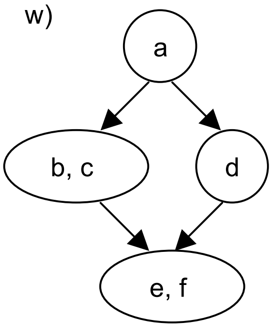
DECISION ANALYSIS – EXERCISES I – INTRODUCTION AND PROMETHEE

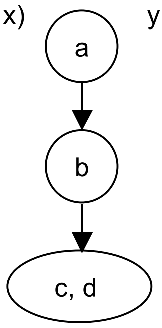
I. Indicate the truth (T) or falsity (F) for the below statements.

a) in multiple criteria choice, one aims at selecting the most preferred subset of alternatives	T	Slide 8
b) in classification problems, classes (categories) need to be preference-ordered and pre-defined	F	Slide 10
c) the ranking presented in figure w) is complete	F	Slide 9
d) the ranking presented in figure x) is complete	T	Slide 9
e) non-dominated alternatives are also weakly non-dominated	T	Slide 11
f) the preference plot presented in figure y) corresponds to a gain-type criterion	T	
g) the preference plot presented in figure z) corresponds to a cost-type criterion	T	
h) one can model incomparability using a preference model in the form of a value function	F	Slide 15
i) among the three families of preference models, decision rules are the most general one	T	Slide 14

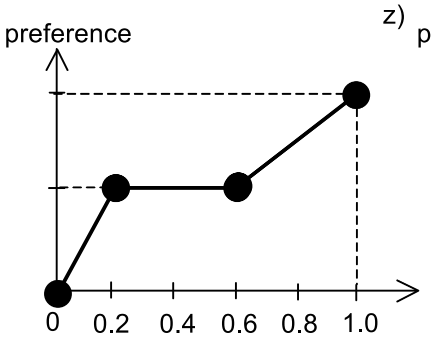
w)



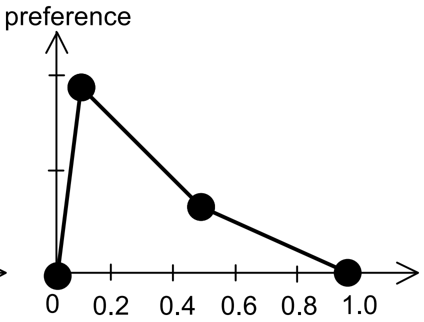
x)



y)



z)



Assume that $g_j(a)$ is the performance of alternative a on criterion g_j, I denotes indifference, and P denotes preference.

II. Which of the below conditions corresponds to the monotonicity of a consistent family of k criteria of gain-type?

a) if $g_j(a) \geq g_j(b)$, j=1,…,k then aPb

c) if aPb and bPc, then aPc

Slide 7 b) if aPb, then ∀c: $g_j(c) \geq g_j(a)$, j=1,…,k ⇒ cPb

d) if aPb, then ∀c: $g_j(c) \geq g_j(a)$, j=1,…,k ⇒ cPa

What would change in the correct answer if we considered a family of k criteria of cost-type? -//- g_j(c) <= g_j(a) -//-

III. Which of the below conditions corresponds to the completeness of a consistent family of k criteria?

a) ∀a,b: $g_j(a) = g_j(b)$, j=1,…,k ⇒ aPb

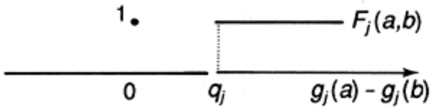
c) if aIb, then ∀c: $g_j(c) = g_j(a)$, j=1,…,k ⇒ aIc

b) if aIb and aIc then bIc

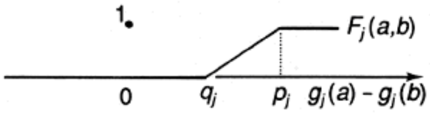
Slide 7 d) ∀a,b: $g_j(a) = g_j(b)$, j=1,…,k ⇒ aIb

IV. Consider the marginal preference functions for the three criteria: g_1, g_2, and g_3 of gain type. Each criterion's intra- and inter-criteria preference information is provided above the respective plots (q_i – indifference threshold, p_i – preference threshold, w_i – weight). First, compute the marginal preference indices π_j for two pairs of alternatives: A and B as well as C and D. Then, compute the comprehensive preference indices π. Note: Slide 27 - formulas for different types of graph

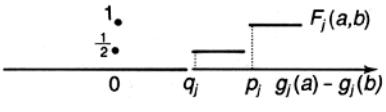
g_1 : $q_1 = 10$, $w_1 = 3$



g_2 : $q_2 = 10$, $p_2 = 20$, $w_2 = 5$



g_3 : $q_3 = 2$, $p_3 = 5$, $w_3 = 2$



	g_1 ↑	g_2 ↑	g_3 ↑
A	10	18	10
B	15	0	20

	g_1	g_2	g_3
$\pi_j(A,B)$	0	0.8	0
$\pi_j(B,A)$	0	0	1

π(A,B)	(3·0 + 5·0.8 + 2·0)/10 = 0.4
π(B,A)	(3·0 + 5·0 + 2·1)/10 = 0.2

	g_1 ↑	g_2 ↑	g_3 ↑
C	20	10	17
D	0	25	20

Slide 25

	g_1	g_2	g_3
$\pi_j(C,D)$	1	0	0
$\pi_j(D,C)$	0	0.5	0.5

15-10 / 20-10

Slide 33

π(C,D)	(3*1 + 5*0 + 2*0) / (3+5+2) = 0.3
π(D,C)	(3*0 + 5*0.5 + 2*0.5) / (3+5+2) = 0.35

Repeat the computations for pair (A,B), while assuming that all criteria are of cost-type. Slide 26

V. Indicate the truth (T) or falsity (F) for the below statements.

Slide

a) the PROMETHEE methods use an outranking-based preference model Slide 20 T
b) PROMETHEE I provides a partial ranking of alternatives Slide 36 T
c) the sum of comprehensive flows of all alternatives in PROMTEHEE II is equal to zero F
d) for alternatives a and b, the sum of marginal preference indices $\pi_j(a,b)$ and $\pi_j(b,a)$ can be equal to zero T 29
e) for alternatives a and b, the sum of comprehensive preference indices $\pi(a,b)$ and $\pi(b,a)$ is never greater than one T 33
f) to select the most preferred subset of alternatives, PROMETHEE V requires pre-computed comprehensive flows T 40

VI. Using the PROMETHEE method, one derived a matrix of comprehensive preference indices $\pi(a,b)$ for all pairs of alternatives. Compute the positive $\Phi^+(a)$, negative $\Phi^-(a)$, and comprehensive flows $\Phi(a)$ for all alternatives. Draw the rankings obtained with PROMETHEE II and PROMETHEE I. Recall that PROMETHEE I admits incomparability.

