MA668: Algorithmic and High Frequency Trading
Lecture 36

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A Prelude

1. Execution algorithms are designed to minimize the market impact of large orders.
2. Recall: Slicing and dicing parent orders into child orders is the main principle upon which most algorithms are devised.
3. One source of uncertainty which determines the market impact of each child order is the volume of the child order relative to the volume that the market can bear at that point in time.
4. In order to see why, we consider executing one child order.
   - (A) If it is small, then the order will not walk beyond the best quotes in the limit order book (LOB) and it will have little or no temporary market impact.
   - (B) If the order size is considerable, then it may walk through several layers of the LOB and therefore, receive poor execution prices relative to the mid-price.
5. Furthermore, to complete this description of order size and volume we must also ask whether any other orders are reaching the market at the same time or just prior to the arrival of the child order.

A Prelude (Contd ...)

1. Over short-time scales (seconds), the impact of a market order (MO) depends on many factors where size, relative to what is displayed in the LOB, is the key.
2. But what traders see on the LOB might change by the time their orders reach the market.
3. Even traders with access to ultra-fast technology are exposed to the risk of changes in the quantity and prices displayed by limit orders (LOs) because there is a delay between sending an MO and its execution.
4. These changes are due to modifications in the provision of liquidity and the activity of liquidity takers.
5. LOs may be cancelled or more may be added and thus the best quotes and/or depth of the LOB change.
6. Similarly, other MOs may arrive just before the agent's MO and consequently deplete the liquidity that was sitting in the LOB.
7. Thus, the size of the agent's child order is relative to what the LOB can bear when all MOs amalgamate with that of the agent's on the liquidity taking side of the market.

A Prelude (Contd ...)

1. Over long-time scales (minutes/hours), the accumulated orders sent by the agent can exert unusual one-sided pressure which may result in further adverse market impact.
2. Ideally, an agent's strategy may avoid adverse over-tilting of the market order flow by devising algorithms that camouflage her/his orders.
3. One way to do this is to choose a rate of trading which targets a predetermined fraction of the total volume traded over the time horizon of the strategy.
4. Here are two strategies that aim at executing a number of shares equivalent to a fraction of:
   - (A) The rate at which other participants are sending MOs.
   - (B) The total volume that has been traded over the entire time horizon.
5. The rate and the total volume quantities are connected because total volume is the sum over the rate of trading, but the optimal execution strategy could be quite different in both cases

A Prelude (Contd ...)

1. One simple approach to targeting (A) (previous slide) is to observe the volume traded over the last several seconds or minutes, and then trade a percentage of this volume over the next several seconds or minutes.
2. Obviously this approach is not optimal because it does not address the problem of market impact when the agent's orders amalgamate with other orders.
3. Targeting (B) (previous slide) is difficult because total volume traded, over the planned execution horizon, is not known ahead of time.
4. Naturally, trading a percentage of the volume that has been traded over the last several seconds will also target (B) (previous slide), although it may not be optimal.
5. Moreover, neither (A) nor (B) (both from previous slide) is entirely compatible with the objective of completing the acquisition or liquidation of an order in full by the end of the trading horizon, because there is no guarantee that the sum of the fractions of volume traded will add up to the number of shares that the agent set out to acquire or liquidate.

A Prelude (Contd ...)

1. Trading algorithms that target benchmarks based on volume are extensively used.
2. One of the most popular benchmarks is the Volume Weighted Average Price, known as VWAP.
3. This benchmark consists, as it name clearly suggests, in calculating:

$$VWAP(T_1, T_2) = \frac{\int_{T_1}^{T_2} S_t dV_t}{\int_{T_1}^{T_2} dV_t}, \tag{1}$$

   where V_t is the total volume executed up to time t, S_t is the mid-price and $[T_1, T_2]$ is the interval over which VWAP is measured.
4. Targeting VWAP is challenging for it is difficult to know ahead of time how many shares will be traded over a period of time.

A Prelude (Contd ...)

