MA 668 | Quiz-1
Algorithmic and High Frequency Trading | 10:05 AM–10:50 AM
IIT Guwahati | 31^{st} January 2024

Instructions

1. Write your name and roll number on the answerscript.
2. Your writing should be legible and neat.
3. This Quiz has 3 questions, for a total of 15 marks.

Questions

[5^{marks}] 1. State TRUE or FALSE:

(A) Liquidity traders post Limit Orders in the Limit Order Book.
(B) NASDAQ being a completely lit market is obliged to publish complete information about the status of their Limit Order Books.
(C) Iceberg orders completely show their quantity.
(D) Larger spread implies more liquidity for the asset.
(E) If the price listed in the Limit Order Book is 474100, then the actual execution price is \$47.4100.

Answer:

(A) FALSE ...(1 mark)
(B) TRUE ...(1 mark)
(C) FALSE ...(1 mark)
(D) FALSE ...(1 mark)
(E) TRUE ...(1 mark)

[5^{marks}] 2. Write the maximization problem formulation for the Grossman-Miller model, for the exponential utility function, at times $t = 1$ and $t = 2$, along with all the corresponding constraints.

Answer:
At $t = 2$, agent j chooses q_2^j, so as to maximize the expected utility, knowing the realization of ϵ_2:

$$\max_{q_2^j} E\left[U\left(X_3^j\right)|\epsilon_2\right],$$

where it can be shown that:

$$E\left[U\left(X_3^j\right)|\epsilon_2\right] = -\exp\left\{-\gamma\left(X_2^j + q_2^j E\left[S_3|\epsilon_2\right]\right) + \frac{1}{2}\gamma^2\left(q_2^j\right)^2\sigma^2\right\}.$$

...(2 marks)

This is subject to the constraints: $X_3^j = X_2^j + q_2^j S_3$ and $X_2^j + q_2^j S_2 = X_1^j + q_1^j S_2$. ...(1 mark)

At time $t = 1$, the portfolio decision is given by:

$$\max_{q_1^j} E\left[U\left(X_2^j\right)\right].$$

...(1 mark)

This is subject to the constraints: $X_2^j = X_1^j + q_1^j S_1$ and $X_1^j + q_1^j S_1 = X_0^j + q_0^j S_1$. ...(1 mark)

[5 marks] 3. Write (step-by-step) the problem of market making using Limit Orders, with the distribution of Limit Orders being described by an exponential distribution. Hence determine the optimal depth (in terms of the parameter of the exponential distribution) at which the Limit Orders are to be posted.

Answer:

The MM's problem is to choose the distance from the mid-price, the depth $\delta^{\pm}$. The uncertainty from MO's come from the probability that an MO arrives ($p_{\pm}$). ...(1 mark)
The probability that once it arrives it walks the book up to where the MM's are resting ($\delta^{\pm}$ away from the mid-point), which is described by the CDF $P_{\pm}$. Therefore, the probability that the LO will be filled is $p_{\pm}P_{\pm}(\delta^{\pm})$. ...(1 mark)
If we assume that the distribution of the other LO's in the LOB is described by an exponential distribution, with parameter $\kappa^{\pm}$, then we have:

$$p_{\pm}P_{\pm}(\delta^{\pm}) = p_{\pm}e^{-\kappa^{\pm}\delta^{\pm}}.$$

...(1 mark)

Let Π denote the MM's profit per trade. Then the MM's optimization problem is given by the following expression:

$$\max_{\delta^+,\delta^-} E\left[\Pi\left(\delta^+,\delta^-\right)\right] = \max_{\delta^+,\delta^-}\left[p^+e^{-\kappa^+\delta^+}\delta^+ + p^-e^{-\kappa^-\delta^-}\delta^-\right].$$

...(1 mark)

The solution os that the LO's are posted at the following depths:

$$\delta^{\pm,*} = \frac{1}{\kappa^{\pm}}.$$

...(1 mark)