MA 668 | Mid-Sem

Algorithmic and High Frequency Trading | 09:00 AM–11:00 AM

IIT Guwahati | 24th February 2024

Instructions

1. Write your name and roll number on the answerscript.
2. Your writing should be legible and neat.
3. This Mid-Sem has 5 questions, for a total of 30 marks.

Questions

[6 marks] 1. State TRUE or FALSE:

(A) Latency is contingent on the amount of orders in the network.

(B) "BA"– type of transition can be interpreted as a sequence of $\Uparrow$ and $\Downarrow$.

(C) Depth in the Limit Order Book indicates the liquidity in the market.

(D) The direct trading cost of a market sell order is the same as that of a market buy order, when the price S_t is the mid-price.

(E) The visible price offered in a Limit Order is usually better than the current market price.

(F) Market maker is a professional trader who profits from intermediary between different liquidity traders.

Answer:

(A) TRUE ...(1 mark)

(B) FALSE ...(1 mark)

(C) TRUE ...(1 mark)

(D) TRUE ...(1 mark)

(E) FALSE ...(1 mark)

(F) TRUE ...(1 mark)

[4 marks] 2. (A) Consider the following Table:

$t/t+1$	Ask Uptick ($\Uparrow$)	Ask Downtick ($\Downarrow$)	Bid Uptick ($\Uparrow$)	Bid Downtick ($\Downarrow$)
Uptick ($\Uparrow$)	43.0	57.0	36.5	63.5
Downtick ($\Downarrow$)	61.8	38.2	55.3	44.7

(i) What is the probability that an uptick ($\Uparrow$) is followed by an ask-price downtick ($\Downarrow$).

(ii) What is the probability that a downtick ($\Downarrow$) is followed by a bid-price downtick ($\Downarrow$).

(B) Recall that: $A :=\Uparrow\Uparrow$, $B :=\Uparrow\Downarrow$, $C :=\Downarrow\Uparrow$ and $D :=\Downarrow\Downarrow$. Now, consider the following Table on the Ask side:

	Up $(t+1, t+2)$ ($\Uparrow\Uparrow$)	Reversal $(t+1, t+2)$ ($\Uparrow\Downarrow, \Downarrow\Uparrow$)	Down $(t+1, t+2)$ ($\Downarrow\Downarrow$)
Uptick (t) ($\Uparrow$)	17.1	59.5	23.4
Downtick (t) ($\Downarrow$)	19.6	59.1	21.3

(i) What are the probabilities of AA and BD for ask.

(ii) What are the alphabetical combinations (in terms of A, B, C, D) for $\Uparrow\Uparrow\Downarrow\Uparrow$ and $\Downarrow\Downarrow\Downarrow$.

(iii) What are the impossible alphabetical combinations of length three (in terms of A, B, C, D).

(C) Consider the following Table:

$(\tau_{i+1} - \tau_i)$ (In mins)	0.1	0.2	0.3	0.4
QS_{τ_i}	33.2	32.8	32.9	33.1

(i) What is the time weighted average quoted spread.

(D) Consider the following Table for the number of changes in the ask or bid:

Asset	Mean	Std Dev	P01	Q1	Median	Q3	P99
ISNS	2	29	0	0	0	0	16
FARO	11	25	0	0	3	13	100
MENT	6	18	0	0	2	7	75
AAPL	150	149	7	64	109	185	709

(i) State TRUE or FALSE: ISNS has no price changes in at least 75% of the quotes.

(ii) State TRUE or FALSE: AAPL is the least volatile of the four assets in the Table.

Answer:

(A) (i) 0.57.

(ii) 0.447. ...(1 mark)

(B) (i) AA: 0.171 and BD: 0.234.

(ii) ABC and DD.

(iii) AC, BA, CC, DA, AD, BB, CD, DB. ...(1 mark)

* *The question was supposed to be for length two*

(C) 32.99. ...(1 mark)

(D) (i) TRUE.

(ii) FALSE. ...(1 mark)

[8marks] 3. Construct the problem of trading on informational advantage by taking into account the following considerations:

(A) The insider observes v.

(B) On observing v, the insider chooses $x(v)$.

(C) u is realized.

