MA668: Algorithmic and High Frequency Trading
Lecture 12

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Figure 3.1

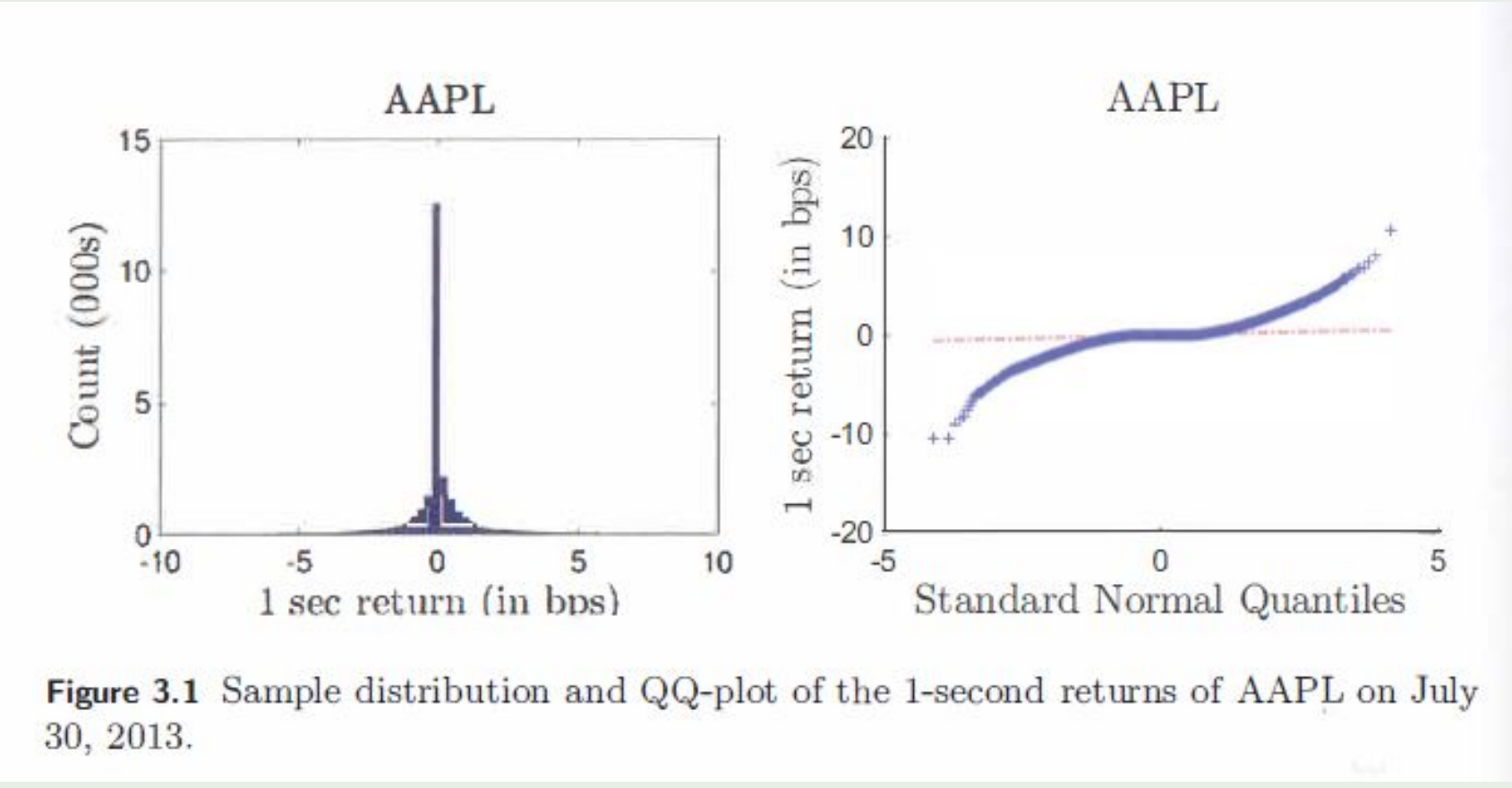

Figure 3.1 Sample distribution and QQ-plot of the 1-second returns of AAPL on July 30, 2013.

