DECISION ANALYSIS – EXERCISES VII – SOLUTION CONCEPTS IN STRATEGIC GAMES

I. Indicate the truth (T) or falsity (F) for the below statements.

a) In the normal-form strategic games, players take action sequentially	
b) The pure Nash equilibrium is not always Pareto efficient	
c) For a normal-form strategic game involving two players and two actions for each of them, there is always at most one pure Nash equilibrium	
d) For the coordination games, all Nash equilibria are always Pareto efficient	
e) There is no pure Nash equilibrium for rock-paper-scissors	
f) A mixed strategy is fully mixed if its support contains at least two actions	
g) Every normal-form game has at least one Nash equilibrium	
h) Various orders of eliminating strictly dominated strategies can lead to different reduced games	
i) The equilibrium in dominant strategies is non-empty for all normal-form games	
j) The correlated equilibrium is non-empty for all normal-form games	

II. Consider the below normal-form strategic game involving two payers, A and B, and solve the following five sub-tasks:

a) identify dominated/dominating strategies, if any, for players A and B

the dominating strategy for player A is … \ is not existing (B,L)

the dominating strategy for player B is … \ is not existing

b) find all pure Nash equilibria; the pure Nash equilibrium is (B , R)

c) justify why (T,L) is not a pure Nash equilibrium (hint: analyze actions of player A)

Because it makes sense for player are to change strategy to (B,L), whereas for player B (T,L) is the best possible outcome

d) justify whether (T,L) or (B,R) or (B,L) is a Pareto efficient action profile

Because you cannot improve things for one player (A) without harming the other one (B)

e) change the least number of utilities in the pay-off matrix to transform the normal-form game into a coordination game

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A \ B	L	R
T	2 \ 2	0 \ 1
B	1 3 \ 0	1 \ 1

III. Consider the below normal-form strategic game involving two payers, A and B, and solve the following four sub-tasks:

a) find all Nash equilibria (including the mixed ones) by drawing a diagram with the best responses for players A and B

hint: for player A, consider $u_A(T,q) \geq u_A(B,q)$, i.e., $0q - 10(1-q) \geq -1q - 6(1-q)$

for player B, consider $u_A(L,p) \geq u_A(R,p)$, i.e., $0p + 0(1-p) \geq 10p - 90(1-p)$

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b) compute an expected utility for player A / B for the following mixed strategy profile s=((0.5, 0.5), (0.1, 0.9)) Slide 20

$u_A(s) = 0 \cdot 0.5 \cdot 0.1 + (-10) \cdot 0.5 \cdot 0.9 + (-1) \cdot 0.5 \cdot 0.1 + (-6) \cdot 0.5 \cdot 0.9 =$ -7.25

$u_B(s) =$ 0*0.5*0.1 + 0*0.5*0.9 + 10*0.5*0.1 + (-90)*0.5*0.9 = -40

c) justify why the above mixed strategy profile is not a Nash equilibrium

d) change the least number of utilities in the below pay-off matrix to transform the normal-form game into a zero-sum game

A \ B	L (q)	R ($1-q$)
T (p)	0 \ 0	-10 \ 10
B ($1-p$)	-1 \ 0 1	-6 \ -90 6

IV. Consider the below normal-form strategic game involving two payers, A and B, and eliminate the strictly dominated strategies to identify the maximally reduced game. Slide 31, 33

