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CALCULUS ASSIGNMENT

SC107 Calculas Fall 2023

WHEN WILL EVERYONE KNOW?

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PROBLEM STATEMENT

The problem statement is to find out the rate of rumor and to determine how many people had heard and believed the rumor at any particular time and to predict the state of equilibrium during the spread.

Abstract

Our Calculus Project delves into the study of how rumors spread within a stable population group. To accomplish this we are using mathematical modeling to represent the spread of information. The model aims to improve understanding on how to track the spread of rumors and further on how to control it. The factors influencing the spread of rumors will be defined in the project.
The project will incorporate graph theory and differential equations to model interactions within a stable population.
The findings and conclusions of this project have implications for diverse fields, the project aims to close the gap between a real-world social phenomena and its mathematical model .
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1 Introduction

In this era of globalization, information about everything and everyone is present at our fingertips. This creates a breeding ground for rumors to take place. Rumors are pieces of unproven,unverified information. Rumors can emerge from a variety of sources, and if they grow unchecked, they can become more important than the truth. In today's hyper connected world,a rumor can spread like wildfire, growing from a small group of people to millions in just a matter of hours.

Rumors exert a very powerful effect on human affairs. They can change how people think, communicate, and behave. They can cause disagreements between people based on their differences such as race, religion, nationality, etc. They can spark violence between ethnic groups. They have the ability to create fear and anxiety in communities. False rumors can damage the reputation of people,corporations, influential personalities, even if they are proven to be untrue later, the damage cannot be undone. Rumors can also have a lot of economic consequences. Rumors about a corporation might cause stock market fluctuations and lead to economic instabilities.

Why do rumors spread?

It's because of the human need for information, social connection and the desire to spread it. Spreading rumors allows people to confirm their biases and fill gaps in their knowledge.

Confirmation bias plays an interesting role in the spread of rumors. When individuals come across information that aligns with their previous beliefs or expectations. They accept the information more positively and are less resistant to propagate it to others, even though the information they have come across might not be verified. This creates a tendency for people to create and spread rumors. People start selectively exposing themselves to information sources that align with their biases, effectively creating an echo chamber.

This tendency to accept information consistent with one's beliefs, coupled with the rapid dissemination of unverified content through digital platforms, contributes significantly to the widespread and persistent acceptance of rumors in society.

Sometimes rumors are created to make sense of things that aren't easily understood or explainable. These rumors thrive on the fact that they make the audience feel like they can rationalize or cope.

For example, during the outbreak of the COVID -19 pandemic, several rumors spread within the general population such as- COVID will not spread when winter arrives, COVID is not more deadly/infectious compared to common cold, etc.

These false rumors, because of the limited knowledge during the start of the pandemic, became the truth for many people as it allowed them to make sense of what COVID was.

These types of rumors keep spreading unchecked because they invoke emotions in people. Similarly these type of rumors can be use for fear mongering, For example, rumors spread that 5G towers can cause the immune system to weaken, therefore lead to increase in spread of COVID, this lead to slowering vaccines aren't safe because they were developed quickly, global warming is not real and more recently during the 2023 Israel-Hamas war, rumors spread quickly about how a hospital was bombed by Israel reportedly killing about 500 people. This news spread everywhere and eventually became the truth for the most, despite being false.

According to several studies, rumors that are relevant to people's lives are more likely to spread. For example, when nuclear waste water from the Fukushima plant was discharged in 2023, rumors spread that it would contaminate the salt obtained from the ocean. This despite not being true, led to panic buying of salt in neighboring countries. Because people want what's best for them, they don't take chances and believe in every information that comes across which might affect them in a negative way.

2 Background

Before making this model, we have studied and go through various report papers and mathematical models, and we came across some already existing solution for this problem, which built up the base of our model and the mathematical models we came across are:

2.1 SI MODEL

SI Model is the simplest and basic model for the spread of rumor, and taking in consideration some other parameters provides us with the rest of the models. SI Model took in consideration of two parameters.Total population is divided into two parts the Spreader and the ignorants.

2.1.1 Spreader

The people who are aware (heard) of the rumor, assuming that all of them will believe the rumor ,and they all will be the next spreaders.

2.1.2 Ignorant

The people who are not aware(heard) of the rumor, they won't be an active parameter for the spread of rumor. Number of ignorant people is equal to the total population minus the Spreader people.

The model works on the idea that when an ignorant person gets the rumor he will be converted to a Spreader person, and this statement can be expressed in the form of a differential equation. And further solving that equation we get the number of Spreader people as a function of time.

2.2 SIR MODEL

The SIR model stands for (Spreader - Ignorant - Removed). It is an improvised version of the SI Model. In the SIR model a new parameter R- Removed people has been added.Total population is divided into three parts the Spreader, the ignorants and the removed.

2.2.1 Spreader

Spreader are those who are aware of rumor and they will spread the rumor further to the ignorant group of people

2.2.2 Ignorant

Ignorant are those who are unaware of rumor and they will be the next target group for the spreaders.

2.2.3 Recovered

Recovered people are those who initially believed the rumor but later on they lose interest and get out from the topic of discussion.

The SIR model is based on the idea that when an ignorant person hears the rumor he will get converted to the spreader and if a person who loses the interest in the rumor will get converted to the recovered section.Converting this into mathematical equations will result into a differential equation involving three variables, and solving that equation we get the the number of people who have heard the rumor as a function of time.

2.3 ISS MODEL

The ISS Model stands for (Ignorant Spreader Stifler). It has been developed by adding up the consideration that the person who knows the rumor but may not spread it . The total population is divided into three parts:

2.3.1 Spreader

Spreader are those who are aware of rumor and they will spread the rumor further to the ignorant group of people.

2.3.2 Ignorant

Ignorant are those who are unaware of rumors and they will be the next target group for the spreaders.

2.3.3 Stiflers

Stiflers are those people who are aware of the rumor but willingly don't want to spread the rumor further.

The ISS model is based on the idea that when an ignorant person hears the rumor and believes it then he may or may not spread the rumor further (i.e, he may or may not be the spreader).Converting this into mathematical equations results into a differential equation involving three variables, and solving that equation we get the the number of people who have heard the rumor as a function of time.

3 About Our Model

In our model, The spread of the rumor starts with a small group of people. With time the small group spreads the rumor to the entire population, a point at which everyone has at least heard the rumor once .

People make random interactions with each other and make unpredictable choices, through our model we can represent how these random interactions and choices lead to a predictable timeline. We have get to the our final model by making considerations and introducing new parameters .

4 Proposed Model - 1

4.1 Brief Introduction

This is our basic model on the spreading of rumors . In this model we assume Ao as the number of people who are initial spreaders. U(t) is the number of people who are unaware of the rumor at any time t.So when an aware person(say A) meets an unaware person(say U),then it will spread rumor ,and the unaware person will be now an aware person and will be next spreader.We consider that the number of people that are aware are spreaders. That is the people who are made aware of the rumor shall spread it.

Figure 1: Flowchart

4.2 Assumptions

i) The total population in this model is considered to be constant .

ii) Every person has either heard the rumor or not . There is no case of forgetting the rumor.

iii) Interactions between aware and unaware people spread the rumor. Interaction between aware and aware do not spread the rumor. Interaction between unaware and unaware do not spread the rumor.

iv) People interact with each other at random.

v) People interact with the same number of people per unit time.

Quantity	Definition	Role
N	Total Population	independent variable
t	Time	independent variable
Ao	Initial Spreaders(Initial Believer)	independent variable
U(t)	No. of people who are unaware of the rumor at any time t	dependent variable
A(t)	No.of people who are aware of the rumor at any time t	dependent variable
μ	Average Rate of contact between a spreader and unawared	independent variable

Table 1: Parameter Table

4.3 Parameters

HOW WILL EVERYONE KNOW? An unaware person hears the rumor form a spreader and becomes an aware person.This event simultaneously result in decrease in U and increase in A.

4.4 Word Equation

Now we are ready to formulate our word equation. Now either a person is aware or unaware ,hence the rate of increment of the aware people is equal to the rate of decrement of the unaware people .