Figure: Figure 3.1: Distribution is single-peaked and seems to have fat tails, which is confirmed by the QQ-plot.

Asset Prices and Intraday Returns (Contd ...)

1. Fat tails exhibited by the asset returns occur often when one is sampling at short intervals.
2. However they may persist even at longer frequencies.
3. One can assume that tails follow a power-law with a probability distribution function given by $f(r) = \frac{\alpha - 1}{r_{\min}} \left(\frac{r}{r_{\min}} \right)^{-\alpha}$.
4. The Maximum Likelihood Estimator (MLE) for α can be shown to be: $\widehat{\alpha} = 1 + T \left[\sum_{t=1}^{T} \log \left(\frac{r_t}{r_{\min}} \right) \right]^{-1}$.
5. For the data that was considered $\widehat{\alpha}_{\text{right}} = 3.35$ and $\widehat{\alpha}_{\text{left}} = 3.38$.

Figure 3.2

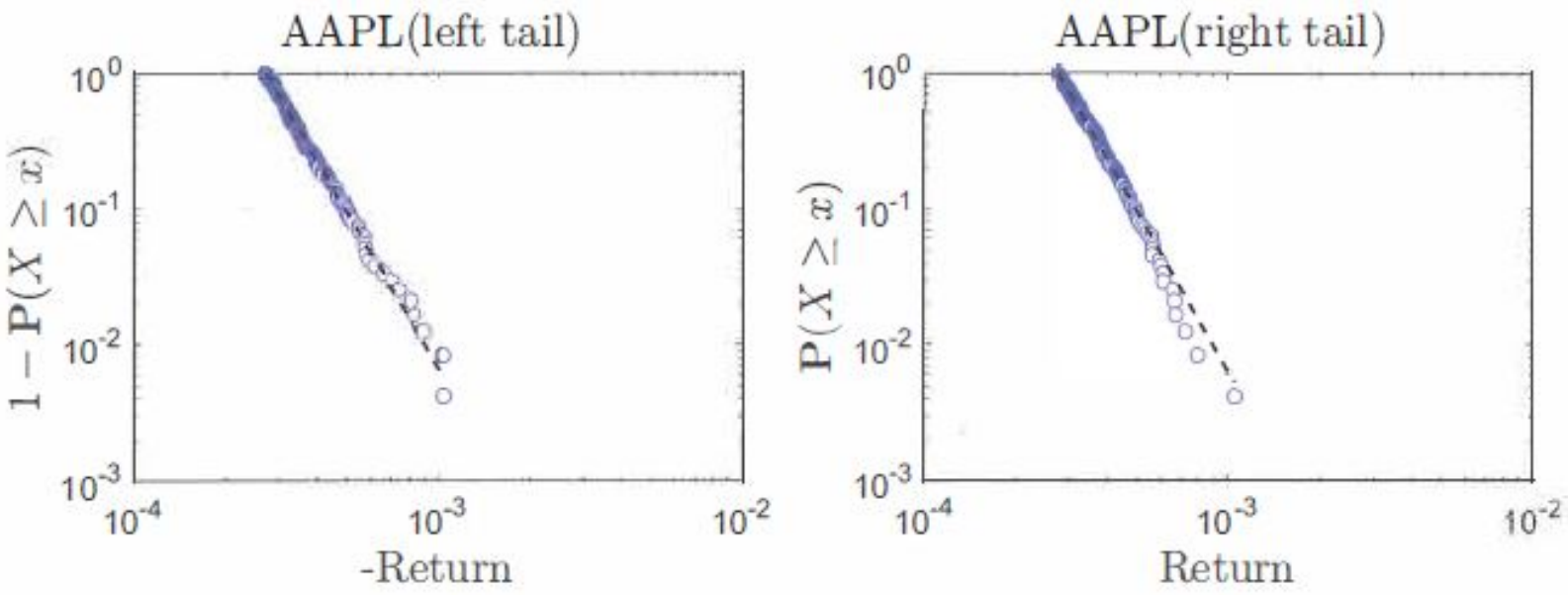

Figure 3.2 Plot of the tails (log scales) for AAPL on July 30, 2013.