A \ B	L	~~C~~	R
T	2 \ 3	~~2 \ 1~~	2 \ 0
M	3 \ 0	~~1 \ 1~~	0 \ 3
~~B~~	~~1 \ 3~~	~~1 \ 1~~	~~1 \ 0~~

Decision Analysis – Exercises VIII – Congestion and Extensive Games

I. Indicate the truth (T) or falsity (F) for the below statements.

a) Every congestion game has at least one pure Nash equilibrium
b) The matching pennies games is not a potential game
c) It is not guaranteed that each potential game has at least one pure Nash equilibrium
d) Every congestion game is a potential game
e) An action profile that does not admit a better response is a pure Nash equilibrium
f) Every congestion game has the finite improvement property
g) Every normal-form game has at least one Nash equilibrium
h) The definition of An extensive-form game involves, e.g., a set of choice nodes, the turn function, and the successor function
i) Every norm-form game can be translated into an extensive-form game
j) Every finite extensive-form game has at least one pure Nash equilibrium
k) The backward induction was originally proposed in the context of tic-tac-toe
l) Every finite extensive-form game has at least one subgame-perfect equilibrium
m) The vackward induction does not guarantee finding a subgame-perfect equilibrium

II. Consider the below strategic games involving two payers, A and B. Are they potential games? If so, define the underlying potential function with P(T,L) = 10. Slide 48

A \ B	L	R
T	2 \ 2	1 \ 1
B	3 \ 0	1 \ 1

The game is not potential:

P(T,L) = 10
P(T,R) = 9
P(B,L) = 11
P(B,R) = 12 and 9

A \ B	L	R
T	2 \ 2	1 \ 1
B	3 \ 0	1 \ 1

III. Consider the below strategic game involving two payers, A and B. Does it have a finite improvement property? If so, identify a pure Nash equilibrium through better response dynamics (mark the path in the table starting in the top-left cell).

Yes, it has the FIP.

(T,L) -> (T,C) -> (T,R)
OR
(T,L) -> (T,C) -> (B,C) -> (B,R) -> (T,R)

Therefore, pure Nash equilibrium: (T,R)

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A \ B	L	C	R
T	2 \ 1	1 \ 3	4 \ 4
B	0 \ 0	2 \ 1	3 \ 3

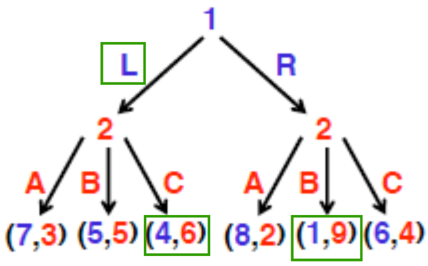
IV. Consider the following congestion game: 5 people need to get from A to B. Everyone can choose between the top and the bottom route. Via the top route, the trip takes 5 minutes (in the other scenario, consider – 6 minutes). Via the bottom route, it depends on the number of fellow travelers: it takes as many minutes x as there are people using this route. Slide 55

5 people overall
action space = [top, bottom]
top delay = 5 min
bottom delay = no. of people on route

a) Define the underlying congestion game (players, resources, action space, delay functions).
b) What are the pure Nash equilibria? Please explain.
c) What is the price of anarchy?

b) sw(x) = -[x*x + (5-x)*5] = -[x^2 - 5x + 25]
maximal for x = 2 and x = 3, minimal for x = 0 and x = 5
pure Nash equilibria: x = 5
c) PoA = sw(5) / sw(2) = -25 / -19 = 1.32

IV. Consider the below presented extensive game involving two players, 1 and 2. Slide 71



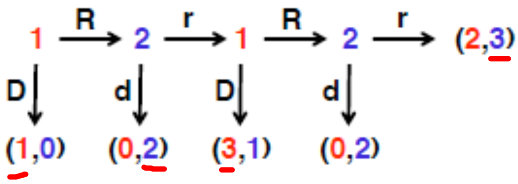
a) Find a pure Nash equilibrium using the backward induction. (L, C-B)

b) Is (L, C-B) the only Nash equilibrium? Yes

c) Is (L, C-C) a subgame perfect-equilibrium? No. Considering node R choice of player R, C=4 is not optimal in this subgame (B leads to much higher utility of 9)

d) Transtorm the extensive game to the normal-form game. Slide 65

V. Consider the below presented centipede game and find a pure Nash equilibrium using a backward induction/



Pure Nash equilibrium: D -> d -> D -> r

Decision Analysis – Exercises IX – Social Choice Theory – Voting Rules

I. Indicate the truth (T) or falsity (F) for the below statements.

a) The winner in the plurality rule does not need to attain more than 50% votes

b) First-past-the-post produces a proportional distribution of seats

c) The elected president in the United States needs to attain more votes in the popular vote than all her opponents