$\implies$ (Total Population) = (Unaware People) + (Aware People) = Constant

Differentiating on both sides with respect to time

$\implies$ (Rate of change of aware people) = -(Rate of change of unaware people)

Hence both of the variables are related to each other , we only need to derive the equation of one of the variables. So, let us derive the rate of change of aware people.

$\implies$ (Rate of change of aware people) $\propto$(Probability that one is aware and the other is unaware will meet)

$\implies$ (Rate of change of aware people) = (Rate of communication)(Probability that one is aware and the other is unaware)

The probability of occurrence of two favorable events simultaneously is the product of the probability of the occurrence of one favorable event with the probability of occurrence of the other favorable events.

$\Longrightarrow$ (Probability that one is aware and the other is unaware meet) = (Probability that one is aware)(Probability that one is unaware)

$$\Longrightarrow \text{(Probability that one is Unaware)} = \frac{Number\ of\ Unaware\ People}{Total\ Population}$$

$$\Longrightarrow \text{(Probability that one is Aware)} = \frac{Number\ Of\ Aware\ People}{Total\ Population}$$

4.5 Translations from Word equations to Mathematical equations

$\Longrightarrow$ Total Population = Unaware People + Aware People = Constant

$\therefore$ N = U(t) + A(t)

$$\Longrightarrow \text{Probability that one is unaware} = \frac{Number\ of\ unaware\ people}{Total\ Population} = \frac{U(t)}{N}$$

$$\Longrightarrow \text{Probability that one is aware} = \frac{Number of aware people}{Total Population} = \frac{A(t)}{N}$$

$\Longrightarrow$ (Probability that one is aware and the other is unaware meet) = (Probability that one is aware) (Probability that one is unaware)

$$\Longrightarrow \text{(Probability that one is aware and the other is unaware meet)} = \left(\frac{U(t)}{N} \cdot \frac{A(t)}{N}\right)$$

The rate of change of aware people is directly proportional to the probability that one is aware and the other is unaware.

$\Longrightarrow$ (Rate of change of aware people) $\propto$(Probability that one is aware and the other is unaware)

$$\Longrightarrow \frac{dA(t)}{dt} \propto \left(\frac{U(t)}{N} \cdot \frac{A(t)}{N}\right)$$

$\Longrightarrow$ (Rate of change of Aware people) = (Rate of Communication) (Probability that one is Aware and the other is Unaware will meet)

$$\implies \frac{dA(t)}{dt} = \mu \left(\frac{U(t)}{N} \cdot \frac{A(t)}{N} \right)$$

Rate of communication or μ is the proportionality constant which is the average rate of contact between a spreader and an unaware person.

Since , U(t) = N - A(t)

$$\implies \frac{dA(t)}{dt} = \mu \left(\frac{(N - A(t))}{N} \cdot \frac{A(t)}{N} \right)$$

Now we have our differential equation in terms of only one dependent variable, i.e., A(t). Let us integrate this differential equation to find the dependence of the aware people on time t.

4.6 Solution

$$\therefore \frac{dA}{dt} = \mu \, \frac{(N-A)(A)}{N^2}$$

$$\therefore \frac{\mu \, dt}{N^2} = \frac{dA}{(N-A)(A)}$$

$$\therefore \frac{\mu}{N^2} \int dt = \frac{1}{N} \int \frac{(N - A + A)}{(N-A)(A)} \, dA$$

$$\therefore \frac{\mu}{N^2} \, N \int dt = \int \frac{(N-A)}{(N-A)(A)} \, dA + \frac{A}{(N-A)(A)} \, dA$$

$$\therefore \frac{\mu}{N} \int_0^t dt = \int_{A_0}^{A(t)} \frac{1}{A} \, dA + \int_{A_0}^{A(t)} \frac{1}{(N-A)} \, dA$$

$$\therefore \frac{\mu}{N} \, [\ t\ ]_0^t = [lnA - ln(N-A)]_{A_0}^{A(t)}$$

$$\therefore \frac{\mu}{N}\, t = \left[ln \left(\frac{A}{(N-A)} \right) \right]_{A_0}^{A(t)}$$

$$\therefore \frac{\mu\, t}{N} = ln \frac{A(t)}{(N-A(t))} - ln \frac{A_0}{(N-A_0)}$$

$$\therefore \frac{\mu\, t}{N} = ln\ \frac{A(t)\ (N-A_0)}{A_0\ (N-A(t))}$$

$$\therefore e^{\frac{\mu\, t}{N}} = \frac{A(t)\ (N-A_0)}{A_0\ (N-A(t))}$$

Applying limit such that, initially at $t \to 0$,the number of Aware people are A_0 and at any time $t \to t$, the number of Aware people are A(t).

$$\therefore \frac{(N-A(t))}{A(t)} = \frac{(N-A_0)}{A_0}\ e^{\frac{-\mu\, t}{N}}$$

$$\therefore \frac{N}{A(t)} = 1 + \frac{(N-A_0)}{A_0}\ e^{\frac{-\mu\, t}{N}}$$

$$\therefore \frac{N}{A(t)} = \frac{A_0 + (N-A_0)\ e^{\frac{-\mu\, t}{N}}}{A_0}$$

$$\therefore A(t) = \frac{N\ A_0}{A_0 + (N-A_0)\ e^{\frac{-\mu\, t}{N}}} \tag{1}$$

4.7 Equilibrium

Equilibrium is the state at which the rate of change of aware people is zero.Any value of the dependent variable A, for which $\frac{dA(t)}{dt} = 0$ is called an equilibrium.

$$\implies \frac{dA(t)}{dt} = \mu\ \left(\frac{(N-A(t))}{N} \cdot \frac{A(t)}{N} \right)$$

As we can see $\frac{dA(t)}{dt} = 0$ when A is equal to N or when A is equal to 0.

Hence, the points A = 0 and A = N are the points of Equilibrium.

However, the initial number of aware people is given by Ao where Ao cannot be equal to 0,thus A = N is the only possible condition when the rate of change of the number of aware people is equal to zero.

At A = N the whole population comes under the category of aware people that is every person has been made aware of rumor.

4.8 Analysis

Initially we observe that the number of aware people is very lesser than the number of unaware people. But as the rumor spreads, the number of aware people with respect to time increases as the unaware people come under the category of aware people.

At a time $t_{1/2}$ the number of aware people equals the number of unaware people.

$\therefore$ N = A(t) + U(t)

Since at $t_{1/2}$, A(t) = U(t) and A(t) = $\dfrac{N}{2}$

$$\therefore A(t) = \frac{N\ A_0}{A_0 + (N - A_0)\ e^{\frac{-\mu\ t}{N}}}$$

$$\therefore \frac{N}{2} = \frac{N\ A_0}{A_0 + (N - A_0)\ e^{\frac{-\mu\ t}{N}}}$$

$$\therefore 2A_0 = A_0 + (N - A_0)\ e^{\frac{-\mu\ t}{N}}$$

$$\therefore A_0 = (N - A_0)\ e^{\frac{-\mu\ t}{N}}$$

$$\therefore e^{\frac{-\mu\ t}{N}} = \frac{A_0}{(N - A_0)}$$

$$\therefore t_{1/2} = \frac{N}{\mu}\ ln\left(\frac{N}{A_0} - 1\right) \tag{2}$$

Hence , $t_{1/2}$ is the time when half of the population has been affected by the rumor.

4.9 Drawbacks of model 1

i) In this model we have assumed that the people who are made aware of the rumor are expected to spread it too; however it depends from person to person whether they believe the rumor or not.

ii) It is not necessary that people who have heard the rumor will spread it.

4.10 Proposed Solution

To overcome the drawbacks of model 1 we are proposing the model 2 in which the people who are made aware of the rumor ,first decide whether they believe the rumor or not , the people who believe the rumor are expected to spread the rumor and people who don't believe the rumor do not spread the rumor.

5 Proposed Model - 2

5.1 Brief Introduction

This is our modified model on the spreading of the rumor.In this model we assume Ao, the number of people which are initial spreaders , U(t) as the number of people who are unaware of the rumor at any time t and A(t) are the number of the people who are aware of the rumor at any time t.Furthermore A(t) is divided into two parts, one part represents the number of people who believe the rumor hence spread it, while the other part represents the number of people who do not believe the rumor , hence do not spread it.