(D) The MM's observe the net order flow $x(v) + u$.

(E) Based on the net order flow, the MM's compete to set the asset price S.

Hence prove that:

$$x^*(v) = \beta(v - \mu), \ \beta = \frac{1}{2\lambda}.$$

[*Note that:* $S(x + u) = \mu + \lambda(x + u)$ *and* $x(v) = x \cdot v$].

Answer:

We begin with the observation that in equilibrium the insider chooses $x(v)$ to maximize her/his expected profit and they anticipate that MM's will set their prices on the basis of what they learn from observing the order flow and what they know about the informed trader's decision problem. ...(1 mark)

MM's choose their prices taking into account the strategy of the insider (in particular they anticipate the functional form of $x(v)$) and the properties of the uninformed order flow that comes from liquidity traders. Accordingly, the MM's set the market price as a function of net order flow: $S(x+u)$. ...(1 mark)

The average price for per unit traded, namely, S moves with the net order flow of $x+u$. In equilibrium, the insider will anticipate the functional form of $S(x+u)$, that is she/he will incorporate price impact when choosing $x(v)$. The equilibrium is a fixed point in the optimization of x, given a functional form of S, and of S, given the functional form of x.

Now, the insider will sell if $v < E[v] = \mu$ and buy if $v > \mu$.

Consequently, the MM's, anticipating the demand as a function of the realization of v, behave optimally and set prices that incorporates all information on v in $x(v)$. ...(1 mark)

Now, the zero (expected) profit condition forces prices to have a very specific property (known as *semi-strong efficiency*): $S = E[v|\mathcal{F}]$, where $\mathcal{F}$ represents all information available to MM's. We can identify a fundamental property of the MM's equilibrium strategy as:

$$S(x+u) = E[v|x+u],$$

and we need to find an $x(v)$ that is optimal, that is, it maximizes the insider's expected trading profits, conditional on the pricing rule. ...(2 marks)

Since v and u are normally distributed, we present the hypothesis that $S(x+u)$ is linear in net order flow. More specifically:

$$S(x+u) = \mu + \lambda(x+u),$$

where λ is an unknown parameter, which can be interpreted as representing the linear sensitivity of the market price to order flow. Consequently, we consider the insider's problem as:

$$\max_x E\left[x\left(v - S\left(x+u\right)\right)\right] \ldots .\text{(2 marks)}$$

Substituting for $S(x+u) = \mu + \lambda(x+u)$ and taking expectation with respect to u, we obtain that the objective function is concave and the first order condition yields:

$$x^*(v) = \beta(v-\mu), \;\; \beta = \frac{1}{2\lambda} \ldots .\text{(1 mark)}$$

[6 marks] 4. Using the Grossman-Miller Market Making model incorporating trading costs and exponential utility, prove that:

$$q_1^{LT1,*} = \frac{i}{n+1} + 2\left(\frac{n}{n+1}\right)\frac{\eta}{\gamma\sigma^2}.$$

Answer:

Suppose that trader pays $\eta > 0$ per share, regardless of whether they are buying or selling the asset. Further, assume that any remaining inventories after $t=2$ are liquidated at $t=3$ and that LT1 wants to sell $|i|$ units ($i > 0$) and LT2 wants to buy the same amount. ...(1 mark)

At $t=2$, since MM's and LT1 enter the period with positive inventory, their optimal final holdings are:

$$q_2^j = \frac{E\left[(S_3 - \eta)|\epsilon_2\right] - (S_2 - \eta)}{\gamma\sigma^2}, \ j \in \{MM, LT1\}$$

and the demand for shares by LT2 is:

$$q_2^{LT2} = \frac{E\left[(S_3 + \eta)|\epsilon_2\right] - (S_2 + \eta)}{\gamma\sigma^2}.$$

As everyone anticipates that their trading positions need to be liquidated anyway, the trading fees do not affect the price at $t = 2$, and we obtain $S_2 = E\left[S_3|\epsilon_2\right] = \mu + \epsilon_2$ (as before when $\eta = 0$ (no fees)). ...(1 mark)

