Exam retake 23.09.21
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ex. 1.

Let $f(x) = x^3$ on $[0, a]$, $a > 0$.

a) L and U Riemann sums - ?

Hint: $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

~~Form~~ Partition $P_n : 0 < \frac{a}{n} < \frac{2a}{n} < \frac{3a}{n} <$

$< \dots < \frac{n \cdot a}{n} = a$.

Therefore:

$x_{k-1} = \frac{(k-1)a}{n}$. length of step $= \frac{a}{n}$.

$x_k = \frac{k \cdot a}{n}$.

$\Rightarrow [x_{k-1}, x_k] = \left[\frac{(k-1)a}{n}; \frac{ka}{n}\right]$.

$\Rightarrow L(f, P_n) = \sum_{k=1}^{n} \left[\frac{(k-1)a}{n}\right]^3 \cdot \frac{a}{n} =$

$= \frac{a^4}{n^4} \cdot \sum_{k=1}^{n} (k-1)^3 = \frac{a^4}{n^4} \cdot$

$\cdot \frac{n^2(n-1)^2}{4}$.

ex. 1 continue.

$$U(f, P_n) = \sum_{k=1}^{n} \left[\frac{ka}{n}\right]^3 \cdot \frac{a}{n} =$$

$$= \frac{a^4}{n^4} \cdot \sum_{k=1}^{n} k^3 = \frac{a^4}{n^4} \cdot \frac{n^2(n+1)^2}{4}.$$

=?

=?

ex. 1. Critical points:

$$f(x,y) = \frac{1}{x} + \frac{4}{y} + \frac{9}{4-x-y}.$$

$$f_{xx} = -\frac{1}{x^2} + \left(9 \cdot (4-x-y)^{-1}\right)' =$$

$$= -\frac{1}{x^2} + \left(-\frac{-9 \cdot 1}{(4-y-x)^2}\right) = \frac{1}{x^2} \cancel{\ldots}$$

$$\cancel{\ldots} = \frac{1}{x^2} + \frac{9}{(4-x-y)^2}.$$

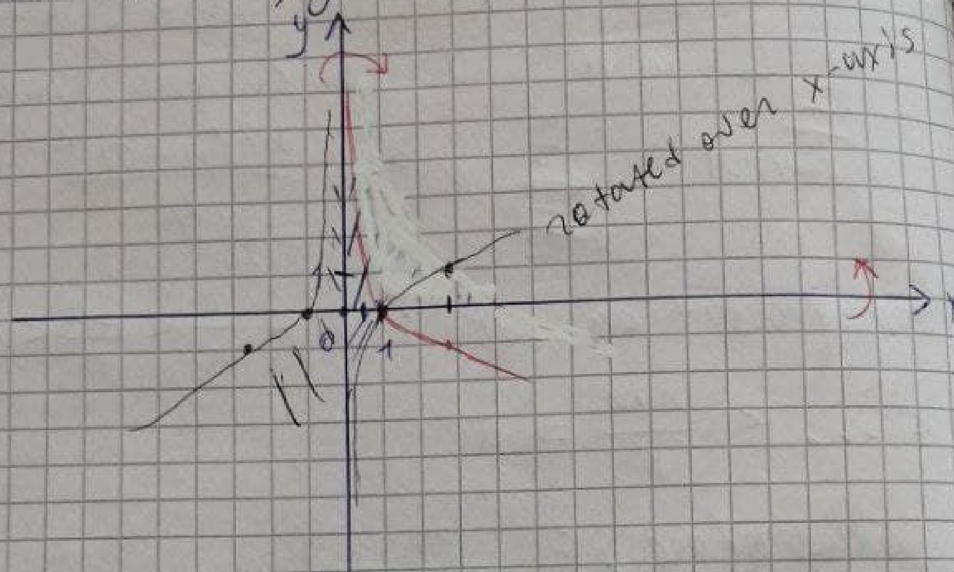
$$f_{yy} = -\frac{4}{y^2} + \frac{1 \cdot 9}{(-x-y+4)^2} = -\frac{4}{y^2}.$$

$$\cancel{\ldots} = -\frac{4}{y^2} + \frac{9}{(4-x-y)^2}.$$

$$\cancel{\begin{cases} \frac{1}{x^2} \cdot \frac{5}{4} = 0 \\ -\frac{7}{4} \cdot \frac{1}{y^2} = 0. \end{cases}} \quad =? \quad \cancel{\begin{cases} \frac{5}{4x^2} = 0 \\ -\frac{7}{4y^2} = 0 \end{cases}} \quad \begin{cases} \cancel{x} \\ \cancel{y} \end{cases}$$

ex. 2.

a) Draw the plane region R bounded by $y = -\ln x$, $0 < x \le 1$ and $x = 0$, $y = 0$.



b) Find the area R – ?

c) The region R is rotated around x-axis. Find volume – ?

d) – " – y-axis.