$\pi(a,b)$	W	X	Y	Z	$\Phi^+(a)$	$\Phi^-(a)$	$\Phi(a)$
W	0	0	0	0.3	0.3	0.9	-0.6
X	0	0	0.2	0.7	0.9	0.1	0.8
Y	0.9	0	0	0.7	1.6	0.3	1.3
Z	0	0.1	0.1	0	0.2	1.7	-1.5

Slide 35
Φ+(a) - sum of row
Φ-(a) - sum of column
Φ(a) = Φ+(a) - Φ-(a)

VII. Consider six alternatives (S, V, W, X, Y, Z) with the following comprehensive flows: $\Phi(S) = 0.9$, $\Phi(V) = -0.9$, $\Phi(W) = -0.6$, $\Phi(X) = 0.8$, $\Phi(Y) = 1.3$, and $\Phi(Z) = -1.5$. Formulate the binary linear program according to the assumptions of PROMETHEE V that would allow selecting a subset of two alternatives that respect the constraints on the maximal budget of 100 and the minimal projected gain of 300. The budgets and gains for all alternatives are provided in the below table. Use the following binary variables: x_S, x_V, x_W, x_X, x_Y, and x_Z, corresponding to the six alternatives.

Φ	0.9	-0.9	-0.6	0.8	1.3	-1.5
	S	V	W	X	Y	Z
budget	40	30	60	50	70	20
projected gain	140	100	150	170	200	120

What would be the optimal subset of alternatives selected by PROMETHEE V? Slide 41

max 0.9*S + (-0.9)*V + (-0.6)*W + 0.8*X + 1.3*Y + (-1.5)*Z

subject to:
140*S + 100*V + 150*W + 170*X + 200*Y + 120*Z >= 300
40*S + 30*V + 60*W + 50*X + Y*70 + 20*Z <= 100
S, V, W, X, Y, Z ∈ {0, 1}

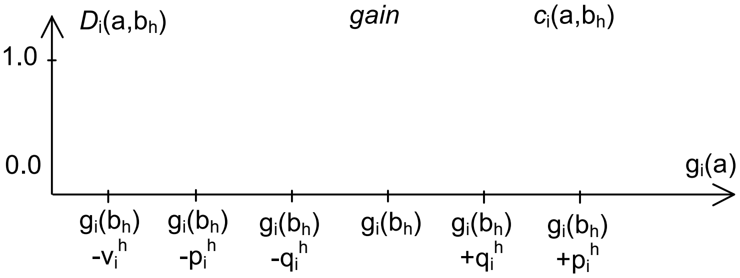
Optimal solution: select S and X
S = 1, V = 0, W = 0, X = 1, Y = 0, Z = 0
objective function = 0.9 + 0.8 = 1.7
cardinality = 2 (S = 1, X = 1)
comprehensive budget = 40 + 50 = 90 < 100
comprehensive projected gain = 140 + 170 = 310 > 300

DECISION ANALYSIS – EXERCISES II – ELECTRE TRI-B

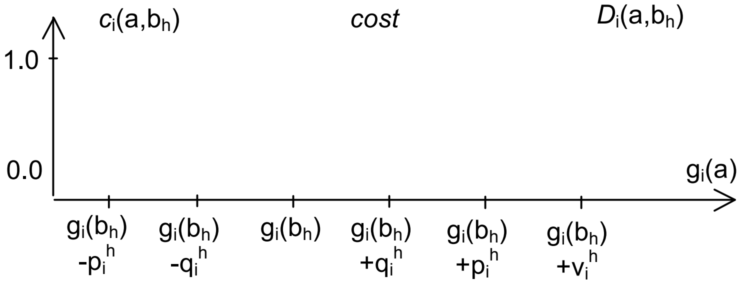
I. Indicate the truth (T) or falsity (F) for the below statements.

		Slide
a) ELECTRE TRI-B allows dealing with multiple criteria sorting problems	T	44
b) ELECTRE TRI-B requires the Decision Maker to specify characteristic class profiles	T	44
c) ELECTRE TRI-B employs a preference model in the form of an outranking relation	T	49
d) Weights used in ELECTRE TRI-B represent importance coefficients rather than trade-offs between criteria	T	63
e) if alternative *a* and profile b_h have the same performances on criterion g_j, then always $c_j(a,b_h)=0$ and $c_j(b_h,a)=0$	F	61
f) when marginal concordance $c_j(a,b_h)$ is greater than zero, then marginal discordance $D_j(a,b_h)$ is always equal to 0	T	58
g) when marginal discordance $D_j(a,b_h)$ is equal to zero, then marginal concordance $c_j(a,b_h)$ is always greater than 0	F	58
h) outranking credibility $\sigma(a,b_h)$ cannot be greater than comprehensive concordance $C(a,b_h)$	T	69
i) if *a* outranks b_h and b_h outranks *a*, then *a* and b_h are incomparable	F	71
j) the class assignment suggested by the optimistic rule of ELECTRE TRI-B is always at least as good as the class assignment suggested by the pessimistic rule	T	77
k) if alternative *a* is indifferent with at least one boundary class profile, then the indications of the pessimistic and optimistic assignment rules of ELECTRE TRI-B differ	F	77

