• 1

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Lim 3.

$$\lim_{x\to 0^+} \frac{\sqrt{1-\cos x}}{\sin x} = \lim_{x\to 0^+} \frac{\sqrt{1-\cos x}\cdot}{\sin x \cdot} \frac{\cdot\sqrt{1+\cos x}}{\cdot\sqrt{1+\cos x}} = \lim_{x\to 0^+} \frac{|1-\cos^2 x|}{\sin x\cdot\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0^+} \frac{|\sin x|}{\sin x\cdot\sqrt{1+\cos x}} =$$

$$= \lim_{x\to 0^+} \frac{1}{\sqrt{1+\cos x}} = \frac{1}{\sqrt{1+\cos 0}}$$

$$= \frac{1}{\sqrt{2}}.$$

$$\lim_{x\to 0^-} \frac{\sqrt{1-\cos x}}{\sin x} = \lim_{x\to 0^-} \frac{\sqrt{1-\cos x}\cdot}{\sin x\cdot} \frac{\cdot\sqrt{1+\cos x}}{\cdot\sqrt{1+\cos x}} = \lim_{x\to 0^-} \frac{|1-\cos^2 x|}{\sin x\cdot\sqrt{1+\cos x}}$$

$$= \lim_{x\to 0^-} \frac{|\sin x|}{\sin x\cdot\sqrt{1+\cos x}} = \frac{-1}{\sqrt{1+\cos 0}}$$

$$= -\frac{1}{\sqrt{2}}.$$

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Der 3.

$f(x) = \sqrt{\ln(1+x^2)}$ is different. at $x = 0$?

$$f'_{+}(0) = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h} = ?$$

$$\sqrt{\ln(1+x^2)} = \frac{\sqrt{\ln(1+(0+h)^2)} - \sqrt{\ln(1+0^2)}}{h} =$$

$$= \frac{\sqrt{\ln(1+h^2)} - \sqrt{\ln 1}}{h} = \frac{\sqrt{\ln(1+h^2)}}{h}$$

$$= \frac{-0}{} = \frac{\sqrt{\ln(1+h^2)}}{h}.$$

$$\lim_{h \to 0^+} \frac{\sqrt{\ln(1+h^2)}}{h} = \sqrt{\lim_{h \to 0^+} \frac{\ln(1+h^2)}{h^2}} =$$

$$= \frac{[0]}{[0]} = \sqrt{\lim_{h \to 0^+} \frac{\frac{2h}{h^2+1}}{2h}} = \sqrt{\lim_{h \to 0^+} \frac{1}{h^2+1}}$$

$$= \sqrt{\frac{1}{0+1}} = 1.$$

$$\lim_{h \to 0^-} \frac{\sqrt{\ln(1+h^2)}}{h} =$$

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$$= -\sqrt{\lim_{h\to 0^-} \frac{\ln(1+h^2)}{h^2}} = \frac{[0]}{[0]} =$$

$$= -\sqrt{\lim_{h\to 0^-} \frac{\frac{2h}{h^2+1}}{2h}} = -\sqrt{\lim_{h\to 0^-} \frac{1}{h^2+1}}$$

$$= -\sqrt{\frac{1}{0+1}} = -1.$$

• $\lim_{h\to 0^+} f(h) \neq \lim_{h\to 0^-} f(x) \Rightarrow f(x)$

is not differentiable at

$x = 0$.

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TaL 5.

TLE to the curve: $y\cos x = x^3 + x\sin y + \frac{\pi}{2}$ going through the point $(0, \frac{\pi}{2})$.

TLE: $y = y(a) + y'(a)(x-a)$

$a = 0$, $y(a) = \frac{\pi}{2}$.

$y'(a) = 1$.

$y = \frac{\pi}{2} + 1(x-0)$.

st.
3)

TLE: $y = \frac{\pi}{2} + x$.

$$y'(x) = \frac{3x^2 + \sin y + x\cdot\cos y + \sin x}{\cos^2 x} =$$

$$y'(a) = \frac{0+1+0+0}{1} = 1.$$

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4

Alp 9.

Taylor polynomial of degree 4 for $f(x)=\cos x$ about 0.

$$P_4(x) = f(0) - f'(0)\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f''''(0)}{4!}x^4.$$

$f(0) = \cos 0 = 1.$

$f'(0) = -\sin 0 = 0.$

$f''(0) = -\cos 0 = -1.$

$f'''(0) = \sin 0 = 0.$

$f''''(0) = \cos 0 = 1.$

$$P_4(x) = 1 - 0 + \frac{-1}{2}\cdot x^2 + 0 + \frac{1}{24}\cdot x^4.$$

$$P_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4.$$

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SkG 3.

$$f(x) = x \cdot e^{-x^2/2}$$

a) $D_f = \mathbb{R}$

b) • $\lim\limits_{x \to +\infty} x \cdot e^{-x^2/2} =$

$$= \lim_{x \to +\infty} \frac{x}{e^{x^2/2}} = \frac{[\text{large}]^{\infty}}{[\text{large}]_{\infty}} \overset{LH}{=} \lim_{x \to +\infty} \frac{1}{e^{x^2/2} \cdot x} = \frac{1}{\infty} = 0.$$

• $\lim\limits_{x \to -\infty} x \cdot e^{-x^2/2} = \lim\limits_{x \to -\infty} \frac{x}{e^{x^2/2}}$

$$= \frac{[\infty]}{[\infty]} \overset{LH}{=} \lim_{x \to -\infty} \frac{1}{e^{x^2/2} \cdot x} =$$

$$= \frac{1}{-\infty} = 0.$$

• $a = \lim\limits_{x \to \infty} \frac{f(x)}{x} = \lim\limits_{x \to \infty} e^{-x^2/2} =$

$$= \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0.$$

=> no oblique asymptots.

c) CP a
$f'(x)$
$= 1 \cdot e$
$\cdot (- \frac{1}{2} \cdot$
$- \cdot x^2 \cdot e$
$\cdot (1 -$

CP: f
$x = -1$
SP: no
d) ~~x~~
$f'($
~~f(~~

• $f(x) >$
=> $\frac{1-}{e}$

=> f(

c) CP and SP:

$$f'(x) = \left(x\cdot e^{-x^2/2}\right)' =$$

$$= 1\cdot e^{-x^2/2} + x\cdot e^{-x^2/2}\cdot\left(-\frac{1}{2}\cdot 2x\right) = 1\cdot e^{-x^2/2} - x^2\cdot e^{-x^2/2} = e^{-x^2/2}\cdot(1-x^2) = \frac{1-x^2}{e^{x^2/2}}.$$

CP: $f'(x)=0 \iff 1-x^2=0$

$x=-1$ or $x=+1$.

SP: no.

d) ~~x $f'(x)$ $f(x)$~~

• $f(x)>0$ where $f'(x)>0$

$$\Rightarrow \frac{1-x^2}{\underbrace{e^{x^2/2}}_{>0}}>0. \qquad 1-x^2>0.$$

$$(1-x)(1+x)>0$$

$$x<1 \quad x>-1$$

$\Rightarrow f(x)>0$ at $x\in(-1;+1)$.

• $f(x) < 0$ at $x \in (-\infty; -1) \cup$
$\cup (1; +\infty)$.

e) $f''(x) = \left(\dfrac{1-x^2}{e^{x^2/2}}\right)' =$

$= \dfrac{-2x \cdot e^{x^2/2} - (1-x^2) \cdot e^{x^2/2} \cdot \frac{1}{2} \cdot 2x}{(e^{x^2/2})^2} = \dfrac{e^{x^2/2}(-2x - (1-x^2) \cdot x)}{(e^{x^2/2})^2} = \dfrac{-2x - x + x^3}{e^{x^2/2}} =$

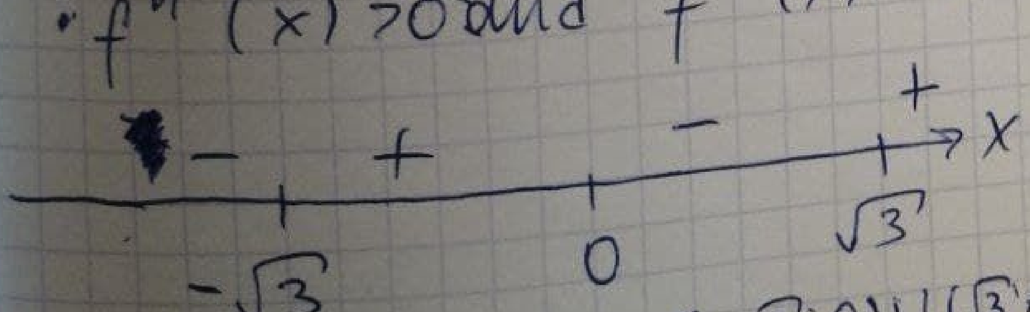
$= \dfrac{x^3 - 3x}{e^{x^2/2}}$.

• $f''(x) = 0$: $x^3 - 3x = 0$.

$x(x^2 - 3) = 0$.

$x = 0 \quad x = \pm\sqrt{3}$.

• $f''(x) > 0$ and $f''(x) < 0$:



• $f''(x) > 0$ at $x \in (-\sqrt{3}; 0) \cup (\sqrt{3}; +\infty)$.

• $f''(x) < 0$ at $x \in (-\infty; -\sqrt{3}) \cup (0; \sqrt{3})$.

Sketch the graph!

x	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$
$f'(x)$	$-$	$-$ 0	$+$ $+$	0 $-$	$-$
$f''(x)$	$-$ 0	$+$	$+$ 0 $-$	$-$	0 $+$
$f(x)$	$\searrow$	$\searrow$ $-\frac{1}{2}$	$\nearrow$ 0 $\nearrow$	$\searrow$	$\frac{1}{2}$ $\searrow$