ME 620: Fundamentals of Artificial Intelligence

Lecture 5: Uninformed Search - I

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Uninformed Search

- Review Problem Solving as Search
- State Spaces
- Graph Searching
- Generic Searching Algorithm
- Uninformed Search Strategies
  - Breadth-First Search
  - Depth-First Search
  - Iterative Deepening
  - Uniform-Cost Search
  - Bidirectional Search

Formal Description of a Problem

State Space	Define a state space that contains all the possible configurations of the relevant objects.
Initial States	Specify one or more states within that space as possible situations from which the problem-solving can start.
Goal States	Specify one or more states that would be acceptable as solutions to the problem
Operators	Specify a set of rules that describes the actions (operators) available; information on what must be true, for the action can take place.

State Space

The set of all possible configurations of the relevant objects is the space of problem states or the problem space. This is also called the state space.

For Example – 8-Puzzle

Each tile configuration is a problem state

8-puzzle have a relatively small space

362,880 i.e., 9! different configurations.

Playing 8-Puzzle

Problem Solving as State Space Search

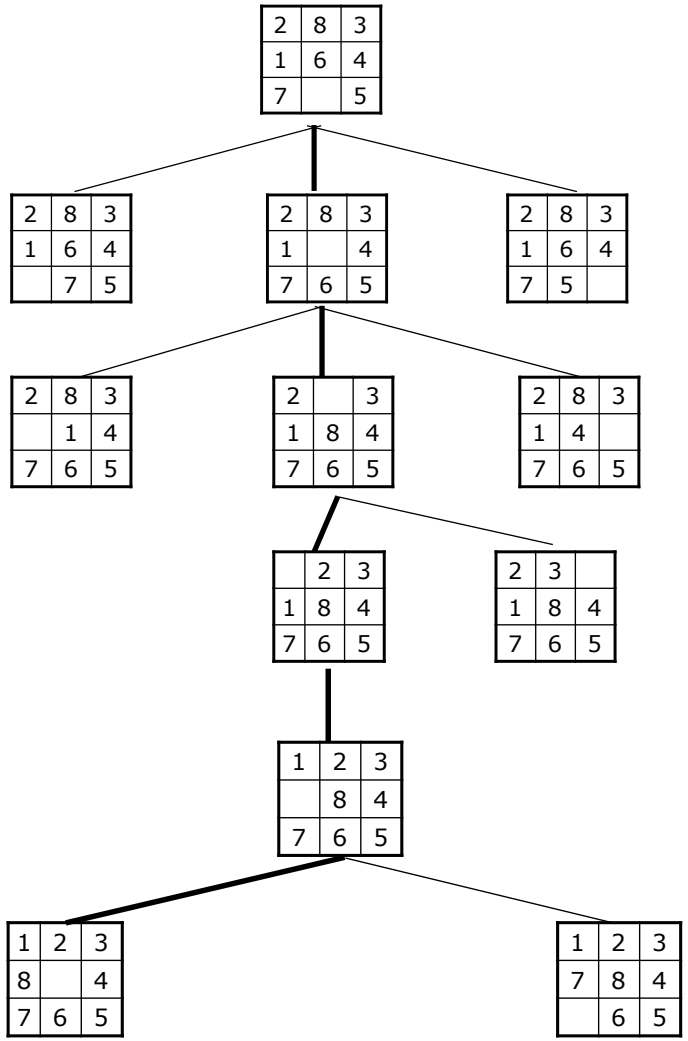
s ; Start Node

2	8	3
1	6	4
7		5

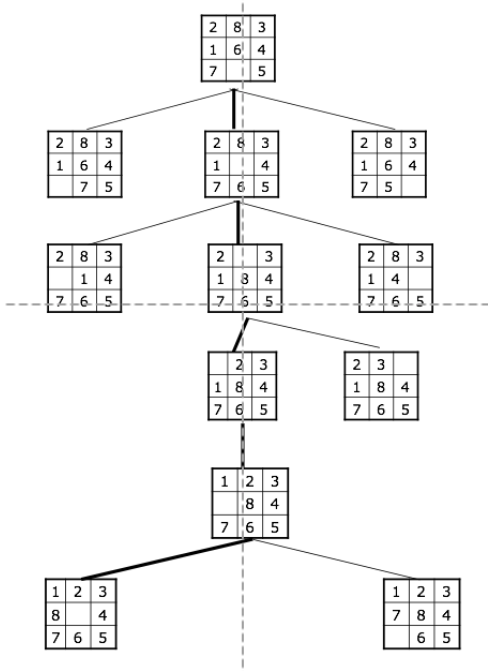
g ; Goal Node

1	2	3
8		4
7	6	5

Problem Solving as State Space Search



Problem Solving as State Space Search



Can be abstracted to the mathematical problem of finding a path from the start node to a goal node in a directed graph.

Graphs: Basic Definitions

- A *graph*, G, comprises a set V of vertices and a set E of edges.
  - The vertex set can be anything, but is most commonly a collection of letters or numbers.
  - The set of edges is a set of doubleton subsets of V. That is $E \subseteq \{\{a,b\}: a,b \in V \text{ and } a \neq b\}$.
  - We denote the graph by G(V,E).
- If G(V,E) is a graph and $\{a,b\} \in E$, then we say vertices a and b are *adjacent* and the edge {a,b} joins them or connects them or is *incident on* them. We call a and b the *endpoints* of the edge.
  - Two edges that share one vertex, such as {a,b} and {b,c} with a≠c, are *adjacent* to each other.