II. Draw the plots of marginal concordance $c_i(a,b_h)$ and discordance $D_i(a,b_h)$ for criterion g_i that is of either gain (to the left) or cost (to the right) type. Then, repeat the exercise while drawing the plots for $c_i(b_h,a)$ and $D_i(b_h,a)$, i.e., with an inverse order of profile b_h and alternative *a* in the considered pair. Slide 61



Slide 58



Slide 60

III. Consider two alternatives *a* and *e*, and a single boundary class profile b_t. They are evaluated on two criteria g_1 and g_2 (the performances are provided in the below table) with the following specification of preference orders as well as intra- and inter-criteria parameters:

- g_1 – gain, weight $w_1=2$, indifference threshold $q_1(b_t)=10$, preference threshold $p_1(b_t)=50$, and veto threshold $v_1(b_t)=100$;
- g_2 – cost, weight $w_2=3$, indifference threshold $q_2(b_t)=0$, preference threshold $p_2(b_t)=10$, and veto threshold $v_2(b_t)=20$.

For pairs (a,b_t), (b_t,a), (e,b_t), (b_t,e), compute the marginal concordance c_j and discordance D_j indices, comprehensive concordance indices *C*, and outranking credibilities σ. When considering the credibility threshold λ = 0.6, verify the truth of outranking relation *S* for all considered pairs and indicate which relation (>, <, ?, *I*) holds for them.

	g_1 ↑	g_2 ↓
a	145	40
e	240	20
b_t	150	15

$c_1(a,b_t)$ =	1	$D_1(a,b_t)$ =	0	$c_1(b_t,a)$ =	1	$D_1(b_t,a)$ =	0	*Outranking*	
$c_2(a,b_t)$ =	0	$D_2(a,b_t)$ =	1	$c_2(b_t,a)$ =	1	$D_2(b_t,a)$ =	0	aS^Cb_t	b_tSa
$C(a,b_t)$ =	(2·1+ 3·0)/5 = 0.4			$C(b_t,a)$ =	(2·1+ 3·1)/5 = 1			*Relation*	
$\sigma(a,b_t)$ =	0.4·(1-1)/(1-0.4) = 0			$\sigma(b_t,a)$ =	1			$a < b_t$	

90-50/100-50

Slide 70

$c_1(e,b_t)$ =	1	$D_1(e,b_t)$ =	0	$c_1(b_t,e)$ =	0	$D_1(b_t,e)$ =	0.8	*Outranking*	
$c_2(e,b_t)$ =	0.5	$D_2(e,b_t)$ =	0	$c_2(b_t,e)$ =	1	$D_2(b_t,e)$ =	0	e S bt	bt S^c e
$C(e,b_t)$ =	(2*1 + 3*0.5) / 2+3 = 0.7			$C(b_t,e)$ =	(2*0 + 3*1) / 2+3 = 0.6			*Relation*	
$\sigma(e,b_t)$ =	0.7			$\sigma(b_t,e)$ =	0.6*(1 - 0.8) / (1 - 0.6) = 0.3			e > bt	

C -> Slide 67

Slide 69

Slide 71

Note:
- c(a,bh) > 0 => D(a,bh) = 0
- D(a,bh) > 0 => c(a,bh) = 0

c for gain-type criterion:
- (a, b_h) -> Slide 55

c for cost-type criterion:
- (a, b_h) -> Slide 56

D for gain-type criterion:
- (a, b_h) -> Slide 58

D for cost-type criterion:
- (a, b_h) -> Slide 60

IV. Assume alternative a is compared with a boundary class profile b_h. The partial results of the concordance and discordance tests are as follows: $C(a,b_h) = 0.6$, $D_1(a,b_h) = 0.5$, $D_2(a,b_h) = 0.0$, $D_3(a,b_h) = 0.9$. Compute the outranking credibility $\sigma(a,b_h)$. Recall the meaning of condition $F = \{ j = 1,\dots,n : D_j(a,b_h) > C(a,b_h)\}$ when taking into account the reasons againsts outranking. Slide 69

$$\sigma(a,b_h) = 0.6 * \frac{1 - 0.9}{1 - 0.6}$$

V. Assume the outranking credibilities for a pair consisting of alternative a and a boundary class profile b_h are as follows: $\sigma(a,b_h) = 0.6$ and $\sigma(b_h,a) = 0.9$. Verify the truth of outranking for pairs (a,b_h) and (b_h,a) when using the credibility threshold $\lambda = 0.8$. Indicate which relation (>, <, *I*, ?) holds for a and b_h. Would the results change for $\lambda = 0.5$ or $\lambda = 0.95$?

yes, aIb_h a?b_h

Slide 71 $0.6 < 0.8$ and $0.9 > 0.8$ => $(a \text{ S} / \boxed{\text{S}^C}\, b_h)$ and $(b_h \boxed{\text{S}} / \text{S}^C\, a)$ imply $(a < b_h)$

VI. Consider the outranking credibilities for pairs consisting of alternative a and a boundary class profile b_h: $\sigma(a,b_h) = 0.75$ and $\sigma(b_h,a) = 0.6$. Indicate the admissible values of the credibility threshold λ for which the following relations hold: aIb_h, $a>b_h$, $a?b_h$, and $a<b_h$. Slide 71

$aIb_h \leftrightarrow$ $\lambda \in [0.5,0.6]$ $a>b_h \leftrightarrow \lambda \in (0.6,0.75]$ $a?b_h \leftrightarrow \lambda > 0.75$ $a<b_h \leftrightarrow$

VII. Consider alternatives a_1 - a_{10} that are compared against boundary class profiles $b_0 – b_4$. Class C_h is defined by a lower profile b_{h-1} and an upper profile b_h. Thus, four classes $C_1 – C_4$ are considered overall, where C_4 is the most preferred class and C_1 is the least preferred. For alternatives $a_1 – a_6$ determine the class assignments according to the pessimistic and optimistic rules of ELECTRE TRI-B. The interpretation of relations in the table is as follows: > (preference), < (inverse preference), *I* (indifference), and ? (incomparability). For alternatives $a_7 – a_{10}$, fill in the relations that would imply the provided assignments.

