Rs 5. Its cost function is  $C(m) = m^2$ . The chemical dyes firm maximizes profit  $\pi_m = 5m - m^2$ , by setting 5 = 2m, so that  $m^* =$ 5/2. Its profit then is Rs 25/4.

Now suppose that the fishing firm is also perfectly competitive and faces a price of Rs 4. Its cost function is  $C(f) = f^2 + mf$ . Since  $m = m^* = 5/2$ , the fishing firm's profit function is  $\pi_f = 4f - f^2 - 2.5f$ . It maximizes profit by setting  $f = \frac{3}{4}$ . Its profit is then 9/16.

Instead, if both firms were jointly owned, then the owner would be interested in maximizing the joint profit

$$\pi = \pi_m + \pi_f = 5m + 4f - m^2 - f^2 - mf$$
.

The first order conditions are  $\delta \pi / \delta m = \delta \pi / \delta f = 0$  and the solutions then are f = 1 and m = 2. We can see that the joint profit  $\pi = 7$ . This is higher than the aggregate profit of 25/+9/16 = 6.8that the firms make when they take decisions independently. Hence, if the firms merge, total profit will increase and the extra profit can be shared between them. Therefore, when a negative externality exists, the goal of profit-maximization itself will encourage firms to internalize the externalities.

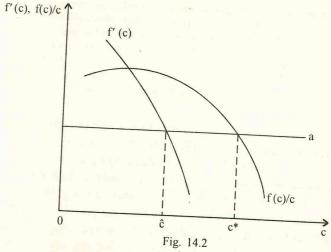
#### 14.3 THE TRAGEDY OF THE COMMONS

The 'tragedy of the commons' highlights what happens when property rights are not well-defined. Suppose that a village has a field in which all villagers graze their cows. The cost of purchasing a cow is Rs a. The number of cows is c and f(c) is the total values of yield of milk from the cows when c cows graze on the field. We assume that f'(c) > 0, but f''(c) < 0. Each extra cow reduces the grass available for the other cows and therefore the average milk yield. In other words, f(c)/c diminishes as c increases.

Private ownership If the field is owned by only one villager, this person will buy cows to maximize profit S = f(c)—ac. The first order condition is f'(c) = a. The value of marginal product is equated to the input cost. Let the solution be  $\hat{c}$ .

Common ownership Now suppose that the field is held in common and no villager can be prevented from grazing cows on

it. Given any c, the value of the yield of milk per cow is f(c)/c. Each villager will compare this return with the price of cows, a. Suppose that there are c cows in the field. The villager contemplating grazing an additional cow will compare f(c+1)/(c+1) with a, and graze the cow if f(c+1)/(c+1) > a. Thus cows will be added to the stock grazing on the field until  $f(c^*)/c^* = a$ . This may also be interpreted as the free-entry, zero profit condition:  $f(c^*)$ 



How does this solution compare with the solution when the grazing area has a single owner? This is seen in Fig. 14.2. We assumed that d[f(c)/c]/dc < 0, so that  $[f'(c)-\{f(c)/c\}]/c < 0$ . This shows that f'(c) < f(c)/c. Hence  $c^* > \hat{c}$ . There will be too many cows grazing on the field. Adding cows to the number already grazing has a negative externality effect on milk production. However, when the land is owned in common, villagers will not take this negative externality into account and too many cows will be allowed to graze.

Similar tendencies are observed with respect to the use of all common property resources. Overfishing is a prime example. In seasons when fishes breed, uncontrolled fishing may have a particularly disastrous effect on the catch of fish during the rest of the year. For sustainability, fishing must be restricted. However, each individual

fisherman neglects this effect. While each fisherman has a negligible effect on the total stock of fish, the accumulated effects of thousands of fishermen result in a serious depletion of fish resources over time.

This can be depicted by a Prisoners' Dilemma game:

		Others	
		Restraint	No restraint
Individual	Restraint	(8,8)	(10,2)
	No restraint	(2,10)	(5,5)

An individual fisherman's profits depend partly on the actions of other fishermen. If others exercise restraint, one individual's dominant strategy is not to exercise restraint, because the individual catch is too small to have any effect on sustainability of the yield, and if others do not exercise restraint, the individual has no incentive to do so.