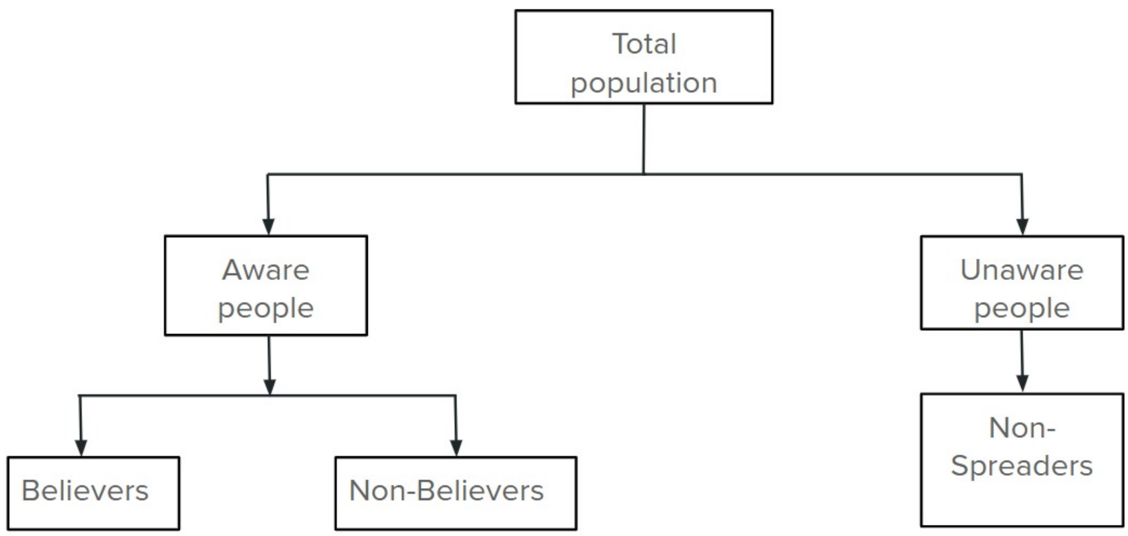


Figure 2: Flowchart

5.2 Assumptions

i) The total population in this model is considered to be constant .

ii) Every person has either heard the rumor or not . There is no case of forgetting the rumor.

iii) Not every interaction between an aware and unaware person spreads the rumor .Interactions between believer and unaware spread the rumor . Interactions between believer and believer do not spread the rumor. Interactions between unaware and unaware do not spread the rumor.People who do not believe the rumor do not spread the rumor.

iv) People interact with each other at random.

v) People interact with the same number of people per unit time.

5.3 Parameters

Quantity	Definition	Role
N	Total Population	independent variable
t	Time	independent variable
Ao	Initial Spreaders(Initial Believer)	independent variable
U(t)	No. of people who are unaware of the rumor at any time t	dependent variable
A(t)	No.of people who are aware of the rumor at any time t	dependent variable
μ	Average Rate of contact between a spreader and unawared	independent variable
P	Probability that a person believe the rumor	independent variable

Table 2: Parameter Table

5.4 Word Equations

Now, we are ready to formulate our word equation.Either a person is aware or unaware of the rumor. The people who are aware decide whether they believe the rumor or not . The people who believe the rumor will spread it . The rate of increment of aware people is equal to decrement of unaware people.

$\implies$ (Total Population) = (Unaware people) + (Aware people) = Constant

Differentiating on both sides with respect to time

$\implies$ (Rate of change of aware people) = - (Rate of change of unaware people)

Hence both of the variables are related to each other , we only need to derive the equation of one of the variables.

However this time the spreaders are not equal to the number of aware people. Spreaders are the people who believe the rumor.

Number of believers is equal to the product of the probability that the person believes the rumor with the number of aware people.

Probability that an aware person believes the rumor is a constant dependending upon the believability of the rumor , nature of people, and persuasive power of the spreader and it is equal to P.

$\implies$ (Number of believer at any time t) = (Probability that the person believes the rumor)(Number of aware people at any time t)

Now let us find the number of aware people at any time t.

The rate of change of aware people is directly proportional to the probability that one is a believer and the other is unaware.

$\implies$ (Rate of change of aware people) $\propto$ (Probability that one is believer and the other is unaware)

$\implies$ (Rate of change of aware people)$=$ (Rate of communication)(Probability that one is believer(Spreader) and the other is unaware)

$\implies$ Rate of communication or μ is the proportionality constant which is the average rate of contact between a spreader and an unaware person.

The probability of occurrence of two favorable events simultaneously is the product of the probability of the occurrence of one favorable event with the probability of occurrence of the other favorable events.

$\implies$ (Probability that one is believer and the other is unaware meet) $=$ (Probability that one is believer)(Probability that one is unaware)

$$\implies Probability\ that\ one\ is\ unaware = \frac{Number\ of\ unaware\ people}{Total\ population}$$

$$\implies Probability\ that\ one\ is\ believer$$

$$= \frac{Number\ of\ believer}{Total\ population}$$

$$= \frac{(Number\ of\ aware\ people)(Probability\ that\ a\ person\ believe\ the\ rumor)}{(Total\ population)}$$

$$\implies (Probability\ that\ one\ is\ believer\ and\ the\ other\ is\ unaware\ meet)$$

$$= (Number\ of\ aware\ people)\left(\frac{Number\ of\ unaware\ people}{Total\ population}\right)$$

$$\left(\frac{Probability\ that\ a\ person\ believe\ the\ rumor}{Total\ population}\right)$$

$\implies (Rate\ of\ change\ of\ aware\ people)$

$= (Rate\ of\ communication)\left(\dfrac{Number\ of\ unaware\ people}{Total\ population}\right)$

$\left(\dfrac{(Probability\ that\ a\ person\ believe\ the\ rumor)(Number\ of\ aware\ people)}{Total\ population}\right)$

5.5 Translations from Word equations to Mathematical equations

$\implies$ Total Population = Unaware People + Aware People = Constant

$\therefore N = U(t) + A(t)$

$\implies$ (Rate of change of aware people) = - (Rate of change of unaware people)

$$\implies \frac{dA(t)}{dt} = -\frac{dU(t)}{dt}$$

$$\implies (Probability\ that\ one\ is\ unaware) = \frac{Number\ of\ unaware\ people}{Total\ population} = \frac{U(t)}{N}$$

$\implies$ Number of believer at any time t

=(Probability that the person believe the rumor)(Number of aware people at any time t)

= PA(t)

$\implies$ *Probability that one is believer*

$$= \frac{(Number\ of\ aware\ people)(Probability\ that\ aperson\ believe\ the\ rumor)}{Total\ population}$$

$$= \frac{A(t)\ P}{N}$$

$\implies (Probability\ that\ one\ is\ believer\ and\ the\ other\ is\ unaware\ meet)$

$$= (Number\ of\ aware\ people)\left(\frac{Probability\ that\ a\ person\ believe\ the\ rumor}{Total\ population}\right)$$

$$\left(\frac{Number\ of\ unaware\ people}{Total\ population}\right)$$

$$= A(t)\left(\frac{P}{N}\right)\left(\frac{U(t)}{N}\right)$$

$\implies$ (Rate of change of aware people)$\propto$(Probability that one is believer and the other is unaware)

$$\implies \frac{dA(t)}{dt} \propto \left(\frac{P\ A(t)}{N}\right)\left(\frac{U(t)}{N}\right)$$

$\implies$ (Rate of change of aware people) = (Rate of communication)(probability that one is believer(spreader) and the other is unaware)

$$\implies \frac{dA(t)}{dt} = \mu\left(\frac{A(t)\ P}{N}\right)\left(\frac{U(t)}{N}\right)$$

Rate of communication or $\mu$is the proportionality constant which is the average rate of contact between a spreader and an unaware person.

Since total population N is constant U(t) can be expressed as ,

$\therefore$ U(t) = N - A(t)

$$\implies \frac{dA(t)}{dt} = \mu\left(\frac{A(t)\ P}{N}\right)\left(\frac{(N - A(t))}{N}\right)$$

Now we have our differential equation in terms of only one dependent variable, i.e., A(t). Let us integrate this differential equation to find the dependence of the aware people on time t.