Slide 74 Slide 76

Alternative	Profiles					Class assignments	
	b_0	b_1	b_2	b_3	b_4	Pessimistic	Optimistic
a_1	>	>	<	<	<	C_2	C_2
a_2	>	?	<	<	<	C_1	C_2
a_3	>	>	*I*	*I*	<	C4	C4
a_4	>	?	?	?	<	C1	C4
a_5	>	>	>	>	<	C4	C4
a_6	*I*	<	<	<	<	C1	C1
a_7	>	>	>	<	<	C_3	C_3
a_8	>	>	<	<	<	C_2	C_2
a_9	>	?	?	<	<	C_1	C_3
a_{10}	?	>	*I*	/	<	C_4	C_4

Pessimistic:

Go from right to left, stop at / or >, assign to C h+1

Optimistic:

Go from left to right, stop at <, assign to C h

DECISION ANALYSIS – EXERCISES III – UTA

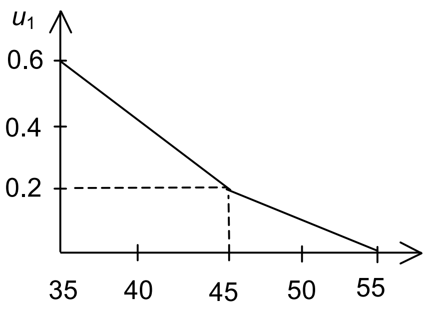
I. Indicate the truth (T) or falsity (F) for the below statements.

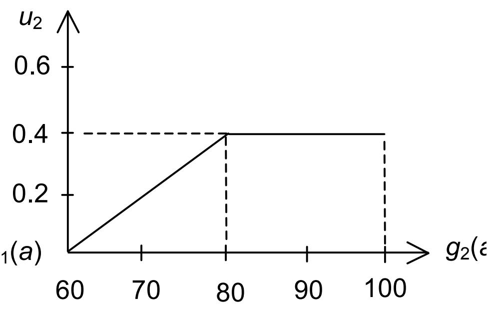
		Slide
a) UTA uses an outranking-based preference model	F	84
b) UTA accepts indirect preference information in the form of holistic judgments	T	94
c) The marginal value functions in UTA are piecewise linear	T	100
d) The marginal value functions in UTA need to be strictly monotonic	F	101
e) The range of a comprehensive value computed in UTA is between -1 and 1	F	97
f) The ordinal regression problem solved by UTA is linear	T	103
g) An additive value model admits indifference	T	85
h) The maximal Kendall's distance between two rankings involving four alternatives is 12 (1/2 * 12 = 6)	F	110
i) The minimal value of Kendall's tau is 0	F	110
j) The alternatives with the same performances on all criteria may attain different comprehensive values	F	102
k) Different alternatives may attain the same comprehensive values	T	
l) If the set is of value functions compatible with the Decision Maker's comparisons is non-empty, there is always only one compatible value function	F	106

II. Consider the performances of alternatives $a_1 - a_5$ and plots of marginal value functions for two criteria: g_1 (cost) and g_2 (gain). Read off the marginal values for all alternatives and compute their comprehensive values. Verify if the reference ranking $a_3 \; P \; a_4 \; P \; a_5$ is satisfied, where P is the preference relation. What is the ranking of all five alternatives?

U(a) = sum of w_i * u_i   Slide 88

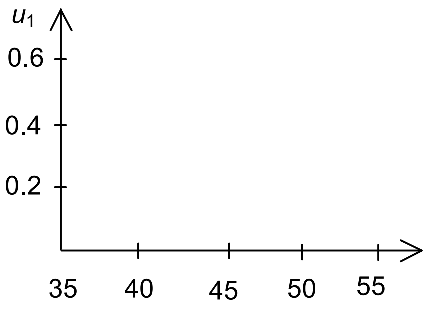
	$g_1 \downarrow$	$g_2 \uparrow$	$u_1(a)$	$u_2(a)$	$U(a)$
a_1	45	80	0.2	0.4	0.6
a_2	50	90	0.1	0.4	0.5
a_3	40	70	0.4	0.2	0.6
a_4	35	60	0.6	0	0.6
a_5	55	100	0	0.4	0.4

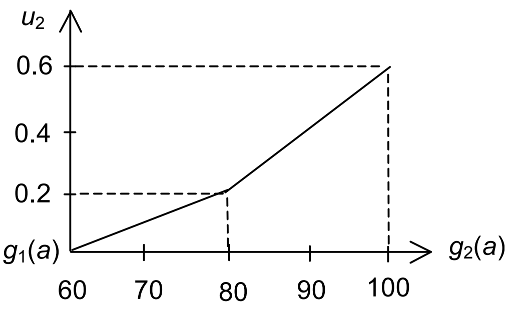




III. Consider the performances of alternatives $a_1 - a_5$ and two criteria: g_1 (cost) and g_2 (gain). The plot of marginal value function u_2 is already given. Draw the plot of marginal value function u_1 with three equally distributed characteristic points (one point in the middle of the performance range 35-55) so that all assumptions of UTA are satisfied and the following reference ranking is reproduced: a_2 ***P*** a_1 ***I*** a_3. What is the ranking of all five alternatives?