d) In cumulative voting, it is possible to assign more than one vote to a single candidate

e) The presidential elections in Poland are currently decided using the plurality with run-off rule

f) Instant-runoff voting requires voters to participate in a few rounds of the election

g) Hamilton method belongs to the largest remainder approaches

h) The elections to the European Parliament in Poland are decided using the Sainte-Lague method

i) The mixed compensatory system always leads to the proportional distribution of seats

j) The Condorcet winner does not exist for all social profiles

k) Kemeny rule is the Condorcet extension

l) Copeland rule exploits a weighted majority graph

m) The maximal Borda score with m candidates and n voters is $m \cdot (n-1)$

II. Consider the provided social profiles and determine the winner according to the specified voting rules.

Profile (12 voters):

5: A > B > C

4: B > C > A

3: C > B > A

a) Plurality rule:

A: 5 B: 4 C: 2

Winner: A

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b) Anti-plurality rule:

A: 5 B: 12 C: 7

Winner: B

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c) Plurality with run-off:

5: A > B

4: B > A

3: B > A

because A, B have the highest Plurality

Winner: B (B: 7, A: 5)

first place: *2, second: *1

d) Borda count

$BSc(A) = 5 \cdot 2 + 4 \cdot 0 + 3 \cdot 0 = 10$

BSc(B) = 5*1+4*2+3*2=16

BSc(C) = 0+4*1+3*2=10

Ranking: B > A=C

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e) Condorcet method

Pairwise comparisons

	A	B	C
A	-	5	5
B	7	-	9
C	7	3	-

Condorcet winner: B

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f) Bucklin voting

1st count:

A: 5 B: 4 C: 3

2nd count:

A: 5 B: 12 C: 7

Winner: B

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Approved candidates (12 voters):

5: A

4: B, C

3: C

g) Approval voting:

A: 5 B: 4 C: 7

Winner: C

Profile (23 voters):

5: A > B > C > D

7: B > D > C > A

7: C > B > A > D

4: D > C > B > A

Single non-transferable vote for 2 seats

Winners: B, C

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h) Instant-runoff voting:

1st round

A: 5 B: 7 C: 7 D: 4

IRV 2nd round:

5: A > B > C

7: B > C > A

7: C > B > A

4: C > B > A

A: 5
B: 7
C: 11

IRV 3rd round:

5: B > C

7: B > C

7: C > B

4: C > B

C: 11
B: 12

Winner: B

Pairwise comparisons

	A	B	C	D
A	-	5	5	12
B	18	-	12	19
C	18	11	-	12
D	11	4	11	-

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i) Copeland rule:

CSc(A) = -1-1+1 = -1

CSc(B) = 1+1+1 = 3

CSc(C) = 1-1+1 = 1

CSc(D) = -1-1-1 = -3

Ranking: B > C > A > D

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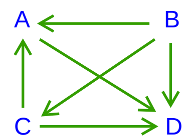
j) Slater rule:

Draw majority graph



Slater ranking:

B -> C -> A -> D

III. Consider the below social profile for 20 voters. Indicate the winners/rankings according to the specified rules.

Profile:
7 : M > W > B
9 : W > B > M
4 : B > M > W

Pairwise comparisons:

	M	W	B
M	-	11	7
W	9	-	16
B	13	4	-

PC (margins):

	M	W	B
M	-	+2	-6
W	-2	-	+12
B	+6	-12	-

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a) Kemeny rule:
M W B - (M vs. W = 11) + (M vs. B = 7) + (W vs. B = 16) = 34
M B W - (M vs. B = 7) + (M vs. W = 11) + (B vs. W = 4) = 22
W M B - (W vs. M = 9) + (W vs. M = 16) + (M vs. B = 7) = 32
W B M - 16 + 9 + 13 = 38
B M W - 13 + 4 + 11 = 28
B W M - 4 + 13 + 16 = 33
Kemeny ranking: W B M

