# MA668: Algorithmic and High Frequency Trading Lecture 17

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Table 4.6

Asset	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	2	29	0	0	0	0	16
FARO	11	25	0	0	3	13	100
MENT	6	18	0	0	2	7	75
AAPL	150	149	7	64	109	185	709

Table 4.6 Number of Changes in the ask or bid.

Figure: Table 4.6

## Table 4.6 (Contd ...)

- Recall: The discussion on non-Markovian nature of price changes.
- **②** Observation: There is a significant tendency (at least for AAPL on July 30th, 2013) for price movements to reverse themselves.
  - Thus, looking at the volatility of one-minute return misses many such price movements.
- To account for all price changes, we construct yet another measure of volatility: We count the number of changes in the bid within a one-minute period and report the statistics in Table 4.6.

#### Table 4.6 (Contd ...)

- We find that the less frequently traded assets (ISNS, FARO and MENT) also have more stable prices (bid and ask): At least 25% of the time, we see no price changes at all.
- ② In case of ISNS, this is even more marked, as it happens at least 75% of the time.
  - Nevertheless, the average is 2 per minute, suggesting that price changes occur frequently but when they do, there are a lot of them.
  - For MENT and FARO: We see more price movements than for ISNS, which is consistent with what we found earlier.
- MENT sees fewer price changes than FARO even though there is more trading in the former stock, but this is linked to the issue of minimum tick size, as discussed previously.

#### Table 4.6 (Contd ...)

- Statistics in Table 4.6: AAPL displays almost two orders of magnitude of price changes than MENT or FARO.
  - We However, the realized volatility of its return is lower than that of FARO and similar to that of MENT.
- Interpretation: Reflects the interaction of small relative tick size and large frequency of trading.
- A one-cent price change for AAPL (with an average price of around \$500 in 2013) is 0.2 bps.
- An asset with such a small relative tick size and with such a large trading activity is bound to have a price level that is very sensitive (and hence generates many changes in the bid/ask within a minute), but most of the resulting changes will be rapidly reversed, generating a relatively low realized volatility.

# Figure 4.7

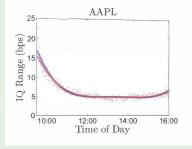


Figure 4.7 Intraday volatility: interquartile range for one-minute returns.

Figure: Figure 4.7

## Figure 4.7 (Contd ...)

- Figure 4.7: We look at how volatility (in terms of the inter-quartile range of one-minute returns) changes over the course of one trading day.
- ② The figure uses dots to display the inter-quartile range for each minute of the day, estimated using all trading days in 2013 (390 observations: one per minute).
- The lines represent the fitted quartic curves.
- The blue line is fitted with standard OLS while the red line is fitted using robust OLS, which controls for outliers by reducing the weight of the more extreme observations at the beginning and at the end of the day.
- Comparison of Figure 4.7 (Intraday volatility pattern) with Figure 4.2 (Intraday volume pattern) we see a common pattern: High at the beginning of the day, lower as the day progresses until it reaches a plateau between 12:00 and 15:00, followed by an increase until market close.
  - While Figure 4.7 shows a large left-slanted smile for the volatility pattern, Figure 4.2 shows a more symmetric volume smile and, if anything, slanted to the right.

## Market Depth and Trade Size: Table 4.7

Asset •	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	619	787	51	150	300	750	3,250
FARO	142	125	14	86	122	171	484
MENT	661	694	117	351	527	784	2,852
AAPL	189	169	64	127	161	210	662

Table 4.7 Average Depth at the Bid and Ask (number of shares).

Figure: Table 4.7

#### Market Depth and Trade Size

- Market quality is not just about the informational content of prices or the cost of executing a small order, it is also about depth.
- By depth, we mean the volume posted in the LOB and available for immediate execution.
- We focus mostly on the volume at-the-touch, that is at the bid and ask price levels.
- Mutual and pension funds manage a large fraction of people's wealth and they need markets to adjust their positions, pay their investors, and evaluate their performance.
- These funds move large quantities of shares. In any one day a fund may want to buy or sell thousands, tens of thousands, or even more shares of any one company.

#### Table 4.7

- Table 4.7: Illustrates how unreasonable it is to think that the market will match those trades at the published bid/ask prices.
- The table shows the distribution of the one-minute time-weighted average of the quantity of shares available at the ask and bid on NASDAQ for every day in 2013.
- This number does not exceed 1,000 shares in 75% of cases for any one of the four assets (not even AAPL).
- Recall (Table 3.1) that in 2013, NASDAQ intraday trading represents around 22 percent of total volume.