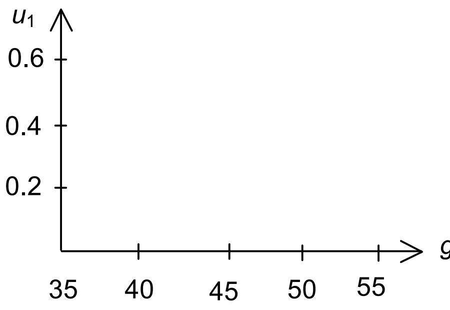
	$g_1 \downarrow$	$g_2 \uparrow$	$u_1(a)$	$u_2(a)$	$U(a)$
a_1	45	80		0.2	
a_2	50	90		0.4	
a_3	40	70		0.1	
a_4	35	60		0	
a_5	55	100		0.6	

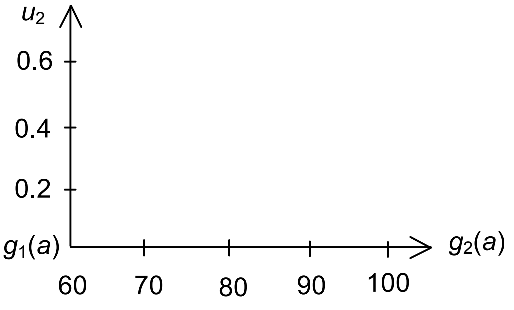




IV. Consider the performances of alternatives $a_1 - a_5$ and two criteria: g_1 (cost; range 35-55) and g_2 (gain; range 60-100). Draw the **linear** value functions on both criteria (without breaking them anywhere) to reproduce the following pairwise comparison: a_5 *P* a_1. Compute the comprehensive value of reference alternatives according to the drawn marginal value functions to prove that they allow reproducing the above pairwise comparison.

	$g_1 \downarrow$	$g_2 \uparrow$	$u_1(a)$	$u_2(a)$	$U(a)$
a_1	45	80			
a_2	50	90			
a_3	40	70			
a_4	35	60			
a_5	55	100			





Hint: a_5 ***P*** a_1 *iff* $U(a_5) > U(a_1)$ *iff* $u_1(a_5) + u_2(a_5) > u_1(a_1) + u_2(a_1)$ *iff* $u_1(55) + u_2(100) > u_1(45) + u_2(80)$ *iff*
iff $0u_1(35) + u_2(100) > 0.5u_1(35) + 0.5u_2(100)$ *iff* $0.5u_2(100) > 0.5u_1(35)$ *iff* $u_2(100) > u_1(35)$
$u_2(100) + u_1(35) = 1$, so $u_2(100) > u_1(35)$ *iff* $u_2(100) > 1 - u_2(100)$ *iff* $u_2(100) > 0.5$

V. Consider three reference alternatives: *x*, *y*, and *z* that by the DM has ordered in the following way: ***x P y I z***. The alternatives are evaluated in terms of two gain-type criteria: g_1, g_2. An additive value function has the following form: $U(a) = u_1(g_1(a)) + u_2(g_2(a))$. For g_1, we consider two characteristic points α_1 and β_1 (thus, $g_1(a) \in [\alpha_1, \beta_1]$), and for g_2 we consider three characteristic points α_2, γ_2 and β_2, where $\gamma_2 = (\alpha_2 + \beta_2)/2$ (thus, $g_2(a) \in [\alpha_2, \beta_2]$). The performances of reference alternatives are as follows: $g_1(x)=\alpha_1$, $g_2(x)=\beta_2$, $g_1(y)=\beta_1$, $g_2(y)=\gamma_2$, $g_1(z)=\beta_1$, $g_2(z)=\alpha_2$. Formulate an ordinal regression problem that would verify if there exists at least one value function compatible with the assumptions of UTA and the DM's preferences. Denote the over- and under-estimation errors for alternative *a* by $\sigma^+(a)$ and $\sigma^-(a)$.

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min $F = \sigma^+(x) + \sigma^-(x) + \sigma^+(y) + \sigma^-(y) + \ldots$ σ+(z) + σ-(z)

subject to: $u_1(\alpha_1) + u_2(\beta_2) - \sigma^+(x) + \sigma^-(x) > u_1(\beta_1) + u_2(\gamma_2) - \sigma^+(y) + \sigma^-(y)$

… u1(B1) + u2(y2) - σ+(y) + σ-(y) = u1(B1) + u2(α2) - σ+(z) + σ-(z)

$u_1(\beta_1) + u_2(\beta_2) = 1$

… u1(α1) = 0, u2(α2) = 0

$u_1(\alpha_1) \le u_1(\beta_1)$

… u2(α2) <= u2(B2)

$u_1(\alpha_1), u_1(\beta_1), u_2(\alpha_2), u_2(\gamma_2), u_2(\beta_2) \ge 0$

$\sigma^+(x), \sigma^-(x), \sigma^+(y), \sigma^-(y), \sigma^+(z), \sigma^-(z) \ge 0$

For which values of *F* there exists a compatible value function?

VI. Fill in the matrices of pairwise relations for the three rankings given to the right: R_I, R_{II}, and R_{III}. Assume the following order of alternatives in rows and columns: a, b, c, d. Compute Kendall's distance and Kendall's tau coefficients for rankings R_I and R_{II} as well as R_I and R_{III}.

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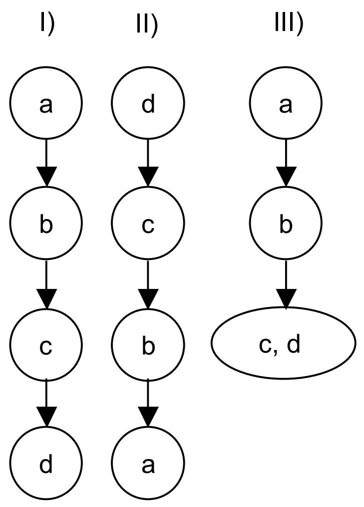
$$R_I = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 1 \\ b & 0 & 0 & 1 & 1 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array} \quad R_{II} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad R_{III} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$



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$d_k(R_I,R_{II})=$ 1/2 * (6 + 6) = 6

$d_k(R_I,R_{III})=$ 1/2 * (0.5 + 0.5) = 0.5

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$\tau(R_I,R_{II}) =$ 1 - 4 * (6 / 4*3) = -1

$\tau(R_I,R_{III}) =$ 1 - 4 * (0.5 / 4*3) = 0.83(3)

VII. The Decision Maker provided the following four pairwise comparisons: $a_1 > a_4$, $a_5 > a_3$, $a_2 > a_6$, $a_3 > a_1$. Change the formulation of the below mathematical programming model to identify the minimal subset of pairwise comparisons underlying inconsistency of preference information. Select the preference direction (min or max), write down the objective function, change the below conditions by adding appropriate formulations, denote the binary variables (if you use them), do not change *CONSTRAINTS* denoting a constraint set modeling the monotonicity, normalization, and non-negativity constraints.

min / *max* V = v_a1,a4 + v_a5,a3 + v_a2,a6 + v_a3,a1

s.t.

