocity and the position measurements with noise.
- The code initializes the estimate of x at time k-1 and sets the error estimate to 1000 (10 times the variance of measurement noise).
- The code calls a function that iteratively updates the state estimate and calculates the Kalman gain and mean squared error.
- The results are plotted using different figures to display the true state value, noisy value, estimate, and Kalman gain for both position and velocity.
- The scaling of the graphs is set to match the figures in the textbook.
- The code outputs the true state vector values of position and velocity, the position measurements with noise, the Kalman gain, mean squared error, and the estimate of x at time k-1.
- The function allows for easy modifications of the inputs to explore the effects on the Kalman filter's performance.

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