MA668: Algorithmic and High Frequency Trading
Lecture 09

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Trading on an Informational Advantage (Contd ...)

1. In previous models: MM's just needed a liquidity premium to cover the expected cost from future price uncertainty.
2. Presence of informed traders: MM's will be adversely selected, buying (selling) when informed traders know it would be better to sell (buy).
3. This adverse selection requires a higher premium borne by other (more impatient liquidity) traders.
4. In this model (Kyle: 1985): Additional premium takes the form of price adjustment to order flow, as described by Kyle's lambda (λ, as given in the preceding slide).
5. This premium does not account for the risk from (random) future price movements (as described earlier), but for adverse selection faced by MM's, since prices on an average will move against the MM's because they trade with the better informed traders in the market.

Market Making with an Informational Disadvantage

1. Kyle (1985): Focused on the problem of an informed trader, while using competition to characterize the MM's decision.
2. To study the problem from MM's perspective, we look at the model due to Glosten and Milgrom (1985).
3. Glosten and Milgrom (1985): The model puts the MM at the centre of the problem of trading with counterparties who have superior information.
4. We again look at a simplified and (essentially) static version of the model that allows us to capture the nature of the MM's decision problem.
5. Situation: As before, there are liquidity traders, informed traders and a competitive group of MM's.
6. The MM is risk-neutral and has no explicit cost from carrying inventory.

Figure 2.3

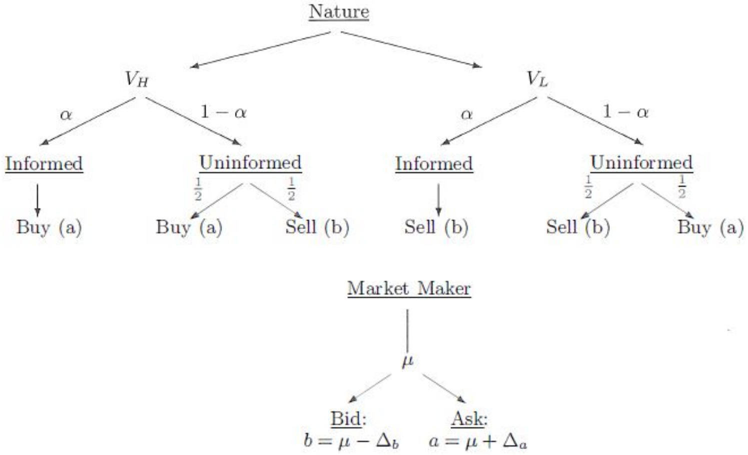

Figure 2.3 The Glosten-Milgrom model.

Figure: Figure 2.3

Market Making with an Informational Disadvantage (Contd ...)

1. The Simple Model: Figure 2.3.
2. Has a future cash value of the asset equal to v which we limit to two possible values:
$$V_H \text{ (High) } > V_L \text{ (Low)}.$$
3. The unconditional probability of $v = V_H$ is p.
4. All orders are of one unit:
   - A. MM's post an LO to sell one unit at price a.
   - B. MM's post an LO to buy one unit at price b.
5. Further, we assume that liquidity traders are price insensitive and want to buy with probability $\frac{1}{2}$ and sell with probability $\frac{1}{2}$.
6. There are many informed traders, all of whom know v but are limited to trade a single unit, which simplifies their decision:
   - A. When $v = V_H$ they buy one unit if $a < V_H$, and do nothing otherwise.
   - B. When $v = V_L$ they sell one unit if $b > V_L$, and do nothing otherwise.

Market Making with an Informational Disadvantage (Contd ...)

1. The total "population" of liquidity and informed traders is normalized to one, and of those, a proportion α are informed and a proportion $(1-\alpha)$ are uninformed liquidity traders.
2. Figure 2.3 captures the probabilistic structure of the model: "Nature" determines whether the underlying state is V_H or V_L.
3. Independent of the state, a trader is picked at random from the population, so that she/he is informed with probability α and is uninformed with probability $(1-\alpha)$.
4. An informed trader will always buy at the asked price of a, when the asset value is V_H AND sell at the bid price of b, when the asset value is V_L.
5. On the other hand, an uninformed trader will buy or sell with equal probability, independent of the true (unknown) value of the asset.

Market Making with an Informational Disadvantage (Contd ...)

1. What is MM's problem: Choose a and b in this setting.
2. Since liquidity traders are price-insensitive, the optimal solution is trivial: Set $a = V_H$ and $b = V_L$.
3. Since MM's compete for business, prices will be set to their *semi-strong efficient* levels, since MM's only use public information (which includes order flow).
4. Competition between MM's drive their expected profits to zero.
5. Therefore a and b are determined by the no-profit condition.
6. Rather than solve for a and b directly, we define the following:
   - A. Ask half-spread: Δ_a.
   - B. Bid half-spread: Δ_b.
   - C. Sum of the two: $\Delta_a + \Delta_b$, represents the (quoted) spread.

Market Making with an Informational Disadvantage (Contd ...)

1. Let the expected value of the asset be $\mu = E[v|\mathcal{F}]$, where $\mathcal{F}$ represents all public information prior to trading.
2. MM's will choose $a = \mu + \Delta_a$ and $b = \mu - \Delta_b$, optimally.
3. Now, in order to determine the effect of choosing a and b on the expected profit and loss, consider what happens when the order comes in:
   - A. If the buy order comes from an uninformed liquidity trader: She/he (MM) makes an expected profit of $a - \mu = \Delta_a$.
   - B. If the buy order comes from an informed trader: She/he (MM) makes an expected loss of $|a - V_H| = |\Delta_a - (V_H - \mu)|$
4. From the point of view of the MM:
   - A. The probability that the liquidity trader wants to buy is $\frac{1}{2}$.
   - B. The probability that an informed trader wants to buy is p (Informed traders will buy only when $v = V_H$, which has probability p).

Market Making with an Informational Disadvantage (Contd ...)

1. Recall: There are $(1-\alpha)$ uninformed liquidity traders and α informed traders.
2. Therefore, the expected profit from posting a price of $a = \mu + \Delta_a$ is given by:
$$\frac{(1-\alpha)/2}{\alpha p + (1-\alpha)/2}\Delta_a + \frac{\alpha p}{\alpha p + (1-\alpha)/2}\left(\Delta_a - (V_H - \mu)\right).$$
3. Setting the expected profit to be equal to zero, we obtain:
$$\Delta_a = \frac{\alpha p}{\alpha p + (1-\alpha)/2}\left(V_H - \mu\right) = \frac{1}{1 + \frac{1-\alpha}{\alpha}\frac{1/2}{p}}\left(V_H - \mu\right).$$
4. Similarly, we can obtain:
$$\Delta_b = \frac{\alpha(1-p)}{\alpha(1-p) + (1-\alpha)/2}\left(\mu - V_L\right) = \frac{1}{1 + \frac{1-\alpha}{\alpha}\frac{1/2}{1-p}}\left(\mu - V_L\right).$$

Market Making with an Informational Disadvantage (Contd ...)

1. Question: How to interpret the equations?
2. We can think of asymmetric information as "toxicity".
3. Accordingly, we can regard α as an indicator of the "prevalence of toxicity".
   - A. p and $V_H - \mu$: Magnitude of buy-toxicity.
   - B. $1 - p$ and $\mu - V_L$: Magnitude of sell-toxicity.
4. Trading algorithms are built to either take advantage of informational advantage or to adjust the depth at which LO's are posted so as to recover losses experience by trading agents from more informed traders.
5. The model will now be extended to two different and complementary ways:
   - A. Incorporation of a time dimension.
   - B. Making liquidity traders price-sensitive.