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b) Minimax (MM) rule:
MM(M): 13 MM(W): 11 MM(B): 16
Minimax ranking: W > M > B

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c) Ranked pairs: (W,B) 12 (B,M) 6 (M,W) 2
Ordered pairs: I) (W,B) 12 II) (B,M) 6 cycle (M,W) 2
Ranked pairs ranking: W -> B -> M

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d) Coombs method:
I stage – winner: yes/no?

	first	last
M	7	9
B	4	7
W	9	4

we eliminate: M
II stage: winner yes/no?

	first	last
B	4	16
W	16	4

Winner: W

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e) Baldwin method:
I stage - Borda scores:
B: 17 M: 18 W: 25
Eliminate: B

II stage: 7: M > W 9: W > M 4: M > W M: 11 W: 9
Ranking: M > W > B

IV. Consider the 750 votes for the three parties: A (240 votes), B (360 votes), and C (150 votes). Distribute the 8 seats between the parties using the indicated methods.

Party	A	B	C
N=1	240	360	150
N=2	120	180	75
N=3	80	120	50
N=4	60	90	37.5
N=5	48	72	30

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a) D'Hondt method
360B, 240A, 180B, 150C, 120B, 120A, 90B,80A
A - 3 seat(s)
B - 4 seat(s)
C - 1 seat(s)

Party	A	B	C
N=1	240	360	150
N=3	80	120	50
N=5	48	72	30
N=7	34,28	51,43	21,43
N=9	26,67	40	16,67

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b) Sainte-Lague method:
360B, 240A, 150C, 120B, 80A, 72B, 51.43B, 50C
A - 2 seat(s)
B - 4 seat(s)
C - 2 seat(s)

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c) Hamilton method (750 / 8 = …)93.75			
Party	A	B	C
Votes	240	360	150
Fair share	2.56	3.84	1.6
Integer	2	3	1
Remainder	0.56	0.84	0.6
Final seats	2	4	2

no. of seats * votes for a party / total votes

V. Consider the mixed non-compensatory and compensatory systems to distribute the seats in a 200-seat parliament to three parties: A, B, and C, for which the shares in the popular vote and the number of seats attained in the 100 FPTP districts are provided in the below tables. Provide the PR seats (out of 100 seats) assigned to each party by the Hamilton method and the total number of seats.

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Non-compensatory				
	% votes	FPTP	PR	Total
A	50	60	25	85
B	30	10	15	25
C	20	30	10	40

PR = % votes / 2
Total = FPTP + PR

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Compensatory				
	% votes	FPTP	PR	Total
A	50	60		50
B	30	10	20	30
C	20	30		20

PR = % votes - FPTP
Total = % votes

VI. Use the Hare-Clark (STV) method to allocate 3 seats based on the incomplete preferences of 20 voters given to the left.
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a) Preferences:

4: A

2: B > A

8: C > D

4: C > E

1: D

1: E

quota q = floor(20/(3+1)) + 1 = 6

Results:

A: 4, B: 2, C: 12, D: 1, E: 1

Seat assigned to: C

b) Transfer of surplus votes (from C):

transfer value = (12 - 6)/12 = 1/2

value for D = 8·1/2 = 4

value for E = 4·1/2 = 2

4: A

2: B > A

4: D

2: E

1: D

1: E

Results:

A: 4, B: 2, D: 1+4 = 5, E: 1+2 = 3

(nobody attains the quota)

c) We eliminate the worst candidate (B)

transfer value = 1

value for A = 2·1 = 2

4: A

2: A

4: D

2: E

1: D

1: E

Results:

A: 4 + 2 = 6, D: 5, E: 3

Seat assigned to: A

d) We eliminate A (no transfer since there are no surplus votes)