Graphs: Paths

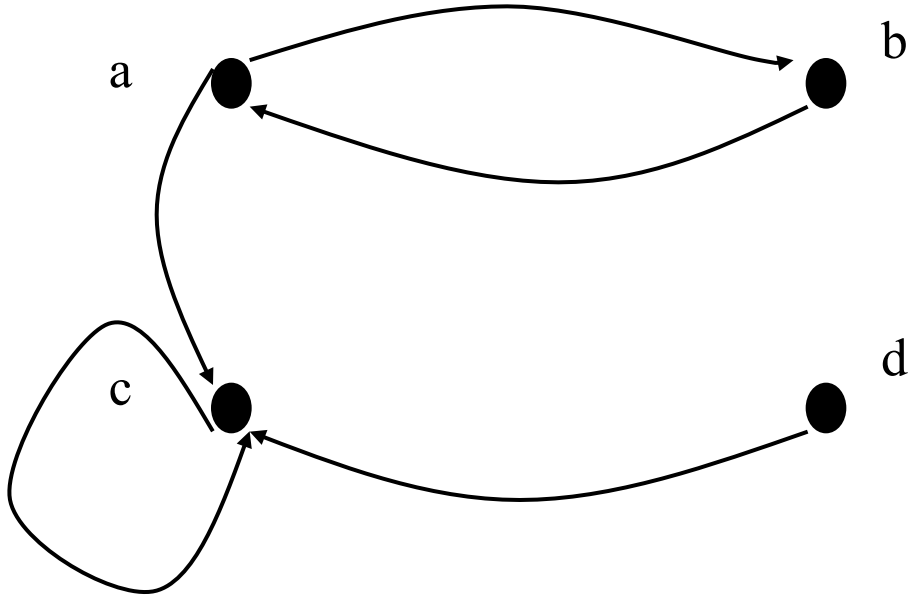
- The notion of a path in a graph is intuitively clear but a little hard to pin down formally.
  - Suppose $G(V,E)$ is a graph with vertices $v_0,v_1,\ldots,v_k$ (not necessarily distinct) and edges $e_1,e_2,\ldots,e_k$ (not necessarily distinct) in which edge $e_i=\{v_{i-1},v_i\}$ for $i=1,\ldots,k$.
  - The alternating sequence of vertices and edges $v_0e_1v_1e_2v_2\ldots e_kv_k$ is *a path from v_0 to v_k of length k* (The length is the number of edges, not vertices).

Directed Graphs (Digraphs)

- A *directed graph* (*digraph*, for short) G consists of a set V of vertices and a set E of *directed edges*.
  - A directed edge is an ordered pair of elements of V. Put another way, the pair G(V,E) with E ⊆V×V is a digraph
  - Digraphs allow for loops of the form (a,a), It is also possible to have two edges (a,b) and (b,a) between vertices a and b.
  - We take the ordering of the pairs in E to give each edge a direction: namely the edge (a,b) goes from a to b.
  - When we draw the digraph, we draw the edge (a,b) as an arrow from a to b.

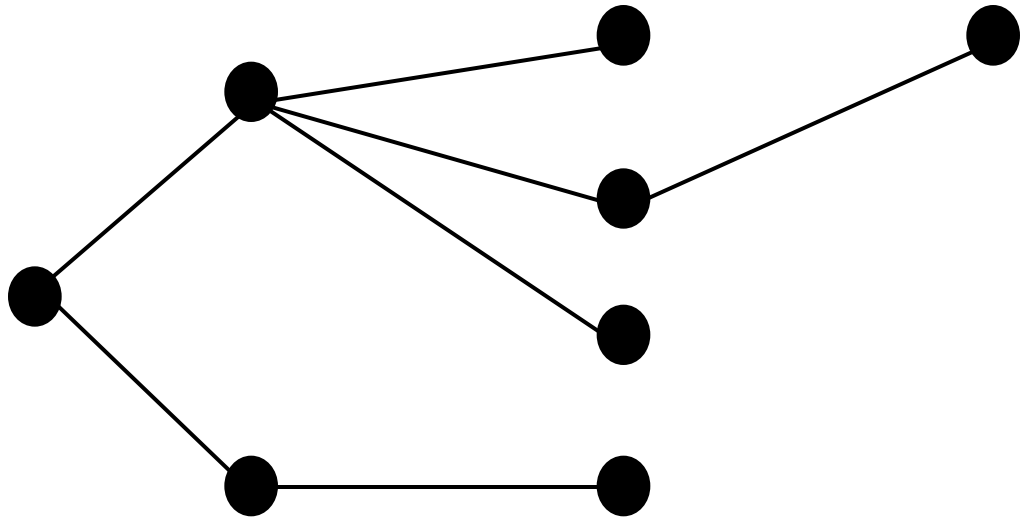
Digraphs: Example

- Example:
  - V={a,b,c,d}
  - E={(a,b),(a,c),(b,a),(c,c),(d,c)}



Trees

- A *tree* is a connected graph with no cycles.
- Trees come up in many contexts: tournament brackets, family trees, organizational charts, and decision trees, being a few examples.



Graph Searching

- The problem of finding a sequence of actions to achieve a goal is abstracted as searching for paths in directed graphs.
- To solve a problem, define the underlying search space and apply a search algorithm to that search space.
  - Searching in graphs provides an appropriate abstract model of problem solving independent of a particular domain.

Formalizing search in a state space

- A state space is a **graph** (V, E) where
  - V is a set of nodes and
  - E is a set of arcs, and each arc is directed from a node to another node
- Each node is a data structure that contains a state description plus other information such as the parent of the node, operator that generated the node from that parent, and other bookkeeping data
- Each arc corresponds to an instance of one of the operators.

Formalizing search in a state space

- Each arc has a fixed, **positive cost** associated with it corresponding to the cost of the operator.
- Each node has a set of **successor nodes** corresponding to all of the legal operators that can be applied at the source node's state.
  - The process of expanding a node means to generate all of the successor nodes and add them and their associated arcs to the state-space graph
- One or more nodes are designated as **start nodes.**
- A **goal test** predicate is applied to a state to determine if its associated node is a goal node.

Formalizing search in a state space

- A **solution** is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The **cost of a solution** is the sum of the arc costs on the solution path.
  - If all arcs have the same (unit) cost, then the solution cost is just the length of the solution (number of steps / state transitions)

Formalizing search in a state space

- **State-space search** is the process of searching through a state space for a solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node.
  - For large state spaces, it isn't practical to represent the whole space.
  - Initially V={S}, where S is the start node;
    - when S is expanded, its successors are generated and those nodes are added to V and the associated arcs are added to E.
    - This process continues until a goal node is found.

Formalizing search in a state space

- **State-space search** is the process of searching through a state space for a solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node.
- Each node implicitly or explicitly represents a partial solution path (and cost of the partial solution path) from the start node to the given node.
  - In general, from this node there are many possible paths (and therefore solutions) that have this partial path as a prefix.

Evaluating Search Strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity**
  - How long (worst or average case) does it take to find a solution? Usually measured in terms of the number of nodes expanded
- **Space complexity**
  - How much space is used by the algorithm? Usually measured in terms of the maximum size of the "nodes" list during the search
- **Optimality/Admissibility**
  - If a solution is found, is it guaranteed to be an optimal one? That is, is it the one with minimum cost?