$U(a_1) > U(a_4)$ - v_a1,a4

$U(a_5) > U(a_3)$ - v_a5,a3

$U(a_2) > U(a_6)$ - v_a2,a6

$U(a_3) > U(a_1)$ - v_a3,a1

CONSTRAINTS

v_a1,a4, v_a5,a3, v_a2,a6, v_a3,a1 ∈{0,1}

Assume that the optimal solution of the problem to the left indicated $a_5 > a_3$ and $a_2 > a_6$ as the minimal subset of pairwise comparisons underlying inconsistency. Which condition must one add in the next iteration to find another (different) minimal subset underlying inconsistency? Refer to the variables you have previously introduced to the left.

v_a5,a3 + v_a2,a6 <= 2 - 1 Slide 113

DECISION ANALYSIS – EXERCISES IV – ANALYTIC HIERARCHY PROCESS AND CHOQUET INTEGRAL

I. Indicate the truth (T) or falsity (F) for the below statements.

		Slide
a) AHP uses a rule-based preference model	F	
b) The preference model of AHP is formed by priorities of all elements at all hierarchy levels	T	146
c) The minimal number of hierarchy levels in AHP is three	T	127
d) AHP enforces cardinal consistency condition ?	T	
e) The typical Saaty's scale is between 1 and 5	F	131
f) AHP requires pairwise comparisons of all pairs of elements with a common predecessor	T	132
g) Pairwise comparisons in AHP are based on the nominal scale	F	132
h) To rank five criteria with the common predecessor, in AHP, it is required to make 10 pairwise comparisons	T	132
i) AHP estimates the priorities by computing the principal eigenvector of a pairwise comparison matrix	T	136
j) AHP is vulnerable to the rank reversal phenomenon	T	149
k) AHP maintains the condition of order preservation	F	148
l) When consistency ratio CR is greater than 0.1, the consistency is judged satisfactory	F	144

II. Given the below incomplete pairwise comparison matrix, make it complete to satisfy the consistency condition of pairwise comparisons (CCPC) and cardinal consistency condition (CCC).

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	a_1	a_2	a_3	a_4
a_1	1	3	6	1/3
a_2	1/3	1	2	1/9
a_3	1/6	1/2	1	1/18
a_4	3	9	18	1

a13 = a12*a23 = 3*2 = 6
a42 = a41*a12 = 3*3 = 9
a43 = a41*a13 = 3*6 = 18

III. Consider the hierarchy consisting of four levels, with two major criteria, each consisting of two sub-criteria, and two alternatives, A and B. Fill in the hierarchy by replacing the question marks (?) so that the hierarchy becomes consistent with the assumptions of AHP. Then, compute the comprehensive scores of A and B. Without computing the exact values, what is the sum of *Sc*(A) and *Sc*(B)?

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Sc(A) = 0.7 * (0.5*0.4 + 0.4*0.6) + 0.3 * (0.75*0.25 + 0.25*0.6) ~= 0.4

Sc(B) = 0.7 * (0.5* 0.6 + 0.4*0.4) + 0.3 * (0.75*0.75 + 0.25*0.4) = 0.52

sum of weights should be equal to 1.0

IV. Consider the below entirely consistent pairwise comparison matrix. Compute the priorities (w_1 – w_4) corresponding to the compared alternatives.

Slide 135

	a_1	a_2	a_3	a_4
a_1	1	1/2	1	3
a_2	2	1	2	6
a_3	1	1/2	1	3
a_4	1/3	1/6	1/3	1
col_j	13/3	13/6	13/3	13

col_j - sum of columns

w_1 = 1 / 13/3 + 1/2 / 13/6 + 1 / 13/3 + 3 / 13
w_2 =
w_3 =
w_4 =

V. Consider the below inconsistent pairwise comparison matrix. Compute the priorities ($w_1 - w_4$) corresponding to the compared alternatives by approximating the principal eigenvector using the methods based on the arithmetic mean of the normalized matrix.

Slide 137

	a_1	a_2	a_3	a_4
a_1	1	1/3	1	3
a_2	3	1	2	5
a_3	1	1/2	1	3
a_4	1/3	1/5	1/3	1
col_j	16/3	61/30	13/3	12

col_j - sum of columns

w_1 = $\overbrace{\text{1 / 16/3 + 1/3 / 61/30 + 1 / 13/3 + 3 / 12}}^{\text{w (as in previous task)}}$ => $\overbrace{\text{1/n * w, where n - number of alternatives}}^{\text{w'}}$

w_2 =

w_3 =

w_4 =

VI. The maximal eigenvalue of the above matrix is 4.034. Compute the consistency index CI and consistency ratio CR. Is the inconsistency level of this matrix acceptable according to a default rule of AHP? Note: consistent if CR <= 0.1 (10%) - Slide 144

Slides 144-145

$$CI = \frac{\text{max - n}}{\text{n - 1}} \qquad CR = \frac{CI}{RI}$$

VII. Consider four alternatives X, Y, W, and Z evaluated in terms of three criteria g_1, g_2, g_3 of gain type. For each statement, indicate its truth (T) or falsity (F) (> denotes a preference relation).