Figure: Figure 3.2: Zooming into the tail.

Interarrival Times

1. As the sampling frequency becomes smaller, it becomes more difficult to try and model the observed prices as a continuous process and then we need to consider a discrete process.
2. Accordingly, we start by looking at the interarrival times between movements at either the bid or the ask.
3. Let τ_i be the times at which there is change either in the bid or in the ask and we look at the frequency of the interarrival times $X_i := \tau_{i+1} - \tau_i$.
4. For the data considered the mean is 10.4 ms and the median is 3 ms.

Figure 3.4

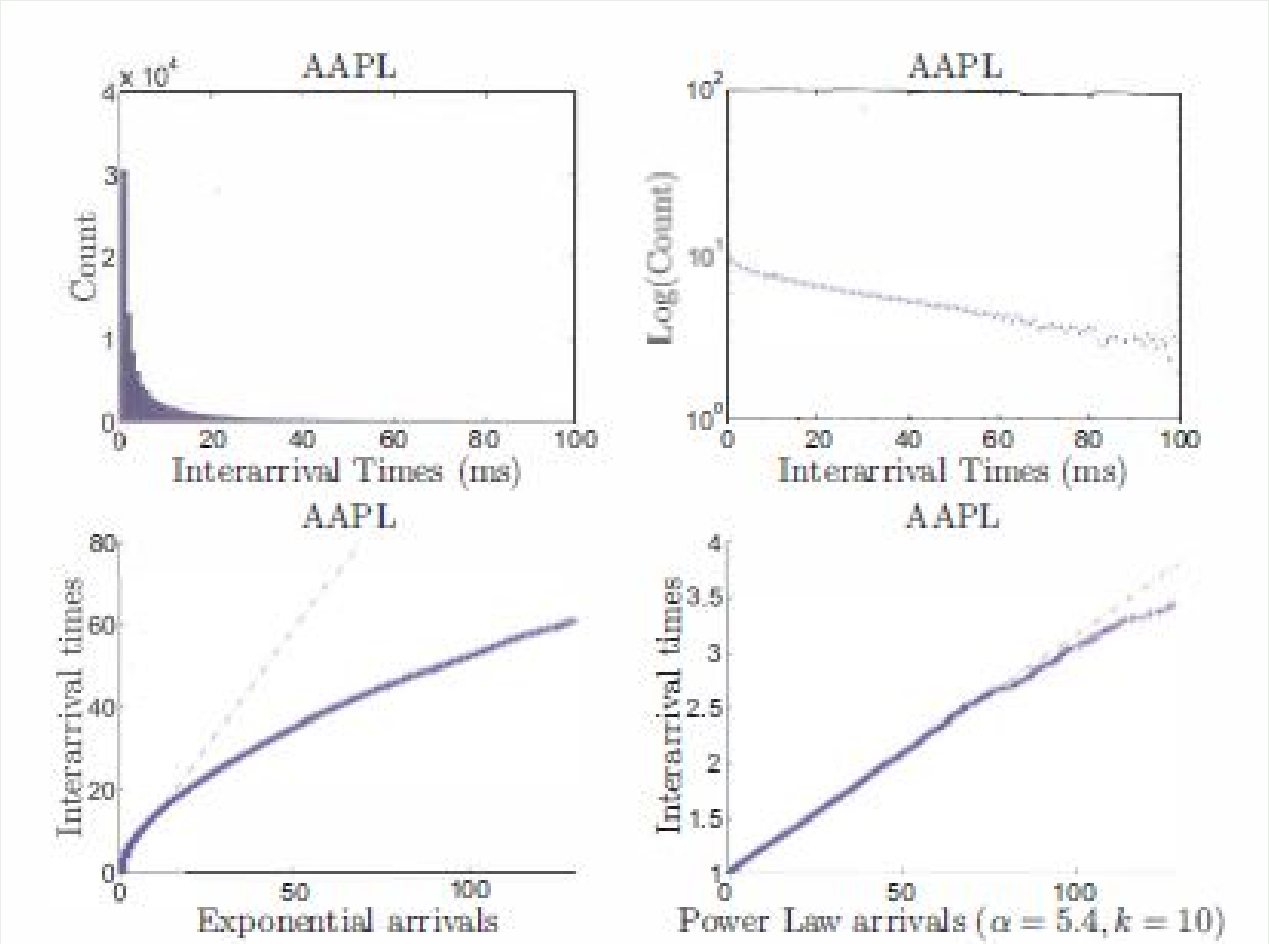

Figure 3.4 Plot of the frequency of interarrival times (X) in absolute and log scales for AAPL on July 30, 2013. The bottom panel describes the QQ-plot relative to the exponential distribution and the power law distribution: $\mathbb{P}(X_i \leq x) = 1 - (k/x)^{\alpha}$.

Figure: Figure 3.4

Latency and Tick Size

1. Goal: Suppose that we wish to execute a trade, say buy 1000 shares of AAPL.
2. Trade-off dilemma: Immediacy VERSUS Cost of execution.
3. If immediacy is of primary concern, then the fastest way to execute a trade is to cross the spread and aggressively execute as much as possible using a MO.
4. However, for an agent driven by immediacy, it is important to account for a new issue, namely, "message latency".
5. Latency: Refers to the delay between sending a message to the market and it being received and processed by the exchange.
6. Sometimes, the time it takes for the exchange to acknowledge receipt of the message is also added in.

Latency and Tick Size (Contd ...)

1. Latency is random and is contingent on many factors:
   A. Distance between sender and exchange.
   B. Structure and type of network.
   C. Amount of orders in the network (which could generate congestion).
2. From the literal meaning perspective, the word "latency" is more generally used to express the time it takes for a message to travel from one point to another.
3. Example:
   A. Time it takes for a message to go from the Chicago Mercantile Exchange (CME) colocation center in Chicago to NASDAQ's processing center in New Jersey.
   B. Estimates of this latency are [6.7 ms, 7.5 ms] when traveling by fiber optic cable and [4.2 ms, 5.2 ms] when traveling by microwave (on a clear day)

Latency and Tick Size (Contd ...)

1. Important: If an agent is trading from home, through a broker, then she/he must be aware of the substantial delay between the moment she/he asks her/his broker to execute the order and the time in which it reaches the market.
2. During this period, the market conditions may have changed a great deal.
3. If however, the agent is trading directly through a broker's feed or has their own feed into the market, the latency will be much shorter, but relatively more to the centers which are co-located.
4. In principle: Players who are co-located face similar latency amongst themselves, though there would still be some latency dependent on their software and hardware configuration.

Latency and Tick Size (Contd ...)

1. Latency is an issue for agents for several reasons:
   - (A) An agent who is trading frequently and on narrow margins needs to be aware of the state of the market and be able to adapt her/his orders (posting a new MO or canceling an existing LO or submitting an MO).
   - (B) When executing an MO, an agent needs to be aware of the relationship between her/his choice of routing strategy and how other traders may react to the information that may be extracted from observing the strategy's outcomes.
   - (C) A large, poorly routed MO will telegraph its progress through the several exchanges leading to poor execution quality and high execution costs, as other traders reposition themselves to absorb the order at more favorable prices (to them).
2. For certain assets, circumstances change very fast and while for others, the market is quite stable.
3. (A) Table 3.3: Slow changes in seldomly traded assets.
   (B) Table 3.4: Faster changes in more frequently traded assets.

