MA668: Algorithmic and High Frequency Trading
Lecture 11

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Daily Asset Prices and Returns

1. When trading, the first variable of interest is the price level:
   - (A) If we have to acquire/liquidate a position, we want to know at what price we can get it if we aggressively execute it.
   - (B) If we are providing liquidity we want to know at what price shares are being bought and sold.
2. Each investor is in the market to meet some objective and will participate for as long as she/he is not losing too much money in pursuit of her/his objective (e.g., transaction costs do not consume the expected price gains or market adjusting prices to eliminate the original mispricing form which she/he wanted to profit from).

Daily Asset Prices and Returns (Contd ...)

1. The observed price process is the outcome of the interaction between these investors.
2. In electronic markets, we see these prices continuously as traders change their positions to meet their objectives, in response to changes in market conditions and information flows.
3. Market efficiency theories tell us that the resulting price process is not predictable and any positive expected return that one can predict, is there as a compensation for bearing risk.
    - (A) Long-term investors receive a compensation for risk, be it market risk, risk from monetary policy changes, or just compensation for future price fluctuations and dividend uncertainty.
    - (B) Liquidity traders require compensation for leaving offers at the bid and ask, and will continue to post orders as long as their trades are sufficiently profitable.
    - (C) Other traders pursue strategies aimed at exploiting deviations from market efficiency, such as keeping prices of similar assets close to each other.

Daily Asset Prices and Returns (Contd ...)

1. Analyze: Properties of the price process for a selection of assets from equity markets.
2. Primary focus: 2013 prices of AAPL (Apple Inc.), as being representative of a highly liquid and a very highly traded asset.
3. In order to illustrate differences across assets, we look at three other assets with tickers ISNS, FARO and MENT.
   - (A) ISNS is the company Image Sensing Systems (Application Software).
   - (B) FARO is the company FARO Technologies Inc. (Scientific and Technical Instruments).
   - (C) MENT is the company Mentor Graphics Corp. (Technical and System Software).

Outstanding shares are the total number of shares of a company's stock that are owned by investors. This includes share blocks held by institutional investors, restricted shares owned by the company's officers and insiders, and shares owned by the general public.

Daily Trading Activity: Table 3.1

Asset	N	V (\$) $(\times 10^3)$	$V(Q)$ $(\times 10^3)$	Total $V(Q)$ $(\times 10^3)$	ShrOut $(\times 10^6)$	Turnover
ISNS	14	18	3	12	5	0.62
FARO	315	1,396	34	137	17	2.04
MENT	908	3,964	204	694	112	1.56
AAPL	24,582	1,505,175	3,208	14,516	941	3.89

Table 3.1 Daily Average Volume in 2013 for selected assets.

Figure: Table 3.1

Daily Trading Activity (Contd ...)

1. Table 3.1 presents the different measures of trading activities for the aforesaid assets:
   - (A) Average number of transactions per day on NASDAQ (N).
   - (B) Average total daily dollar value of shares traded on NASDAQ (V($) in 000s).
   - (C) Average number of shares traded daily on NASDAQ (V(Q), in 000s).
   - (D) Total number of shares traded in all markets (Total V(Q), in 000s).
   - (E) Outstanding shares (ShrOut) is given in millions, as on December 30, 2012.
   - (F) Share turnover (Turnover) represents the total number of shares traded during 2013 divided by the number of outstanding shares.
2. Considering that the market is open for 6.5 hours (09:30 to 16:00):
   - (A) ISNS is rarely traded: Once every half hour.
   - (B) FARO and MENT are regular small assets: Average 1 to 2 trades per minute.
   - (C) AAPL is one of the most highly traded equity stocks: 1 trade per second [a].

[a] These 2013 numbers are not re-scaled for the AAPL June 2, 2014 "7-for-1" split

Daily Price Predictability

1. Examine: Properties of the price process by considering returns constructed from changes in prices from market open to market close for each day in 2013.
2. *Efficient Market Hypothesis*: Daily returns should be close to unpredictable and reflect information in the market.
3. Run: Ordinary Least Squares (OLS) regression for intra-day (market open to market close) returns for the four assets.
4. Accordingly, we include a number of variables related to market efficiency and market forces

Daily Price Predictability: The Variables

1. Variable 1 (Return on SPY): SPY is an Exchange Traded Fund (ETF) that tracks the S&P 500 index. Buying SPY serves the objective of tracking S&P 500 at the lowest possible cost (cheaper than acquiring all the 500 assets in the index and removes the the cost associated with constant re-balancing of one's portfolio to match the weight changes in S&P 500).
2. Variable 2 (Volatility Index VIX): VIX is an index continuously published by the Chicago Board of Options Exchange (CBOE) which is designed to measure the market's expectation on future short-term volatility in the S&P 500 index. It is computed by taking a certain weighted average of short term opportunities on the S&P 500 index. It is used a proxy for market uncertainty and is an indicator of market risk-aversion.
3. Variable 3 (Order flow): Difference between the number of shares aggressively bought and shares aggressively sold. If a transaction is the result of a passive limit sell (buy) order being lifted (hit) by an aggressive market buy (sell) order, we refer to it as an aggressive buy (sell) order. Thus the order flow is a proxy for the net demand for the asset.

