# Fundamentals of Artificial Intelligence

## Decision Network

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# Reasoning under Uncertainty

- Uncertainty – the quality or state of being not clearly known
  - distinguishes *deductive* knowledge from *inductive* belief

The agent's knowledge can at best provide only a degree of belief in the relevant sentences.

- Sources of uncertainty
  - Ignorance

Uncertainty can arise because of incompleteness and incorrectness in the agent's understanding of the properties of the environment.

  - Complexity
  - Physical randomness
  - Vagueness

# Combining beliefs and desires

- We can make **decision based on probabilistic reasoning** (Belief Networks), but it does not include what an gent wants.
- An agent's **preferences between world states are captured by a utility function** - it assigns a single number to express the desirability of a state.
- **Utilities are combined with the outcome probabilities for actions** to give an expected utility for each action.




# Bayesian Decision Theory

Bayesian decision theory refers to a decision theory which is informed by Bayesian probability. It is a statistical system that tries to quantify the tradeoff between various decisions, making use of probabilities and costs

- **Decision making under uncertainty**

  What action to take when the state of the world is unknown?

- **Bayesian answer**

  Find the utility of each possible outcome (action-state pair), and take the action that maximizes **expected utility**

# Maximum Expected Utility

- Expected utility

Prior to execution of A, the agent assigns probability P(Resulti(A)\Do(A),E) to each outcome, where E summarizes the agent's available evidence about the world, and Do(A) is the proposition that action A is executed in the current state. Then we can calculate the expected utility of the action given the evidence, EU(A\E), using the following formula:.d

$$EU(A|E) = \sum P(Result_i(A)|E, Do(A))U(Result_i(A))$$

- Maximum expected utility - a rational agent should choose an action that maximizes the agent's EU.
- Simple decisions are one-shot decisions.

Single or one-shot decisions.





# The Basis of Utility theory

- **Why should maximizing the average utility be so special?**

Intuitively, the principle of Maximum Expected Utility (MEU) seems like a reasonable way to make decisions, but it is by no means obvious that it is the only rational way.

- Constraints on rational preferences
  1. Orderability
  2. Transitivity
  3. Continuity
  4. Substitutability
  5. Monotonicity,
  6. Decomposability.

Can be answered by writing down some constraints on the preferences that a rational agent should have, and then showing that the MEU principle can be derived from the constraints.

# The Basis of Utility theory

The **complex scenarios are called lotteries**; to emphasize that the different attainable outcomes are like different prizes, and that the outcome is determined by chance.

1. **Orderability**
   - Given any two states, a rational agent must either prefer one to the other or else rate the two as equally preferable.
2. **Transitivity**
   - Given any three states, if an agent prefers A to B and prefers B to C, then the agent must prefer A to C.
3. **Continuity**
   - If some state B is between A and C in preference, then there is some probability p for which the rational agent will be indifferent between getting B for sure and the lottery that yields A with probability p and C with probability 1 — p.




# The Basis of Utility theory

4. **Substitutability**
   - If an agent is indifferent between two lotteries, A and B, then the agent is indifferent between two more complex lotteries that are the same except that B is substituted for A in one of them. This holds regardless of the probabilities and the other outcome(s) in the lotteries.
5. **Monotonicity**
   - Suppose there are two lotteries that have the same two outcomes, A and B. If an agent prefers A to B, then the agent must prefer the lottery that has a higher probability for A (and vice versa).
6. **Decomposability**
   - Compound lotteries can be reduced to simpler ones using the laws of probability. This has been called the "no fun in gambling" rule because it says that an agent should not prefer (or disprefer) one lottery just because it has more choice points than another

# The Basis of Utility theory

- The six constraints form the axioms of utility theory.
- The existence of a utility function follows from the axioms of utility:

Notice that the axioms of utility theory do not say anything about utility. They only talk about preferences. Preference is assumed to be a basic property of rational agents.