Table 3.3

Asset	Var	$\Delta X \neq 0$ (%)	Stats (for $\Delta X \neq 0$) P01	Q1	Q2	Q3	P99
ISNS	Bid	4.2	-43.0	-4.0	1.0	5.0	40.0
	Ask	3.4	-51.0	-4.0	-1.0	3.0	51.0
	Midprice	6.7	-25.8	-2.0	0.5	2.0	26.8
	Quoted Spread	6.7	-45.0	-5.0	-1.0	4.0	46.0
FARO	Bid	48.6	-21.0	-2.0	1.0	3.0	19.0
	Ask	49.0	-19.0	-3.0	-1.0	3.0	22.0
	Midprice	63.4	-16.0	-1.5	0.5	2.0	16.0
	Quoted Spread	60.8	-18.0	-2.0	-1.0	2.0	18.0
MENT	Bid	44.2	-5.0	-1.0	1.0	1.0	6.0
	Ask	44.1	-5.0	-1.0	-1.0	1.0	5.0
	Midprice	52.8	-4.5	-1.0	0.5	1.0	5.0
	Quoted Spread	31.1	-5.0	-1.0	-1.0	1.0	4.0

Table 3.3 One minute changes in bid, ask, midprice, and quoted spread.

Figure: Table 3.3

Table 3.4

Asset	Var	$\Delta X \neq 0$ (%)	Stats (for $\Delta X \neq 0$) P01	Q1	Q2	Q3	P99
AAPL	Bid	3.84	-17.0	-3.0	-1.0	3.0	18.0
	Ask	4.00	-18.3	-3.0	1.0	3.0	17.0
	Midprice	6.75	-11.5	-1.5	0.5	1.5	11.0
	Quoted Spread	6.69	-16.0	-3.0	-1.0	3.0	18.0
ORCL	Bid	0.41	-2.0	-1.0	-0.5	1.0	2.0
	Ask	0.40	-2.0	-1.0	1.0	1.0	2.0
	Midprice	0.47	-2.0	-1.0	0.5	1.0	2.0
	Quoted Spread	0.16	-3.0	-1.0	-1.0	1.0	2.0

Table 3.4 One hundred ms changes in bid, ask, midprice, and quoted spread.

Figure: Table 3.4

Table 3.3 (Contd...)

1. Bid, ask, mid-price, quoted spread from the end of one minute to the next: ISNS, FARO and MENT.
2. Note: One minute delay is a very long time for a trader and one would not expect such a delay, unless one is trading through a very slow connection to the exchange (or the order does not go through the exchange).
3. In Column 1, we have the percentage of minutes for which bid/ask/mid-price/quoted spread is different.
4. For bid (ISNS) there is movement of +5 cents (Q3) to -4 cents (Q1).
5. For mid-price (ISNS) there is movement of +2 cent (Q3) to -2 cents (Q1).
6. This is an unconditional analysis for all months of 2013 and does not take into account that market participants react to order flow.

Table 3.4 (Contd ...)

1. The picture is quite different when one examines the most popular stocks (in terms of activity).
2. Assets such as AAPL and ORCL will most certainly experience changes in prices within one minute, but for these assets a one minute delay is unreasonably long by any standard.
3. Table 3.4 reflects changes after a 100 ms delay rather than one minute and for the last 3 months in 2013, rather than the whole year.
4. Reporting: Instead of statistics for the whole sample, we report the median values for the statistics computed at the daily level.
5. AAPL: After computing the daily percentage for 100 ms intervals with non-zero bid price changes for AAPL for each day from October-December 2013, the median is reported to be 3.84%.
6. Noticeable difference in frequency and magnitude of price changes for AAPL and ORCL.

Non-Markovian Nature of Price Change

1. Question: How successive price changes are interrelated?
2. Examine first: Whether the sign of the current price change can predict the sign of the next (non-zero) price change.
3. AAPL on July 30, 2013: Table 3.5 shows how an increase in the price (bid or ask) is more often than not followed by a reversal and similarly for a fall in the price.