5.6 Solution

The differential equation of aware people is as follows

$$\therefore \frac{dA}{dt} = \mu\left(\frac{U}{N}.\frac{PA}{N}\right)$$

$$= \frac{\mu P}{N^2}(N - A).A$$

$$= \frac{\mu P}{N^2} NA - \frac{\mu P}{N^2} A^2$$

$$\therefore \frac{dA}{dt} = \frac{\mu P}{N} A - \frac{\mu P}{N^2} A^2$$

$$\therefore \frac{dA}{dt} = \frac{\mu PAN - \mu PA^2}{N^2}$$

$$\therefore \frac{dt}{N^2} = \frac{dA}{\mu PAN - \mu PA^2}$$

$$\therefore \frac{dt}{N^2} = \frac{dA}{\mu P(A)(N-A)}$$

$$\therefore \frac{dt}{N} = \frac{N}{\mu P(A)(N-A)} \quad dA$$

$$\therefore \int_0^t \frac{dt}{N} = \frac{1}{\mu P} \int_{A_0}^{A(t)} \frac{(N-A)+A}{A(N-A)} \, dA$$

$$\therefore \int_0^t \frac{dt}{N} = \frac{1}{\mu P} \int_{A_0}^{A(t)} \frac{1}{A} + \frac{1}{(N-A)} \quad dA$$

$$\therefore \frac{t}{N} = \frac{1}{\mu P} \quad [lnA + ln(N-A)]_{A_0}^{A(t)}$$

$$\therefore \frac{t}{N} = \frac{1}{\mu P} \quad \left[ln \left(\frac{A}{N-A} \right) \right]_{A_0}^{A(t)}$$

$$\therefore \frac{t}{N} = \frac{1}{\mu P} \quad \left[ln \left(\frac{N-A}{A} \right) \right]_{A(t)}^{A_0}$$

$$\therefore \frac{t}{N} = \frac{1}{\mu P} \quad \left[ln \left(\frac{N-A_0}{A_0} \right) - ln \left(\frac{N-A(t)}{A(t)} \right) \right]$$

$$\therefore \frac{\mu Pt}{N} = ln \left(\frac{N}{A_0} - 1 \right) - ln \left(\frac{N}{A(t)} - 1 \right)$$

$$\therefore \frac{\mu P t}{N} = ln\left(\frac{\frac{N}{A_0} - 1}{\frac{N}{A(t)} - 1}\right)$$

$$\therefore e^{\frac{\mu P t}{N}} = \frac{\frac{N}{A_0} - 1}{\frac{N}{A(t)} - 1}$$

$$\therefore e^{\frac{\mu P t}{N}} - 1 = \frac{\frac{N}{A_0} - \frac{N}{A(t)}}{\frac{N}{A(t)} - 1}$$

$$\therefore \frac{N}{A_0} - \frac{N}{A(t)} = (e^{\frac{\mu P t}{N}} - 1)\left(\frac{N}{A(t)} - 1\right)$$

$$\therefore \frac{N}{A_0} = \frac{N}{A(t)} + (e^{\frac{\mu P t}{N}} - 1)\left(\frac{N}{A(t)} - 1\right)$$

$$\therefore \frac{N}{A_0} = \frac{N}{A(t)} + \frac{Ne^{\frac{\mu P t}{N}}}{A(t)} - e^{\frac{\mu P t}{N}} - \frac{N}{A(t)} + 1$$

$$\therefore \frac{N}{A_0} = \frac{Ne^{\frac{\mu P t}{N}}}{A(t)} - e^{\frac{\mu P t}{N}} + 1$$

$$\therefore \frac{Ne^{\frac{\mu P t}{N}}}{A(t)} = \frac{N}{A_0} + e^{\frac{\mu P t}{N}} - 1$$

$$\therefore \frac{Ne^{\frac{\mu P t}{N}}}{A(t)} = \frac{N + A_0\left(e^{\frac{\mu P t}{N}} - 1\right)}{A_0}$$

$$\therefore A(t) = \frac{N A_0 \; e^{\dfrac{\mu P t}{N}}}{N + A_0 \left(e^{\dfrac{\mu P t}{N}} - 1 \right)} \tag{3}$$

5.7 Example

Let us take a example so we can understand the equation better.

Let , the total population of the people be equal to 1000.
N = 1000

Initially people who knows the rumor i.e Ao is equal to 10.
Ao = 10

The rate at which people communicate with each other is equal to μ.
$\mu = 2$ people/sec

The probability at which people believe the rumor is equal to P.
P = 0.75

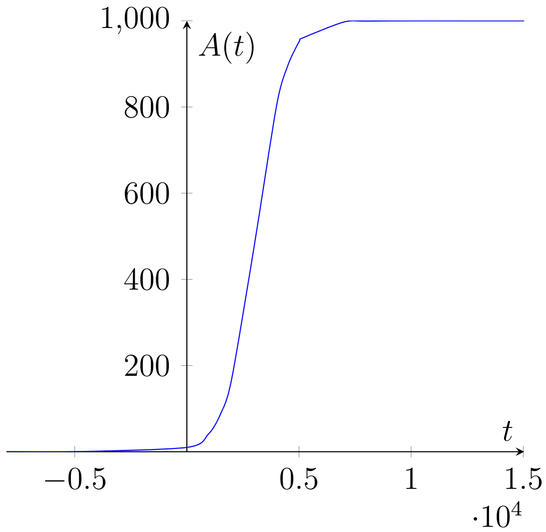
So,now let us find the final equation of A(t).

$$\therefore A(t) = \frac{N A_0 \; e^{\dfrac{\mu P t}{N}}}{N + A_0 \left(e^{\dfrac{\mu P t}{N}} - 1 \right)}$$

$$\therefore A(t) = \frac{(10)(1000) \; e^{\dfrac{(3/4)\ (2)\ t}{1000}}}{1000 \ + \ 10 \left(e^{\dfrac{(3/4)\ (2)\ t}{1000}} - 1 \right)}$$

$$\therefore A(t) = \frac{1000 \ e^{0.0015t}}{100 \ + \ (e^{0.0015t} - 1)}$$

So, now by plotting the graph of this equation we can find the value of A(t) at any time t.



5.8 Equilibrium

Equilibrium is the state at which the rate of change of aware people is zero.Any value of the dependent variable A, for which $\frac{dA}{dt} = 0$ is called an equilibrium.

$$\Longrightarrow \frac{dA(t)}{dt} = \mu \left(\frac{A(t)\ P}{N} \right) \left(\frac{(N - A(t))}{N} \right)$$

As we can see $\frac{dA}{dt} = 0$ when A is equal to N or when A is equal to 0.

Hence, the points A = 0 and A =N are the points of Equilibrium.

However, the initial number of aware people is given by A_0 where A_0 cannot be equal to 0,thus A = N is the only possible condition when the rate of change of the number of aware people is equal to zero.

At A = N the whole population comes under the category of aware people that is every person has been made aware of rumor.

5.9 Analysis

Initially we observe that the number of aware people is very lesser than the number of unaware people. But as the rumor spreads, the number of aware people with respect to time increases as the unaware people come under the category of aware people.

At a time $t_{1/2}$ the number of aware people equals the number of unaware people.

$\therefore N = A(t) + U(t)$

Since at $t_{1/2}$, A(t) = U(t) and A(t) = $\dfrac{N}{2}$

$$\therefore A(t) = \frac{N A_0 \, e^{\frac{\mu P t}{N}}}{N + A_0 \left(e^{\frac{\mu P t}{N}} - 1 \right)}$$

$$\therefore \frac{N}{2} = \frac{N A_0 \, e^{\frac{\mu P t}{N}}}{N + A_0 \left(e^{\frac{\mu P t}{N}} - 1 \right)}$$

$$\therefore 2A_0 \, e^{\frac{\mu P t}{N}} = N + \left(e^{\frac{\mu P t}{N}} - 1 \right) A_0$$

$$\therefore N - A_0 = A_0 \, e^{\frac{\mu P t}{N}}$$

$$\therefore \frac{(N - A_0)}{A_0} = e^{\frac{\mu P t}{N}}$$

$$\therefore ln \left(\frac{N - A_0}{A_0} \right) = \frac{\mu P t}{N}$$

$$\therefore t_{1/2} = \frac{N}{\mu P} \, ln \left(\frac{N - A_0}{A_0} \right) \tag{4}$$

Hence , $t_{1/2}$ is the time when half of the population has been affected by the rumor.