  - Utility principle
    - If an agent's preferences obey the axioms of utility, then there exists a real-valued function U that operates on states such that U(A) > U(B) if and only if A is preferred to B, and U(A) = U(B) if and only if the agent is indifferent between A and B.
  - Maximum Expected Utility principle
    - The utility of a lottery is the sum of the probabilities of each outcome times the utility of that outcome.

$$U([p_1, S_1; \ldots ; p_n, S_n]) = \sum_i p_i U(S_i)$$

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# Utility Functions

- Utility functions map states to real numbers.
- **Utility theory has its roots in economics**; the utility of money
  - Risk averse
    - prefer a sure thing with a payoff that is less than the expected monetary value of a gamble.
  - Risk seeking
  - Certainty equivalent
    - The value an agent will accept in lieu of a lottery is called the certainty equivalent of the lottery.
  - Risk neutral
    - Gambles with small sums, we expect risk neutrality.

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# Multiattribute Utility Theory

- A given state may have multiple utilities
  - because of multiple evaluation criteria
  - because of multiple agents (interested parties) with different utility functions.
- Problems like these, in which outcomes are characterized by two or more attributes, are handled by **multiattribute utility theory**.
  - The basic approach adopted in multiattribute utility theory is to identify regularities in the preference behavior we would expect to see.
  - Use what are called representation theorem.

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# Maximizing expected utility

- Decision theory: A rational agent should choose the action that maximizes the agent's expected utility
- Maximizing expected utility (MEU) is a normative criterion for rational choices of actions
- Must have **complete** model of:
  - Actions
  - States
  - Utilities
- Even if you have a complete model, will be computationally **intractable**

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# Decision Networks

- **Decision Networks combine Bayesian Networks with decision theory**
  - Bayesian network represents probabilistic model of the current and the state resulting from a given decision in terms of attributes
    - Chance nodes represent attributes
    - Connections represent conditional effects
  - **Additional nodes** introduce decisions and utilities
    - Decision node represent possible decisions
    - Utility node calculates the utility of the decision

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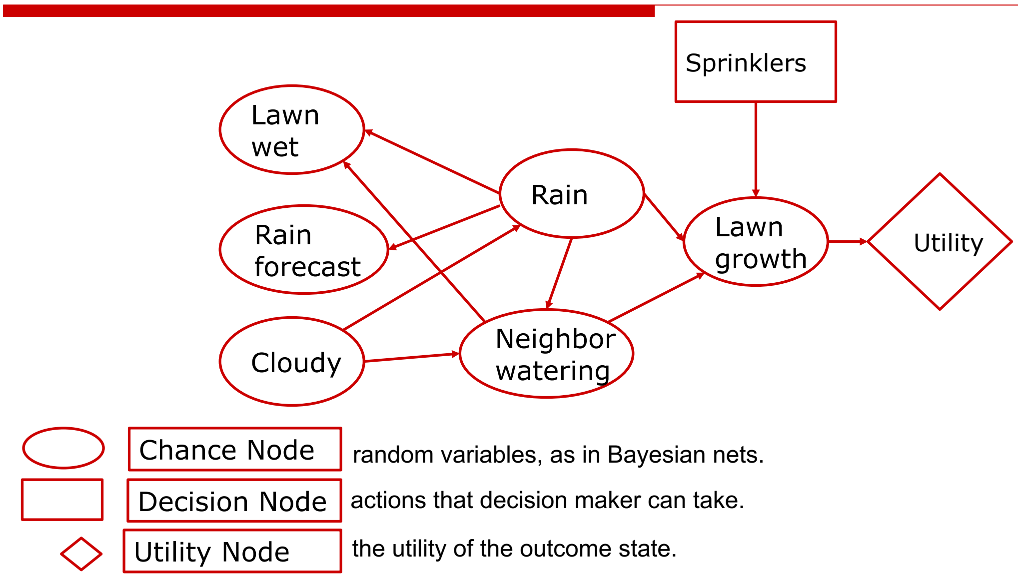
# Decision Network Example




random variables, as in Bayesian nets.

actions that decision maker can take.

the utility of the outcome state.