1. Investors target VWAP because of their desire to ensure that when acquiring or liquidating a large position they obtain an average price close to what the market has traded over the same period of time.
2. One way to target VWAP is to follow strategy (B) because targeting a fraction of the rate of trading at every instant in time ensures that the investor is tracking the average price.
3. Ideally, if the investor's strategy smoothes the execution of the number of shares she/he wishes to execute over the planned time horizon and at the same time adamantly targets a fixed proportion of the rate at which other market participants are trading, then the average cost of the shares she/he executes will be close to VWAP.
4. One can formulate and solve the agent's liquidation problem for (A) and (B) in a way that is consistent with the overall goal of full (or partial) liquidation: The acquisition problem is very similar.
5.
   - (A) Strategies that target (A) are often called percentage of volume (POV).
   - (B) Strategies that target (B) are labeled as percentage of cumulative volume (POCV).

Figure 9.1

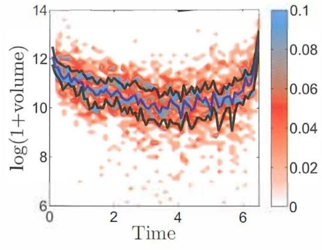


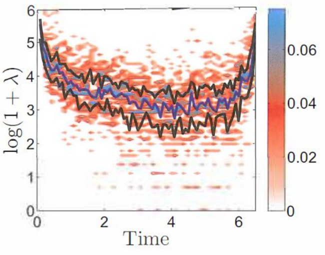
Figure 9.1 Trading volume, for both buy and sell orders, for INTC for Oct-Nov, 2013 using 5 minute windows.

Figure: Figure 9.1

Figure 9.1 (Contd ...)

1. Figure 9.1: Displays the volume of trades (for both buy and sell orders) of INTC (Intel Corporation) using 5 minute windows for every trading day (which consists of 6.5 hours) of the fourth quarter of 2013.
2. The panel on the left shows a heat-map of the data together with the median (second quartile) as well as first and third quartile estimates.
3. Note that we plot $\log(1 + \text{volume})$ because there are 5 minute windows with no trades and so the volume is zero.
4. The panel on the right shows a functional data analysis (FDA) approach to viewing the data whereby the volumes are regressed against Legendre polynomials (the thin lines).
5. The mean of the regression is then plotted as the solid blue line, which represents the expected (or average) trading volume throughout the day for this ticker.
6. The data are also shown using the dots.
7. From the two pictures one observes that although volume exhibits a "U"-shaped pattern, high volumes at the start and end of the trading day and lower volume in the hours in between, there are days where realized traded volumes deviate from this intraday pattern.

Figure 9.2



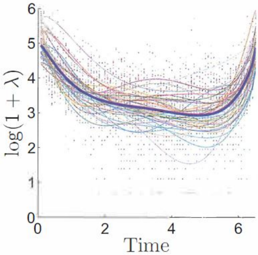


Figure 9.2 Trading intensity, for both buy and sell orders, for INTC for Oct-Nov, 2013 using 5 minute windows.

Figure: Figure 9.2

Figure 9.2 (Contd ...)

1. Figure 9.2: Uses the same data as Figure 9.1, but instead of the volume it shows the intensity of trades for both buy and sell orders.
2. For each 5 minute window we calculate the intensity λ as the number of trades that were made over that time window.
3. The figure shows $\log(1 + \lambda)$ because there are 5 minute intervals where no MOs were sent.
4. The panel on the left shows a heat-map of the data together with the median as well as first and third quartile estimates.
5. The panel on the right shows an FDA approach to viewing the data whereby the intensities are regressed against Legendre polynomials (the thin lines).
6. The mean of the regression is then plotted as the solid blue line, which represents the expected (or average) trade intensity through the day.
7. The data are also shown using the dots.
8. As expected, the trading intensity follows a similar pattern to that of the volume shown in Figure 9.1.