SUM	Alternative	g_1	g_2	g_3
19	X	8	4	7
19	Y	8	6	5
14	W	3	4	7
14	Z	3	6	5

Relations X > W and Y > Z can be represented using a weighted sum model	T
Relations W > X and Z > Y can be represented using a weighted sum model	T
Relations X > Y and W > Z can be represented using an additive value function	T
Relations X > Y and Z > W can be represented using an additive value function	F
Relations X > Y and Z > W can be represented using the Choquet integral	T

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VIII. Consider the capacities for all subsets of criteria: u(ø) = 0, u({g_1}) = 0.3, u({g_2}) = 0.4, u({g_3}) = 0.5, u({g_1, g_2}) = 0.8, u({g_1, g_3}) = 0.6, u({g_2, g_3}) = 0.7, u({g_1, g_2, g_3}) = 1. Compute the Choquet integral for alternative A = [3, 6, 5].

sorted A = [6, 5, 3]
g2,g3,g1

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Ch(A) = [3 - 0] * u({g1, g2, g3}) + [5 - 3] * u({g2,g3}) + [6 - 5] * u({g2})

IX. Consider the capacities for various subsets of criteria: u(ø) = 0, u({g_1}) = 0.3, u({g_2}) = 0.4, u({g_3}) = 0.5, u({g_1, g_2, g_3}) = 1. Provide the example capacities for pairs of criteria so that g_1 and g_2 interact positively, g_1 and g_3 interact negatively, and there is no interaction between g_2 and g_3.

u({g_1, g_2}) = 0.3 + 0.4 = 0.7

u({g_1, g_3}) = 0.3 + 0.5 = 0.8

u({g_2, g_3}) = 0.4 + 0.5 = 0.9

X. Consider the weights for individual criteria and criteria pairs: m(ø) = 0.3, m({g_1}) = 0.3, m({g_2}) = 0.4, m({g_3}) = 0.5, u({g_1, g_2}) = 0.1, u({g_1, g_3}) = -0.1, u({g_2, g_3}) = -0.2. Compute the Choquet integral for alternative A = [3, 6, 5] using the Möbius representation. Verify if these weights satisfy the normalization and monotonicity constraints.

Ch(A) = 3 * m({g1}) + 6 * m({g2}) + 5 * m({g3}) + min{3, 6}*m({g1, g2}) + min{3, 5}*m({g1, g3}) + min{6, 5}*m({g2, g3})

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DECISION ANALYSIS – EXERCISES V – ROUGH SET APPROACH

I. Indicate the truth (T) or falsity (F) for the below statements.

a) The lower approximation of a given class contains all objects that possibly belong to this class	F	179
b) The boundary region of a crisp set is empty	T	181
c) The upper approximation of set X is always a proper superset of set X	T	182
d) Rough membership measure takes a value between 0 and 1	T	183
e) For a given set X, its quality of approximation is always at least as high as the accuracy of approximation	F	184
f) When considering two subsets of condition attributes *P*, *P*', such that *P*' ⊂ *P*, the quality of approximation of classification for *P*' is always not higher than for *P*	T	
g) The reduct is defined as the intersection of all cores	F	187/9
h) Rough sets are coupled with a value-based preference model	F	
i) For certain decision rules, the certainty factor is always equal to 1	T	203
j) Approximate rules are induced from upper approximations of classes	F	196
k) The LEM2 algorithms finds a local covering of a given set	T	198
l) In the classification algorithm coupled with the rough set approach, the rule's specificity is defined as the number of elementary conditions in the condition part of the rule	T	205

II. Analyze the below decision table involving six objects (A1-A6), a pair of condition attributes *P* = {X1, X2}, and a decision attribute K. The problem concerns three classes: P, R, and S. Analyze the provided results to make sure you understand how they should be computed.

Object	X1	X2	K
A1	8	4	P
A2	5	7	P
A3	2	3	P
A4	5	7	R
A5	2	5	S
A6	8	5	S

Lower and upper approaximations

$$\underline{P}(P) = \{A1, A3\}, \overline{P}(P) = \{A1, A2, A3, A4\}$$

$$\underline{P}(R) = \varnothing, \overline{P}(R) = \{A2, A4\}$$

$$\underline{P}(S) = \{A5, A6\}, \overline{P}(S) = \{A5, A6\}$$

Example class boundary

$$Bn_P(P) = \{A2, A4\}$$

Example accuracy of approximation

$$\alpha_P(P) = \frac{|\underline{P}(P)|}{|\overline{P}(P)|} = 2/4$$

Example quality of approximation

$$\gamma_P(P) = \frac{|\underline{P}(P)|}{|P|} = 2/3$$

Quality of approximation of classification

$$\gamma_P(Cl) = \frac{|\underline{P}(P)| + |\underline{P}(R)| + |\underline{P}(S)|}{|U|} = 4/6$$

Example rough membership degrees

$$\mu_P^P(A2) = \mu_R^P(A2) = 1/2, \ \mu_S^P(A6) = 1, \ \mu_R^P(A6) = 0$$

Example rules and their parameters

- *if* X1=5, *then* P

  $\sup = 1, \sigma = 1/6, cer = 1/2, \text{cov} = 1/3$
- *if* X2=5, *then* S

  $\sup = 2, \sigma = 2/6, cer = 2/2, \text{cov} = 2/2$
- *if* X1=2 and X2=5, *then* S

  $\sup = 1, \sigma = 1/6, cer = 1/1, \text{cov} = 1/2$

Reducts

- {X1} – no becasue A3 and A5 become indiscernible
- {X2} – yes
- {X1,X2} – no (since {X2} is a reduct)

Core: {X2}

III. Assume that *P* = {X, Y, Z} is a set of condition attributes for the below-provided decision table, and DEC is a decision attribute. Compute the lower and upper approximations of classes E, F, and G and their boundaries.