Other examples include the use of common forest land and use of groundwater resources in large metropolitan cities.

Does the preceding discussion imply that property rights to all common property resources (CPRs) should be handed over to individuals? The existence of CPRs in developing countries like India introduce a measure of equity in income distribution. Dependence on common property is greatest among the poor because they do not possess income-generating private property assets and therefore depend on CPRs for fuelwood, crop wastes, cow dung, weeds, fodder, organic manure like dry leaves and forest litter, etc. CPRs also support a variety of income-producing activities like milk production or fishing. The support provided to the poorest sections by the common pooling of resources sometimes serves to redress the bias in favour of larger and richer farmers that most technological advances in agriculture seem to have.

## 14.4 PUBLIC GOODS

Suppose that there is a room full of people and one person starts smoking. The non-smokers all have to endure the smoke from the cigarette in equal degree, even though they may have very different preferences and resources. Similarly, when the street lamps are lighted, the benefit accrues in equal measure to all pedestrians and motorists.

When a private good like bread is consumed, the consumption of a particular piece of bread by one individual excludes another individual from consuming the same piece of bread. Moreover, different individuals can consume different amounts of bread. We say that a good is excludable if people can be excluded from consuming it. A good is a non-rival in consumption if additional units can be provided to another consumer at zero marginal cost (one person's consumption does not reduce the amount left for others, i.e. everyone can consume the same amount). Goods that are both non-excludable and non-rival are called pure public goods. More generally, goods that are non-excludable are called public goods.

There are many commodities that have features of either nonexcludability or non-rivalry or none or both. The Table below provides some examples of each.

	Exclusive	Nonexclusive	
Rival	Most goods Clothes Shoes	Online computer services Fishing Congested highways	
When not at capacity: Airline seats Horse racing		Pure public goods National defence Streetlights	

Most of the goods we consume are private goods, which are both rival and exclusive. When I buy a piece of clothing, I prevent others from buying it, and my purchase affects the cost of producing an additional garment. Note that when a particular flight is taking off and some seats are empty, the marginal costs of filling up these seats will be zero. Seating some people there will not affect the availability of seats to others, therefore we classify this good as non-rival in consumption. A congested highway provides an example of a good that is rival in consumption but nonexclusive. No one can be prevented from driving on

the highway, yet every extra car diminishes the space left for other cars to drive on and makes driving more difficult (reduces the consumption of service from the highway). Similarly, additional subscribers to an online computer service can slow down the access time of existing subscribers.

## 14.5 OPTIMAL PROVISION OF A PUBLIC GOOD

In this section, I discuss the optimal level of a public good whose amount can be varied continuously. Consider an economy with only two goods, one private and the other public. Suppose that there are two agents, whose initial wealth levels are given by  $w_1$  and  $w_2$ . Their respective contributions to the public good are given by  $g_1$  and  $g_2$ , and let  $x_1$  and  $x_2$  denote the consumption of the private good of each person. Let G measure the amount of the public good (in rupees) and c(G) be its cost. If G amount of the public good is provided, then the two individuals have to spend Rs c(G).

The two agents face the constraint that their total initial wealth cannot exceed their total expenditures on the private good and the public good:

$$x_1 + x_2 + c(G) = w_1 + w_2.$$

We consider a Pareto-efficient provision of the public good. The provision is Pareto-efficient if agent 1's utility is maximized given the utility level of agent 2. We must remember that both agents consume the same amount of the public good. The problem can then be written as

$$\max u_1(x_1, G),$$

$$x_1, x_2, G$$

subject to 
$$u_2(x_2, G) = u_2^*$$
 and  $x_1 + x_2 + c(G) = w_1 + w_2$ .

