Introduction to probability: Exercises

1. Classical and geometric probability

2.03.2021

Exercise 1. Basic counting.

(a) In how many ways one can arrange 5 people in a queue?

(b) In how many ways one can form a 5-letter word (even meaningless) from letters A, B, and C

(c) A local telephone number of a 7-digit sequence, but the first digit has to be different from 0 or 1. How many distinct numbers are there?

(d) Count the number of words that consist of four distinct letters (from a 26-letter alphabet)

(e) You have n_1 academic book, n_2 fantasy books and n_3 contemporary books. In how many different ways can you arrange them on a shelf so that the books of the same genre are contiguous?

Exercise 2. We draw three white or black marbles from an urn. Let A denote an event that *there is exactly one while marble in our draw*, B – *at most one white marble*, C – *at least one white marble*. Describe (in words) the following events: $A', B', C', A \cup B, A \cap B, B \cup C, B \cap C, B' \cap C'$.

Exercise 3. How many distinct words can you make from word BABA by simply changing the order of the letters? How about the word BARBARA?

In each of the following exercises, define the sample space Ω and a relevant event

Exercise 4. What are the chances of getting 6,5,4,3,2,1 out of 6 numbers matched in the Lotto game?

Exercise 5. We create a random word of length 8 from 26 Latin letters. What is the probability that every letter will be distinct (no two letters are the same)?

Exercise 6. There are 100 devices in a batch, 5 of which are defective. We randomly select 20 devices to test and accept the whole batch if there is no more than one defective device in our sample. What is the probability of accepting the batch?

Exercise 7. 50 people form a queue in a random order. What is the chance than the oldest person stands next to the youngest one?

Exercise 8. We randomly form a 4-digit number out of digits $0, \ldots, 9$ (with repetitions, but the first digit *cannot* be 0). What is the probability that the number is

(a) Divisible by 3?

(b) Divisible by 10?

Exercise 9. Calculate the probability that a randomly selected binary sequence of length n has exactly k ones?

Exercise 10. In a restaurant, 10 customers ordered 10 different dishes. Unfortunately, the waiter wrote down the dishes only, but not who ordered them. He then decided to give the dishes to the customers in a random order. Calculate the probability that

(a) A given, fixed customer will get his or her own dish.

(b) A given couple sitting at a given table will receive a pair of dishes they ordered.

(c) Everyone will receive their own dishes.

Exercise 11. In a poker game, we draw 5 out of 52 cards. What is the probability of obtaining:

(a) One pair

(b) Three of a kind

(c) Flush (all cards are of the same suit)

Exercise 12. There are 10 girls and 10 boys in a classroom which has 10 double (2-person) desks. The pupils were assign their seats at random. Calculate the probability that

(a) At a given (e.g., first) desk, there sit a girl and a boy.

(b) At a given desk, there sit two girls.

(c) At every desks there sits a mixed couple (a girl and a boy).

Exercise 13. At the round table with 20 chairs, 10 (heterosexual) marriages were assigned seats in a completely random order. Calculate the probability that

(a) A given husband (e.g. Bob) sits next to his wife.

(b) A given wife (e.g. Alice) sits in between two other wives.

(c) No marriage is split, i.e. every husband and wife are sitting next to each other.

Exercise 14. There are 3 buses which arrive at a certain bus-stop in a random time between 10:00 and 10:15 minutes. What is the chance that a passenger (arriving at 10:00) will wait for the first bus less than 5 minutes?

Exercise 15. A number was randomly drawn from a unit interval $[0, 1]$. What is the probability that the number is rational?

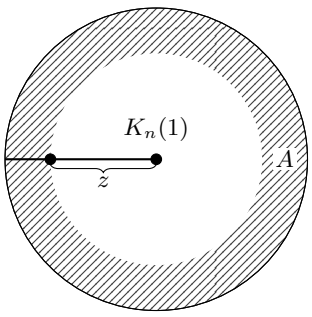
Exercise 16. A stick of length 10 cm is broken at a random point. What is the probability that the area of a rectangle formed from the two parts is less than 10 cm^2?

Exercise 17. We randomly select two points from interval $[0, 1]$, which split the interval into three parts. What is the probability that one can form a triangle out of these parts?

Exercise 18. Alice and Bob go on a date. They arrive independently at a random moment between 10:00 and 11:00. Alice waits for Bob no more than 10 minutes, while Bob waits for Alice no more than 20 minutes. What is the probability that they will meet?

Exercise 19. Define an *n-dimensional ball of radius r*:

$$K_n(r) = \left\{(x_1, \ldots, x_n) \in \mathbb{R}^n \colon \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2} \leq r\right\}$$



What is the probability that a randomly selected point from $K_n(1)$ will be more than $z \in (0, 1)$ away from the origin? *Hint*: The n-volume of the n-dimensional ball of radius r is given by $|K_n(r)| = C_n r^n$, where C_n is a dimension-dependent factor ($C_1 = 2, C_2 = \pi, C_3 = \frac{4}{3}\pi, \ldots$).