MA668: Algorithmic and High Frequency Trading
Lecture 05

Prof. Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology Guwahati

The Problem Solution (Contd ...)

1. Now at date t, the total quantity of assets available was equal to the quantity of assets that LT1 brought to the market.
2. Therefore LHS is given by:

$$nq_1^{MM} + q_1^{LT1} + q_1^{LT2} = i + q_1^{LT2} = i - i = 0.$$

3. Hence substituting and solving, we obtain that:

$$S_2 = E[S_3 | \boxed{\epsilon_2} = \mu + \epsilon_2 + E(\epsilon_3) = \mu + \epsilon_2.$$

4. Therefore: $q_2^j = 0$.
5. This makes sense, since at $t = 2$, there are no asset imbalances, and therefore the price of the asset reflects its "fundamental value" (efficient price) and no one will want to hold a non-zero amount of the risky asset.
6. From Figure 2.1 (bottom half), we see the asset holding q_1^j of $j \in \{MM, LT1, LT2\}$, as they enter $t = 2$, and how after trading at a price equal to S_2, they end up with holdings q_2^j equal to zero.

Figure 2.1

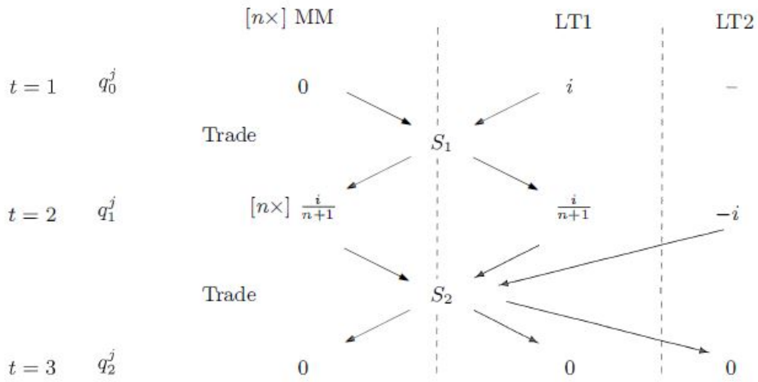

Figure 2.1 Trading and price setting in the Grossman–Miller model.

Figure: Figure 2.1

The Problem Solution (Contd ...)

1. Next we consider, what happens at $t = 1$.
2. Participating agents: n MM's and LT1 (Recall: LT2 will not appear until $t = 2$) anticipate that whatever they do, the future market price will be efficient and they will end up exiting at $t = 2$, with no inventories, so that $X_3 = X_2$.
3. Thus, their portfolio decision is given by:

$$\max_{q_1^j} E\left[U\left(X_2^j\right)\right],$$

subject to $X_2^j = X_1^j + q_1^j S_2$ and $X_1^j + q_1^j S_1 = X_0^j + q_0^j S_1$.

4. Repeating the analysis of $t = 2$, at $t = 1$, we obtain the optimal portfolio solution as:

$$q_1^{j,*} = \frac{E[S_2] - S_1}{\gamma\sigma^2},$$

S2 = mu + epsilon2

for all agents j that are present, namely, n MM's and LT1.

The Problem Solution (Contd ...)

1. At $t = 1$, the demand and supply for an asset have to be equal to each other, so that:
$$nq_0^{MM} + q_0^{LT1} = nq_1^{MM} + q_1^{LT1},$$
where $q_0^{LT1} = i$ (recall that if $i > 0$, LT1 is holding i shares, which she/he wants to sell) and $q_0^{MM} = 0$.

2. This gives us the equation:
$$i = (n+1)\left(\frac{\mu - S_1}{\gamma\sigma^2}\right) \iff S_1 = \mu - \gamma\sigma^2\left(\frac{i}{n+1}\right).$$

3. Figure 2.1 (top half) reflects how the MM's and LT1 enter the market with asset holdings q_0^j and after trading at S_1, they exit $t = 1$ and enter $t = 2$ with q_1^j.

