MA668: Algorithmic and High Frequency Trading
Lecture 07

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Measuring Liquidity (Contd ...)

1. According to the model, at $t = 0$, there are no liquidity traders and no trade, so that $S_0 = E[S_3] = \mu_0$, will be the equilibrium price.
2. According to the model, the subsequent equilibrium prices at $t = 1$ and $t = 2$ are:
$$S_1 = \mu_1 + \lambda q^{LT1} \text{ and } S_2 = \mu_2,$$
respectively.
3. In order to construct the autocovariance of price changes, we let $\Delta_1 = S_1 - S_0$ and $\Delta_2 = S_2 - S_1$.
4. Autocovariance of price changes are given by the following expression:

$$\begin{aligned} Cov\left[\Delta_1, \Delta_2\right] &= Cov\left[\mu_1 + \lambda q^{LT1} - \mu_0, \mu_2 - \mu_1 - \lambda q^{LT1}\right], \\ &= Cov\left[\epsilon_1 + \lambda q^{LT1}, \epsilon_2 - \lambda q^{LT1}\right], \\ &= -\lambda^2 Var\left[q^{LT1}\right] < 0. \end{aligned}$$

Measuring Liquidity (Contd ...)

1. In this simple (essentially) static model, where all the action takes place at $t = 1$, the autocovariance of price changes captures the market liquidity, just like price impact does.
2. Interesting effect: As liquidity increases and $\lambda \to 0$, so the autocovariance of price changes, and the price process converges to the underlying "efficient price" martingale process.

Market Making Using Limit Orders

1. So far we have proposed that MM's participate through the posting of LO's.
2. We now consider why an MM would behave in this way and the simplest solution to how she/he does it.
3. Original model of Ho and Stroll is beyond the scope of the current discussion.
4. Accordingly, we set up a static version of the model that captures some of the basic elements of the MM's problems.
5. Grossman and Miller: MM is a professional trader who profits from intermediating between different liquidity traders.

Market Making Using Limit Orders (Contd ...)

1. For our discussion: We consider a small risk-neutral trader with costless inventory management and infinite patience.
2. Does not require compensation for her/his services, but makes a profit from optimally choosing how to provide liquidity in an uncertain environment populated by other MM's, who do not react to our MM's decision.
3. The uncertainty in this context comes from the timing and size of the large incoming MO's.
4. Information: All information is public so that everyone agrees on the current asset value, S_t, which is referred to as the mid-price.
5. Given that our trader is one of the many MM's, we consider the behavior of other MM's, as known and represented by a fixed LOB (which is unaffected by the decision of our MM).

Market Making Using Limit Orders (Contd ...)

1. How does our MM make money?: By adding her/his LO's to the book and then clearing the resulting inventory at later date(s).
2. Since our MM has no inventory costs, incurs no trading costs, is risk-neutral and infinitely patient, we can assume that she/he liquidates her/his inventory at the mid-price, at no cost.
3. MM's problem:
   - (A) To choose where on the LOB to place her/his LO's so as to maximize her/his profit per trade.
   - (B) Optimally balancing the increase in the per trade received, as she/he increases the distance of LO from the mid-price, WITH the frequency with which she/he will trade, which decreases with that distance from the mid-price.

Market Making Using Limit Orders: Formal Setup

1. Formally, the MM's problem is to choose the distance from the mid-price, the depth $\delta^{\pm}$. Then:
   - A. She/he will post the sell LO's at $S_t + \delta^+$.
   - B. She/he will post the buy LO's at $S_t - \delta^-$.
2. The uncertainty from MO's come from the probability that an MO arrives ($p_{\pm}$) and the probability that once it arrives it walks the book up to where the MM's are resting ($\delta^{\pm}$ away from the mid-point), which is described by the CDF $P_{\pm}$.
3. Therefore, the probability that the buy LO will be filled is $p_- P_-(\delta^-)$.
4. If we assume that the distribution of the other LO's in the LOB is described by an exponential distribution, with parameter κ^-, we have:

$$p_- P_-(\delta^-) = p_- e^{-\kappa^- \delta^-}.$$

5. Similarly, the probability that the sell LO is filled is: $p_+ e^{-\kappa^+ \delta^+}$.

Market Making Using Limit Orders: Formal Setup (Contd ...)

Clearly: As the MM posts her/his LO's deeper in the LOB, the probability that her/his order (once an MO arrives) decreases, though her/his profit per trade ($\delta^{\pm}$) increases.

