## INTRODUCTION TO ARTIFICIAL INTELLIGENCE - LECTURE 6 - NEURAL NETWORKS

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I. Consider two examples  $\mathbf{a}$  and  $\mathbf{b}$  described in terms of three attributes  $x_1$ ,  $x_2$ , and  $x_3$  (see table below). Compute the respective excitations when weights of a unit are as follows:  $w_0 = -2$ ,  $w_1 = -1$ ,  $w_2 = 1$  and  $w_3 = 2$ . Compute the activations obtained with two different types of functions: threshold and Leaky ReLU.

example	<b>X</b> <sub>1</sub>	<b>X</b> <sub>2</sub>	<b>X</b> <sub>3</sub>	excitation	threshold	Leaky ReLU
а	2	1	1	2*(-1) + 1*1 + 1*2 + (-2) = -1	0	-0.01
b	1	2	1	1*(-1) + 2*1 + 1*2 + (-2) = +1	+1	+1

II. Compute the Mean Squared Error  $E_{\text{MSE}} = \frac{1}{n} \sum_{j=1,\dots,n} [z_j - y_j]^2$  (please use 1/n, not 1/2) based on the obtained  $z^j$  and desired  $y^j$  results for the three examples: **a**, **b** and **c**. When computing the error  $\mathbf{E}^j$  for an individual example, please use:  $\mathbf{E}^j = \frac{1}{2} [z_j - y_j]^2$ 

j	<b>z</b> <sup>j</sup>	y <sup>j</sup>	Ej
а	1	1	0
b	1	2	1/2
С	4	1	4.5

Answer:  $E_{MSE} = (1/3) * 1 * 1 * 9 = 3$ 

III. Consider a neuron with an excitation function  $exc = 3x_1 + 1x_2 + 2$  ( $w_1=3$ ;  $w_2=1$ ;  $w_0=2$ ) and a **sigmoid activation function**. Assume we optimize the Minimal Square Error  $E^j_{MSE} = 1/2[z^j - y^j]^2 = 1/2(\delta^j)^2$  (assume ½ is used when computing  $E^j_{MSE}$ , not 1/n). Propagate example  $x_i^j$  (given below) with desired output of  $z^j = 0.3$  through the unit and apply the gradient descend algorithm to optimize the weights with the learning rate  $\eta=1$ . Hint: you can use the equations you know from the lecture for the gradient descend algorithms and a sigmoid activation function; no need to derive them again.

$\mathbf{x_1}^{j}$	$\mathbf{x}_2^{j}$	z <sup>j</sup>	exc	y <sup>j</sup>	δ <sup>j</sup>
-1	1	0.3	3*(-1) + 1*1 + 2 = 0	1 / (1 + e^0) = 1/2	0.3 - 0.5 = -0.2

$$\Delta w_1^{j} = 1 * (-0.2) * (-1) * (1/2) * (1 - 1/2) = 0.05$$
 $w_1^{j'} = w_1^{j} + \Delta w_1^{j} = 3 + 0.05 = 3.05$ 

$$\Delta w_2^{j} = 1 * (-0.2) * 1 * (1/2) * (1 - 1/2) = -0.05$$
 $w_2^{j'} = w_2^{j} + \Delta w_2^{j} = 1 + (-0.05) = 0.95$ 

$$\Delta w_0^{j} = 1 * (-0.2) * 1 * (1/2) * (1 - 1/2) = -0.05$$
 $w_0^{j'} = w_0^{j} + \Delta w_0^{j} = 2 + (-0.05) = 1.95$ 

IV. Given the gray  $3\times3$  matrix of inputs, perform **zeropadding** of size 1. Then, **convolve** the resulting matrix with the  $2\times2$  filter defined by the green matrix (bias = 0) with a stride of 1.

Original matrix and matrix after zeropadding

0	0	0	0	0
0	-2	1	-1	0
0	0	-1	2	0
0	1	0	-2	0
0	0	0	0	0

Filter

0	1
-1	0

Matrix after convolution

0	2	-1	1
-2	1	0	-2
0	-2	2	2
1	0	-2	0

V. Apply **MAX pooling** with filter of size 2×2 and a stride of 2 on the gray matrix given to the left.

Original matrix

4	2	5	1
3	1	0	0
0	3	4	3
5	6	0	7

Matrix after MAX pooling

4	5
6	7