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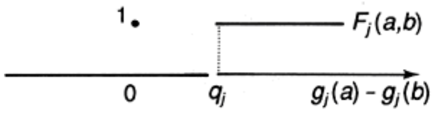
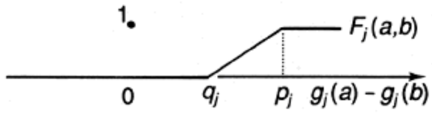
DECISION ANALYSIS – SHORT EXERCISES I – INTRODUCTION AND PROMETHEE

I. Consider the marginal preference functions for the three criteria: g_1, g_2, and g_3 of gain type. Each criterion's intra- and inter-criteria preference information is provided above the respective plots (q_i – indifference threshold, p_i – preference threshold, w_i – weight). First, compute the marginal preference indices π_j for two pairs of alternatives: A and B as well as C and D. Then, compute the comprehensive preference indices π.

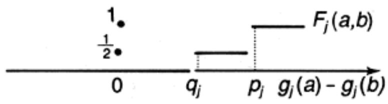
g_1 : $q_1 = 10$, $w_1 = 3$



g_2 : $q_2 = 10$, $p_2 = 20$, $w_2 = 5$



g_3 : $q_3 = 2$, $p_3 = 5$, $w_3 = 2$



	$g_1\uparrow$	$g_2\uparrow$	$g_3\uparrow$
A	10	18	10
B	15	0	20

	g_1	g_2	g_3
$\pi_j(A,B)$	0	0.8	0
$\pi_j(B,A)$	0	0	1

$\pi(A,B)$	$(3\cdot0 + 5\cdot0.8 + 2\cdot0)/10 = 0.4$
$\pi(B,A)$	$(3\cdot0 + 5\cdot0 + 2\cdot1)/10 = 0.2$

	$g_1\uparrow$	$g_2\uparrow$	$g_3\uparrow$
C	20	10	17
D	0	25	20

	g_1	g_2	g_3
$\pi_j(C,D)$	1	0	0
$\pi_j(D,C)$	0	0.5	0.5

$\pi(C,D)$	(3*1 + 5*0 + 2*0)/10 = 0.3
$\pi(D,C)$	(3*0 + 5*0.5 + 2*0.5)/10 = 0.35

II. Using the PROMETHEE method, one derived a matrix of comprehensive preference indices $\pi(a,b)$ for all pairs of alternatives. Compute the positive $\Phi^+(a)$, negative $\Phi^-(a)$, and comprehensive flows $\Phi(a)$ for all alternatives. Draw the rankings obtained with PROMETHEE II and PROMETHEE I. Recall that PROMETHEE I admits incomparability.

$\pi(a,b)$	W	X	Y	Z	$\Phi^+(a)$	$\Phi^-(a)$	$\Phi(a)$
W	0	0	0	0.3	0.3	0.9	-0.6
X	0	0	0.2	0.7	0.9	0.1	0.8
Y	0.9	0	0	0.7	1.6	0.3	1.3
Z	0	0.1	0.1	0	0.2	1.7	-1.5

III. Consider six alternatives (S, V, W, X, Y, Z) with the following comprehensive flows: $\Phi(S) = 0.9$, $\Phi(V) = -0.9$, $\Phi(W) = -0.6$, $\Phi(X) = 0.8$, $\Phi(Y) = 1.3$, and $\Phi(Z) = -1.5$. Formulate the binary linear program according to the assumptions of PROMETHEE V that would allow selecting a subset of two alternatives that respect the constraints on the maximal budget of 100 and the minimal projected gain of 300. The budgets and gains for all alternatives are provided in the below table. Use the following binary variables: x_S, x_V, x_W, x_X, x_Y, and x_Z, corresponding to the six alternatives.

Φ	0.9	-0.9	-0.6	0.8	1.3	-1.5
	S	V	W	X	Y	Z
budget	40	30	60	50	70	20
projected gain	140	100	150	170	200	120

What would be the optimal subset of alternatives selected by PROMETHEE V?

max 0.9*x_s + (-0.9)*x_v + (-0.6)*x_w + 0.8*x_x + 1.3*x_y + (-1.5)*x_z
subject to:
40*x_s + 30*x_v + 60*x_w + 50*x_x + 70*x_y + 20*x_z <= 100
140*x_s + 100*x_v + 150*x_w + 170*x_x + 200*x_y + 120*x_z >= 300
x_s, x_v, x_w, x_x, x_y, x_z ∈ {0, 1}
Optimal solution: select S and X
x_s, x_x = 1, x_w, x_v, x_y, x_z = 0
objective function = 0.9 + 0.8 = 1.7
budget = 40 + 50 = 90 < 100
projected gain = 140 + 170 = 310 > 300