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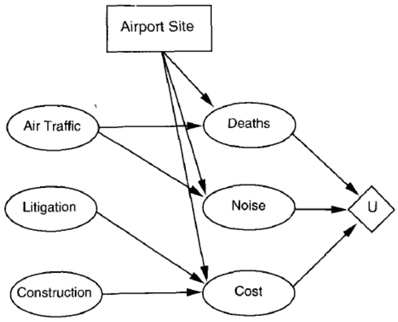
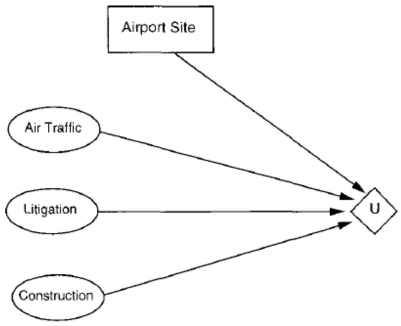
# Decision networks

## Action-utility tables

- Notice that because the Noise, Deaths, and Cost chance nodes refer to future states, they can never have their values set as evidence variables.



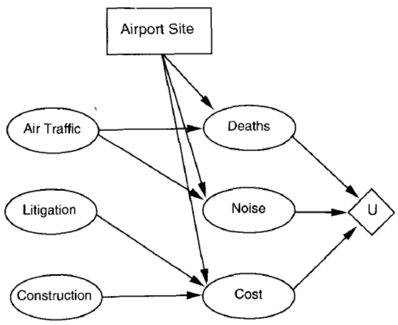



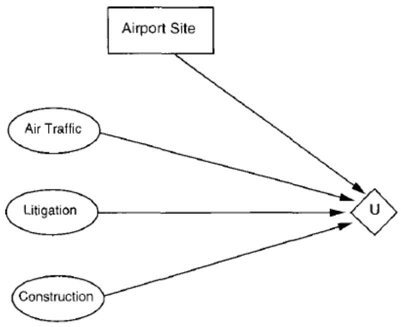



# Decision networks

## Action-utility tables

- Simplified version omits these nodes. Omission of an explicit description of the outcome of the siting decision means that it is less flexible with respect to changes in circumstances.

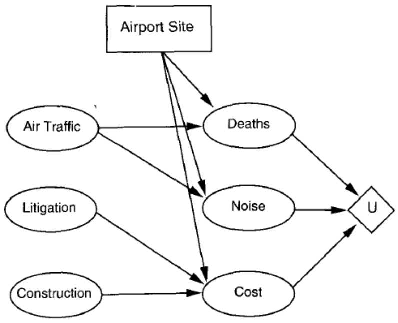


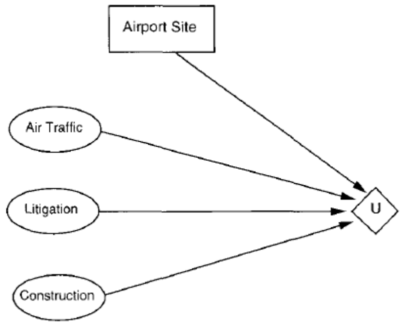



# Decision networks

## Action-utility tables

- Rather than representing a utility function on states, the table associated with the utility node represents the expected utility associated with each action.



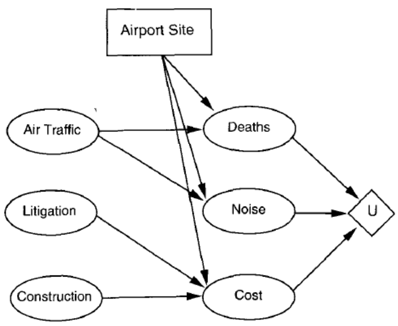


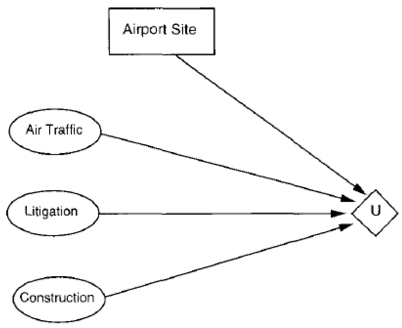

# Decision networks

## Action-utility tables

In the original DN, a change in aircraft noise levels can be reflected by a change in the conditional probability table associated with the Noise node, whereas a change in the weight accorded to noise pollution in the utility function can be reflected by a change in the utility table. In the action-utility diagram, on the other hand, all such changes have to be reflected by changes to the action-utility table.