Continuing the above example , on substituting the values of all the parameters in the equation of $t_{1/2}$, we can obtain the value of $t_{1/2}$.

N = 1000
A_0 = 10
μ = 2 people/sec
P = 0.75

$$\therefore t_{1/2} = \frac{N}{\mu P}\, ln\left(\frac{N - A_0}{A_0}\right)$$

Substituting all the values in the above equation,

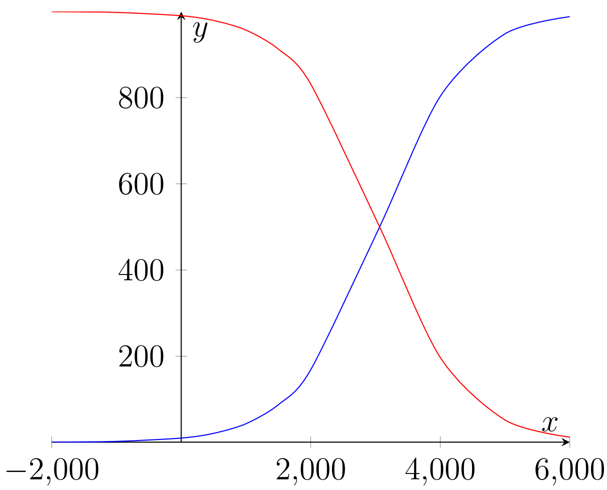
$$\therefore t_{1/2} = \frac{1000}{(2)(0.75)}\, ln\left(\frac{1000 - 10}{10}\right)$$

$$\therefore t_{1/2} = \frac{1000}{1.5}\, ln\left(\frac{990}{10}\right)$$

$$\therefore t_{1/2} = \frac{1000}{1.5}\, ln(99)$$

$$\therefore t_{1/2} = 3063.41$$

We can verify our results of $t_{1/2}$ by plotting the graph of U(t) and A(t) with respect to time and the point where both the graph intersect is the print of $t_{1/2}$



5.10 Drawbacks of model 2

i) In this model we have assumed that the people who believe the rumor, spread the rumor. However, the people who believe the rumor may not spread the rumor .

ii) It is also possible that the people who don't believe the rumor may choose to spread the rumor or not spread it .

5.11 Proposed Solution

To overcome the drawbacks of model 2 we are proposing the model 3 in which the people who are made aware of the rumor ,first decide whether they believe the rumor or not , the people who believe the rumor may or may not spread the rumor . Also the people who do not believe the rumor may or may not spread the rumor too.

6 Proposed Model-3

6.1 Brief Introduction

This is our most accurate model on the topic WHEN WILL EVERYONE KNOW. In this model we assume Ao, the number of people which are initial spreaders , U(t) as the number of people who are unaware of the rumor at any time t and A(t) are the number of the people who are aware of the rumor at any time t.Furthermore A(t) is divided into two parts, one part represents the number of people who believe the rumor,i.e,B(t) and the other part represents the number of people who do not believe the rumor,i.e,C(t). Furthermore B(t) which is the number of people who believe the rumor is divided into two parts where one part represents the number of people who spread the rumor and the other part represents the number of people who do not spread the rumor. Also C(t) which is the number of people who do not spread the rumor is divided into two parts where one part represents the number of people who spread the rumor and the other part represents the number of people who do not spread the rumor.
Hence the number of spreaders are equal to the number of people who believe the rumor and spread it and the number of people who do not believe the rumor and spread it.

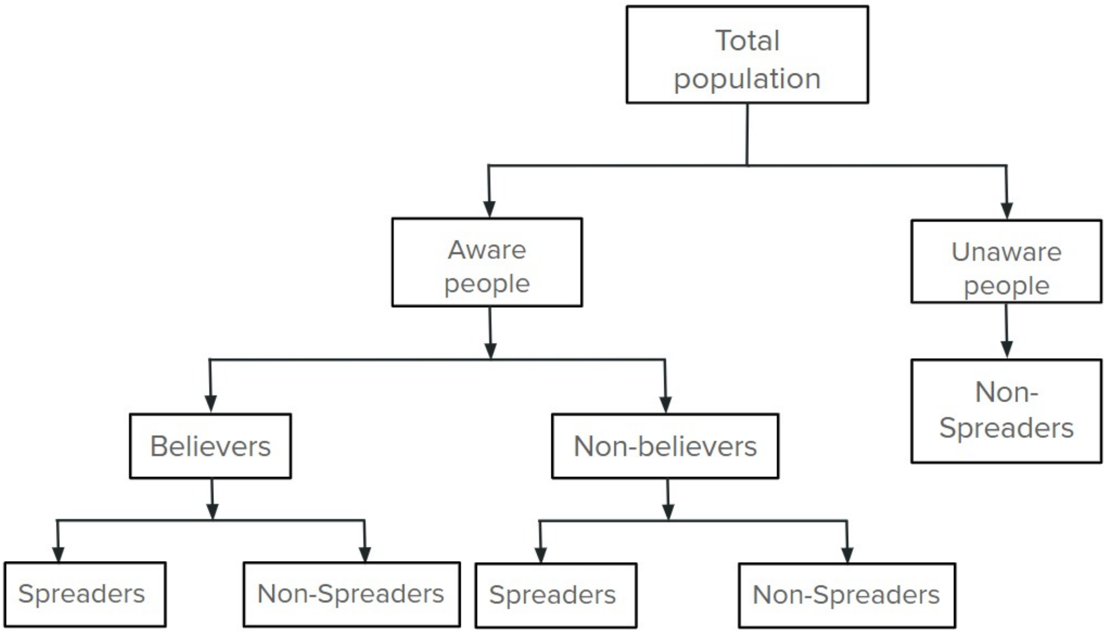


Figure 3: Flowchart

6.1.1 Assumptions

i) The total population in this model is considered to be constant.

ii) Every person has either heard the rumor or not . There is no case of forgetting the rumor.

iii) Not every interaction between an aware and unaware person spreads the rumor .Not every interaction between a believer and unaware spreads the rumor. Even some interactions between a non believer and an unaware person can also spread the rumor.Not every interaction between believer and an unaware person spreads the rumor. Interactions between unaware and unaware do not spread the rumor.

iv) People interact with each other at random.

v) People interact with the same number of people per unit time.

6.2 Parameters

Quantity	Definition	Role
N	Total Population	independent variable
t	Time	independent variable
Ao	Initial Spreaders(Initial Believer)	independent variable
U(t)	No. of people who are unaware of the rumor at any time t	dependent variable
A(t)	No.of people who are aware of the rumor at any time t	dependent variable
S(t)	Number of people who spread the rumor	dependent variable
α	Probability that a person believing in rumor spreads it	independent variable
β	Probability that a person not believing in rumor spreads it	independent variable
P	Probability that a person believes the rumor	independent variable

Table 3: Parameter Table

6.3 Word Equations

Now, we are ready to formulate our word equation.Either a person is aware or unaware of the rumor. The people who are aware firstly decide whether they believe the rumor or not and then the people who believe the rumor may or may not spread it . Also the people who decide not to believe a rumor may spread it .The rate of increment of aware people is equal to decrement of unaware people.

$\implies$ (Total population) = (Unaware people) + (Aware people) = Constant

Differentiating on both sides with respect to time

$\implies$ (Rate of change of aware people) = - (Rate of change of unaware people)

Hence both of the variables are related to each other , we only need to derive the equation of one of the variables.

However the number of believers is not equal to the number of spreaders this time . The number of spreaders are equal to the number of people who believe the rumor and spread it and the number of people who do not believe the rumor and spread it.