Important Parameters

- Maximum number of successors of any state of the search tree
  - branching factor b of the search tree
- Minimal length of a path in the state space between the initial and a goal state
- depth d of the shallowest goal node in the search tree
- Maximum depth m of the state space.

Graph Search

Procedure **GRAPHSEARCH**

Create a search graph **G** consisting solely of the start node **s**.
Put **s** on a list called OPEN

Create a list called CLOSED that is initially empty.

Loop: If OPEN is empty, exit with failure

Select first node on OPEN, remove it from OPEN and put it on closed. Call this node n.

If n is goal node, exit successfully with the solution obtained by tracing a path along the pointers from n to s in G

Graph Search

Expand node n generating the set M of its successors and install them as successors of n in G.

Establish a pointer to n from those members of M that were not already in G. Add these members of M to OPEN.

For each member of M already in G decide whether or not to redirect its pointer to n.

For each member of M already on CLOSED, decide for each of its descendants in G whether or not to redirect its pointer.

Reorder the list OPEN, either according to some arbitrary scheme or according to heuristic merit.

Go LOOP.

Uninformed vs. informed search

Uninformed search strategies

- Also known as "blind search," use no information about the likely "direction" of the goal node(s)
- Uninformed search methods: Breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional

Informed search strategies

- Also known as "heuristic search," informed search strategies use information about the domain to (try to) (usually) head in the general direction of the goal node(s)
- Informed search methods: Hill climbing, best-first, greedy search, beam search, A, A*

Uninformed Search Strategies

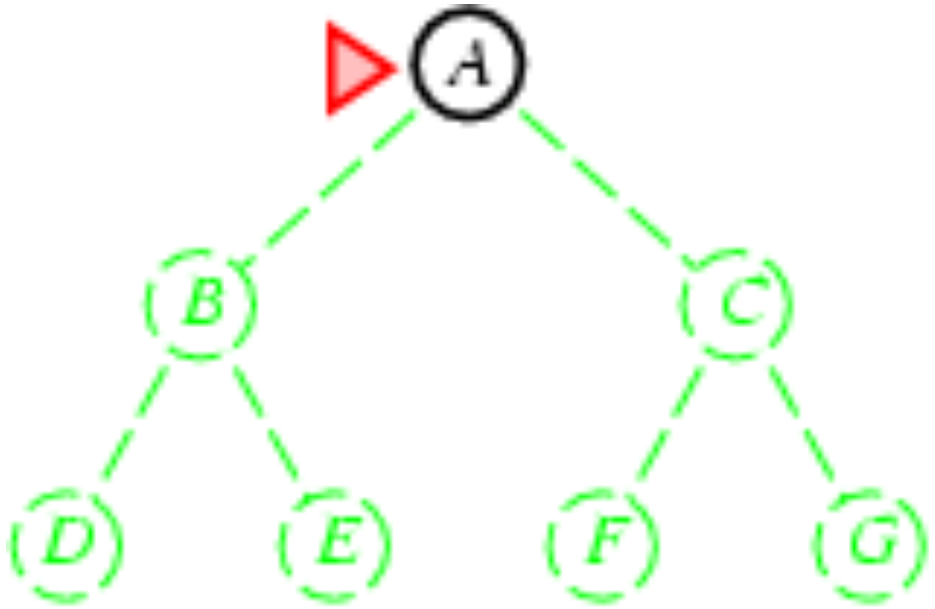
No information from the problem domain is used in ordering the nodes on OPEN, some arbitrary scheme is used.

- Breadth-First Search
- Depth-First Search
- Iterative Deepening
- Uniform-Cost Search
- Bidirectional Search

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end.

Is A a goal state?

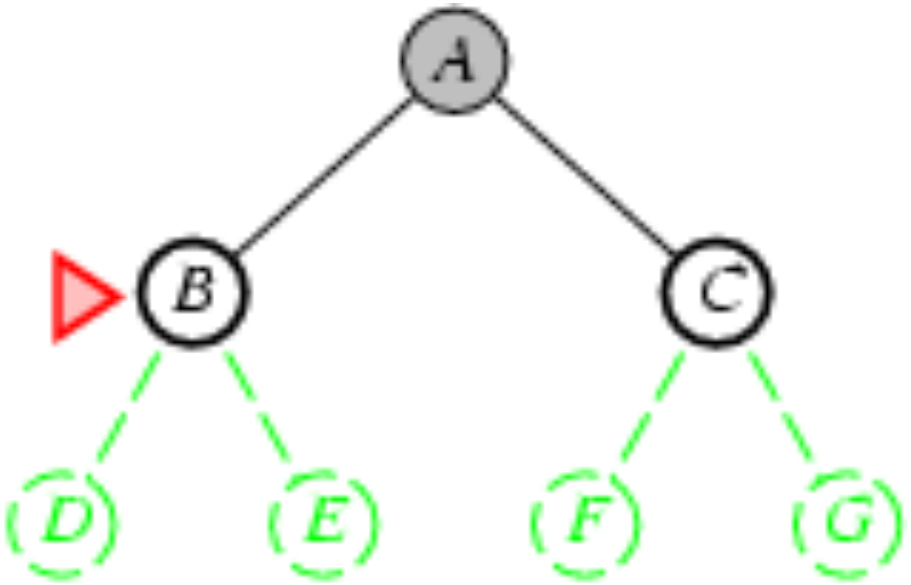


Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe = [B,C]

Is B a goal state?

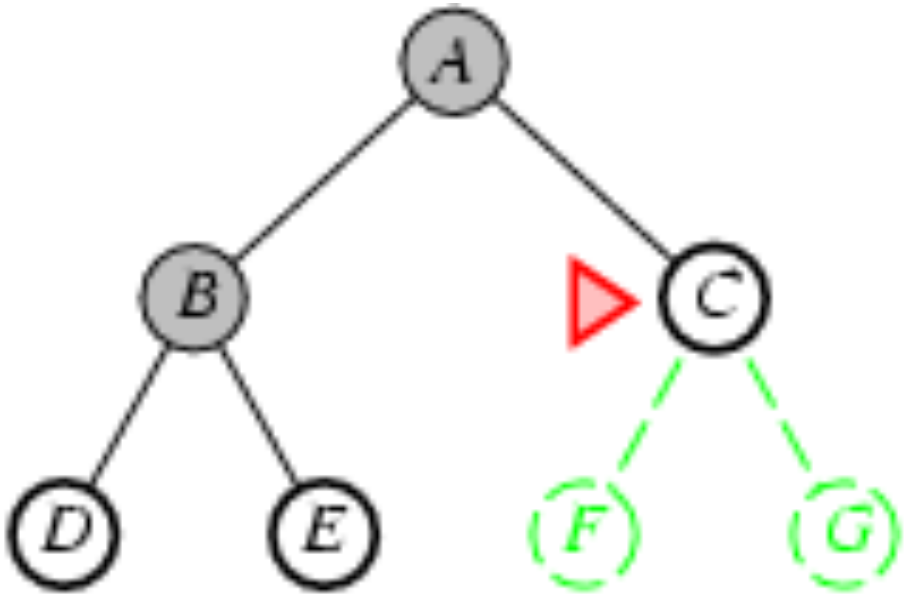


Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[C,D,E]

Is C a goal state?

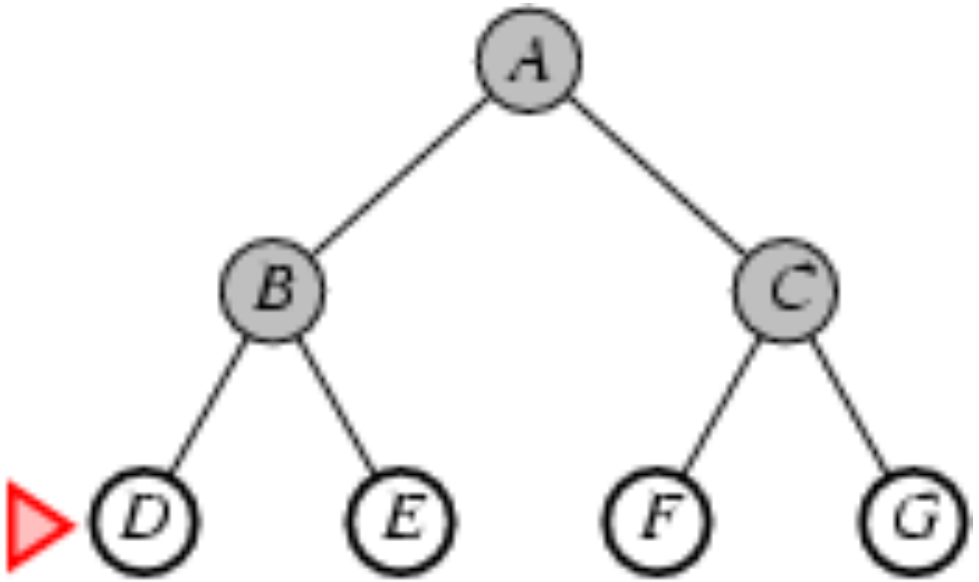


Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *fringe* is a FIFO queue, i.e., new successors go at end

Expand:
fringe=[D,E,F,G]

Is D a goal state?



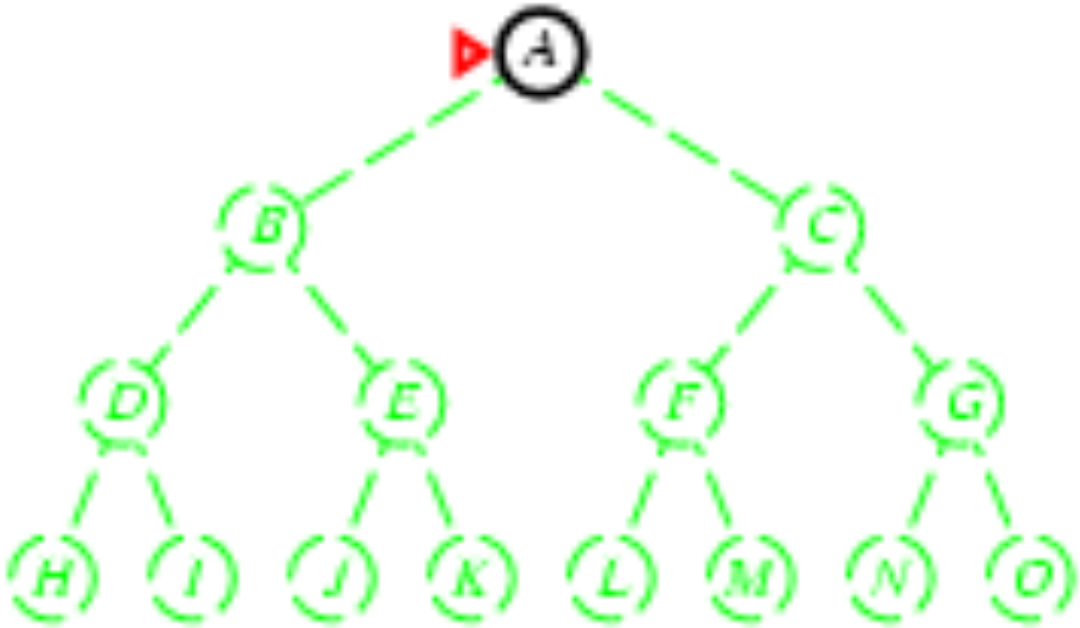
Properties of Breadth-first search

- Complete?
  - Yes it always reaches goal (if *b* is finite)
- Time?
  - $1+b+b^2+b^3+\ldots+b^d = O(b^d)$
  - This is the number of nodes we generate
- Space?
  - $O(b^d)$
  - keeps every node in memory, either in fringe or on a path to fringe.
- Optimal?
  - Yes (if we guarantee that deeper solutions are less optimal, e.g. step-cost=1).

Depth-first search

- Expand *deepest* unexpanded node
- Implementation:
  - *fringe* = Last In First Out (LIPO) queue, i.e., put successors at front

Is A a goal state?

