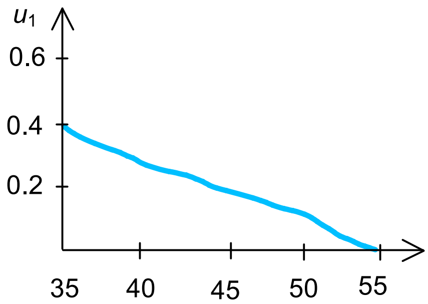
Sofya Aksenyuk, 150284.

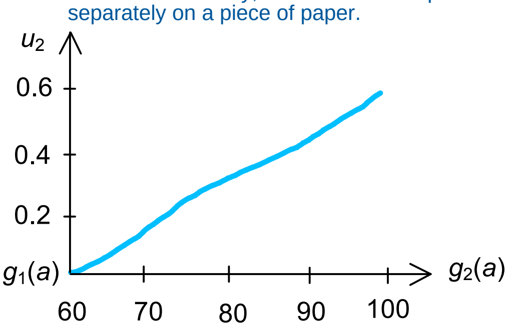
DECISION ANALYSIS – SHORT EXERCISES III – UTA

I. Consider the performances of alternatives a_1 – a_5 and two criteria: g_1 (cost; range 35-55) and g_2 (gain; range 60-100). Draw the **linear** value functions on both criteria (without breaking them anywhere) to reproduce the following pairwise comparison: a_5 *P* a_1. Compute the comprehensive value of reference alternatives according to the drawn marginal value functions to prove that they allow reproducing the above pairwise comparison.

P.S.: Sorry, my PDF-editor doesn't have a line option, so I tried my best. To determine the values correctly, I have drawn the plots separately on a piece of paper.

	$g_1\downarrow$	$g_2\uparrow$	$u_1(a)$	$u_2(a)$	$U(a)$
a_1	45	80	0.2	0.3	0.5
a_2	50	90	0.1	0.45	0.55
a_3	40	70	0.3	0.15	0.45
a_4	35	60	0.4	0.0	0.4
a_5	55	100	0.0	0.6	0.6





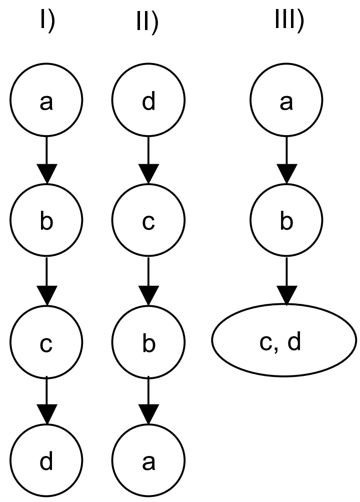
Hint: a_5 **P** a_1 iff $U(a_5) > U(a_1)$ iff $u_1(a_5) + u_2(a_5) > u_1(a_1) + u_2(a_1)$ iff $u_1(55) + u_2(100) > u_1(45) + u_2(80)$ iff

iff $0u_1(35) + u_2(100) > 0.5u_1(35) + 0.5u_2(100)$ iff $0.5u_2(100) > 0.5u_1(35)$ iff $u_2(100) > u_1(35)$

$u_2(100) + u_1(35) = 1$, so $u_2(100) > u_1(35)$ iff $u_2(100) > 1 - u_2(100)$ iff $u_2(100) > 0.5$

II. Fill in the matrices of pairwise relations for the three rankings given to the right: R_I, R_{II}, and R_{III}. Assume the following order of alternatives in rows and columns: a, b, c, d. Compute Kendall's distance and Kendall's tau coefficients for rankings R_I and R_{II} as well as R_I and R_{III}.

$$R_I = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_{II} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \quad R_{III} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$



$d_k(R_I,R_{II}) = \frac{1}{2}(3 + 3 + 3 + 3) = 6$

$\tau(R_I,R_{II}) = 1 - 4\frac{6}{4*(4-1)} = -1$

$d_k(R_I,R_{III}) = \frac{1}{2}(0 + 0 + 0.5 + 0.5) = 0.5$

$\tau(R_I,R_{III}) = 1 - 4\frac{0.5}{4*(4-1)} = \frac{5}{6}$

III. The Decision Maker provided the following four pairwise comparisons: $a_1 > a_4$, $a_5 > a_3$, $a_2 > a_6$, $a_3 > a_1$. Change the formulation of the below mathematical programming model to identify the minimal subset of pairwise comparisons underlying inconsistency of preference information. Select the preference direction (min or max), write down the objective function, change the below conditions by adding appropriate formulations, denote the binary variables (if you use them), do not change *CONSTRAINTS* denoting a constraint set modeling the monotonicity, normalization, and non-negativity constraints.

min / *max* V = v_a1,a4 + v_a5,a3 + v_a2,a6 + v_a3,a1

s.t.

$U(a_1) > U(a_4)$ - v_a1,a4

$U(a_5) > U(a_3)$ - v_a5,a3

$U(a_2) > U(a_6)$ - v_a2,a6

$U(a_3) > U(a_1)$ - v_a3,a1

CONSTRAINTS

v_a1,a4, v_a5,a3, v_a2,a6, v_a3,a1 ∈ {0,1}

Assume that the optimal solution of the problem to the left indicated $a_5 > a_3$ and $a_2 > a_6$ as the minimal subset of pairwise comparisons underlying inconsistency. Which condition must one add in the next iteration to find another (different) minimal subset underlying inconsistency? Refer to the variables you have previously introduced to the left.