4: D

2: E

1: D

1: E

Results: D: 5, E: 3

(nobody attains the quota)

e) We eliminate the worst candidate (E)

4: D

1: D

Results: D: 5

Seat assigned to: D

(even if it did not reach the quota, it was the only candidate standing)

I. Indicate the truth (T) or falsity (F) for the below statements.

a) The winner-turns-loser paradox violates the monotonicity property of voting rules	
b) Anonymity of voting rules is defined as symmetry with respect to candidates	
c) For three individuals, there is no unrestricted domain voting rule that is liberal and Paretian	
d) The dictatorship system is Paretian	
e) The plurality rule is independent with respect to irrelevant alternatives	
f) All cup rules are the Condorcet extensions	
g) The Borda rule for four players is strategyproof	
h) The Shapley-Shubik power index assumes that all coalitions have the same probability to form	
i) The Banzhaf power index satisfies the null player property	

II. Consider the three below social profiles. For each of them, determine the winner using the plurality with run-off. Name and explain the paradoxes occurring between the first and second profiles (4 voters change their preferences from A > B > C to C > A > B) and between the second and third profiles (additional 2-46 voters submit the following ballot: A > B > C).

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1st profile:			2nd profile:			3rd profile:		
27: A > B > C	27: A > C 42: C > A 24: C > A	A: 27 C: 66	**23**: A > B > C	23: B > C 46: C > B 24: B > C	B: 47 C: 46	**25-69**: A > B > C	25-69: A > C 46: C > A 24: C > A	A: 25-69 C: 70
42: C > A > B			**46**: C > A > B			46: C > A > B		
24: B > C > A			24: B > C > A			24: B > C > A		
Winner: C			Winner: B			Winner: C		

III. Consider the social profiles given below. The group is divided into two sub-groups. For each of them, indicate the winner using the instant-runoff voting. Then, verify if the winner for the entire group aligns with the consistency principle. Slide 138

It does not. According to the consistency principle, A should have been a winner for the entire group.

1st sub-group	2nd sub-group	IRV winners	
4: A > B > C	4: A > B > C	1st sub-group	A
2: B > A > C	6: B > A > C	2nd sub-group	A
4: C > B > A	3: C > A > B	Entire group	B

IV. Consider the two social profiles. In the second one, all voters raise D over B and C without changing the order of A and D. Compute the results of the Copeland method and say if it is independent of irrelevant alternatives. Slide 135 Slide 114

It is not independent of irrelevant alternatives, since A or B have not remained the winner.

1st profile	PC	A	B	C	D	Scores
1: A > B > C > D	A	-	4	4	2	CS(A) = 2-1=1
1: A > D > B > C	B	2	-	4	3	CS(B) = 1
2: B > D > A > C	C	2	3	-	3	CS(C) = 0
2: C > D > A > B	D	4	1	3	-	CS(D) = 0

2nd profile	PC	A	B	C	D	Scores
1: A > D > B > C	A	-	4	4	2	CS(A) = 1
1: A > D > C > B	B	2	-	3	0	CS(B) = -2
2: D > B > A > C	C	2	3	-	0	CS(C) = -2
2: D > C > A > B	D	4	6	6	-	CS(D) = 3

V. Use the Hamilton method to determine the number of seats for the three parties based on the preferences of 250 voters: A (24 votes), B (113 votes), and C (113 votes). First, assume there are 25 seats to fill in the parliament, and then consider 26 seats. Do the results confirm the Alabama paradox? Slide 140 The results confirm Alabama paradox (violate common sense), since when there are 25 seats A gets 3 seats, but when 26 - only 2.

Party	Votes	Fair share (25)	Seats (25)	Fair share (26)	Seats (26)
A	24	25·24/250 = 2.4	3	26·24/250= 2.496	2
B	113	11.3	11	11.752	12
C	113	11.3	11	11.752	12

VI. Consider the following preferences of 13 voters: 6: A > C > B, 3: B > C > A, 1: B > A > C, **3**: **C > B > A**. Determine the results using the plurality rule and the Borda count. How to manipulate the preferences of the three voters in bold to make candidate B the winner.