Let us form the Lagrangean

$$L = u_1(x_1, G) + \lambda_1 \{ u_2^* - u_2(x_2, G) \} + \lambda_2 \{ w_1 + w_2 - x_1 - x_2 - c(G) \}.$$

The first order conditions are

1. 
$$\delta L/\delta x_1 = \delta u_1/\delta x_1 - \lambda_2 = 0$$

2.  $\delta L/\delta x_2 = -\lambda_1 \delta u_2/\delta x_2 - \lambda_2 = 0$ 

3. 
$$\delta L/\delta G = \delta u_1/\delta G - \lambda_1 \delta u_2/\delta G - \lambda_2 c'(G) = 0$$

4. 
$$\delta L/\delta \lambda_1 = u_2^* - u_2(x_2, G) = 0$$

5. 
$$\delta L/\delta \lambda_2 = w_1 + w_2 - x_1 - x_2 - c(G) = 0$$

From (1), we get  $\lambda_2 = \delta u_1/\delta x_1$  and from (1) and (2), eliminating  $\lambda_2$ , we get  $\lambda_1 = -(\delta u_1/\delta x_1)/(\delta u_2/\delta x_2)$ . Using these values in (3), we get the condition

$$(\delta u_1/\delta G)/(\delta u_1/\delta x_1) + (\delta u_2/\delta G)/(\delta u_2/\delta x_2) = c'(G).$$

This condition for the optimal provision of a public good can be written more succintly as

$$MRS_{G1} + MRS_{G2} = MC_G,$$

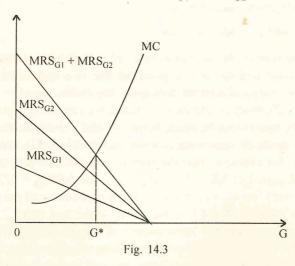
i.e. the *sum* of the marginal rates of substitution between the private good and the public good for the two individuals must equal the marginal cost of providing the public good.

If the efficiency condition is violated, we can show that at least one of the agents can be made better off and nobody made worse off. The mode of reasoning follows that developed in chapter 4. Suppose, for example, that the sum of the MRSs is less than the marginal cost. Let MC = 1,  $MRS_{G1} = \frac{1}{2}$  and  $MRS_{G2} = \frac{1}{3}$ . Then agent 1 would be willing to accept half a rupee more of the private good for the loss of Re 1 of the public good and agent 2 would be willing to accept  $\frac{1}{3}$  more rupees of the private good for the loss of Re 1 of the public good. Suppose we reduce the amount of the public good by Re 1, then we can compensate the two agents by giving them Rs  $\frac{5}{6}$ , and still have Rs  $\frac{1}{6}$  left to distribute to the two individuals and make them better off. Thus if the sum of the MRSs is less than the MC, less of the public good and more of the private good should be provided.

Another way of interpreting the Pareto-efficiency condition is to think of the MRS as measuring the marginal willingness to pay for an extra unit of the public good. Then the efficiency condition simply says that at the margin, the sum of the willingnesses to pay, must be equal to the cost of providing an extra unit of the public good.

The efficiency condition for a private good is that the MC should be equal to the MRS of each person separately. People can consume different amounts of the private good, but they must all value it the same at the margin for efficiency. In the case of a public good, all individuals have to consume the same amount of the good, but they can value it differently at the margin.

The public good efficiency condition is illustrated in Fig. 14.3. The MRS curves are added vertically because both agents must consume the same amount of the public good. The efficient provision of the public good, G\*, is obtained at the point of intersection of the MC and the  $MRS_{G1} + MRS_{G2}$  curve.



# 14.6 PRIVATE PROVISION OF A PUBLIC GOOD

The efficiency condition tells us how much of the public good should be provided. The public good is provided in equal amounts to all agents and it might seem natural to seek contributions from each individual to fund the public good. Yet, if it is left to the self-interest motive of individuals in a society to finance public goods, it seems likely that too little of it would be provided.

Let us consider a model similar to that developed in Section 14.5. Individual i can spend her initial wealth w, on either the private good  $(x_i)$  or as contribution to the public good  $(g_i)$ . The total amount of the public good will be the result of the contributions from the two agents,  $G = g_1 + g_2$ . To simplify matters, let c(G) = G, so that c'(G) = 1.