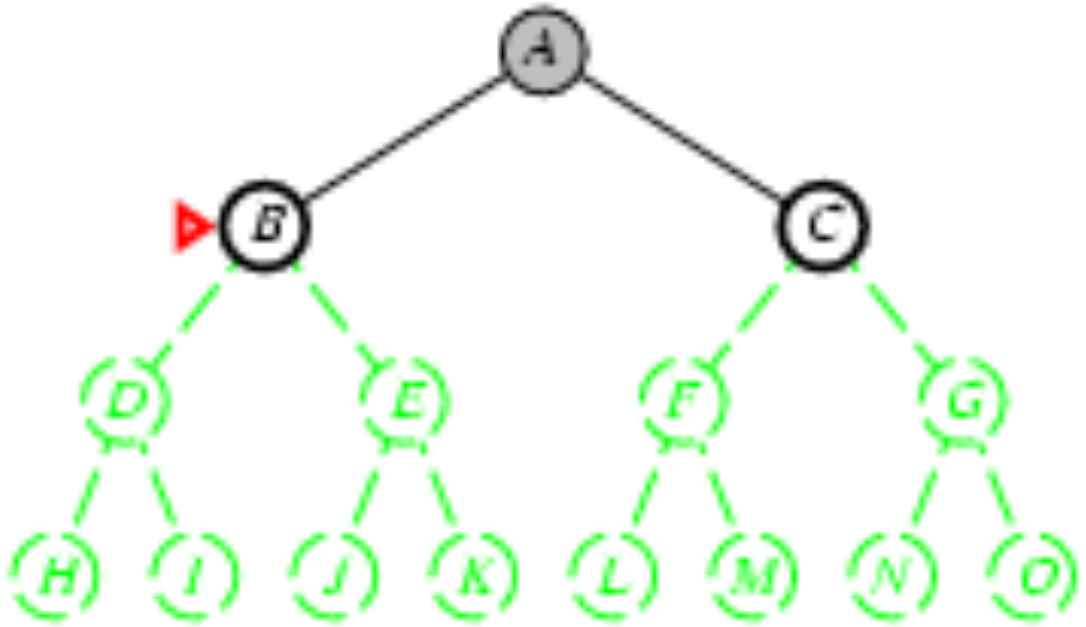


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[B,C]

Is B a goal state?

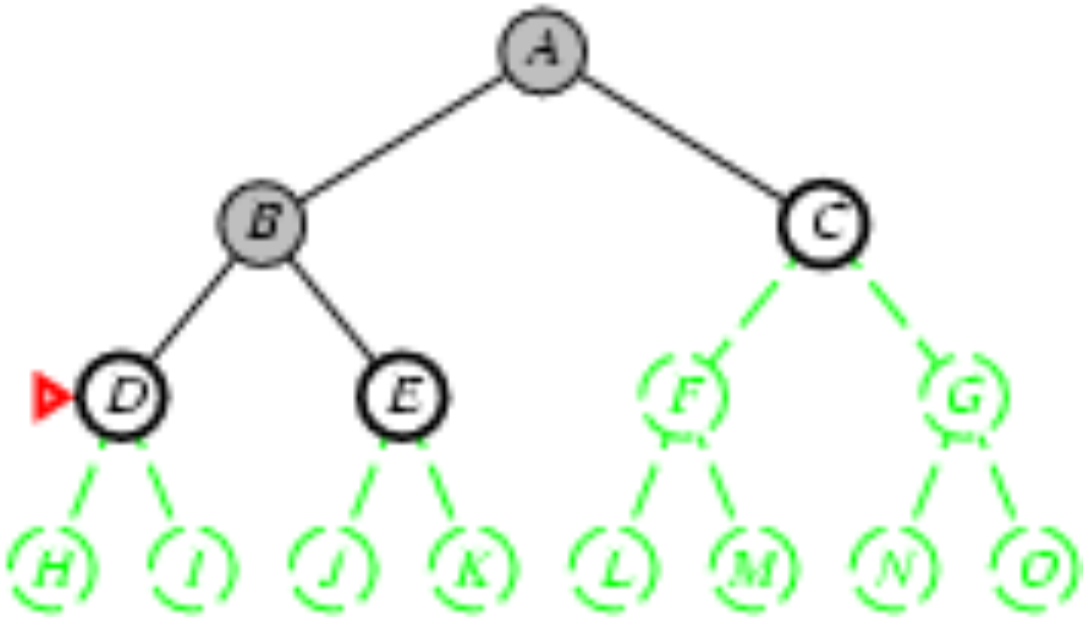


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[D,E,C]

Is D = goal state?

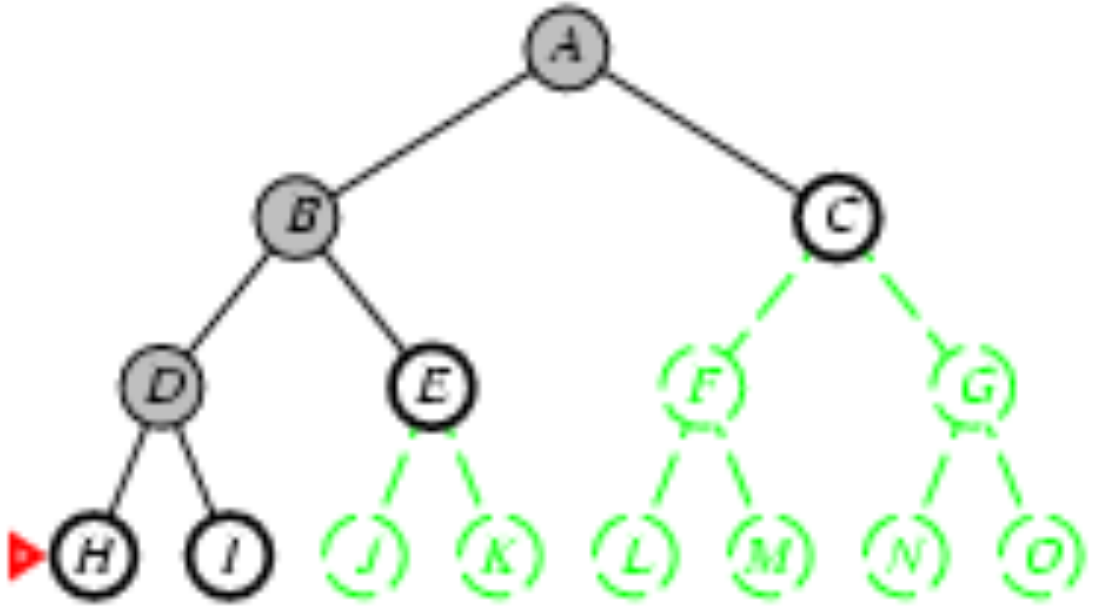


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[H,I,E,C]

Is H = goal state?