$\implies$ (Number of spreaders at any time t) = (Number of people who believe the rumor and spread it) + (Number of people who do not believe the rumor and spread it)

$\implies$ (Number of people who believe the rumor) = (Probability that a person believes the rumor)(Number of the aware people at any time t)

$\implies$ (Number of people who do not believe the rumor) = (1 - Probability that a person believes the rumor)(Number of the aware people at any time t)

$\implies$ (Number of people who believe the rumor and spread it) = (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time t)

$\implies$ (Number of people who do not believe the rumor and spread it) = (One - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t)

Now let us find the final equation of the number of spreaders at any time t.

$\implies$ (Number of spreaders at any time t) = (Number of people who believe the rumor and spread it) + (Number of people who do not believe the rumor and spread it)

$\implies$ (Number of spreaders at any time t) = (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time t) + (One - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t)

Let us find the number of aware people at any time,

The rate of change of aware people is directly proportional to the probability that one is a spreader and the other is unaware.

$\implies$ (Rate of change of aware people) $\propto$(probability that one is spreader and the other is unaware)

$\implies$ (Rate of change of aware people) = (Rate of communication)(probability that one is spreader and the other is unaware)

Rate of communication or μ is the proportionality constant which is the average rate of contact between a spreader and an unaware person.

$\implies$ (Rate of change of aware people) = (Rate of communication)(Probability that one is spreader and the other is unaware)

The probability of occurrence of two favorable events simultaneously is the product of the probability of the occurrence of one favorable event with the probability of occurrence of the other favorable events.

$\implies$ (Probability that one is spreader and the other is unaware meet) = (Probability that one is a spreader)(Probability that one is unaware)

$\implies$ Probability that one is a spreader

$$=\frac{Number\ of\ spreader}{Total\ population}$$

= (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time t) + (one - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t) / (Total population)

$$\implies Probability\ that\ one\ is\ unaware = \frac{Number\ of\ unaware}{Total\ population}$$

$\implies$ (Probability that one is spreader and the other is unaware meet) =[(Probability that a person believes the rumor) (Probability that a person believing in rumor spreads it) (Number of the aware people at any time t) + (one - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)](Number of the aware people at any time t) / (Total population)(Number of unaware) / (Total population)

$\implies$ (Rate of change of aware people) = (Rate of communication) (Probability that one is spreader and the other is unaware)

$\implies$ (Rate of change of aware people) = (Rate of communication)[(Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it) (Number of the aware people at any time t) + (one - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)](Number of the aware people at any time t) / (Total population) (Number of unaware) / (Total population)

6.4 Translations from Word equations to Mathematical equations

Now, we are ready to formulate our word equation.Either a person is aware or unaware of the rumor. The people who are aware firstly decide whether they believe the rumor or not and then the people who believe the rumor may or may not spread it . Also the people who decide not to believe a rumor may spread it .The rate of increment of aware people is equal to decrement of unaware people.

$\Longrightarrow$ (Total population) = (Unaware people) + (Aware people) = Constant

$\therefore$ N = U(t) + A(t)

Differentiating on both sides with respect to time

$\Longrightarrow$ (Rate of change of aware people) = - (Rate of change of unaware people)

$$\Longrightarrow \frac{dA(t)}{dt} = -\frac{dU(t)}{dt}$$

However the number of believers is not equal to the number of spreaders this time . The number of spreaders are equal to the number of people who believe the rumor and spread it and the number of people who do not believe the rumor and spread it.

$\Longrightarrow$ (Number of spreaders at any time t) = (Number of people who believe the rumor and spread it) + (Number of people who do not believe the rumor and spread it)

$\Longrightarrow$ (Number of people who believe the rumor) = (Probability that a person believes the rumor)(Number of the aware people at any time t)

$\therefore$ B(t) = P $\cdot$ A(t)

$\Longrightarrow$ (Number of people who do not believe the rumor) = (1 -Probability that a person believes the rumor)(Number of the aware people at any time t)

$\therefore$ C(t) = (1-P) $\cdot$ A(t)

$\Longrightarrow$ (Number of people who believe the rumor and spread it)
= (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time)
= P α A(t)

$\Longrightarrow$ (Number of people who do not believe the rumor and spread it)
= (One - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t)
= (1-P) β A(t)

Now let us find the final equation of the number of spreaders at any time t.

$\implies$ (Number of spreaders at any time t)
= (Number of people who believe the rumor and spread it) + (Number of people who do not believe the rumor and spread it)
= S(t)

$\implies$ (Number of spreaders at any time t)
= (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time t) + (One - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t)
= P α A(t) + (1-P) β A(t)

Let us find the number of aware people at any time t,

The rate of change of aware people is directly proportional to the probability that one is a spreader and the other is unaware.

$\implies$ (Rate of change of aware people) $\propto$ (Probability that one is spreader and the other is unaware)

$\implies$ (Rate of change of aware people) = (Rate of communication)(Probability that one is spreader and the other is unaware)

Rate of communication or μ is the proportionality constant which is the average rate of contact between a spreader and an unaware person.

$\implies$ (Rate of change of aware people) = μ (Probability that one is spreader and the other is unaware)

The probability of occurrence of two favorable events simultaneously is the product of the probability of the occurrence of one favorable event with the probability of occurrence of the other favorable events.

$\implies$ (Probability that one is spreader and the other is unaware meet) = (Probability that one is a spreader)(Probability that one is unaware)

$\implies$ (Probability that one is a spreader)

$$= \frac{Number\ of\ spreader}{Total\ population}$$

$$= \frac{S(t)}{N}$$

= (Probability that a person believes the rumor)(Probability that a person believing in rumor spreads it)(Number of the aware people at any time t) + (one - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)(Number of the aware people at any time t) / (total population)

$$= \frac{P\alpha A(t) + (1-P)\beta A(t)}{N}$$

$\implies$ (Probability that one is unaware) = $\dfrac{Number\ of\ unaware}{Total\ population} = \dfrac{U(t)}{N}$

$\implies$ (Probability that one is spreader and the other is unaware meet) = [(Probability that a person believes the rumor) (Probability that a person believing in rumor spreads it)(Number of the aware people at any time t) + (one - probability that a person believes the rumor)(Probability that a person not believing in rumor spreads it)](Number of the aware people at any time t) / (total population) * (number of unaware) / (total population)

$$= \left(\frac{P\alpha A(t) + (1-P)\beta A(t)}{N}\right)\left(\frac{U(t)}{N}\right)$$

$\implies$ (Rate of change of aware people) = (Rate of communication)(Probability that one is spreader and the other is unaware)

$\implies$ (rate of change of aware people) = (rate of communication)[(probability that a person believes the rumor) (probability that a person believing in rumor spreads it)(number of the aware people at any time t) + (one - probability that a person believes the rumor)(probability that a person not believing in rumor spreads it)] * (number of the aware people at any time t) / (total population) * (number of unaware) / (total population)

$$\therefore \frac{dA(t)}{dt} = \mu\left(\frac{P\alpha A(t) + (1-P)\beta A(t)}{N}\right)\left(\frac{U(t)}{N}\right)$$

Since total population N is constant U(t) can be expressed as ,

$\therefore$ U(t) = N - A(t)

$$\therefore \frac{dA(t)}{dt} = \mu\left(\frac{P\alpha A(t) + (1-P)\beta A(t)}{N}\right)\left(\frac{N - A(t)}{N}\right)$$

6.5 Solution

$$\therefore \frac{dA}{dt} = \frac{\mu}{N^2}\ [\alpha P + (1-P)\ \beta](N-A)A$$

$$\therefore \frac{dA}{(N-A)A} = \frac{\mu}{N^2}\ [\alpha P + (1-P)\ \beta]\ dt$$

Integrateing on both side

$$\therefore \frac{1}{N}\int \frac{N-A+A}{(N-A)(A)}\,dA = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ \int dt$$

$$\therefore \frac{1}{N}\left[\int \frac{1}{A}\,dA + \int \frac{1}{N-A}\,dA\right] = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ \int dt$$

On applying limit, i.e at time $t = 0$, *the number of people aware of rumor are Ao and at time* $t = t$, *the number of people aware of rumor are* $A(t)$.