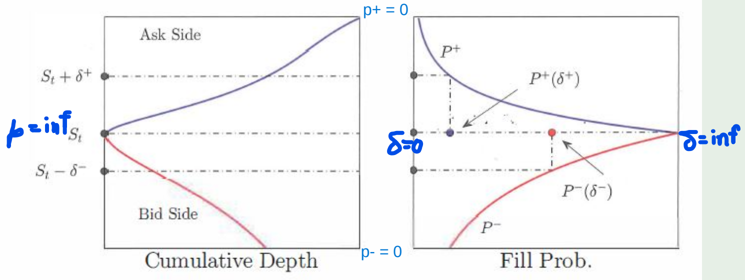

Figure 2.2 The LOB and the probability of execution.

Figure: Figure 2.2

Market Making Using Limit Orders: Formal Setup (Contd ...)

1. The left panel of Figure 2.2 illustrates a hypothetical LOB around a mid-price of S_t and two possible limit orders:
   - A. A sell LO on the ask side at $S_t + \delta^+$.
   - B. A buy LO on the bid side at $S_t - \delta^-$.
2. The right panel describes the corresponding distribution $P^+(P^-)$ of execution of the order posted at a distance $\delta^+(\delta^-)$ from the mid-price, conditional on the arrival of a buy (sell) MO.
3. *Optimization Problem*: Let Π denote the MM's profit per trade. Then the MM's optimization problem is given by the following expression:

$$\max_{\delta^+,\delta^-} E\left[\Pi\left(\delta^+,\delta^-\right)\right] = \max_{\delta^+,\delta^-}\left[p^+ e^{-\kappa^+\delta^+}\delta^+ + p^- e^{-\kappa^-\delta^-}\delta^-\right]. \quad (1)$$

4. *Solution*: Post LO's at the following depths:

$$\delta^{\pm,*} = \frac{1}{\kappa^{\pm}}.$$

5. Given our parametric choice of $P_{\pm}$, the optimal depth is equal to the mean depth in the LOB.

Trading on an Informational Advantage

1. So far, we have not yet dwelled on a major aspect of trading, namely informational differences.
2. Many trades originate not because someone needs cash (has extra cash) and accordingly sells an (invests in an) asset: But because one party has (or believes she/he has) better information about what the price is going to do than is reflected in current prices.
3. So we move onto the next step beyond modeling based on publicly available information.
4. Question: How to exploit an informational advantage while taking into account one's price impact.
5. Kyle (1985): Examines the decision problem of a trader who has a strong informational advantage.
6. Kyle (1989): Case of several competing informed traders.

Trading on an Informational Advantage (Contd ...)

1. Kyle (1985): Studies how the informed trader adjusts her/his trading strategy to take into account the market reaction, particularly the price impact that her/his trade generates in equilibrium.
2. Before studying the model: We need to first define what we mean by:
   A. A strong informational advantage.
   B. Price efficiency in this context.
3. For simplicity: We only consider the investor's static decision problem.
4. The same basic idea extends to the dynamic setting.
5. Formal setting of the static model: There is a market for an asset that opens at one point in time.
6. The asset is traded at price S, and after trading, the asset has a cash value equal to v.

Trading on an Informational Advantage (Contd ...)

1. Note that the future cash value v, of the asset, is uncertain.
2. In particular, v is assumed to be normally distributed with mean μ and variance σ^2.
3. Now the market has three types of traders:
   - A. An informed trader.
   - B. An anonymous mass of price insensitive liquidity traders (traders who need to execute trades whatever the cost).
   - C. A large number of MM's who observe and compete for the order flow (flow of incoming buy and sell orders from the informed and the liquidity traders).
4. In contrast to Grossman and Miller (1988): MM's are risk neutral and therefore they do not need liquidity premium to compensate for the price risk from holding inventory.

Trading on an Informational Advantage (Contd ...)

1. Therefore, any liquidity premium that arises will come from the need to compensate MM's for their informational disadvantage.
2. This will be borne by the price-insensitive liquidity traders.
3. These liquidity traders, will have an aggregate net demand of represented by a random quantity u.
4.
   A. $u > 0$: Aggregate liquidity traders want to by u units.
   B. $u < 0$: Aggregate liquidity traders want to sell $|u|$ units.
5. Assumption: u is normally distributed with mean 0 and variance σ_u^2, and is independent of v.
6. In principle, since liquidity traders are not sensitive to the price (u does not depend on S) MM's could charge a very large liquidity premia.
7. However, competition for order flow between MM's drives the liquidity premium to zero, so that (when there are only MM's and liquidity traders) $S = E[v]$.