MA668: Algorithmic and High Frequency Trading
Lecture 40

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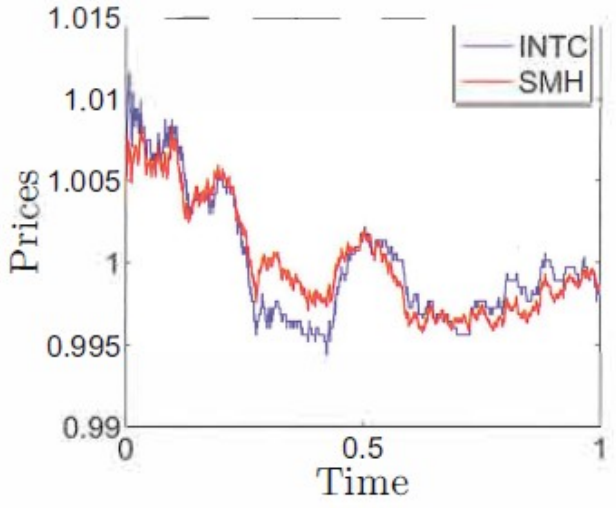
An Overview

1. The success of many trading algorithms depends on the quality of the predictions of stock price movements.
2. Predictions of the price of a single stock are generally less accurate than predictions of a portfolio of stocks.
3. A classical strategy which makes the most of the predictability of the joint (rather than the individual) behaviour of two assets is pairs trading, where a portfolio consisting of a linear combination of two assets is traded.
4. At the heart of the strategy is how the two assets co-move.
5. An example: We take two assets whose spread, that is the difference between their prices, exhibits a marked pattern and deviations from it are temporary.
6. Then: Pairs trading algorithms profit from betting on the empirical fact that spread deviations tend to return to their historical or predictable level.
7. Thus, pairs trading fall under the class of strategies sometimes labeled as statistical arbitrage (or StatArb for short). They are not true arbitrages (which are strategies that produce returns in excess of the risk-free rate with zero risk), but rather are strategies which bet off of the typical behaviour of asset prices and hence are not risk-free.

Figure 11.1



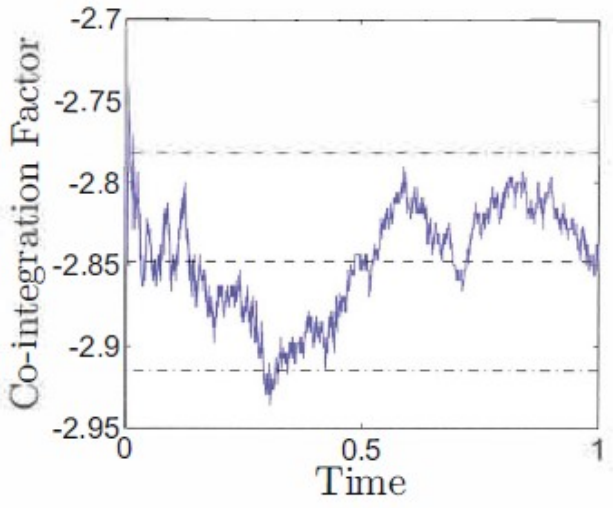


Figure 11.1 INTC and SMH on November 1, 2013 for the whole day of trading: (left panel) midprice relative to mean midprice; (right panel) co-integration factor. The dashed line indicates the mean-reverting level, the dash-dotted lines indicate the 2 standard deviation bands.

Figure: Figure 11.1

Figure 11.1 (Contd ...)

1. Figure 11.1: We show an example with Intel Inc. (INTC) and the Market Vectors Semiconductor ETF (SMH) for November 1, 2013.
2. The left panel shows the mid-price paths of INTC and SMH (scaled by the mean mid-prices).
3. It is clear that the two assets tend to move together and in the same direction.
4. Thus, a portfolio consisting of long one asset and short the other will exhibit a less volatile and more predictable behaviour than that of the individual assets.
5. In this case, since the assets tend to co-move in the same direction, a simple pairs trading strategy is to buy the portfolio if its value is less than a threshold and sell it if its value is greater than the threshold.
6. This strategy will deliver profits as long as the value of the portfolio fluctuates about and reverts to the threshold.

An Overview (Contd ...)

1. A more sophisticated approach is to look at the co-integration factor of the prices of the two stocks.
2. The right panel of Figure 11.1 shows the path of a co-integration factor:

$$\epsilon_t = AS_t^{INTC} + BS_t^{SMH},$$

   where A and B are estimated (from the data that day) to be $A \sim 0.95$ and $B \sim -0.63$.
3. Thus, if the mean-reverting behaviour we have observed is persistent, then we expect the value of a portfolio long 0.95 shares in INTC and short 0.63 shares in SMH to hover around the mean of the co-integration factor which is zero.
4. Question: How can we profit from the mean-reverting to zero value of this portfolio.
5. Answer: A pairs trading strategy which consists of going long the portfolio when it is "cheap" and then closing the position when the portfolio's value increases, or going short the portfolio when it is "dear" and closing the position when the portfolio's value decreases.