a) $\ln e = 1$, $\ln 1 = 0$, $\ln 0$ doesn't exist.

b) $R = \int_0^1 (-\ln x)\, dx =$

$$= -\int_0^1 (\ln x)\, dx =$$

$$= \left\{ \begin{matrix} u = \ln x & v' = dx \\ u' = \frac{1}{x} dx & v = x \end{matrix} \right\} =$$

$$= \lim_{a \to 0^+} \left(-x \ln x \Big|_a^1\right) + \int_0^1 1\,dx =$$

$$= \lim_{a \to 0^+} \left(-\overset{=0}{\ln 1} + a \ln a\right) +$$

$$+ \int_0^1 1\,dx = \lim_{a \to 0^+} (\underbrace{a \ln a}_{\to 0}) +$$

$$+ \int_0^1 1\,dx = 0 + \int_0^1 1\,dx =$$

$$= x \Big|_0^1 = 1 - 0 = 1.$$

c) over x-axis:

$$V = \pi \cdot \int_0^1 \ln x\,dx =$$

$$= \pi \cdot (-1) = \pi \cdot (-1) = -\pi.$$

from (b)

d) over y-axis:

$$V = \pi \cdot \int_{-1}^0 (-\ln(-x)) = \pi \cdot \int_{-1}^0 (-\ln(-x)) =$$

$$= -\pi \cdot \int_{-1}^0 \ln(-x) = \left\{ \begin{matrix} u = \ln(-x) & v' = 1 \\ u' = \frac{1}{x}\,dx & v = x \end{matrix} \right\}$$

$$= \lim_{b \to 0^-} \left(-x \ln(-x) \Big|_{-1}^b\right) +$$

$$+ \int_{-1}^0 1\,dx = \lim_{b \to 0^-} \left(-\underbrace{b \ln(-b)}_{\to 0} - \overset{=0}{\ln 1}\right) + x \Big|_{-1}^0 = 0 + 1 = 1.$$

ex. 1. conv / div - ?

$$\sum_{k=1}^{\infty} \frac{\ln k}{k^{5/4}} = \sum_{k=1}^{\infty} \frac{\ln k}{\sqrt[4]{k^5}}.$$

Using comparison test:

Since smaller sum:

$$\sum_{k=1}^{\infty} \frac{\ln k}{k^2} \text{ converges}$$

$$\left(\sum_{k=1}^{\infty} \frac{\ln k}{k^2} < \sum_{k=1}^{\infty} \frac{\ln k}{k^{5/4}} \right)$$

then bigger sum converges as well.

ex. 1.

Solve IVP:

$$\begin{cases} y'''(x) - y'(x) = x \\ y(0) = 1,\ y'(0) = 2,\ y''(0) = 3 \end{cases}$$

1) homogen. eq.:

$$y'''(x) - y'(x) = 0$$

$$\lambda^3 - \lambda^1 = 0$$

$$\lambda(\lambda^2 - 1) = 0$$

$$\lambda_1 = 0 \quad \text{or} \quad (\lambda - 1)(\lambda + 1) = 0$$

$$\lambda_2 = \pm 1$$

$$\Rightarrow y_H = c_1 e^0 + c_2 e^{-x} + c_3 e^x$$

$$= c_1 + c_2 e^{-x} + c e^x$$

2) particular:

guess: $y_p = Ae^x$ – no.

$$y_p = Ax \cdot e^x$$

$$\Rightarrow y_p' = A \cdot e^x + e^x \cdot Ax = e^x(A + Ax)$$

$$y_p'' = e^x \cdot A + e^x \cdot Ax + A \cdot e^x = e^x(A + Ax + A) = e^x(2A + Ax)$$

$$y_p''' = e^x \cdot A + e^x \cdot Ax + A \cdot e^x + e^x A = e^x(A + Ax + A + A) = e^x(3A + Ax)$$

$\cancel{\Rightarrow e^x(3A+Ax) - e^x(A+Ax) =}$
$= x.$
$\cancel{e^x(3A+Ax-A-Ax) = e^{...}}$

~~2) particular:~~

$\cancel{\text{guess: } y_p = Ax.}$

$\cancel{y_p' = A.}$

$\cancel{y_p''' = 0.}$

$\cancel{3A + Ax = 0.}$

2) particular:

$$y_p = x(A_1 + A_2x^2).$$

$$y_p' = A_1 + 2A_2x.$$

$$y_p''' = 0.$$

$$\Rightarrow 0 - (A_1 + 2A_2x) = x$$

$$-A_1 - 2A_2x = x.$$

$$\Rightarrow -A_1 = 0 \Rightarrow A_1 = 0.$$

$$\Rightarrow -2A_2 = 1 \Rightarrow A_2 = -\frac{1}{2}.$$

$$\Rightarrow y_p = -\frac{x^2}{2}.$$

$$\Rightarrow y = y_H + y_p = c_1 + c_2e^{-x} + ce^x - \frac{x^2}{2}.$$