

MA668: Algorithmic and High Frequency Trading

Lecture 18

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Table 4.9

Event	<i>n</i>	Buy Market Orders				Sell Market Orders			
		<i>n</i> = 0	Q1	Q2	Q3	<i>n</i> = 0	Q1	Q2	Q3
Benchmark	2,340,000	99.6	1	1	2	99.7	1	1	2
Buy Order	6,852	70.7	1	2	3	94.3	1	1	2
Buy Order (n-o)	5,707	71.5	1	2	3	94.4	1	1	2
Buy Order-Large	532	57.5	1	2	5	92.1	1	1	2
Sell Order	7,358	94.5	1	1	2	74.9	1	2	3
Sell Order (n-o)	6,269	94.7	1	1	2	75.7	1	2	3
Sell Order-Large	347	92.2	1	1	2	62.2	1	2	4

Table 4.9 Market Impact of an execution on MOs (AAPL 20130730).

Figure: Table 4.9

Table 4.9 (Contd ...)

- ① With the information we have from NASDAQ we now look at what happens after orders are executed there.
- ② Table 4.9: Looks at the executions for AAPL on NASDAQ on July 30th, 2013.
- ③ Recall: We do not observe MOs, but rather, what happens to existing posted orders.
- ④ To illustrate the kind of analysis that can be done, we make the simplistic assumption that all orders executed at the same time (same millisecond) and at the same price level, are all part of the same MO, and are aggregated accordingly.

Table 4.9 (Contd ...)

- ① Table 4.9: Looks at what happens when different types of buy and sell orders are executed, *i.e.*, we look at the arrival of MOs under different circumstances.
- ② The row labeled “Benchmark” captures what happens on average by including every 10ms interval during the trading day (2.34 million observations).
- ③ This serves as benchmark with which to compare what happens in the 10ms time interval AFTER an MO comes in.
- ④ We compare this benchmark with what happens after the following six events:
 - Ⓐ Buy [Sell] Order: An order is executed against orders posted on the ask (sell-side) [bid (buy-side)] of the book.
 - Ⓑ Buy [Sell] Order ($n = 0$): Same as buy [sell] order but ignoring buy [sell] orders within 10ms of a previous buy [sell] order.
 - Ⓒ Buy [Sell] Order-Large: Buy [sell] orders for strictly more than 300 shares.

Table 4.9 (Contd ...)

- 1 Table 4.9: We include the statistics on order arrival on the bid and ask side in the benchmark case, as well as after these six events.
- 2 The columns labelled " $n = 0$ " describe the percentage of cases in which we do not observe an MO arriving on the bid/ask side after an execution.
- 3 In the benchmark case we see that despite the high level of trading activity in AAPL, our interval size is sufficiently small so that in roughly 99.4 percent of cases we do not observe an order arriving.
- 4 After any type of execution, this proportion falls on both sides of the book.
- 5 We observe that after a Buy MO there is at least a second MO arriving in 30 percent of cases, and at least one sell MO in 5 percent of cases.
- 6 A similar pattern is observed after a Sell MO: In at least 25 percent of cases a sell MO is followed by at least one other MO, while in at least 5 percent of cases a sell MO is followed by a buy MO.

Table 4.9 (Contd ...)

- ① We also see very similar patterns if we exclude MO arrivals that occur just after other MOs.
- ② The number of arrivals, however, increases substantially after a large order: a large buy (sell) is followed by other buys in 43 (38) percent of cases, and buy (sell) in 8 (8) percent of cases.
- ③ If we look at the quartiles conditional on there being at least one MO (the columns labeled "Q1", "Q2" and "Q3"), the arrival of a market buy (sell) order seems to have no clear effect on the distribution of arriving sell (buy) orders, but we do see evidence suggesting that incoming orders on the same side may be more frequent.
- ④ Accordingly, our very preliminary and limited analysis suggests that order arrival seems to be followed by further order arrival on both sides of the book, and more on its own side than on the other side of the book.

Table 4.10

Ticks	Ask	Changes in ASK			Bid	Changes in BID		
		Buys	$\Delta \neq 0$	$\Delta > 3c$		Sells	$\Delta \neq 0$	$\Delta > 3c$
Obs		6,852	3,259	1,165		7,358	4,052	1,910
0	99.5	28.2	10.7	9.7	99.5	22.7	7.2	6.1
≥ -5	12.8	0.1	0.0	0.0	17.2	0.3	0.4	0.2
-4	4.5	0.1	0.1	0.3	4.5	0.0	0.0	0.0
-3	5.6	0.1	0.1	0.0	5.6	0.1	0.1	0.0
-2	8.6	0.1	0.1	0.1	7.0	0.1	0.1	0.1
-1	22.6	1.1	0.4	0.7	17.9	0.8	0.4	0.5
1	14.2	19.5	20.5	0.9	13.3	15.7	14.3	0.9
2	7.4	13.6	13.6	0.9	7.1	11.8	11.3	0.9
3	5.6	12.3	11.9	1.9	5.5	10.4	10.2	0.9
4	4.4	9.8	9.2	15.3	4.3	9.5	9.4	12.2
≤ 5	14.3	43.4	44.2	80.0	17.5	51.3	53.9	84.4

Table 4.10 Market Impact of an execution on the best price – own side (AAPL 20130730).

