MA668: Algorithmic and High Frequency Trading
Lecture 08

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Trading on an Informational Advantage (Contd ...)

1. Now we consider the possibility that a new trader enters the market, and that this trader (the "insider") knows the exact value of v.
2. Further, the insider is the only one who knows v and chooses how much to trade.
3. Let $x(v)$ be the number of shares traded by the insider.
4. On the other hand, the MM's know that there is an informed trader in the market, but do not know who this trader is.
5. For the formal analysis, we structure the model as follows:
   - (A) The insider observes v.
   - (B) On observing v, the insider chooses $x(v)$.
   - (C) u is realized.
   - (D) The MM's observe the net order flow $x(v) + u$.
   - (E) Based on the net order flow, the MM's compete to set the asset price S.

Trading on an Informational Advantage (Contd ...)

1. In order to solve the model, we use the concept of (Bayesian) Nash equilibrium (without going into all the details).
2. Means: All agents optimize given the decisions of all other players, according to their beliefs (which are updated according to Baye's rule wherever possible).
3. Require: In equilibrium the insider chooses $x(v)$ to maximize her/his expected profit, taking into account the dynamics of the game (her/his order will be mixed in with those of the liquidity traders).
4. Further, they anticipate that MM's will set their prices on the basis of what they learn from observing the order flow and what they know about the informed trader's decision problem.
5. Also require:
   - (A) MM's choose their prices taking into account the strategy of the insider (in particular they anticipate the functional form of $x(v)$) AND
   - (B) The properties of the uninformed order flow that comes from liquidity traders.

Trading on an Informational Advantage (Contd ...)

1. MM's set the market price as a function of net order flow: $S(x+u)$.
2. Important: Model naturally tells us that prices are affected by the order flow, so that trading automatically generates a price impact.
3. The average price for per unit traded, namely, S moves with the net order flow of $x+u$.
4. In equilibrium: The insider will anticipate the functional form of $S(x+u)$, that is she/he will incorporate price impact when choosing $x(v)$.
5. The equilibrium is a fixed point in the optimization of x, given a functional form of S, and of S, given the functional form of x.

Trading on an Informational Advantage (Contd ...)

1. Let us now try to examine what the insider should do.
2. Most natural response: Sell if $v < E[v] = \mu$ AND Buy if $v > \mu$.
3. Further whether selling or buying, do as much as possible to leverage his informational advantage.
4. While this seems natural, one must take into consideration that MM's will adjust their prices to the order flow they observe.
5. Accordingly, even if $v < \mu$, the insider cannot expect $S = \mu$.
6. Extreme case: When there are no liquidity traders, everyone knows that any trade that comes is from the insider.
7. Consequently, the MM's, anticipating the demand as a function of the realization of v, behave optimally and set prices that incorporates all information on v in $x(v)$.
8. Fortunately (for the insider): $\exists$ liquidity traders who add noise to the order flow and allow the insider to camouflage her/his trade to gain positive expected profits.

Trading on an Informational Advantage (Contd ...)

1. Question: How do MM's set their prices.
2. Since MM's compete for order flow, any profits they could extract are competed away.
3. Thus: Whatever the price strategy, it will lead to zero expected profits for our (risk-neutral) MM's.
4. However, there will never be negative profits, as they can always choose not to trade.
5. The zero (expected) profit condition forces prices to have a very specific property (known as *semi-strong efficiency*): $S = E[v|\mathcal{F}]$, where $\mathcal{F}$ represents all information available to MM's.
6. Interpretation: Prices reflect all publicly available information.

Trading on an Informational Advantage (Contd ...)

1. We can identify a fundamental property of the MM's equilibrium strategy:

$$S(x+u) = E[v|x+u].$$

2. To solve the model: We need to find an $x(v)$ that is optimal, that is, it maximizes the insider's expected trading profits, conditional on the pricing rule.
3. Since v and u are normally distributed, we present the hypothesis that $S(x+u)$ is linear in net order flow. More specifically:

$$S(x+u) = \mu + \lambda(x+u).$$

4. Here λ is an unknown parameter, which can be interpreted as representing the linear sensitivity of the market price to order flow.
5. Taking this functional form as give, we consider the insider's problem as:

$$\max_x E\left[x\left(v - S\left(x+u\right)\right)\right].$$

Trading on an Informational Advantage (Contd ...)

1. Substituting for $S(x+u) = \mu + \lambda(x+u)$ and taking expectation with respect to v, we obtain that the objective function is concave and the first order condition yields:

$$x^*(v) = \beta(v-\mu),\ \beta = \frac{1}{2\lambda}.$$

2. We know that: $S = E[v|x+u]$.
3. From the optimal x, we know that:

$$x + u = \beta(v-\mu) + u.$$

4. Since v and u are independent and normal, so $x+u$ is normal with mean 0 and variance $\beta^2\sigma^2 + \sigma_u^2$.
5. One can derive that:

$$S = \mu + 2(x+u)\frac{\sigma_u}{\sigma}.$$

6. Hence the linear sensitivity parameter is $\frac{2\sigma_u}{\sigma}$.