#### Market Depth and Trade Size (Contd ...)

- Depth and trade size are not independent of one another.
- The decision of how much to trade depends on the expected availability of shares resting as LOs for immediate execution.
- Similarly, the decision of how much to offer will depend on the expected flow of incoming MOs.
- If the depth is thin (few orders resting in the LOB), MOs will be small, which implies that in thin markets, relatively urgent large orders that would walk the LOB need to be broken up into smaller MOs which are then sequentially executed over a period of time.
- The institutional, legal, technological and economic changes of the last 15 years have produced a steady decline of the average trade size (the number of shares in a single trade).

# Figure 4.8

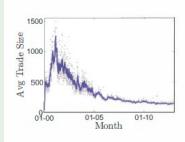


Figure 4.8 Average monthly trade size, AAPL 2000-2013.

Figure: Figure 4.8

#### Figure 4.8 (Contd ...)

- Figure 4.8: For AAPL we observe the monthly average trade size, computed from CRSP data.
- ② The dots are the monthly averages and the dark line is a smoothed representation (a moving average).
- It shows a sharp increase in the early 2000s to a peak of around 1,300 shares per trade, and a steady decline to around 200 shares by the end of 2013.
- Nowadays, with the 7:1 split, the numbers should be different, though a similar pattern is observable in most stocks.

#### Table 4.8

Table 4.8: Gives the statistics on NASDAQ trade sizes (total number of shares traded, Q, divided by the number of trades, n) for 2013. As can be seen there, trade sizes in AAPL are smaller than for the other three assets, though not substantially so.

Asset	Mean	StdDev	P01	Q1	Median	Q3	P99
ISNS	206.2	348.7	1.0	100.0	100.0	200.0	1600.0
FARO	99.8	76.4	1.0	64.0	100.0	100.3	374.8
MENT	199.9	172.9	6.0	100.0	150.0	240.3	867.1
AAPL	121.1	40.5	52.2	95.7	115.4	139.0	252.8

Table 4.8 Average Size of a Trade (Q/n).

Figure: Table 4.8

#### Market Depth and Trade Size (Contd ...)

- When developing algorithms that provide liquidity to the market, the depth, captured by the shape of the LOB, is critical because this dictates where a trader should post her/his LOs.
  - 2 We already discussed a simple model of market making.
- These ideas are developed further in the context of optimizing market making and optimal execution algorithms.
- Later: Optimal execution (buying or selling large positions) when the agent employs LOs and possibly also MOs.
- Stater: The shape of the LOB plays a critical role in the optimal posting of liquidity for a market maker, and we consider how different assumptions about the shape of the book affect the optimal posting strategy.

Figure 4.9

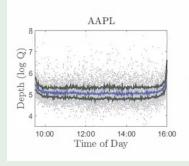


Figure 4.9 Intraday depth: (log) average quantity of shares posted at the bid and at the ask.

Figure: Figure 4.9

## Figure 4.9 (Contd ...)

- Figure 4.9: To close this discussion we look at the intraday patterns of depth.
- 2 This figure shows the intraday pattern of the average quantity of shares posted at the bid and ask every minute of 2013.
- As expected, there is the usual sharp increase at the end of the trading day.
- This is consistent with greater market quality in the form of narrower spreads and the increased desire of traders to close positions at the end of the day.
- Somewhat, though only marginally, surprising is that depth is also higher at the beginning of the day, where we hypothesized that high price uncertainty leads to wider spreads.
- Figure suggests that the theoretical trade-off between the benefits of market making from increased order arrival and the cost from higher uncertainty discussed, is resolved in practice in favor of providing greater liquidity.

#### **Price Impact**

- A major concern for participants who wish to execute a large order is that the order will have an adverse price impact.
- ② This means: Increasing the price when buying aggressively and lowering it when selling.
- There are several variables that can be used to measure the price impact of an order.
- Measuring depth, as we have just done, gives us a measure of price impact, in the sense that the depth at the bid/ ask tells us how large a market order can be with a zero price impact, i.e., without walking the LOB.
- Sut, in a single market, a large order would consume all the volume at the best quote and work its way through to the next tick and so on until the order is filled.
- Thus, whenever an MO walks the LOB, the average price per share is worse than the best quote at the moment the MO was sent to the LOB.

#### Price Impact (Contd ...)

- So far we have only considered what happens to the MO as it reaches the LOB, but executing a (relatively large) trade can be quite complex.
- ② In the US, with 11 public exchanges, regulators have felt a need to regulate what should happen to orders that consume all existing liquidity in one venue at the best bid/ask, in order to protect investors.
- This regulation and the multiplicity of exchanges raise related issues of time delays between different venues.
- So, when sending an MO one has to design the routing strategy very carefully: To which exchanges and when to submit the order, and what will happen if that order consumes all available liquidity at some point.
- So, an agent needs to know what happens as the order is executed, but also what happens in the aftermath of a trade.