	Ask			Bid	
	Uptick	Downtick		Uptick	Downtick
$t/t+1$	($\Uparrow$)	($\Downarrow$)		($\Uparrow$)	($\Downarrow$)
Uptick ($\Uparrow$)	43.0	57.0	Uptick ($\Uparrow$)	36.5	63.5
Downtick ($\Downarrow$)	61.8	38.2	Downtick ($\Downarrow$)	55.3	44.7

Table 3.5 Empirical Transition Rates: Single Price Change.

Figure: Table 3.5

Table 3.6

	Ask		
	$\text{Up}(t+1,t+2)$ ($\Uparrow\Uparrow$)	$\text{Reversal}(t+1,t+2)$ ($\Uparrow\Downarrow$, $\Downarrow\Uparrow$)	$\text{Down}(t+1,t+2)$ ($\Downarrow\Downarrow$)
$\text{Uptick}(t)$ ($\Uparrow$)	17.1	59.5	23.4
$\text{Downtick}(t)$ ($\Downarrow$)	19.6	59.1	21.3
Unconditional	18.3	59.3	22.4

	Bid		
	$\text{Up}(t+1,t+2)$ ($\Uparrow\Uparrow$)	$\text{Reversal}(t+1,t+2)$ ($\Uparrow\Downarrow$, $\Downarrow\Uparrow$)	$\text{Down}(t+1,t+2)$ ($\Downarrow\Downarrow$)
$\text{Uptick}(t)$ ($\Uparrow$)	24.0	60.3	15.7
$\text{Downtick}(t)$ ($\Downarrow$)	23.7	58.1	18.2
Unconditional	23.9	59.1	17.0

Table 3.6 Empirical Transition Rates: Pairs of Tick Changes.

Figure: Table 3.6

Table 3.6 (Contd ...)

1. Investigate: Whether the sign of the current price change can predict the sign of the price changes in the next two periods.
2. Table 3.6 describes the relative frequency of the price reversals ($\Uparrow\Downarrow$ or $\Downarrow\Uparrow$) relative to consecutive move in the same direction ($\Uparrow\Uparrow$ of $\Downarrow\Downarrow$) conditioned on the current price change being $\Uparrow$ or $\Downarrow$.
3. Table 3.6 shows both conditional or unconditional probability of the future two price moves (little difference between both).

Table 3.7

	Ask			
$(t+1)$	A ($\Uparrow\Uparrow$)	B ($\Uparrow\Downarrow$)	C ($\Downarrow\Uparrow$)	D ($\Downarrow\Downarrow$)
A(t) ($\Uparrow\Uparrow$)	54.4	45.6	-	-
B(t) ($\Uparrow\Downarrow$)	-	-	70.0	30.0
C(t) ($\Downarrow\Uparrow$)	34.4	65.6	0.0	0.0
D(t) ($\Downarrow\Downarrow$)	-	-	48.6	51.4

	Bid			
$(t+1)$	A ($\Uparrow\Uparrow$)	B ($\Uparrow\Downarrow$)	C ($\Downarrow\Uparrow$)	D ($\Downarrow\Downarrow$)
A(t) ($\Uparrow\Uparrow$)	43.0	57.0	-	-
B(t) ($\Uparrow\Downarrow$)	-	-	62.2	37.8
C(t) ($\Downarrow\Uparrow$)	32.8	67.2	-	-
D(t) ($\Downarrow\Downarrow$)	-	-	46.9	53.1

Table 3.7 Empirical Transition Rates: Pairs of Ticks.

Figure: Table 3.7

Table 3.7 (Contd ...)

1. Longer sequence of price changes: Can signs of two past price changes predict the next one?
2. Accordingly, we define four states, one for each possible pair of signs of price changes as follows: A: $\Uparrow\Uparrow$, B: $\Uparrow\Downarrow$, C: $\Downarrow\Uparrow$ and D: $\Downarrow\Downarrow$.
3. Price change signs are then used to generate a sequences of A,B,C,D's with overlapping observations, e.g., $\Uparrow\Downarrow\Downarrow\Uparrow$: BDC.
4. There are several transitions that are not possible, e.g., BA.
5. The estimation transition frequencies can be seen in Table 3.7.