MA668: Algorithmic and High Frequency Trading
Lecture 16

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Table 4.3

Asset	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	12.56	45.00	-42.00	3.50	9.23	19.00	65.00
FARO	7.63	8.61	-10.00	3.33	6.50	10.76	32.84
MENT	1.23	1.57	0.00	1.00	1.00	1.03	5.38
AAPL	9.32	3.85	2.67	6.61	8.83	11.47	20.20

Table 4.3 Quantity-weighted Effective spread (in cents).

Figure: Table 4.3

Table 4.3 (Contd ...)

1. Table 4.3: Construction made using one-minute buckets and for each we have computed the ~~quality~~ quantity-weighted effective spreads qES for the four spreads:

$$qES_t = \sum_{j=1}^{m} \frac{q_j}{\sum_{s=1}^{m} q_s} ES_j,$$

where:

A. $j \in \{1, 2, \ldots, m\}$ indexes the trades that took place during minute t.
B. q_j denotes the number of shares in trade j.
C. ES_j is the effective spread for trade j.

Spreads (Contd ...)

1. "Effective spreads" differ from "Quoted spreads" in several ways.
2. Recall that: "Effective spread" is equal to the "Quoted half-spreads", provided a trade executes against a visible LO and does not walk the LOB.
3. Dataset: Trades are recorded via the execution of the posted LO and so we do not have information on the MO that was sent to the market.
4. This implies that none of our executions walk the LOB.
5. This biases our measure of the ES downwards, but the bias is small, as we see very few executions of LO's away from the bid/ask during previous milliseconds (a necessary condition for an MO to walk the LOB at NASDAQ, as any remaining quantity may need rerouting in search of best execution in all markets).
6. Another reason why this bias is small, is that in the fragmented US market, when an MO comes into the market and it is greater than the depth at the bid/ask, the part that is not executed is usually routed to other markets and only under very special circumstances will it literally walk the LOB.

Spreads (Contd ...)

1. Note that, in general this rerouting makes it virtually impossible to reconstruct the quantity of a large incoming MO without specific information from the agent who sent it.
2. Thus, our measured "effective spread" has to be equal to or lower than the (current) "quoted spread".
3. A visible trade will generate $ES = \frac{QS}{2}$.
4. As not all posted LOs are visible, some trades will be executed at prices better than the bid/ask.
5. This will generate an ES that is strictly smaller than the QS and may even produce a *negative* ES.
6. One obtains a negative "effective spread" if an incoming market buy (sell) order meets an hidden sell (buy) order that is below (above) the mid-price.
7. Refer to Table 4.3: P01 entries for ISNS and FARO.

Spreads (Contd ...)

1. There is another difference between ES and QS, namely, that ES can only be measured when there is a trade, while QS are always observable.
2. Therefore, it may be possible that quoted spreads different from effective spreads if market conditions around trades are systematically different from those without trades.
3. Refer to Tables 4.2 and 4.3: Inter-quartile range and standard deviation shows that ES is less volatile than QS.
4. We have seen that assets with greater trading frequency have better market quality in the sense that execution costs for small trades (the quoted spread) is smaller.
5. If we look at the intraday pattern of trades we find further evidence that lower execution costs occur when trading is high.

Figure 4.6

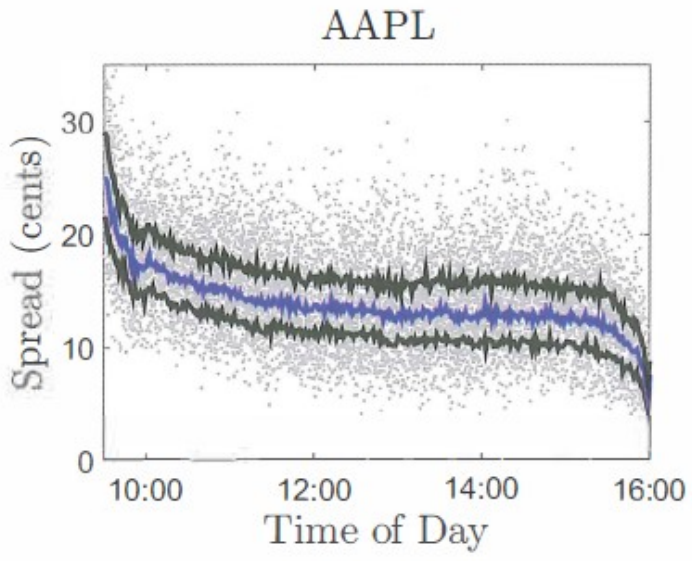

Figure 4.6 Intraday spread pattern: interquartile range for one-minute returns.

Figure: Figure 4.6

Figure 4.6 (Contd ...)

1. Figure 4.6: Plot of one-minute time-averaged quoted spreads for AAPL in 2013, as well as the first quartile, median and third quartile.
2. Observation: Quoted spreads are initially high, decline rapidly during the first half-hour of trading, and are mostly constant throughout the remainder of the day until the last (half) hour of trading, when the spread rapidly declines.
3. This pattern in quoted spreads is also seen in the effective spread.
4. Recall from Figure 4.2: Afternoon is associated with increased trading, and hence we find, that during a period with a constant flow of information, more trading and lower spreads occur together.
5. However: This connection between trading volume and spreads fails during the morning where the situation is completely reversed, *i.e.*, declining volumes go hand-in-hand with declining spreads.
6. This can be explained to the other factor affecting volume, namely, information.