Targeting Percentage of Market's Speed of Trading

1. We assume that the agent's execution strategy targets a percentage of the speed at which other market participants are trading and we focus on the liquidation strategy with MOs only.
2. The setup for optimal acquisition, as opposed to liquidation, is very similar.
3. In the liquidation problem, the agent searches for an optimal liquidation speed, which we denote by ν_t, to target a fraction ρ of the speed at which the overall market (excluding the agent) is trading.
4. This is different from a strategy which caps the optimal liquidation speed to be at most a fraction of other market participant's speed of trading, which will become clear when we write down the agent's performance criteria.
5. The agent's inventory Q^ν, satisfies the SDE:

$$dQ_t^\nu = -\nu_t dt, \;\; Q_0^\nu = \mathfrak{R}.$$

Targeting Percentage of Market's Speed of Trading (Contd ...)

1. Let μ_t denote the speed at which all other market participants are selling shares using MOs.
2. This rate of selling can be estimated by summing up all the shares that are executed over a small time window and dividing by the time window.
3. We assume that the agent's speed of liquidation is not taken into account when calculating μ_t.
4. Therefore, since the agent's objective is to seek an optimal liquidation speed ν_t, which targets the POV $\rho\mu_t$, at every instant in time, with $0 < \rho < 1$, her/his performance criteria and value function are:

$$H^{\nu}(t,x,S,\mu,q) = \mathbb{E}_{t,x,S,\mu,q}\left[X_T^{\nu} + Q_T^{\nu}\left(S_T^{\nu} - \alpha Q_T^{\nu}\right) - \varphi\int_t^T \left(\nu_u - \rho\mu_u\right)^2 du\right], \tag{2}$$

and

$$H(t,x,S,\mu,q) = \sup_{\nu\in\mathcal{A}} H^{\nu}(t,x,S,\mu,q), \tag{3}$$

respectively.

Targeting Percentage of Market's Speed of Trading (Contd ...)

1. Here X_T^ν is terminal cash, $\alpha \geq 0$ is a liquidation penalty and $\varphi \geq 0$ is the target penalty parameter.
2. In this setup, deviations from the target are penalized by $\varphi \int_t^T (\nu_u - \rho\mu_u)^2 \, du$, but this penalization does not affect the cash process.
3. High values of φ constrain the strategy to closely track the target $\rho\mu_t$ at every instant in time and low values of φ result in liquidation strategies which are more lax about tracking the POV target.
4. The agent's speed of trading ν_t has both temporary and permanent impact on the price of the asset. Assuming the impacts are linear in ν_t:

$$dS_t^\nu = -b\nu_t dt + \sigma dW_t, \; S_0^\nu = S, \tag{4}$$

$$\widehat{S}_t^\nu = S_t^\nu - k\nu_t, \; \widehat{S}_0^\nu = \widehat{S}, \tag{5}$$

$$dX_t^\nu = \widehat{S}_t^\nu \nu_t dt, \; X_0^\nu = x, \tag{6}$$

   with $b \geq 0$ and $k \geq 0$.
5. In this setup we assume that the order flow μ_t from other agents does not affect the mid-price process.

Solving the DPE When Targeting Rate of Trading

1. We solve the agent's control problem (3) assuming that order flow of other agents, μ_t, is Markov and independent of all other processes (specifically it is independent of the Brownian motion W_t which drives the mid price). We denote its infinitesimal generator by $\mathcal{L}^\mu$.
2. The dynamic programming principle suggests that the value function should satisfy the DPE:

$$\begin{aligned} 0 &= \left(\partial_t + \frac{1}{2}\sigma^2 \partial_{SS} + \mathcal{L}^\mu\right) H \\ &+ \sup_\nu \left[(S - k\nu)\nu\partial_x H - \nu\partial_q H - b\nu\partial_S H - \varphi(\nu - \rho\mu)^2\right], \end{aligned} \tag{7}$$

   subject to the terminal condition $H(T, x, S, \mu, q) = x + q(S - \alpha q)$.
3. The supremum is attained at:

$$\nu^* = \frac{S\partial_x H - \partial_q H - b\partial_S H + 2\varphi\rho\mu}{2(k + \varphi)}. \tag{8}$$