How Does the Market Reach a Solution

1. Question: How does the market reach a solution for LT1's liquidity needs.
2. LT1, a trader who wants to sell a total of $i > 0$ units at $t = 1$, finds that there is no one currently in the market with a balancing liquidity need.
3. There are traders in the market, but they will not accept trading at the efficient price of μ, because if they do, they will be taking on risky shares (they are exposed to the price risk from the realization of ϵ_2), without compensation.
4. But, if they receive adequate compensation (which we call a "liquidity discount", as for $i > 0$, $S_1 < E[S_2] = \mu$), the n MM's will accept the LT1's shares.
5. However LT1 being price sensitive, if she/he has to accept a discount on the shares, then she/he will not sell all the i shares at once.
6. In equilibrium:
   - (A) Both the n MM's and LT1 end up holding $q_1^{j,*} = \dfrac{i}{n+1}$ units of the asset, each.
   - (B) In other words LT1 sells $\dfrac{n}{n+1} i$ units to the n MM's, and also holds onto the remaining $\dfrac{i}{n+1}$ units, to be sold later.

How Does the Market Reach a Solution (Contd ...)

1. Trading occurs at a price below the efficient price $S_1 = \mu - \gamma\sigma^2 \left(\frac{i}{n+1}\right)$.
2. The difference between the trading price and the efficient price, namely, $|S_1 - \mu| = \left|\gamma\sigma^2 \frac{i}{n+1}\right|$, represents the (liquidity) discount the MM's receive in order to hold LT1's shares.
3. The size of the discount is influenced by the variables in the model:
   - A. The size of the liquidity demand ($|i|$).
   - B. The amount of competition among MM's (captured by n).
   - C. The market's risk aversion (γ).
   - D. The risk/volatility of the underlying asset (σ^2).

How Does the Market Reach a Solution (Contd ...)

1. Size of liquidity, Risk aversion and Volatility $\rightarrow$ Increased discount.
2. Competition $\rightarrow$ Reduced discount.
3. If LT1 wants to buy ($i < 0$), then the solution would be the same, except that instead of a discount, the agents would receive a premium equal to $|S_1 - \mu|$, per share, when selling to LT1.
4. Finally, as competition (n) increases, the Premium $\rightarrow$ Zero and Price $\rightarrow$ Efficient level, $S_1 = \mu$ and LT1's optimal initial net trade $q_1^{LT1,*} - q_0^{LT1}$ converges to her/his liquidity need (i).

Trading Costs

1. Grossman and Miller framework helps to understand the cost of holding assets (in this case, via the uncertainty it generates to the risk averse MM's) affects the liquidity via the cost of trading ($|S_1 - \mu|$) and the demand of immediacy (at $t = 1$, LT1 only executes $\frac{n}{n+1} i$, rather than i).
2. Also competition between MM's is crucial in determining the trading costs.
3. Question: What drives n?
   Answer: n is driven by the trading costs borne by the MM's.
4. In this context, we must distinguish between participation costs, which are needed, to be present in the market and do not depend on trading activity AND trading costs that do depend on trading activity, such as trading fees (which we ignored in the previous analysis).

Trading Costs (Contd ...)

1. Grossman and Miller link the competition n with participation costs.
2. How is this accomplished: By introducing an earlier stage to the model in which the potential MM's decide whether they want to actively participate in the market and provide liquidity or prefer to do something else.
3. Quantitatively: The decision is determined as a function of a participation cost c, which proxies for the time and investments needed to keep a constant, active and competitive presence in the market and not doing something else.
4. Conclusion (without going into the details): Level of competition decreases monotonically with the supplier's participation costs.
5. In other words, participation costs, proxied by the cost parameter c, increases the size of the liquidity premium (via its effect on competition n).

Trading Costs (Contd ...)

1. In addition to parameter c, which captures the fixed costs of participating in the market, we could also consider inclusion of a cost of trading that depends on the level of activity, such as actual trading fees (which depend on the quantity traded).
2. While exchange trading fees are usually proportional to the dollar-volume, but for simplicity, we use fees proportional to the number of shares traded, and parameterized by η.
3. Since the fees are known, they act like participation costs for liquidity traders.
4. Effect of having $\eta > 0$: Liquidity traders with a desired trade ($|i|$) that is small relative to trading fees, will find trading too expensive and refrain from trading.
5. So we consider the scenario of sufficiently large desired trades so that trading is preferred to not trading by all participants.