

Theory Question

Q1. Using $V(s) = \sum_{a \in \max(a)} P(R + \gamma V(s'))$

Iteration 1

$$V(s_0) = \frac{1}{2}(1 + 0.9(0)) + 0.5(1 + 0.9(0)) = 1$$

$$V(s_1) \text{ (a) } \frac{1}{2}(2 + 0) + \frac{1}{2}(2 + 0) = 2.$$

$$\text{(b) } 1(2 + 0) = 2.$$

$$V(s_2) = 1(3 + 0) = 3.$$

$$V(s_3) = 1 \times 10 = 10$$

Iteration 2

$$V(s_0) = \frac{1}{2}(1 + 0.9 \times 2) + \frac{1}{2}(0.9 \times 3 + 1) = 3.25$$

$$V(s_1) \text{ (a) } \frac{1}{2}(2 + 0.9 \times 2) + \frac{1}{2}(2 + 0.9 \times 10) = 7.4$$

$$\text{(b) } 2 + 0.9 \times 3 = 4.7$$

$$V(s_2) = 3 + 0.9 = 3.9$$

$$V(s_3) = 10 + 0.9 \times 10 = 19.$$

Iteration 3

$$V(s_0) = \frac{1}{2}(1 + 0.9 \times 7.4) + \frac{1}{2}(1 + 0.9 \times 3.9) = 3.8 + 2.2 = 6.08$$

$$V(s_1) \text{ (a) } \frac{1}{2}(2 + 0.9 \times 7.4) + \frac{1}{2}(2 + 0.9 \times 19) = 4.3 + 9.5 = 13.8$$

$$\text{(b) } 2 + 0.9 \times 3.9 = 5.5$$

$$V(s_2) = 3 + 0.9 \times 3.25 = 5.9$$

$$V(s_3) = 10 + 0.9 \times 19 = 27.1$$

(b) For state s_1

$$\text{Action 1} \Rightarrow \frac{1}{2}(2 + 0.9 \times 13.8) + 0.5(2 + 0.9 \times 27.1) \\ = 7.24 + 13.19 = 20.44$$

$$\text{Action 2} \Rightarrow 1(2 + 0.9 \times 5.925) = 7.33$$

According to these results Action 1 will give us the optimal policy.

c) This is False. Suppose you have a cyclic Markov Decision Process then ~~even~~ even if you take γ to be 0.5 after N iterations there will not be any convergence and it'll keep ~~go~~ on the path.

(ii) This is also False. Using $V(s) = R + \gamma V(s)$ considering $P=1$ In this case there can be no convergence.

(iii) True Using $V(s) = R$ and, similar case as (ii), no convergence, \therefore constant for each state.

(IV) True, The markov decision process is cyclic, there'll be a terminal state no matter what. After $n-1$ states a N iterations

(V) False, Noise is zero, since no stochastic actions, and ~~there~~ no ~~absorbing~~ goal states, it would not converge after N steps.

Q2 Consider an image $\rightarrow n^2$ pixels where each pigment is 8 bits. \Rightarrow Bits = 24 (total)

~~the~~ Given k clusters $\rightarrow k$ colors.
(If compressed data is being sent)

Resultant bits = $n^2(\log k)$

$$\text{Compression Ratio} = \frac{24 n^2}{n^2 \log k} = 24 (\log k)^{-1}$$

We need $24k$ extra bits if we are considering the receiver as well. (decompressing data) \Rightarrow Compression Ratio = $\frac{24 n^2}{n^2 \log k + 24k}$