$$\therefore \frac{1}{N}\left[\int_{A_0}^{A(t)} \frac{1}{A}\,dA\ +\ \int_{A_0}^{A(t)} \frac{1}{N-A}\,dA\right] = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ \int_0^t dt$$

$$\therefore \frac{1}{N}[lnA - ln(N-A)]_{A_0}^{A(t)} = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ t$$

$$\therefore ln\left(\frac{A(t)}{N-A(t)}\right)\ -\ ln\left(\frac{A_0}{N-A_0}\right) = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ t$$

$$\therefore ln\left(\frac{(N-A_0)(A(t))}{(N-A(t))(A_0)}\right) = \frac{\mu}{N^2}\,[\alpha P + (1-P)\ \beta]\ t$$

$$\therefore \frac{(N-A_0)(A(t))}{(N-A(t))(A_0)}\ =\ e^{\frac{\mu t}{N}\ [\alpha P + (1-P)\ \beta]}$$

$$\therefore \frac{N-A(t)}{A(t)} = \frac{(N-A_0)}{A_0}\ \ e^{\frac{-\ \mu t}{N}\ [\alpha P + (1-P)\ \beta]}$$

$$\therefore \frac{N}{A(t)} = 1\ +\ \frac{(N-A_0)\ e^{\frac{-\ \mu t}{N}\ [\alpha P + (1-P)\ \beta]}}{Ao}$$

$$\therefore A(t) = \frac{NA_0}{A_0 \ + \ (N - A_0)\ e^{\frac{-\mu t}{N}\ [\alpha P + (1-P)\ \beta]}} \tag{5}$$

$at\ t \to \infty\ ,\ A(t) \to N$

Unaware people:

$\therefore U(t) = N - A(t)$

$$= N \ - \ \frac{NA_0}{A_0 \ + \ (N - A_0)\ e^{\frac{-\mu t}{N}\ [\alpha P + (1-P)\ \beta]}}$$

$$\therefore U(t) = \frac{N\ (N - A_0)\ e^{\frac{-\mu t}{N}\ [\alpha P + (1-P)\ \beta]}}{A_0 \ + \ (N - A_0)\ e^{\frac{-\mu t}{N}\ [\alpha P + (1-P)\ \beta]}}$$

6.6 Example

Let us take a example so we can understand the equation better.

Let , the total population of the people be equal to 1000.
N = 1000

Initially people who knows the rumor i.e Ao is equal to 10.
Ao = 10

The rate at which people communicate with each other is equal to μ.
$\mu = 2$ people/sec

The probability at which people believe the rumor is equal to P.
P = 0.75

The probability that a person believing in rumor spreads it , is equal to α .
$\alpha = 0.75$

The probability that a person not believing in rumor spreads it , is equal to β .
$\beta = 0.25$

So, now let us find the final equation of A(t).

$$\therefore A(t) = \frac{NA_0}{A_0 \; + \; (N - A_0) \; e^{\frac{-\mu t}{N}[\alpha P + (1-P) \; \beta]}}$$

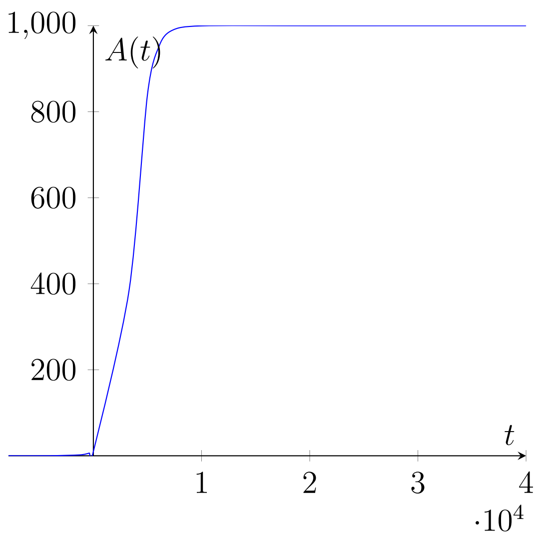
$$\therefore A(t) = \frac{(1000)(10)}{10 \; + \; (1000 - 10) \; e^{\frac{-2t}{1000}[(3/4)(3/4)+(1/4)(1/4)]}}$$

$$\therefore A(t) = \frac{10000}{10 + (990) \; e^{\frac{-t}{500}[(9/16)+(1/16)]}}$$

$$\therefore A(t) = \frac{10000}{10 + (990) \; e^{\frac{-t}{800}}}$$

$$\therefore A(t) = \frac{1000}{1 + (99) \; e^{\frac{-t}{800}}}$$

So, now by plotting the graph of this equation we can find the value of A(t) at any time t.



6.7 Equilibrium

Equilibrium is the state at which the rate of change of aware people is zero.Any value of the dependent variable A, for which $\frac{dA}{dt} = 0$ is called an equilibrium.

$$\implies \frac{dA(t)}{dt} = \mu \left(\frac{P\alpha A(t) + (1-P)\beta A(t)}{N} \right) \left(\frac{U(t)}{N} \right)$$

As we can see $\frac{dA(t)}{dt} = 0$ when A is equal to N or when A is equal to 0.

Hence, the points A = 0 and A = N are the points of Equilibrium.

However, the initial number of aware people is given by Ao where Ao cannot be equal to 0,thus A = N is the only possible condition when the rate of change of the number of aware people is equal to zero.

At A = N the whole population comes under the category of aware people that is every person has been made aware of rumor.

6.8 Analysis

Initially we observe that the number of aware people is very lesser than the number of unaware people. But as the rumor spreads, the number of aware people with respect to time increases as the unaware people come under the category of aware people.

At a time $t_{1/2}$ the number of aware people equals the number of unaware people.

$\therefore N = A(t) + U(t)$

Since at $t_{1/2}$, A(t) = U(t) and A(t) = $\frac{N}{2}$

$$\therefore A(t) = \frac{NA_0}{A_0 \;+\; (N - A_0)\; e^{\frac{-\mu t}{N}\;[\alpha P + (1-P)\;\beta]}}$$

$$\therefore \frac{N}{2} = \frac{NA_0}{A_0 \left(1 \;-\; e^{\frac{-\mu t}{N}\;[\alpha P + (1-P)\;\beta]} \right) \;+\; N\, e^{\frac{-\mu t}{N}\;[\alpha P + (1-P)\;\beta]}}$$

$$Put\ ,\ \frac{-\mu t}{N}\;[\alpha P + (1-P)\;\beta] = x$$

$$\therefore \frac{2A_0}{A_0(1 - e^x) \;+\; Ne^x} = 1$$

$$\therefore 2A_0 = A_0 - A_0 e^x + Ne^x$$

$\therefore A_0 = e^x\ (N - A_0)$

$\therefore e^x = \dfrac{A_0}{N - A_0}$

$\therefore e^{\frac{-\mu t}{N}[\alpha P + (1-P)\ \beta]} = \dfrac{A_0}{N - A_0}$

$\therefore \dfrac{-\mu t}{N}\ [\alpha P + (1-P)\ \beta] = ln\left(\dfrac{N - A_0}{A_0}\right)$

$$\therefore t_{1/2} = \frac{N}{\mu(\alpha P\ +\ \beta(1-P))} ln\left(\frac{N - A_0}{A_0}\right) \tag{6}$$

Hence $t_{1/2}$ is the time when half of the population has been affected by the rumor.