Figure: Table 4.10

Table 4.10 (Contd ...)

- ① Table 4.10: We look at how different events affect the bid and ask prices.
- ② Convention: +1 is a one cent move away from the best price, that is:
 - Ⓐ If the ask price is \$453.02, +1 in the ask is a change from \$453.02 to \$453.03.
 - Ⓑ On the bid side, with the bid price at \$452.96, +1 in the bid is a price drop of one cent, that is a change from \$452.96 to \$452.95.
- ③ With this convention, positive price changes represent moves away from the midprice and negative price changes represent moves towards the midprice, which allows us to provide a more streamlined presentation of the different effects of MOs on bid and ask sides.

Table 4.10 (Contd ...)

- ① Returning to Table 4.10: We consider how different orders affect the best price on their own side of the LOB, that is, the left side of the table describes how the ask price reacts to an aggressive buy MO, and the right describes how the bid side reacts to an aggressive sell MO.
- ② We consider two benchmark cases: The column “Ask” (“Bid”) is the benchmark case that looks at average changes in the ask (bid), that is after every 10 ms interval. The first row tells us the percentage of time for which there was no change in the bid (99.5 percent) and no change in the ask (also 99.5 percent).
- ③ We also look at what happens to the ask (bid) after a “Buy” (“Sell”) order comes in, and the percentage of times when the ask (bid) stays the same drops to 28 (23) percent.
- ④ This percentage falls even further if we only look at executions that sweep the order book ($\Delta \neq 0$), that is after a buy (sell) order that generates an immediate change in the ask (bid).

Table 4.10 (Contd ...)

- 1 Such 'large' executions are more long-lived in the sense that 10 ms after such a change the probability that the ask (bid) has returned to its pre-order arrival level drops to 11 (7) percent.
- 2 The columns labeled " $\Delta > 3c$ " look at the subset of the executions that sweep the order book, and we also observe a large (greater than three cent) change in the ask (bid) price respectively. The likelihood of returning is smaller than that for all sweep orders but not by much.
- 3 The rows of Table 4.10 (except the "Obs" and "0" rows) reflect the distribution of price movements conditional on different non-zero price changes.
- 4 The benchmark distributions for bid and ask price movements are symmetric and very similar, something that is not true for the distributions after MOs arrivals.

Table 4.10 (Contd ...)

- ➊ After a buy (sell) order, the distribution of the ask (bid) clearly shifts away from its previous level and is almost never better (closer to the mid-price) than before the arrival of the MO 10ms later.
- ➋ The difference we observe for a sweep order seems to be centered on the probability of returning to the pre-arrival level, but does not seem to have much effect on the distribution of price changes for non-zero changes.
- ➌ However, large price swings do seem to be followed by changes in the distribution of bid/ask price changes, and we see little evidence that these large price movements are reversed within 10ms.

Table 4.11

Ticks	Bid	Changes in BID			Ask	Changes in ASK		
		Buys	$\Delta \neq 0$	$\Delta > 3c$		Sells	$\Delta \neq 0$	$\Delta > 3c$
Obs		6852	3259	1165		7358	4052	1910
0	99.5	81.6	78.7	75.4	99.5	82.2	80.3	79.1
≥ -5	17.5	6.9	6.5	7.7	14.3	6.0	6.0	8.3
-4	4.3	1.8	1.7	2.8	4.4	1.7	1.8	2.0
-3	5.5	1.8	1.9	3.1	5.6	2.1	2.1	2.8
-2	7.1	3.2	3.9	3.1	7.4	3.4	3.3	4.3
-1	13.3	8.7	8.2	9.1	14.2	8.0	8.1	8.5
1	17.9	29.5	28.7	34.8	22.6	29.2	27.7	28.8
2	7.0	10.8	11.8	11.8	8.6	11.6	12.9	13.5
3	5.6	7.0	6.3	7.3	5.6	6.5	6.4	6.0
4	4.5	6.1	5.0	2.4	4.5	6.8	5.6	4.5
≤ 5	17.2	24.2	25.9	17.8	12.8	24.7	26.1	21.3

Table 4.11 Market Impact of an execution on the best price – other side (AAPL 20130730).

Figure: Table 4.11

Table 4.11 (Contd ...)

- 1 Table 4.11: Repeat of the analysis but looking at the effect of an order arrival on the other side of the book, that is how the arrival of a buy (sell) MO affects the bid (ask). We keep the signs so that a positive move in Table 4.10 is also a positive move on the other side of the book in Table 4.11.
- 2 That is, suppose the ask price is \$453.02 and the bid is \$452.96.
- 3 After a buy order, +1 cent change in the ask is an increase from \$453.02 to \$453.03 (Table 4.10), and +1 cent move in the bid is an increase from \$452.96 to \$452.97 (Table 4.11).
- 4 On the other hand, after a sell order, +1 cent change in the bid results in a decrease from \$452.96 to \$452.95 (Table 4.10), and a +1 cent move in the ask results is a change from \$453.02 to \$453.01 (Table 4.11).

