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QUANTIFIER LAWS

De Morgan's Laws

$$\sim \forall_x P(x) \Leftrightarrow \exists_x \sim P(x)$$

$$\sim \exists_x P(x) \Leftrightarrow \forall_x \sim P(x)$$

Laws of Quantifiers Distribution

$$\forall_x (P(x) \wedge Q(x)) \Leftrightarrow \forall_x P(x) \wedge \forall_x Q(x)$$

$$\exists_x (P(x) \vee Q(x)) \Leftrightarrow \exists_x P(x) \vee \exists_x Q(x)$$

$$\forall_x P(x) \vee \forall_x Q(x) \Rightarrow \forall_x (P(x) \vee Q(x))$$

$$\exists_x (P(x) \wedge Q(x)) \Rightarrow \exists_x P(x) \wedge \exists_x Q(x)$$

Laws of Quantifiers (In)dependence

$$\forall_x \forall_y P(x,y) \Leftrightarrow \forall_y \forall_x P(x,y)$$

$$\exists_x \exists_y P(x,y) \Leftrightarrow \exists_y \exists_x P(x,y)$$

Laws of Quantifier Movement

$$(\Phi \rightarrow \forall_x \Psi(x)) \Leftrightarrow \forall_x (\Phi \rightarrow \Psi(x))$$

provided that x is not free in Φ

$$(\Phi \rightarrow \exists_x \Psi(x)) \Leftrightarrow \exists_x (\Phi \rightarrow \Psi(x))$$

provided that x is not free in Φ

$$\forall_x \Phi(x) \rightarrow \Psi \Leftrightarrow \exists_x (\Phi(x) \rightarrow \Psi)$$

provided that x is not free in Ψ

$$\exists_x \Phi(x) \to \Psi \Leftrightarrow \forall_x (\Phi(x) \to \Psi)$$

provided that x is not free in Ψ

$$(\Phi \wedge \forall_x \Psi(x)) \Leftrightarrow \forall_x (\Phi \wedge \Psi(x))$$

provided that x is not free in Φ

$$(\Phi \wedge \exists_x \Psi(x)) \Leftrightarrow \exists_x (\Phi \wedge \Psi(x))$$

provided that x is not free in Φ

$$(\Phi \vee \forall_x \Psi(x)) \Leftrightarrow \forall_x (\Phi \vee \Psi(x))$$

provided that x is not free in Φ

$$(\Phi \vee \exists_x \Psi(x)) \Leftrightarrow \exists_x (\Phi \vee \Psi(x))$$

provided that x is not free in Φ