Continuting the above example , on substituing the values of all the parameters in the equation of $t_{1/2}$, we can obtain the value of $t_{1/2}$

N = 100
$A_0 = 10$
$\mu = 2$ people/sec
P = 0.75
$\alpha = 0.75$
$\beta = 0.25$

$\therefore t_{1/2} = \dfrac{N}{\mu(\alpha P\ +\ \beta(1-P))} ln\left(\dfrac{N - A_0}{A_0}\right)$

$\therefore t_{1/2} = \dfrac{1000}{2((0.75)(0.75)\ +\ (0.25)(1-0.75))}\ ln\left(\dfrac{1000 - 10}{10}\right)$

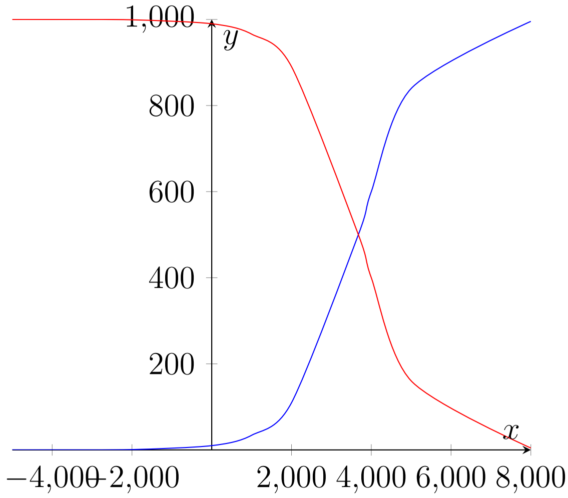
$\therefore t_{1/2} = \dfrac{1000}{2((0.75)(0.75)\ +\ (0.25)(1-0.75))}\ ln\left(\dfrac{1000 - 10}{10}\right)$

$\therefore t_{1/2} = \dfrac{1000}{2((0.75)(0.75)\ +\ (0.25)(1-0.75))}\ ln\left(\dfrac{990}{10}\right)$

$\therefore t_{1/2} = \dfrac{1000}{2(0.5625\ +\ 0.0625)}\ ln99$

$\therefore t_{1/2} = 3672\ sec$

We can verify our results of $t_{1/2}$ by plotting the graph of U(t) and A(t) with respect to time and the point where both the graph intersect is the print of $t_{1/2}$.



6.9 Drawbacks of model 3

i) Here we have assumed that the probability of believing the rumor is a constant which applies equally for all. However it depends from person to person to believe the rumor .

ii) There is a possibility that the people who start spreading the rumor later do not spread it, the people who believed the rumor earlier might feel that it is untrue afterward, the people who did not believe might believe it afterwards and many more psychological aspects of believing of rumor or spreading of rumor have not been considered in our model.

iii) The total population is considered as a constant throughout this model which is not true in reality.

iv) Rate of communication or μ is the proportionality constant which is the average rate of contact between a spreader and an unaware person also depends from person to person more the connections more the spreading of rumor for a person.

v) We have not considered the online aspect of spreading rumors. In today's world online platforms play a major role in spreading information.

6.10 Futuristic Model

This model tries to cover all the psychological aspects of the spread of rumors. Our model fails to deliver the most accurate reactions of a person after hearing a rumor. However this futuristic model can be expected to accurately measure the most natural reaction of people. It has the ability to calculate the spread of a rumor among people based on the idea of dealing with this problem for individuals rather than considering groups of people. This model also takes into account the reality that one person can be more socially connected than the other and hence will spread a rumor at a faster rate than others. In this model the main focus is on the degree of social connection of a person with other people, the reactions of people towards a rumor, and the random changes in believability of the rumor.

The above problems can be solved using graph theory. Graph Theory is used by many social media app. Graph theory helps in knowing how much a person is connected socially.

`https://images.app.goo.gl/xZ7XutsAatdreLHK9`

Graph theory includes nodes and edges. Using this theory we can place people as nodes and they are connected to other people i.e, nodes through edges . The more the edges connected to a node i.e, a person , the more the person is socially connected . By graph theory it is easier to represent the connection of billions of people and analyze it accurately. Once a rumor is passed to a node it is likely to be passed on to all the other nodes through edges. The above applications of graph theory help us focus more on individual thinking rather than communal thinking.That is when a rumor is spread to a single person it is automatically spread to all the nodes connected to it depending on the person's habit of spreading any unknown information.And further it is spread throughout different nodes.

The population is increasing rapidly in today's world.It is affected by both death rate and birth rate. In our futuristic model we can incorporate the factor of population which is increasing with time and is not constant like our previous models.

To overcome the drawback of not considering online sharing of information , our futuristic model intends to add the graph theory to evaluate the rate of online spreading of information and our model 3 for evaluating the rate of offline spread of information.

6.11 Conclusions

In this model we have studied about the spread of the rumor and how it varies with time. This model tells us about how a rumor spreads. It depends extensively on psychological reactions of humans towards rumors which includes the probability of a person to spread the rumor and to believe it or not. Making small changes in the initial parameters can result in a drastic change in the final timeline of the rumor spread . The models developed in our project have widespread applications in various fields.

With each iteration of our models, we arrived at better and more accurate results, this was due to increased parameters and considerations.

Future iterations can lead to more developed models which can be better adapted to real life conditions with lower and practical assumptions. With this digital age rapidly evolving, it becomes even more important to study and model the spread of rumors. It is also important to use this data ethically to make sure there is ill use of the results gathered . As we increased the parameters we got new models which were more accurate than the previous one.

7 Applications

i) The results of this model have diverse applications in multiple fields.

ii) Natural Disaster - The most crucial application of this model is during natural disasters. In the time of natural disasters, it is necessary that all the people are made aware of it . If a natural disaster is predicted then people can be made aware about it ahead of time by placing the time remaining for the natural disaster in our final model and evaluating the number of people who initially need to be made aware of the rumor.This helps in concentrating on the amendments during natural disasters.

iii) Stopping Misinformation from spreading - In today's tech-world , spreading of misinformation is a concerning matter. Misinformation or wrong information or rumors spread within seconds of initiation. These sometimes affect the privacy and respect of citizens . In order to nullify those rumors we can spread truth in a rapid manner with the use of equations by placing the time and evaluating the number of people to start the rumor with.

iv) Media Companies- Media companies can use insights from this model to make sure their editing process tries to minimize the amount of amplification for unverified rumors.

v) Marketing Industry -An important application of information about how rumors/news spreads would be in the marketing industry. Companies can use it to identify segments of their audience should they target based on how they consume and spread information. They can make personalized marketing campaigns for segments which can lead to a faster spread of their message.They can use this information to optimize their timings and frequency of making adverts to maximize the number of people who come in contact with their messages while making sure that costs remain as low as possible.

vi) A regional application of this would be, deciding the time to announce an event, for example in college to optimize the amount of people who are delivered the information while the news is still fresh, so they are more likely to retain the information about the event.

vii) The healthcare sector can use it to timely disseminate information countering misconceptions during times of crisis. The model will be useful to calculate how many people may have been affected by fake news and rumors.

viii) In the financial sector, this model can be used to learn how news about certain events spread in stock markets and to estimate the time between the start of spread of a news and its effect on market fluctuations. Findings of our model can be used to help spread financial literacy among the general public.

ix) Political Usage- Politicians can use this information to make better campaigning decisions.

x) Using relevant parameters they can find out what kind of policies or decision news will spread the fastest in a particular population group.

xi) The model can help us determine the most effective means of spreading information during times of crises, depending on modified parameters such as literacy rates among the population, urban or rural locations, in diverse geographical terrains . For example, in areas with low literacy and internet penetration, physical means of spreading information should be used more such as newspapers, pamphlets , public announcements ,etc. Whereas in areas with high literacy electronic means of communication such as emails. Texts, phone alerts can be used.

xii) Models on how rumors amplify and spread can lead to better moderation policies to control rumors in online platforms. More advanced safeguards can be formed to stop the spread of misinformation. Impact of rumors via social media on a population can be calculated, such as how many people are influenced in a certain amount of time, how changing recommendation algorithms can influence spread of rumors, etc.

[4, 2, 1, 3].

References

[1] Nick Fedewa, Emily Krause, and Alexandra Sisson. Spread of a rumor. *Society for Industrial and Applied Mathematics. Central Michigan University*, 25:977–1002, 2013.

[2] Dubravka Gavric and Armen Bagdasaryan. A fuzzy model for combating misinformation in social network twitter. In *Journal of Physics: Conference Series*, volume 1391, page 012050. IOP Publishing, 2019.

[3] Raúl Isea and Rafael Mayo-Garcia. Mathematical analysis of the spreading of a rumor among different subgroups of spreaders. *arXiv preprint arXiv:1607.05771*, 2016.

[4] Kazuki Kawachi. Deterministic models for rumor transmission. *Nonlinear analysis: Real world applications*, 9(5):1989–2028, 2008.

`https://youtu.be/5a_4GTbAMBM?si=bdVUYCA809FvO3eK`

`https://youtu.be/xLOqs1lB1Wc?si=hrSd_Y5wdQUbGfRt`