Department of Mathematics
Indian Institute of Technology Guwahati

End–Semester Examination November 24, 2021

MA 473 Computational Finance (Part – I)

Time: **09:00 – 10:30 Hrs.** Marks: **17**

There are ***THREE*** questions in this paper. Answer all questions.

---

1. By using the transformation $\xi = \dfrac{S}{S + P_m}$, $\tau = T - t$ and $V(S,t) = (S + P_m)\overline{V}(\xi,\tau)$, where $P_m > 0$ is a constant, transform the following Black-Scholes PDE:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2(S)S^2\dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad 0 \le S,\ t \le T, \\ V(S,T) = V_T(S), \quad 0 \le S, \end{cases}$$

from infinite domain $0 \le S$ to finite domain $0 \le \xi \le 1$. Further, obtain the boundary conditions at $\xi = 0$ and $\xi = 1$. (**6** marks)

2. Consider the American put option, by using the *finite element method*, obtain the minimization problem

$$\min_{v\in\mathcal{K}} I(y;v) = 0, \quad \text{where } I(y;v) = \int_{x_{\min}}^{x_{\max}} \left(\frac{\partial y}{\partial \tau} \cdot (v - y) + \frac{\partial y}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial y}{\partial x} \right) \right) dx,$$

where $\mathcal{K}$ is the family of admissible functions. (**3** marks)

   (a) Further, obtain the following fully discrete version of the above minimization problem:

$$(v^{(n+1)} - w^{(n+1)}))^T \left(Cw^{(n+1)} - r \right) \ge 0,$$

   where $C := B + \triangle\,\tau\,\theta\,A$, and $r := (B - \triangle\,\tau\,(1-\theta)A)w^{(n)}$, here A and B are tri-diagonal matrices obtained from the integrals involving the the basis functions and their derivatives by applying a numerical quadrature formula. (**4** marks)

3. Show that the solution of the following problems are equivalent, *i.e.,* the solution of one satisfies the other and vice-versa: (**4** marks)

$$\textbf{(FEM)} \qquad \begin{cases} \text{Find } w, \text{ such that for all } v \ge g \\ (v - w)^T\,(Cw - r) \ge 0, \quad w \ge g \end{cases}$$

and

$$\textbf{(FDM)} \qquad \begin{cases} \text{For each } w, \text{ the following holds true:} \\ (Cw - r) \ge 0, \quad w \ge g \\ (Cw - r)^T\,(w - g) = 0. \end{cases}$$