Action-utility formulation is a compiled version of the original formulation.



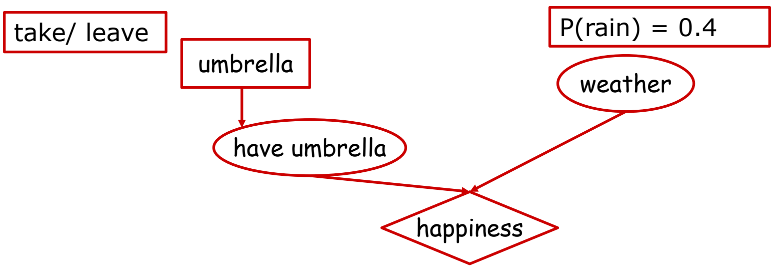



# Decision Networks

- To **determine rational decisions** the network has to be **evaluated and utilities computed**
  - Set evidence variables according to current state
  - For each action value of decision node
    - Set value of decision node to action
    - Use belief-net inference to calculate posterior probabilities for parents of utility node
    - Calculate utility for action
  - Return action with highest utility




# Decision Networks




Consider a simple decision network for a decision of whether the agent should take an umbrella when it goes out. The agent's utility depends on the weather and whether it takes an umbrella.

Domain for each random variable and the domain for each decision variable.
Random variable *Weather* has domain *{shine,rain}*,
Decision variable *Umbrella* has domain *{take,leave}*.

The designer specify the probability of the random variables given their parents.

# Bayesian Decision Theory

## Example

| Action | Rain (p=0.4) | Shine (1-p=0.6) |
|---|---|---|
| Take umbrella | 30 | 10 |
| Leave umbrella | -100 | 50 |

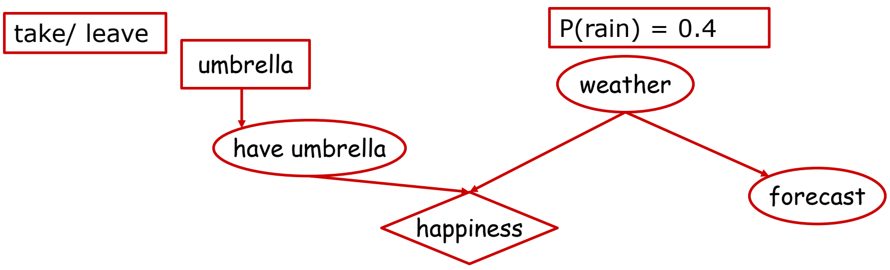
Expected utilities:

- E(Take umbrella) = $30 \times 0.4 + 10 \times 0.6 = 18$
- E(Leave umbrella) = $-100 \times 0.4 + 50 \times 0.6 = -10$





# Decision Networks




| f | w | p(f\|w) |
|---|---|---|
| sunny | rain | 0.3 |
| rainy | rain | 0.7 |
| sunny | no rain | 0.8 |
| rainy | no rain | 0.2 |

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# Value of information

In the preceding analysis, we have assumed that all relevant information, or at least all available information, is provided to the agent before it makes its decision. In practice, this is hardly ever the case

- One of the **most important** parts of decision making is **knowing what questions to ask**.
- To conduct expensive and critical tests or not depends on two factors:
  - Whether the different possible outcomes would make a significant difference to the optimal course of action
  - The likelihood of the various outcomes.
- **Information value theory** enables an agent to **choose what information to acquire**.



# Summary

- Probability theory describes what an agent should believe based on evidence
- **Utility theory describes what an agent wants.**
- Decision theory puts the two together to describe what an agent should do
- **A rational agent should select actions that maximize its expected utility.**
- Decision networks provide a simple formalism for expressing and solving decision problems.