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Plurality rule	Borda count
Results: A: 6 B: 4 C: 3	Results: BSc(A) = 13 BSc(B) = 11 BSc(C) = 15
Manipulation: 3: B > C > A A: 6 + 0 + 0 + 0 = 6 B: 4 + 3 = 7 C: 0 + 0 = 0 after manipulation	Manipulation: 3: B > C > A BSc(A) = 6·2 + 3·0 + 1·1 + 3·0 = 13 BSc(B) = 6·0 + 3·2 + 1·2 + 3·2 = 14 BSc(C) = 6·1 + 3·1 + 1·0 + 3·1 = 12 after manipulation

A B C D

VII. Consider the following simple game: [6; 4, 3, 2, 1] (the numbers in the brackets mean a rule requires 6 votes to pass, and voter A can cast four votes, B three votes, C two, and D one. Slide 161

Compute the Shapley-Shubik power indices for all voters.

Order	Piv.	Order	Piv.	Order	Piv.	Order	Piv.
ABCD	B	BACD	A	CABD	A	DABC	B
ABDC	B	BADC	A	CADB	A	DACB	C
ACBD	C	BCAD	A	CBAD	A	DBAC	A
ACDB	C	BCDA	D	CBDA	D	DBCA	C
ADBC	B	BDAC	A	CDAB	A	DCAB	A
ADCB	C	BDCA	C	CDBA	B	DCBA	B

Party	SS index
A	10/24
B	6/24
C	6/24
D	2/24

Compute the Banzhaf power indices for all voters. Slide 163

Coal.	Win	Coal.	Win	Coal.	Win	Coal.	Win
ø	N	D	N	BC	N	ABD	Y
A	N	AB	Y	BD	N	ACD	Y
B	N	AC	Y	CD	N	BCD	Y
C	N	AD	N	ABC	Y	ABCD	Y

Winning	Critical
AB	AB
AC	AC
ABC	A
ABD	A
ACD	A
BCD	BCD
ABCD	-

sum => Overall: 10 critical votes

Party	Bz index
A	5/10
B	2/10
C	2/10
D	1/10

A B C D
[6; 4, 3, 2, 1]

DECISION ANALYSIS – EXERCISES XI – CLASSICAL METHODS FOR MULTIPLE OBJECTIVE OPTIMIZATION

I. Indicate the truth (T) or falsity (F) for the below statements.

a) The two objectives optimized in portfolio management are expected return and risk

b) Various solutions in the decision space are always translated to different points in the objective space

c) Weakly Pareto optimal solutions are always Pareto optimal

d) The number of solutions contained in the Pareto frontier may be finite

e) The max point attains not better values than the nadir point on all objectives

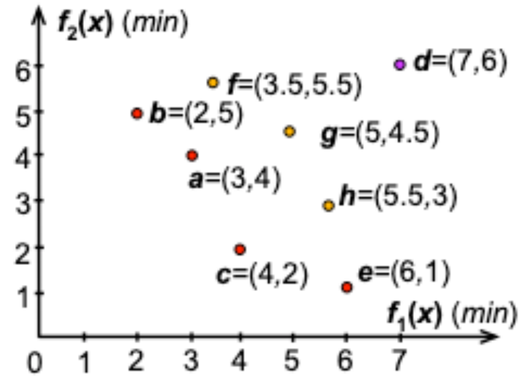
f) Classical optimization methods require multiple runs with different parameter values to approximate the Pareto frontier

g) The weighted sum method (WSM) parameterized with positive weights for all objectives identifies the Pareto optimal solution

h) The epsilon constrain method (ECM) can find non-support efficient solutions

i) In the no-preference model, the computational and decision phases interchange

II. Consider a set of solutions ***a-h*** in the objective space with two minimized objectives (see figure below).



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a) Compute the ideal point $\mathbf{z}^{ideal}$. (2, 1) infeasible