Ad Hoc Bands

1. A simple strategy to profit from the co-integration factor's mean-reversion, as seen in Figure 11.1, is to place bands which are one standard deviation above and below the mean-reverting level, which is zero, and do one of the following:
   A. `Buy one unit of the portfolio if the lower band is hit.`
   B. `Sell one unit of the portfolio if the upper band is hit.`
2. Once the strategy has entered into a position, either long or short, the next step is to close it.
3. To close the position: The strategy waits for the value of the portfolio to be within a small interval, say $\frac{1}{10}$ standard deviation of the mean-reverting level of the co-integration factor.
4. At that point the agent liquidates the position.

Figure 11.2

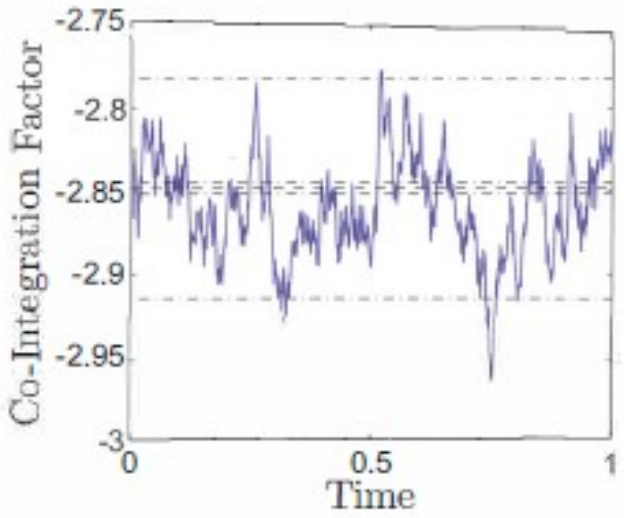
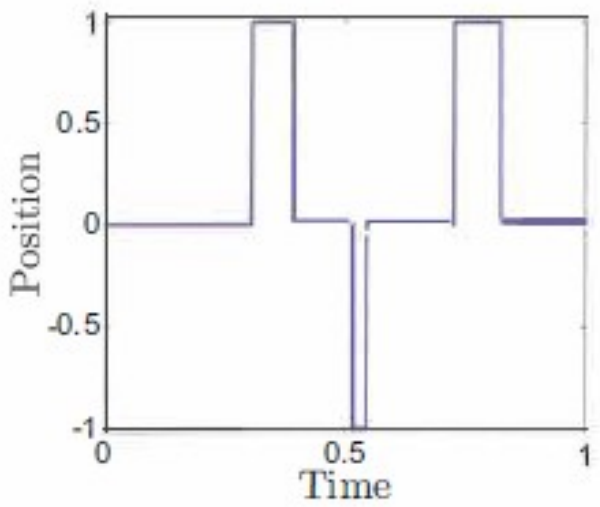


Figure 11.2 A sample path of the co-integration factor, the trading position, and the book value of the trade, using the two standard deviation banded strategy.



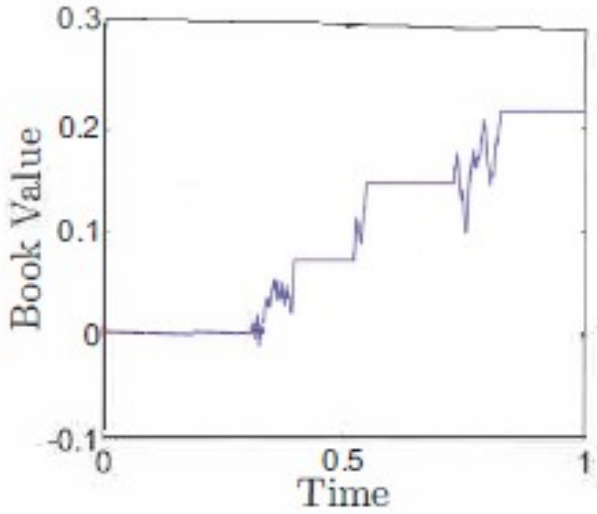


Figure: Figure 11.2

Figure 11.2 (Contd ...)