Figure 4.6 (Contd ...)

1. When the market opens, and during the subsequent hour of trading, the market absorbs all the information that has accumulated since the last market close, which would explain the heavier trading.
2. But: A lot of new information is also associated with a great deal of uncertainty.
3. Observation 1: In presence of greater uncertainty it is optimal to post wider bid-ask spreads.
4. Observation 2: (To be shown later) Greater price uncertainty increases the depths of the quotes that a risk-averse market maker sends to the LOB.
5. Observation 3: More information at the beginning of the day explains the coincidence of wider spreads with greater volume.

Volatility

1. Volatility measures price fluctuations and represents a cost (*i.e.*, low market quality) in the sense that a rapidly changing price makes it difficult to determine the actual market price of the asset.
2. Of course, one may observe price changes because the true market value of the asset is changing, and hence the literature differentiates between fundamental volatility and microstructure noise.
   - (A) Volatility: Captures the fluctuations in the true market price.
   - (B) Microstructure noise: Represents extraneous fluctuations due to the way the market operates.
3. There is a large (and growing) number of measures of raw volatility (unconditional volatility which does not distinguish fundamental volatility from microstructure noise) and of microstructure volatility.

Volatility (Contd ...)

1. For brevity: The term volatility is used to refer to raw volatility and we measure the volatility of asset returns, rather than of asset prices.
2. The simplest measure of volatility is the realized volatility *i.e.*, the standard deviation of a sample of returns.
3. Sometimes volatility is measured using the square (or the absolute value) of the return, which is useful if there are very few observations and we are working on a sufficiently small time scale so that the mean return can be safely assumed to be (essentially) equal to zero.
4. Another common alternative is to use the range of return (or price) *e.g.*, by taking the difference between the maximum and the minimum values of the price over a certain interval and normalizing it by either the minimum value, the mean/median, the initial value, or the average of the minimum and the maximum.

Table 4.4

Asset	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	16.6	54.8	0.0	0.0	0.0	14.4	160.3
FARO	8.3	12.7	0.0	3.8	6.6	10.3	31.3
MENT	5.6	6.6	0.0	3.2	4.6	6.5	20.1
AAPL	5.5	4.2	1.0	3.3	4.7	6.7	18.1

Table 4.4 Realised one-min volatility (15 min samples).

Figure: Table 4.4

Table 4.4 (Contd ...)

1. Table 4.4: Displays the statistical properties of realized volatility measured as σ_t, the standard deviation of one-minute returns over fifteen minute period (for everyday of 2013), that is, for every 15-minute period (each 15-minute period indexed by t):

$$\sigma_t^2 = \frac{1}{15}\sum_{j=1}^{15}\left(r_j - \frac{1}{15}\sum_{s=1}^{15} r_s\right)^2,$$

   where $j \in \{1, 2, \ldots, 15\}$ is the index for each of the individual minutes withing the 15-minute period (t) and r_j is the realized return for minute j in t.

2. Observation: As we go from AAPL to ISNS, we observe that the less frequently traded asset has a higher mean volatility.

Table 4.5

Asset	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	0.10	1.37	0.00	0.00	0.00	0.00	0.00
FARO	3.77	2.33	1.10	2.50	2.84	4.26	12.05
MENT	3.00	3.47	0.00	0.00	2.18	5.04	12.84
AAPL	6.23	2.85	3.72	4.72	5.28	6.45	19.47

Table 4.5 Interquartile Range of one-minute returns.

Figure: Table 4.5

Table 4.5 (Contd ...)

1. Table 4.5: Displays an alternative way to look at the same idea, only now we are looking at the statistical properties of a different variable, sampled over smaller time intervals.
2. We include the statistics for the inter-quartile range of one-minute returns.
3. AAPL: 5.28 bps is the median of 252 observations, one for each trading day, of the inter-quartile range observed for the one-minute returns during that day.
4. ISNS displays a zero inter-quartile range for most days (which is natural, considering that it is an asset that displays very few price movements).
5. By focusing on the median inter-quartile range for each day, this sampling method misses the very large but relatively rare price movements that are responsible for the high volatility numbers for ISNS in Table 4.4.

Table 4.5 (Contd ...)

1. A different effect is responsible for the differences between MENT and FARO.
2. Despite MENT having similar trading activity than FARO, it has lower volatility.
3. MENT has more than 25% of days with an inter-quartile range equal to zero, but it also has lower realized one-minute volatility.
4. The difference between MENT and FARO has probably much more to do with MENT's relative tick size.
5. As we saw above when looking at the quoted spread, the one cent tick size is a binding constraint for MENT most of the time.
6. This leads to an unusual degree of price stickiness, as many small price movements are not sufficient to push the bid or ask a whole cent (5 bps) away from their current levels.
7. Thus, despite having similar activity levels as FARO, its price displays lower volatility.