Price Impact (Contd ...)

- ① With this convention, we see that the effect of an arrival on one side of the LOB is followed by a similar but weaker effect on the other.
- ② The probability of the price remaining/returning to the pre-arrival level drops from 99.5 to 82 for both the bid and the ask after a buy and a sell order arrive, respectively.
- ③ This probability is slightly smaller for (inter-market) sweep orders. We also see a shift in the distribution of non-zero price changes that (weakly) follows that of the changes on the other side of the book.
- ④ So we see how the arrival of a buy order is followed by a shift in the (non-zero) bid price changes away from the mid-price, so the conditional probability of a 1 cent move away from the pre-arrival bid price goes from 17.9 to 29.5 percent after a buy order, and that of a 1 cent move away from the pre-arrival ask price goes from 22.6 to 29.2 percent after a sell order. The pattern is very similar after a buy (sell) order, a sweep buy (sell) order, or a sweep buy (sell) order with a large price move.
- ⑤ Combining this observation with the price moves in Table 4.10, we find evidence that the **quoted spread increases after a buy or sell order, and substantially so after a large sweep order.**

Table 4.12

	Ticks	Changes in ASK				Changes in BID			
		10	30	100	1,000	10	30	100	1,000
Bench	≤ -3	0.1	0.3	1.0	7.7	0.1	0.4	1.2	8.9
	$\{-1, -2\}$	0.2	0.4	1.2	7.7	0.1	0.3	0.7	4.0
	0	99.5	98.7	96.0	72.9	99.5	98.6	95.7	70.3
	$\{1, 2\}$	0.1	0.3	0.7	3.9	0.1	0.4	1.1	6.9
	≥ 3	0.1	0.3	1.0	7.7	0.1	0.4	1.3	9.9
Buys	≤ -3	0.2	0.3	0.4	1.3	6.9	9.8	13.2	26.2
	$\{-1, -2\}$	0.8	1.0	1.4	3.1	7.4	7.8	8.3	11.3
	0	28.2	26.7	24.3	16.0	81.6	77.8	72.4	49.2
	$\{1, 2\}$	23.8	22.4	21.8	18.4	2.2	2.4	2.6	4.1
	≥ 3	47.0	49.6	52.1	61.3	1.9	2.2	3.5	9.2
Sells	≤ -3	6.8	8.3	12.0	24.0	4.1	4.2	4.6	6.8
	$\{-1, -2\}$	7.3	8.3	9.3	13.3	4.1	4.0	4.3	6.2
	0	82.2	78.8	73.4	52.3	47.3	44.6	41.1	28.1
	$\{1, 2\}$	2.0	2.1	2.6	4.0	15.9	15.7	15.0	14.5
	≥ 3	1.8	2.4	2.7	6.4	28.5	31.4	35.0	44.4

Table 4.12 Market Impact of an execution on the midprice over time (AAPL 20130730).

Figure: Table 4.12

Table 4.12 (Contd ...)

- ① Table 4.12: To conclude our look at the impact of MOs, we look at the effect on the changes we observed at the 10ms horizon, we consider longer (30ms, 100ms and 1,000ms) horizons. Table 4.12 is split horizontally into three sections:
 - Ⓐ The first (“Bench”) is the benchmark table that looks at changes in the bid and ask over the corresponding horizons for all such time intervals.
 - Ⓑ The bottom two sections consider the effects of the arrival of a buy and a sell order respectively on bid and ask prices.
- ② For this table we continue to keep the signs matched on the bid and ask sides, but to avoid confusion we keep the sign of changes on the bid (ask) side the same as in the benchmark case, as well as after a buy or a sell order, that is, the interpretation of the sign does not depend on whether it follows a buy or a sell MO, but only on which side of the book we are looking at.

Table 4.12 (Contd ...)

- ① So, suppose the ask price is \$453.02 and the bid is \$452.96.
- ② After a buy order, +1 cent in the ask is a move from \$453.02 to \$453.03, and +1 cent move in the bid is a move from \$452.96 to \$452.95 (a 1 cent move away from the mid-price).
- ③ The same happens after a sell order (and in the benchmark case): +1 cent in the ask results in an increase from \$453.02 to \$453.03, and +1 cent move in the bid is move from \$452.96 to \$452.95 (one cent away from the mid-price).
- ④ Note also, that all percentiles reflect probabilities (we are not conditioning on non-zero price movements in this table).

Table 4.12 (Contd ...)

- ① The first thing to notice in Table 4.12 is the natural effect of time on all prices: as we expand the horizon, prices tend to move more, and the distributions become more dispersed.
- ② We also see that the initial price movements are not followed by quick reversals and that even one second (1,000 ms) after a buy order there is a marked shift of the bid and ask away from its pre-execution level, with worse prices and a hint of a delayed price impact on future executions and wider spreads.
- ③ All these results must be interpreted in context, and not causally.
- ④ As we will now see, MOs do not arrive at random times. They tend to arrive when spreads are narrow, and opportunistically hit orders that are posted closer to the mid-price, so it is only natural that we should observe a wider spread after an execution.