1. Figure 11.2: Shows three pictures:
    1. A simulated sample path of the co-integration factor.
    2. The path of the inventory of the strategy which opens and closes positions a few times during the trading horizon.
    3. Finally, the accumulated cash and marked-to-market value of the strategy.
2. The strategy starts by waiting for the path of the co-integration factor to breach one of the outer bands.
3. At around $t \sim 0.3$ the path hits the lower outer band so the agent longs the portfolio, in anticipation of its value appreciating.
4. Then for a short period of time the agent holds on to the portfolio whose value fluctuates one-to-one with changes in the co-integration factor.
5. The book value of the strategy is given by:

$$BV_t = X_t + \beta_t \left(AS_t^{INTC} + BS_t^{SMH} \right),$$

where X_t is the strategy's accumulated cash position and β_t denotes the units of the portfolio held by the strategy, in this case assuming that $\beta_t \in \{-1, 0, 1\}$.

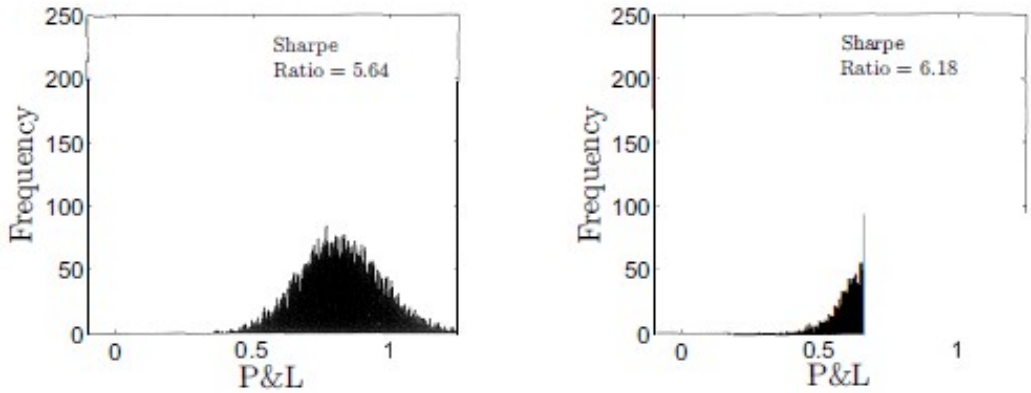
Figure 11.2 (Contd ...)

1. The next step is to wait until the co-integration factor hits the inner band, to close the position.
2. Here this occurs at around $t \sim 0.4$ and the strategy goes back to holding zero units of the portfolio and locks in a profit equal to the difference between the outer and inner bands.
3. Next, the strategy waits for a little while and enters into a short position at around $t \sim 0.51$ and liquidates at around $t \sim 0.55$.
4. Finally, the strategy enters into a long position around $t \sim 0.73$ and liquidates at around $t \sim 0.83$.
5. In all, for this simulated path, the strategy makes a profit of three times the outer-inner band spread.

Figure 11.2 (Contd ...)

1. In the scenario in Figure 11.2, the agent ends the trading horizon with zero inventory.
2. This is not guaranteed by the strategy, and in fact the strategy may have entered a long/short position which never reverted back to the inner band by the end of the trading horizon.
3. This would induce potential losses into the strategy.
4. The wider the trigger bands, the more likely it is to end with inventory.
5. Also, while wider bands have larger profits when the position closes out, the co-integration factor makes fewer outer-inner band transitions when the band size increases.

Figure 11.3



(a) band = 0.25 × *std.dev.*

(b) band = 0.5 × *std.dev.*

(c) band = 1.0 × *std.dev.*

(d) band = 2.0 × *std.dev.*

Figure 11.3 P&L histograms from 10,000 scenarios using the naive strategy with various trigger bands.

Figure: Figure 11.3

Figure 11.3 (Contd ...)

1. Figure 11.3: Shows the profit and loss (P&L) histogram from generating 10,000 scenarios from the estimated model, computing the co-integration factor and placing trades as described above.
2. The figure shows the effect that the band size has on the P&L, as well as the Sharpe Ratio (that is, the mean P&L divided by the standard deviation of the P&L).
3. Notice that the Sharpe Ratio first increases as the band widens, but then starts decreasing.
4. Also notice that when the band size is largest at $2 \times$ std.dev., the distribution is multi-modal.
5. In fact, on close examination, all of the distributions are multi-modal.
6. The reason is because the profit from closing out a long/short position equals approximately the band size (since you enter into a long/short position once the factor hits the band and close it out near the mean).

Figure 11.3 (Contd ...)

1. Hence, the P&L is concentrated near integer multiples of the band size and the weight on a given multiple equals the probability of making that many round trip trades during the trading horizon.
2. The reason the P&L is not concentrated solely on the band is because towards the end of the trading horizon, the co-integration factor may not return to the equilibrium value prior to trading end.
3. Hence, the trader must close out a position that might make less profit than the band size, or in fact may take a loss if the co-integration factor moves away from the equilibrium prior to ending the trading horizon.