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b) Compute a utopian point $\mathbf{z}^{utop}$ for ε=0.1. (1.9, 0.9)

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c) Compute the max point $\mathbf{z}^{max}$. (7,6)

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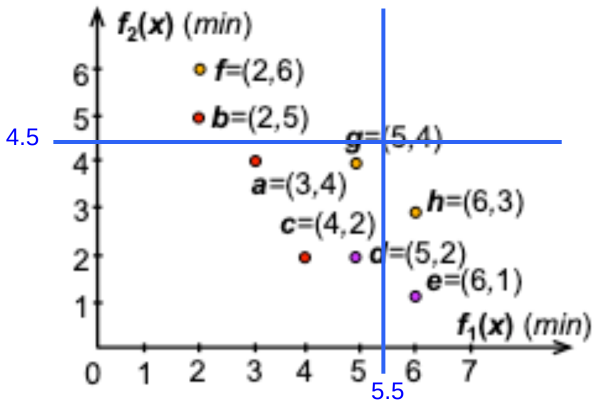
d) Compute the nadir point $\mathbf{z}^{nadir}$. (6,5)

III. Consider a set of solutions ***a-h*** in the objective space with two minimized objectives (see figure below).



a) Identify Pareto optimal and weakly Pareto optimal solutions. Slide 177

PO: b, a, c, e Weakly PO: f, d

b) What would be the solution returned by **WSM** with Slide 187

the following objective function: ***Minimize*** $0.5 \cdot f_1(x) + 0.5 \cdot f_2(x)$?

c=(4,2) 0.5*4 + 0.5*2 = 3

c) What about **WSM** with: ***Minimize*** $2/3 \cdot f_1(x) + 1/3 \cdot f_2(x)$?

b=(2,5) 2/3*2 + 1/3*5 = 4/3 + 5/3 = 3

d) Solution ***a*** is Pareto optimal. Can it be discovered by **WSM**? Yes (Slide 188)

e) What would be the solution returned by **ECM** with the following objective function and constraint: Slide 193

Minimize $f_1(x)$, *s.t.* $f_2(x) \leq 4.5$? a=(3,4)

f) What about **ECM** with: ***Minimize*** $f_2(x)$, *s.t.* $f_1(x) \leq 5.5$? c, d

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How to reformulate the objective function using the augmentation factor to be sure that **ECM** always returns a Pareto optimal rather than a weakly Pareto optimal solution?

Minimize f2(x) + p * (f1(x) + f2(x)) s.t. f1(x) <= 5.5, where p can even be equal to 1.0 in our case

g) What would be the solution(s) returned by the **ASF** method with the following objective function:

Minimize *max*$\{0.5 \cdot f_1(x), 0.5 \cdot f_2(x)\}$? f, h, e

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h) What about **ASF** with: ***Minimize*** *max*$\{2/3 \cdot f_1(x), 1/3 \cdot f_2(x)\}$?

i) Which solution would be selected for the following order of lexicographic optimization $(f_1(x), f_2(x))$? b

I. Indicate the truth (T) or falsity (F) for the below statements.

a) Recombination can introduce new information to the optimization

b) The impact of the mutation on the evolutionary search is exploitative rather than explorative

c) Evolutionary optimization methods require multiple runs with different parameter values to approximate the Pareto frontier

d) Tournament selection belongs to the class of ordinal selection methods

e) The max point attains not better values than the nadir point on all objectives

f) VEGA applies a generation model of managing the population

g) The crowding distance for the non-dominated solutions in NSGA-II is equal to infinity

h) SPEA2 includes the archive members in the selection process

i) SMS-EMOA is an indicator based evolutionary algorithm for multiple objective optimization

II. Given the following chromosome in the binary encoding [1 0 1 1 0 0], representing an example solution for the knapsack problem, present a chromosome obtained after a flip bit mutation: Slide 213