The choices of each agent will depend on the choices of the other. We consider a Nash equilibrium, where each person's choice is optimal against the other person's choices. The problem of individual i is

max 
$$u_i(x_i, G)$$
,  
 $x_i$ ,  $G$   
subject to  $x_i+g_i=w_i$ .  
We then get

$$MRS_i = 1, i = 1, 2.$$

Remembering that MC = 1, we see that the sum of the MRSs is more than MC and hence too little of the public good (compared to the Pareto-efficient amount) will be provided when individuals decide privately how much of the public good is to be provided.

This result is not surprising. In the case of positive externalities, individuals do not take account of the benefits to others and hence the private profit motive leads to a suboptimal output of the good conferring positive externalities. In the case of a public good, everyone can enjoy the good in equal amount, but this benefit is not take into account by individuals.

# THE FREE-RIDER PROBLEM

Once a public good is provided, nobody can be prevented from enjoying its services. If a bridge is built, then all motorists can drive on it. Hence, each person will have an incentive to pay as little as possible towards the construction of the bridge. Each person will hope that the contributions from others will be sufficient to build the bridge.

In general, free-riding refers to the consumption of a good without paying for it. If five students go to watch a movie and the five tickets are paid for by only four students, then the fifth student is free-riding on the others. The problem is especially acute in the case of public goods, because the free-rider cannot

be prevented from consuming the good once it is provided (in the case of cinema tickets, the four students can refuse to pay for the ticket of the fifth one, in which case the latter will not be able to watch the movie).

Free-riding bears a striking similarity to the Prisoner's Dilemma, though the two are not quite the same. Suppose that there are two tenants in a house who are trying to decide whether to construct a collapsible gate at the entrance or not. If the gate is constructed, both tenants will enjoy improved security in equal measure, so this is akin to a public good.

Suppose that each person has a wealth of Rs 5000, each values the gate at Rs 1000 and that the cost of the gate is Rs 1500. Note that the joint valuation of the gate exceeds its cost, and once constructed it will benefit both tenants. Now each tenant must decide whether to buy the gate or not. The pay-offs to the tenants are represented below:

It can be easily checked that the dominant strategy equilibrium for each player is not to buy the gate. If tenant A decides to buy the gate, then it is in the interest of player B to free-ride: to enjoy the increased security, but not to contribute anything for it. If tenant A does not buy, it is not in tenant B's interest to buy either. The situation is slightly different from the Prisoners' Dilemma. In the latter, the optimal situation is for the two players to take the same decision. In the free-riding situation, the optimal solution is for just one person to buy and both to enjoy the increased security.

A Pareto-improvement can be achieved if one of the tenants buys the gate and the other makes a side-payment to her. For example, if tenant A buys the gate and tenant B pays her Rs 501, then tenant A manages to break even and tenant B still enjoys a surplus of Rs 499.

#### DEMAND REVELATION

If individuals can be made to correctly reveal their preferences for a public good, then society will know, (a) whether it is worth providing the public good, and (b) how much to charge individuals for it. However, when individuals are directly asked to reveal their preferences, there is an incentive for people to free-ride. Suppose that individuals know that contributions will be levied upon them in proportion to their stated valuation of the public good. Then, if each individual believes that the contribution from others will be sufficient to provide for the good, she will have the incentive to understate her valuation of the benefits from the public good in order to keep down her subsequent contributions.

On the other hand, if everyone's contribution is predetermined, then each individual will have an incentive to exaggarate her true valuation. If an individual gets even a small positive gain from the public good, it will be in her interest to say that her valuation is very high: this will ensure that the good will be provided, but the individual will not have to bear any extra cost.

To bypass this problem, it has been suggested that each agent will receive a side payment equal to the sum of the other bids if the good is provided. Suppose that the sum of all other bids was negative and that the ith individual's bid made the total positive and ensured that the public good would be provided. Then the ith agent would have to pay an amount equal to the sum of other bids. With such a system in place, each agent always finds telling the truth to be the dominant strategy. Unfortunately, the total side payments may be very large under such a scheme and it might be very costly to induce agents to tell the truth.

#### LINDAHL ALLOCATIONS

Can we develop a price system that will support the efficient allocation of the public good? Erik Lindahl, a Swedish economist, provided an answer to this question in 1919.

Suppose that the cost of the public good is apportioned in the ratio h: (1-h) between the two agents. Agent 1 then solves the problem: