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IT – Lab 3

Mutual Information & Conditional Entropy

I. a) Consider two dices with 6 sides, and their independent throws X and Y. Show by doing calculations that $H(X, Y) = H(X) + H(Y)$.

$H(X) = \log 6$

$H(Y) = \log 6$

$H(X,Y) = \log 36 = 2 \cdot \log 6 = \log 6 + \log 6 = H(X) + H(Y).$

b) What is the value of mutual information between X and Y, that is $I(X, Y)$?

$I(X,Y) = H(X) + H(Y) - H(X,Y) = H(X) + H(Y) - (H(X) + H(Y)) =$

$= H(X) - H(X) + H(Y) - H(Y) = 0.$

II. a) Take the dices from the previous task, but now assume that they are in a quantum entanglement so that they always have the same outcome. Does $H(X, Y) = H(X) + H(Y)$ still hold? Calculate these values.

$H(X) = \log 6.$

$H(Y) = \log 6.$

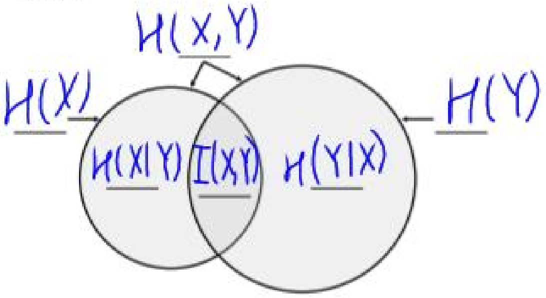
$H(X,Y) = \sum_{(x,y)} p(x,y) \cdot \log \frac{1}{p(x,y)} = 6 \cdot \left(\frac{1}{6} \cdot \log 6\right) = \log 6.$

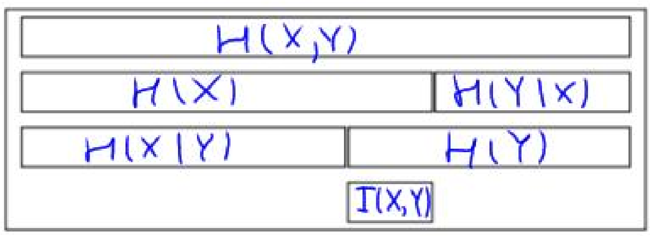
$H(X) + H(Y) = 2 \log 6 \neq \log 6 \Rightarrow H(X) + H(Y) \neq H(X,Y) \Rightarrow$ does NOT hold.

b) What is the value of mutual information between X and Y, that is $I(X, Y)$?

$I(X,Y) = 2 \log 6 - \log 6 = \log 6.$

III. Assign the following expressions: $H(X)$, $H(Y)$, $H(X, Y)$, $H(X|Y)$, $H(Y|X)$, $I(X, Y)$ to the appropriate fields on the two different visualizations of entropy below. Notation: a field connected with arrow(s) corresponds to the area occupied by a full circle(s), while the fields inside circles correspond to the single-colored area they are in.





IV. Below there is a probability distribution of a certain transmission system, where X is the input variable, and Y is the output variable.

	$Y=0$	$Y=1$
$X=0$	$2/8$	$1/8$
$X=1$	$1/8$	$4/8$

a) Compute entropies $H(X)$ and $H(Y)$.

$$H(X) = p(X=0)\cdot\log\frac{1}{p(X=0)} + p(X=1)\cdot\log\frac{1}{p(X=1)} = \frac{3}{8}\cdot\log\frac{8}{3} + \frac{5}{8}\cdot\log\frac{8}{5}.$$

$$H(Y) = p(Y=0)\cdot\log\frac{1}{p(Y=0)} + p(Y=1)\cdot\log\frac{1}{p(Y=1)} = \frac{3}{8}\log\frac{8}{3} + \frac{5}{8}\log\frac{8}{5}.$$

b) Compute joint entropy $H(X,Y)$.

$$H(X,Y) = \frac{1}{4}\cdot\log 4 + \frac{1}{8}\cdot\log 8 + \frac{1}{8}\cdot\log 8 + \frac{1}{2}\cdot\log 2 = \frac{7}{4}\log 2.$$

c) Compute conditional entropy $H(Y|X)$.

$$H(Y|X) = H(X,Y) - H(X) = \frac{7}{4}\log 2 - \frac{3}{8}\log\frac{8}{3} - \frac{5}{8}\log\frac{8}{5}.$$

d) Compute mutual information $I(X,Y)$.

$$I(X,Y) = H(X) + H(Y) - H(X,Y) = \frac{3}{4}\log\frac{8}{3} + \frac{5}{4}\log\frac{8}{5} - \frac{7}{4}\log 2.$$

V. Write **'T'** when a sentence is true, and **'F'** when it is false.

$\forall_{X,Y}\ H(X) \geqslant H(X,Y)$	F
$\forall_{X,Y}\ H(X) \geqslant H(X\|Y)$	T
$\forall_{X,Y}\ H(X) \geqslant I(X,Y)$	T
$\forall_{X,Y}\ I(X,Y) = H(X) + H(Y) - H(X\|Y)$	F
$\forall_{X,Y}\ H(X,Y) = H(X) + H(Y) - I(X,Y)$	T
If X and Y are independent, then $I(X,Y) = 0$	T
If X and Y are independent, then $H(X\|Y) = H(Y\|X)$	F
If X and Y are independent, then $H(X\|Y) = H(X)$	T