At $t = 1$, LT1 has a situation similar to that at $t = 2$, as any quantities not sold now, will have to be sold later. Accordingly,

$$q_1^{LT1} = \frac{E\left[(S_2 - \eta)\right] - (S_1 - \eta)}{\gamma\sigma^2}.$$

...(1 mark)

On the other hand, MM's anticipate that whatever they buy, they will have to sell later, which changes their asset demand function to:

$$q_1^{MM} = \frac{E\left[(S_2 - \eta)\right] - (S_1 + \eta)}{\gamma\sigma^2}.$$

The resulting market equilibrium condition is:

$$i = nq_1^{MM} + q_1^{LT1} = n\left(\frac{\mu - S_1 - 2\eta}{\gamma\sigma^2}\right) + \frac{\mu - S_1}{\gamma\sigma^2}.$$

This gives us:

$$i = (n+1)\left(\frac{\mu - S_1}{\gamma\sigma^2}\right) - \frac{2n\eta}{\gamma\sigma^2} \Rightarrow S_1 = \mu - \gamma\sigma^2\left(\frac{i}{n+1}\right) - 2\left(\frac{n}{n+1}\right)\eta.$$

...(1 mark)

Recall that: For LT1, $i > 0$. Thus we conclude that the presence of trading fees introduces an extra liquidity discount to the initial price S_1. Now, almost all the trading fees are paid for by the liquidity trader initiating the transaction.

(A) She/he pays own trading fee of η, per share PLUS

(B) A substantial fraction $\frac{n}{n+1}$ of the two transaction fees paid by the MM's (2η), though indirectly, via a lower sale price, a lower S_1.

...(1 mark)

This also affects the immediacy that she/he obtains from the market, as her/his holdings at the end of $t = 1$ are no longer $q_1^{LT1,*} = \frac{i}{n+1}$, but $q_1^{LT1,*} = \frac{i}{n+1} + 2\left(\frac{n}{n+1}\right)\frac{\eta}{\gamma\sigma^2}$. ...(1 mark)

[6 marks] 5. For the Glosten-Milgrom model, prove that the half spreads, for price sensitive liquidity traders are given by:

$$\Delta_a = \frac{1}{1 + \frac{1-\alpha}{\alpha}\frac{(1-F(\Delta_a))/2}{p}}\left(V_H - \mu\right),$$

and

$$\Delta_b = \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{(1-F(\Delta_b))/2}{1-p}} \left(\mu - V_L \right).$$

[*You can make use of the result from the basic/static Glosten-Milgrom model*].

Answer:

It is assumed that liquidity traders get an additional (exogenous) value from executing their desired trade. Let the trader i get a cash equivalent utility gain of c_i if she/he manages to execute her/his desired trade. Thus, if the transaction cost imposed by the half spread is too high (higher than c_i), then trader i will prefer not to execute the trade. ...(1 mark)

Assume that the distribution of parameter c_i in the population of liquidity traders is described by the CDF F, that is, $F(c)$ is the proportion of liquidity traders that have $c_i \leq c$. c_i is referred to as the agent's urgency parameter. ...(1 mark)

Now, we can recompute the expected profit of the MM from setting an ask price of $a = \mu + \Delta_a$, which will now be given by:

$$\frac{(1-F(\Delta_a))(1-\alpha)/2}{\alpha p + (1-F(\Delta_a))(1-\alpha)/2} \Delta_a + \frac{\alpha p}{\alpha p + (1-F(\Delta_a))(1-\alpha)/2} \left(\Delta_a - (V_H - \mu) \right).$$

...(2 marks)

In the expression, we have incorporated the fact that whenever a liquidity trader wants to buy $(1-\alpha)/2$, only $(1-F(\Delta_a))$ will have sufficient urgency to execute the trade with a buy-half-spread equal to Δ_a. ...(1 mark)

Introducing the parameter increases the half-spreads, which are now implicitly defined by the expressions:

$$\Delta_a = \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{(1-F(\Delta_a))/2}{p}} \left(V_H - \mu \right),$$

and

$$\Delta_b = \frac{1}{1 + \frac{1-\alpha}{\alpha} \frac{(1-F(\Delta_b))/2}{1-p}} \left(\mu - V_L \right).$$

...(1 mark)