Daily Price Predictability: Model 1

$$r_{t,j} = \alpha + \beta_{1,j} r_{t-1,j} + \beta_{2,j} \text{SPY}_t + \beta_{3,j} \text{VIX}_t + \beta_{4,j} \log(1 + Q_t) + \beta_{5,j} \text{OF}_t + \epsilon_{t,j}. \quad (1)$$

Description of Model 1

1. $r_{t,j} = \left(\frac{P_{\text{close}} - P_{\text{open}}}{P_{\text{open}}} \right)_{t,j}$: Intraday return for each of the four stocks, where $j \in \{\text{ISNS,FARO,MENT,AAPL}\}$.
2. (A) α: Constant which captures mean-daily return and should be close to zero.
   (B) $r_{t-1,j}$: Previous day's intraday return.
   (C) SPY_t: Contemporaneous intraday return on the SPY.
   (D) VIX_t: Contemporaneous intraday return on the VIX.
   (E) $\log(1 + Q_t)$: Log of one plus the number of shares of stock traded in all markets that day.
   (F) OF_t: Order flow for the stock on NASDAQ that day.

Daily Price Predictability: Model 2

$$\begin{aligned} r_{t,j} &= \alpha + \beta_{1,j} r_{t-1,j} + \beta_{2,j} \text{SPY}_t + \beta_{3,j} \text{VIX}_t + \beta_{4,j} \log(1 + Q_t) + \beta_{5,j} \text{OF}_t \\ &+ \beta_{6,j} \text{SPY}_t \; 1_{\{\text{SPT}_t < 0\}} + \beta_{7,j} \text{VIX}_t \; 1_{\{\text{VIX}_t < 0\}} + \epsilon_{t,j}. \end{aligned} \tag{2}$$

Description of Model 2

1. The two additional terms indicate the returns on SPY and VIX multiplied by indicators which equal 1 on days in which the NASDAQ or VIX moved down and 0 otherwise.
2. These two variables allow us to verify if there is an asymmetric reaction to the asset's return to any "good" or "bad" news in the market.

Table 3.2

Variables	ISNS		FARO		MENT		AAPL	
	M1	M2	M1	M2	M1	M2	M1	M2
constant	0.25	0.27	-2.83	-2.92	-2.97	-3.07	1.09	1.18
$r_{t-1,j}$	-0.10	-0.09	0.06	0.06	0.05	0.05	**−0.12**	**−0.12**
SPY (%)	-0.60	**−1.36**	**1.04**	1.03	**1.04**	**1.07**	0.28	0.06
VIX (%)	**−0.08**	-0.01	-0.03	-0.05	0.00	0.01	-0.03	-0.02
Log Q	0.01	0.00	0.25	0.27	0.23	0.24	-0.08	-0.08
Order Flow	0.03	0.02	**0.05**	**0.05**	**0.03**	**0.03**	**0.06**	**0.05**
Negv SPY	—	1.43	—	0.11	—	-0.08	—	0.52
Negv VIX	—	-0.19	—	0.06	—	-0.01	—	-0.01
Adj R	0.01	0.01	0.17	0.17	0.27	0.27	0.31	0.31

Table 3.2 Robust OLS regression of intraday (open-to-close) return. (Bold: 5% significance)

Figure: Table 3.2

Table 3.2 (Contd ...)

1. Standard OLS: Minimization of the sum of squared residuals, that is, the minimization of the sum of squared distance between the observations and the fitted values.
2. Robust OLS: The errors in the estimation are estimated so as to reduce the impact of outliers on the estimated parameters.
3. Some observations:
   - (A) The coefficient of order flow is significant and positive for all the latter three assets indicating that NASDAQ order flow and the asset's return move together.
   - (B) The coefficient of previous days return is not consistently significant.
   - (C) During 2013 our assets' returns were not significantly affected by changes in market sentiment, as measured through VIX.

Asset Prices and Intraday Returns

1. Daily market information is of primary interest only for investors with medium-term to long-term horizons.
2. High frequency trading strategies are executed over very short horizon and so we must look at what goes on in much finer detail.
3. Usage of millisecond-stamped message level data for the NASDAQ market to study prices over several sampling periods.
4. Focus on AAPL: Single day, namely July 30th, 2013, with construction of asset returns over one-second intervals.
5. Choice of date: A day with small positive price gain and positive net order flow.
6. 1.45 million shares bought VERSUS 1.24 million shares sold on NASDAQ.
7. Price increased from market open ($449.96) to market close ($453.32) by $3.36 (+74 bps).

Asset Prices and Intraday Returns (Contd ...)

1. The return of the asset is computed using the microprice (also called weighted mid-price) denoted by S_t^*.
2. It is the weighted average of the best bid (P_t^b) and the best ask (P_t^a), weighted by the relative quantities posted at the bid V_t^b and ask V_t^a:

$$S_t^* = \frac{V_t^b}{V_t^b + V_t^a} P_t^a + \frac{V_t^a}{V_t^b + V_t^a} P_t^b. \tag{3}$$

3. Microprice incorporates information on order imbalance (e.g., a relatively larger quantity of offers on the bid than on the ask indicates greater buying pressure and the "true" price is close to the ask than to the bid).

Asset Prices and Intraday Returns (Contd ...)

1. Choice of sampling frequency is important as it has very significant effect on the properties of the empirical distribution of the asset's return.
2. If the sampling frequency is very short: Many of the observations will be equal to zero.
3. For AAPL on July 30, 2013, sampling at one-second frequency results in 33% of the returns being zero.
4. The analysis is carried out using returns and usage of basis points (bps) as the unit of analysis (that is a value of 1 represents a change in the microprice of $\frac{1}{100}$-th of a percentage point).
5. If the average microprice is \$454.30, so that a positive return of 1 bps represents an increase of 4.5 cents in the microprice, and 0.22 bps is equivalent to 1 tick (1 cent).