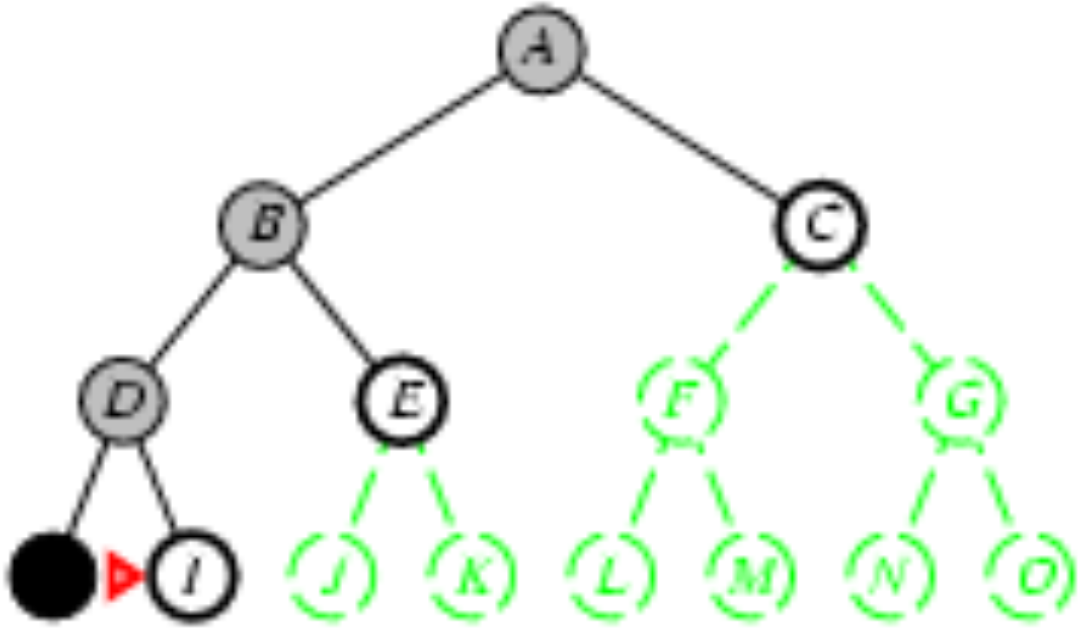


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[I,E,C]

Is I = goal state?

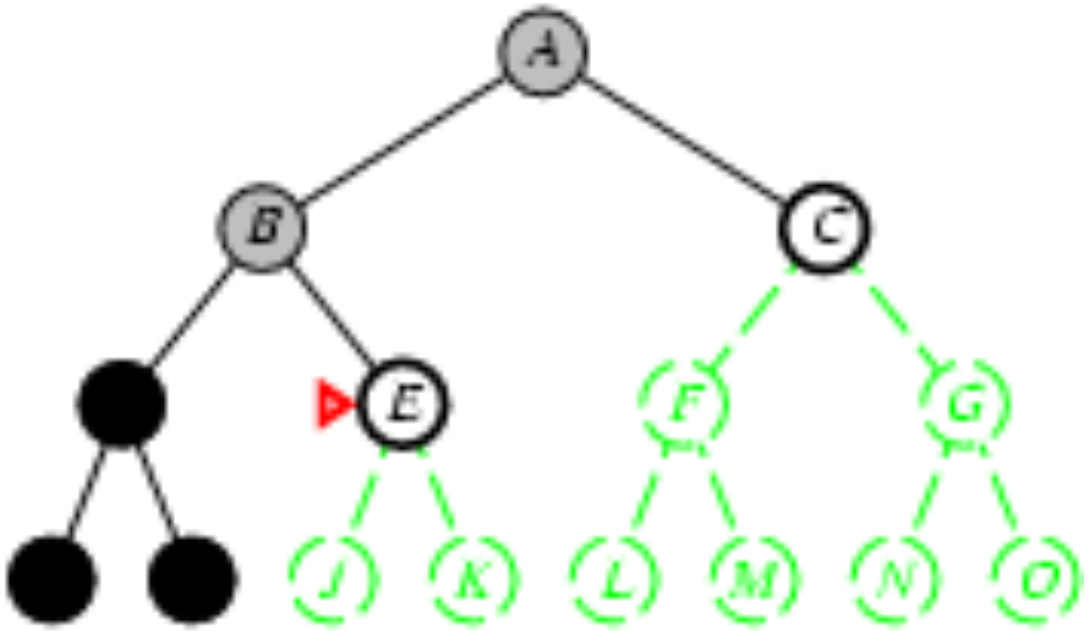


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[E,C]

Is E = goal state?

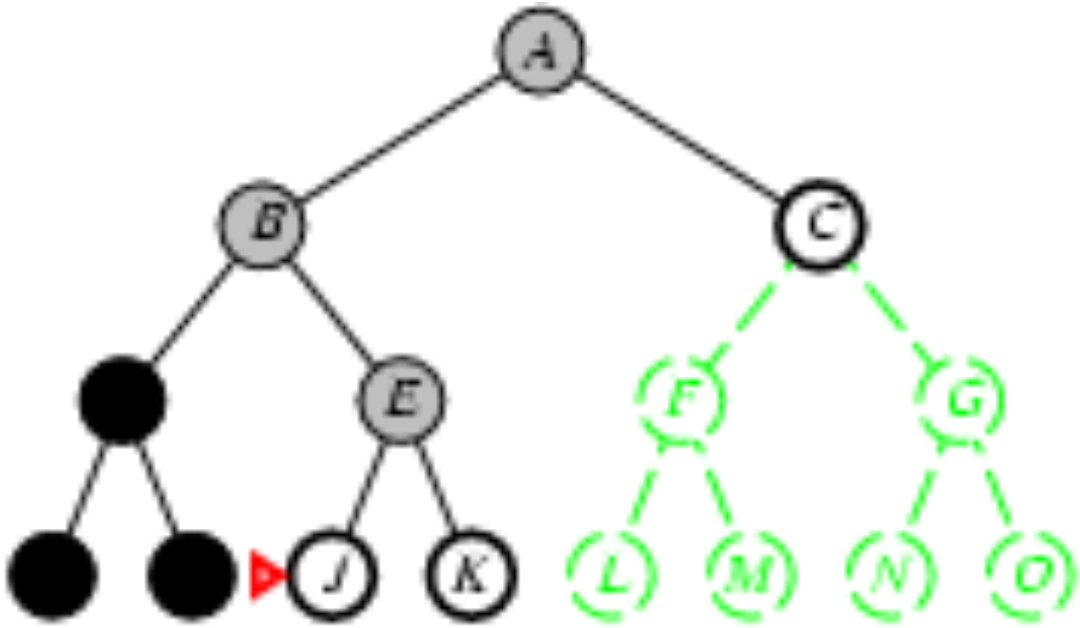


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[J,K,C]

Is J = goal state?