0	1	0	0	1	1

III. Given the two below presented chromosomes in the binary encoding:

1	**0**	**1**	**1**	**0**	**0**

0	**1**	**1**	**0**	**0**	**1**

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present a pair of chromosomes obtained after applying 2-point crossover with crossover points after the second and fifth genes:

1	0	1	0	0	0

0	1	1	1	0	1

IV. Given the following table of fitness values for seven solutions ***a*** **-** ***g*** (fitness F_i to be maximized):

sol	*a*	*b*	*c*	*d*	*e*	*f*	*g*
F_i	3	1	3	2	0.5	1.5	1

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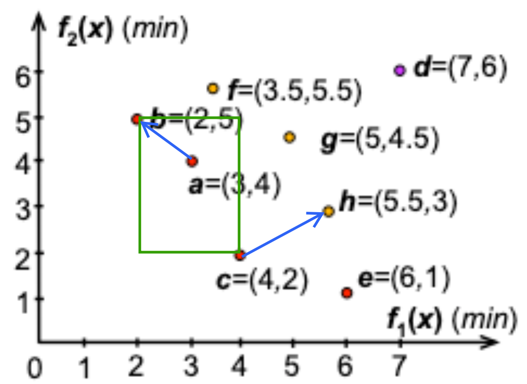
Indicate the parents selected for the recombination operator with the tournament selection of size 4, when the following subsets of solutions participate in each tournament:

i) {*a, d, f, g*} - *Winner*: a

ii) {*b, c, e, f*} - *Winner*: c

V. Consider a set of solutions ***a-h*** in the objective space with two minimized objectives (see figure below).



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a) Use the Kung's method to identify the first non-dominated front.

{b, a, c, e}

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b) Show the Pareto fronts used by NSGA-II as the primary sorting criterion.

Front 1 = {b, a, c, e}
Front 2 = {f, g, h}
Front 3 = {d}

c) For all solutions, compute their raw fitness values (sum of strengths of dominating solutions) according to the rules of SPEA2. Slide 230

S({b, a, c, e, f, g, h, d}) = {2, 3, 3, 1, 1, 1, 1, 0}
R({b, a, c, e, f, g, h, d}) = {0, 0, 0, 0, 5, 6, 3, 12}

d) Which solution: ***a*** or ***c*** would be found more favorable by:

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NSGA-II (draw cuboids related to the respective crowding distances), c

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SPEA2 (draw distances to the k=1 nearest neighbor), or c

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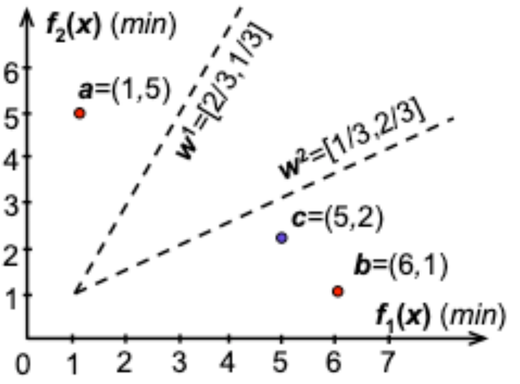
SMS EMOA (draw individual contributions to hypervolume; assume $\boldsymbol{d}=z^{ref}$). c

VI. Consider a population composed of just two solutions ***a*** and ***b***, which is evolved by MOEA/D with the two uniformly distributed weight vectors provided in the figure. The latter ones are used as the parameters in the weighted Chebyshev distance from the reference point. Solutions ***a*** and ***b*** are the only ones contained in the current external archive.



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a) Compute the current reference point according to MOEA/D.

(1, 1) in MOEA/D – z_ref is composed of the min objective values in the population

b) Associate solutions ***a*** and ***b*** with the targets (which solution is the best for which target?).

a for f1, b for f2

c) Assume that by recombining ***a*** and ***b*** (the neighborhood's size T=2) and further mutating the newly obtained solution, we obtain solution ***c***. Will it become the new best solution for some target(s)? Show the new archive. No, and the archive remains the same.