Object	X	Y	Z	DEC	Lower approximation	Upper approximations	Class boundaries
1	2	2	2	E	$\underline{P}$(E) = { 2, 3 } $\underline{P}$(F) = { 5, 6 } $\underline{P}$(G) = { 8, 9, 10 } lower of the concept contains all objects which definitely belong to the concept	$\overline{P}$ (E) = { 1, 2, 3, 4, 7 } $\overline{P}$ (F) = { 5, 6 } $\overline{P}$ (G) = { 7, 8, 9, 10, 1, 4 } upper of the concept contains all objects which possibly belong to the concept	*Bn*$_P$(E) = { 1, 4, 7 } *Bn*$_P$(F) = { ∅ } *Bn*$_P$(G) = { 7, 1, 4 } Bn = upper - lower
2	2	1	2	E			
3	1	1	3	E			
4	2	2	2	E			
5	3	2	2	F			
6	3	2	2	F			
7	2	2	2	G			
8	2	1	3	G			
9	1	2	2	G			
10	2	1	3	G			

IV. While considering data from the previous task, indicate for each statement its truth (T) or falsity (F).

The accuracy of approximation for class F is 1/2, i.e., $\alpha_P(F) = 1/2$	F	
The rough membership for object 7 in view of class G is 1/2, i.e., $\mu_G(7) = 1/2$	F	Slide 183
The quality of approximation of classification is $\gamma_P(Cl) = 7/10$	T	
The coverage factor for rule "*if* X = 3, *then* DEC=F" is 1	T	Slide 193
The certainty factor for rule "*if* Y = 1 and Z = 3, *then* DEC=E" is 1/2	F	
The set of condition attributes {Y, Z} is a reduct	F	

V. Consider the below decision table concerning seven objects (O1-O7) with a set of condition attributes *P* = {C1, C2, C3} and a decision attribute D. Compute the lower and upper approximations of classes A and B and the respective boundaries. Compute the accuracy and quality of approximation for each class. Compute the quality of approximation of classification. Indicate reducts and the core.

Object	C1	C2	C3	D
O1	a	1	+	B
O2	a	3	-	A
O3	a	2	+	A
O4	b	1	-	B
O5	a	2	+	A
O6	b	3	+	B
O7	a	1	+	A

For class A:

$\underline{P}(A) =$ {2,3,5}

$\overline{P}(A) =$ {2,3,5,1,7}

$Bn_P(A) =$ {1,7}

$\alpha_P(A) = \dfrac{|\underline{P}(A)|}{|\overline{P}(A)|} =$ 3/5

$\gamma_P(A) = \dfrac{|\underline{P}(A)|}{|A|} =$ 3/4

For class B:

$\underline{P}(B) =$ {4, 6}

$\overline{P}(B) =$ {4,6,1,7}

$Bn_P(B) =$ {1,7}

$\alpha_P(B) = \dfrac{|\underline{P}(B)|}{|\overline{P}(B)|} =$ 2/4

$\gamma_P(B) = \dfrac{|\underline{P}(B)|}{|B|} =$ 2/3

For two classes (A and B):

$Bn_P(A) = Bn_P(B)$

Quality of approximation of classification:

$\gamma_P(Cl) = \dfrac{|\underline{P}(A) + \underline{P}(B)|}{|U|} =$ 5/7

Reducts: {C1, C2}, {C2, C3}

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Core: {C1, C2} INTERSECTS {C2, C3} in C2

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Analyze the minimal sets of certain, possible, and approximate rules generated with LEM2 to make sure you understand how they should be induced.

Certain rules	Possible rules	Approximate rules
if C2 = 2, *then* D = A	*if* C1 = a, *then* possibly D = A	*if* C2 = 1 and C1 = a, *then* D = A or D = B
if C2 = 3 and C3 = -, *then* D = A	*if* C2 = 1, *then* possibly D = B	
if C1 = b, *then* D = B	*if* C1 = b, *then* possibly D = B	

VI. Consider the below decision table with a set of condition attributes *P* = {C, D, E} and a decision attribute DEC. It is known that $\underline{P}$(F)={*a, b, c, d*}, $\underline{P}$(G)={*f, g*}, $\overline{P}$ (F)={*a, b, c, d, e, i*}, $\overline{P}$ (H)={*e, h, i, j*}, and Bn_P(F)=Bn_P(H)={*e, i*}. Induce the following minimal sets of rules from the above approximations: certain rules for class G, possible rules for class F, and approximate rules for classes F and H. The rules need to have the following syntax: "if (conjunction of elementary conditions), then decision".

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Object	C	D	E	DEC
a	Y	T	A	F
b	Y	Y	A	F
c	Y	Y	B	F
d	K	K	B	F
e	Y	Y	K	F
f	T	Y	Y	G
g	T	Y	T	G
h	T	K	K	H
i	Y	Y	K	H
j	T	K	Y	H

Certain rule for class G: Certain decision rule supported by objects from the lower approximation

Possible rules for class F: Possible decision rule supported by objects from the upper approximation

Approximate ruels for classes F and H: Approximate decision rule supported by objects from the boundary

VII. Consider the below decision table (objects A-F; the condition attributes are: Headache, Myalgia, and Temperature; the decision attribute is Flue) and a set of certain rules for classes ”Yes” and ”No”.

Object	Headache	Myalgia	Temperature	Flu		Certain rules
A	no	yes	high	Yes		R1) *if* Headache = yes *then* Flu = Yes
B	yes	yes	normal	Yes		R2) *if* Myalgia = yes and Temperature = high, *then* Flu = Yes
C	yes	yes	high	Yes		R3) *if* Headache = no and Temperature = normal, *then* Flu = No
D	no	yes	normal	No		R4) *if* Myalgia = no, *then* Flu = No
E	no	no	high	No		
F	no	yes	normal	No		

Classify the previously unseen objects G, H, and I (I is missing evaluation on Headache, which has not been recorded). In the case there are completely matching rules, take only these rules into account. Otherwise, consider all partially matching rules. Indicate their symbols (R1-R4) in the below table. Then, compute the support for each class (Yes or No) and indicate the final class assignments (Yes or No). In case of tied supports, indicate all classes with the maximal support.

Object	Headache	Myalgia	Temperature	Flu	Object	G	H	I
G	yes	no	high	?	(Completely or partially) matching rules:	R1, R4		
H	no	no	normal	?	Slides 205, 207 Support:	Yes: 2*1= 2 No: 3*1 = 3	Yes: No:	Yes: No:
I		yes	normal	?	Assignment:	No		