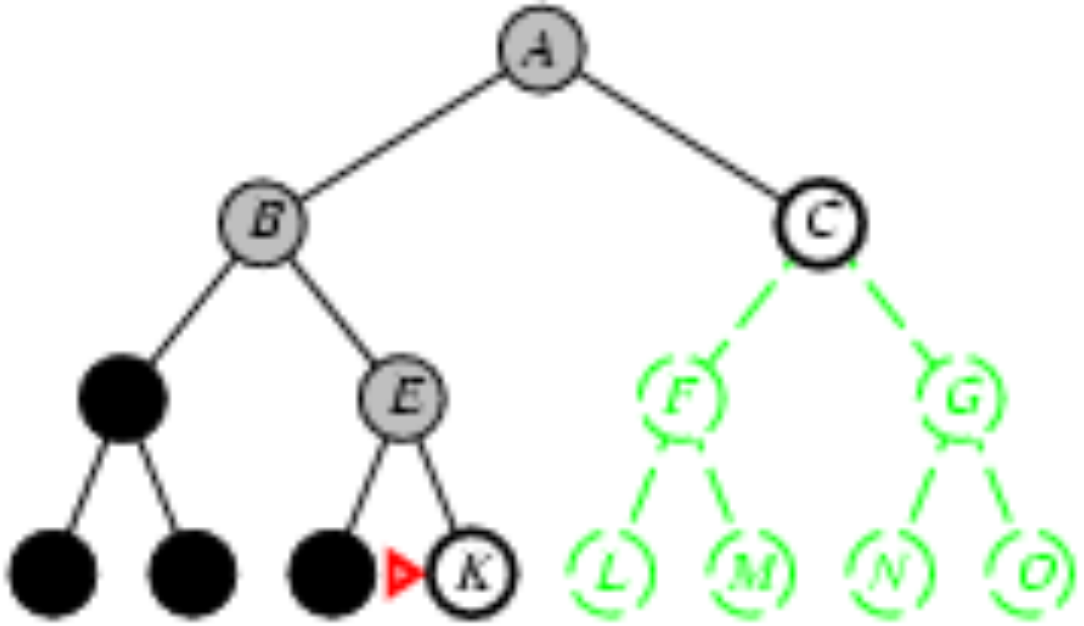


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[K,C]

Is K = goal state?

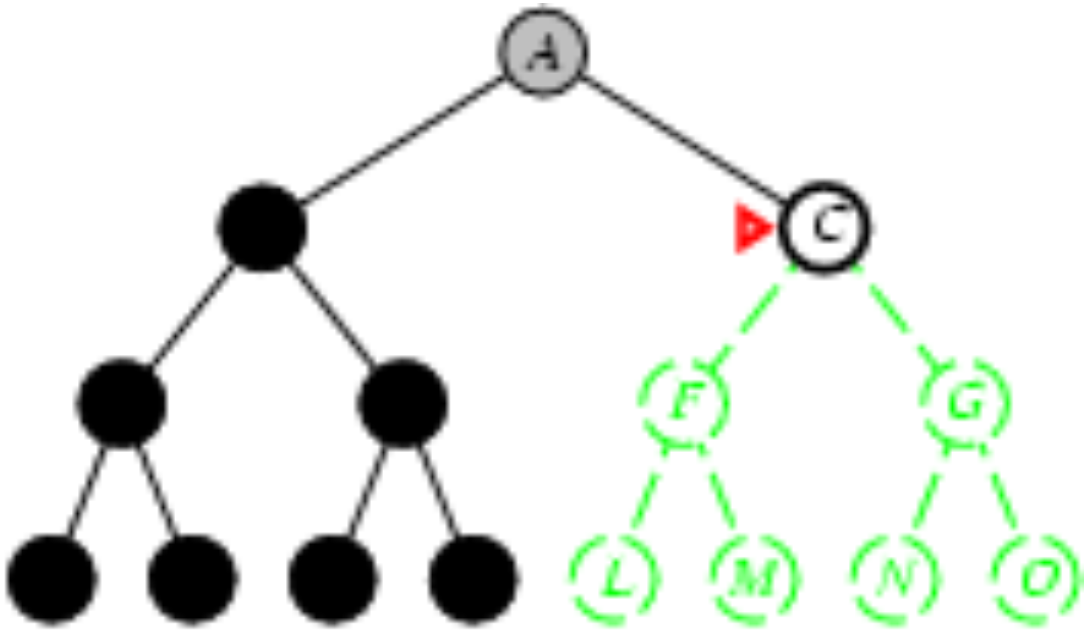


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[C]

Is C = goal state?

