

DECISION ANALYSIS – SHORT EXERCISES IV – ANALYTIC HIERARCHY PROCESS AND CHOQUET INTEGRAL

I. Given the below incomplete pairwise comparison matrix, make it complete to satisfy the consistency condition of pairwise comparisons (CCPC) and cardinal consistency condition (CCC).

	a_1	a_2	a_3	a_4
a_1	1	3	6	$1/3$
a_2	$1/3$	1	2	$1/9$
a_3	$1/6$	$1/2$	1	$1/18$
a_4	3	9	18	1

II. Consider the below entirely consistent pairwise comparison matrix. Compute the priorities ($w_1 - w_4$) corresponding to the compared alternatives.

	a_1	a_2	a_3	a_4
a_1	1	$1/2$	1	3
a_2	2	1	2	6
a_3	1	$1/2$	1	3
a_4	$1/3$	$1/6$	$1/3$	1

col_j $13/3$ $13/6$ $13/3$ 13

$$w_1 = 3/13 + 6/26 + 3/13 + 3/13 = 0.923$$

$$w_2 = 6/13 + 6/13 + 6/13 + 6/13 = 1.846$$

$$w_3 = 3/13 + 6/26 + 3/13 + 3/13 = 0.923$$

$$w_4 = 3/39 + 6/78 + 3/39 + 1/13 = 0.3$$

III. Consider four alternatives X, Y, W, and Z evaluated in terms of three criteria g_1, g_2, g_3 of gain type. For each statement, indicate its truth (T) or falsity (F) ($>$ denotes a preference relation).

	Alternative	g_1	g_2	g_3		
19	X	8	4	7	Relations $X > W$ and $Y > Z$ can be represented using a weighted sum model	T
19	Y	8	6	5	Relations $W > X$ and $Z > Y$ can be represented using a weighted sum model	F
14	W	3	4	7	Relations $X > Y$ and $W > Z$ can be represented using an additive value function	T
14	Z	3	6	5	Relations $X > Y$ and $Z > W$ can be represented using the Choquet integral	F

IV. Consider the capacities for all subsets of criteria: $u(\emptyset) = 0$, $u(\{g_1\}) = 0.3$, $u(\{g_2\}) = 0.4$, $u(\{g_3\}) = 0.5$, $u(\{g_1, g_2\}) = 0.8$, $u(\{g_1, g_3\}) = 0.6$, $u(\{g_2, g_3\}) = 0.7$, $u(\{g_1, g_2, g_3\}) = 1$. Compute the Choquet integral for alternative $A = [3, 6, 5]$.

$$Ch(A) = [3 - 0] * u(\{g_1, g_2, g_3\}) + [6 - 3] * u(\{g_2\}) + [5 - 3] * u(\{g_3\}) = 3*1 + 3*0.4 + 2*0.5 = 3 + 1.2 + 1 = 5.2$$