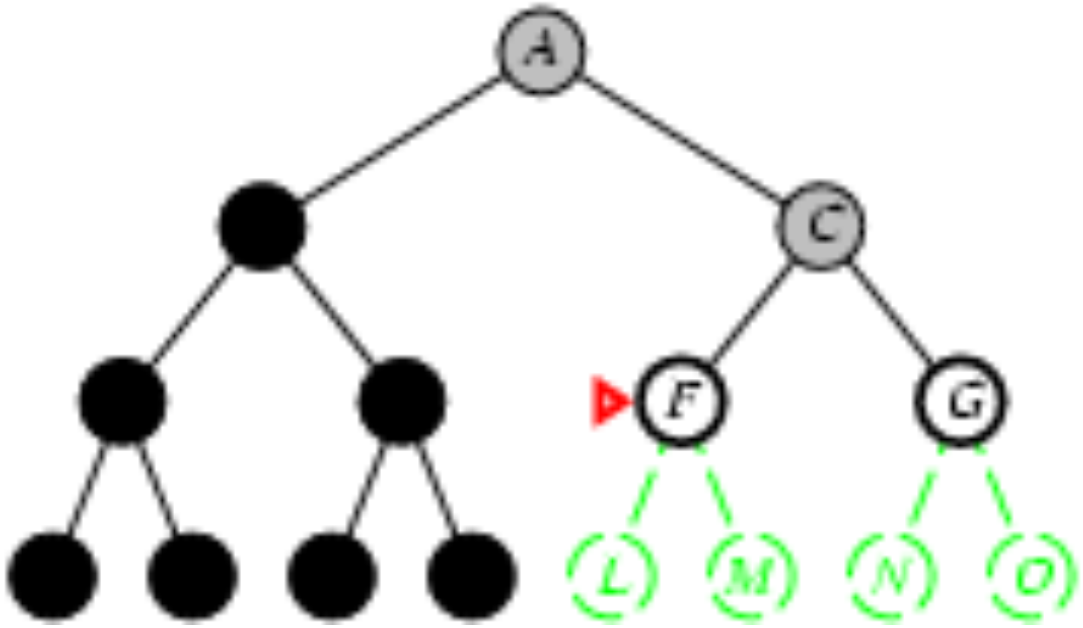


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[F,G]

Is F = goal state?

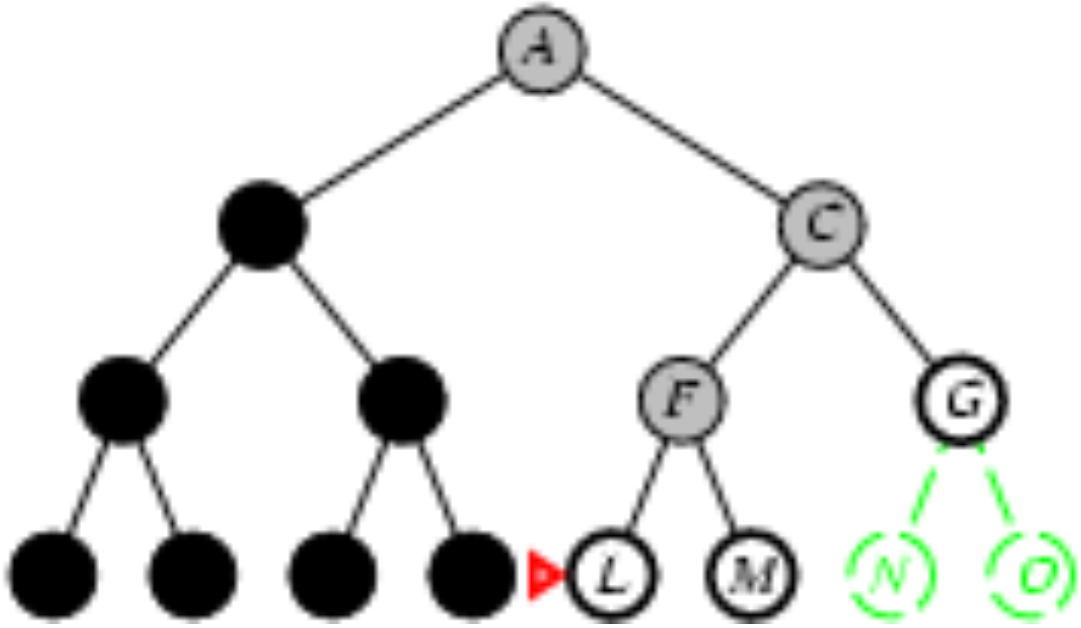


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[L,M,G]

Is L = goal state?

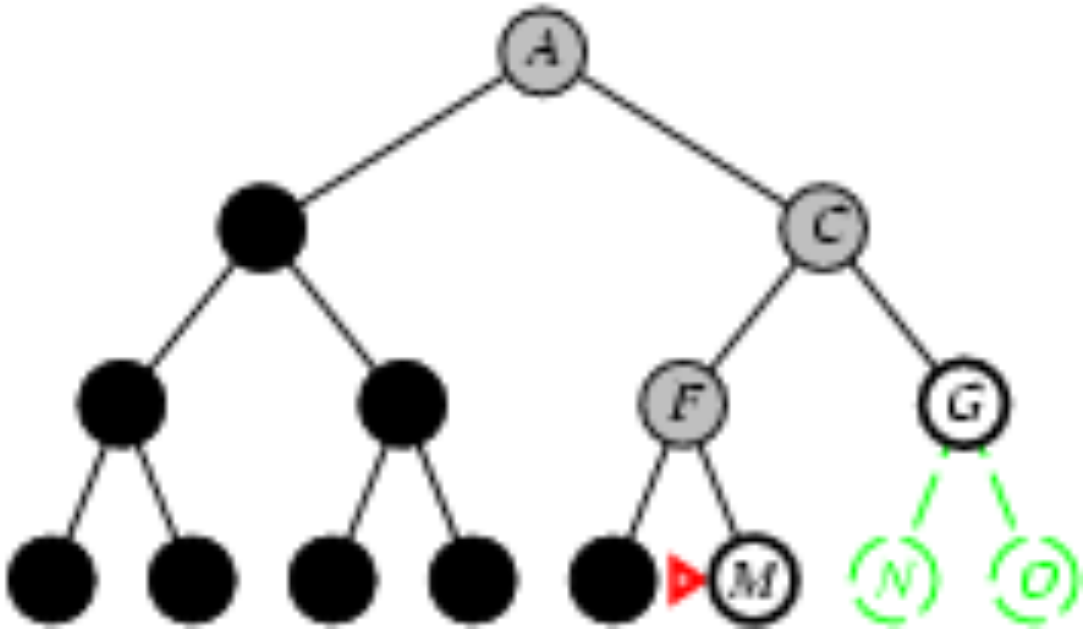


Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *fringe* = LIFO queue, i.e., put successors at front

Expand:
queue=[M,G]

Is M = goal state?



Properties of Depth-first search

- Complete?
  - No: fails in infinite-depth spaces
  - Can modify to avoid repeated states along path
- Time?
  - $O(b^m)$ with m=maximum depth
  - Terrible if *m* is much larger than *d ;* but if solutions are dense, may be much faster than breadth-first
- Space?

  = O(b + b + b + ... (m times)) = O(bm)
  - *O(bm),* i.e., linear space! (we only need to remember a single path + expanded unexplored nodes)
- Optimal?
  - No (It may find a non-optimal goal first)