Summary of academic paper named “A Wavelet-Based ECG Delineator: Evaluation on standard databases”

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I. Introduction

This document is a summary of the academic paper on the usage of wavelet transformation to detect fiducial points on ECG signal (QRS complexes, P and T waves) obtained from a single-lead electrocardiography.

II. Chapters

This summary is composed of four chapters namely Discrete Wavelet Transform, QRS detection, QRS Delineation, P and T waves Detection and delineation Techniques.

A. Discrete Wavelet Transform

Wavelet transform is obtained by multiplying a given signal with a wavelet analysing function.A wavelet is a rapidly decaying wave like oscillation with zero mean IE it exists in finite duration. Wavelet transformation decomposes the signal as a combination of a set of basis functions, obtained through scaling and translation of a prototype wavelet.Wavelet transform of a signal x(t) is

$$W_a x(b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\psi\left(\frac{t-a}{b}\right) dt, a > 0$$

where a is the scaling factor and $\psi(t)$ is the prototype wavelet.
ECG is composed of slopes and local maxima (or minima) at different scales, occurring at different time instants within the cardiac cycle. If the prototype wavelet $\psi(t)$ is made derivative of a smoothing function say $\theta(t)$the detection would be made easier. Wavelet transform of a signal x(t) may now be defined as

$$W_a x(b) = -a\frac{d}{db} \int_{-\infty}^{+\infty} x(t)\theta_a(t-b)dt$$

where $\theta_a(t) = \left(\frac{1}{\sqrt{a}}\right)\theta\left(\frac{t}{a}\right)$, which is the scaled version of smoothing function.
Discrete Wavelet Transform (DWT) is computed by passing a signal successively through a high pass and a low pass filter. For each decomposition level, signal convolve with low pass filter H(z) and with high pass filter G(z). The signals would be continuously separated into low frequencies and high frequencies as shown in Fig.2.

The scaling factor **a** and/or translation parameter **b** may be discretised based on a dyadic grid on time scale plane to obtain dyadic discrete wavelet transform as shown below.Dyadic wavelet transforms

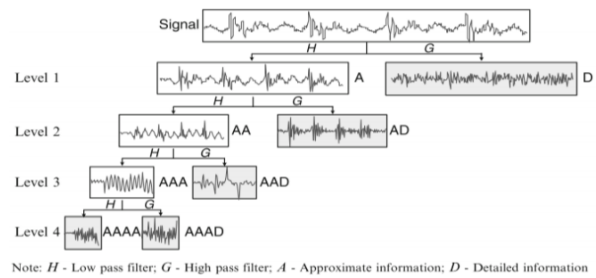

FIG. 1. DWT Process

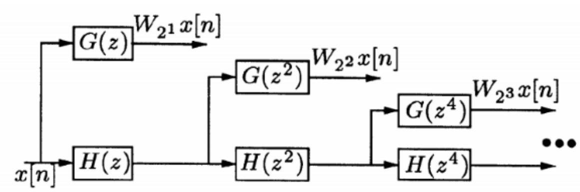

FIG. 2. Algorithme à trous.

are scale samples of wavelet transforms following a geometric sequence of ratio 2. Assuming a = 2^k and b= $2^k l$

$$\psi_{k,l}(t) = 2^{-k/2}\psi\left(2^{-k}t - l\right); k, l \in Z^+.$$

This discrete transformation is equivalent to a filter bank composed of cascaded Finite impulse response filters. The cascading of filters make the signal time-variant and reduce the temporal resolution for increasing scales. To preserve the time-invariance and temporal resolution same sampling rate is used in all scales using a technique called algorithme à trous. Fig 2 represents the algorithm with coefficients $W_2^k x[2^k l]$The equivalent frequency response for kth scale can be represented as

$$Q_k\left(e^{jw}\right) = \begin{cases} G(e^{jw}) & \text{k=1} \\ G(e^{j2^{k-1}w}), \prod_{l=0}^{k-2} H(e^{j2^l w}) & \text{k} \geq 2 \end{cases}$$

The paper suggests usage of 4th degree spline prototype wavelet as follows

$$\theta(\Omega) = \left(\frac{\sin(\Omega/4)}{\Omega/4}\right)^4$$

whose Fourier transform is given by

$$\Psi(\Omega) = j\Omega\left(\frac{\sin(\Omega/4)}{\Omega/4}\right)^4$$

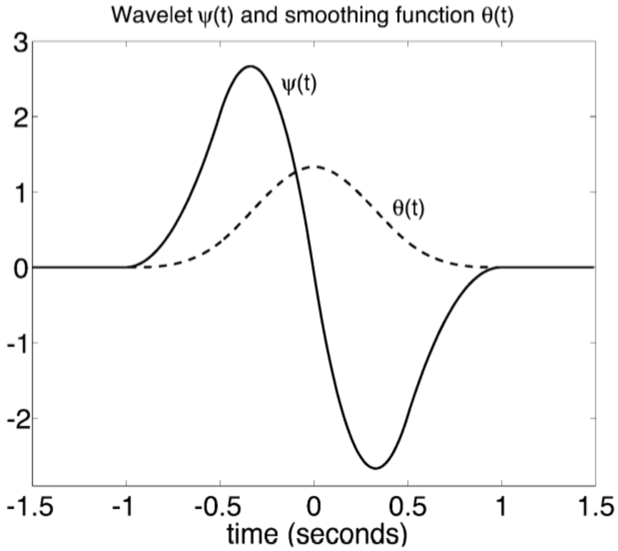

FIG. 3. Prototype wavelet $\psi(t)$ $\theta(t)$

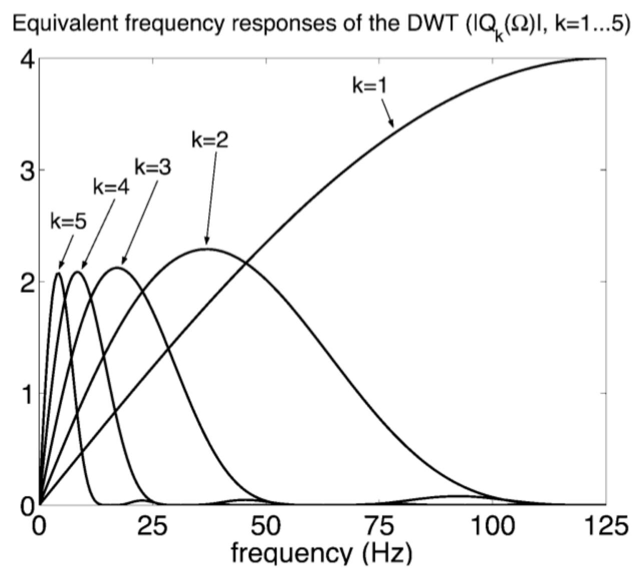

FIG. 4. Equivalent Frequency responses of DWT at scales $2^k, k = 1, .., 5$

Fig 3 represents the prototype wavelet and smoothing function.FIR filters , H(z) and G(z), to implement DWT may be derived as as

$$H(e^{jw}) = e^{jw/2}\left(\cos(w/2)\right)^3$$
$$G(e^{jw}) = 4je^{jw/2}\left(\sin(w/2)\right)$$

Using algorithme à trous the frequency responses of first five scales are represented on Fig 4.To obtain frequency responses for sampling rate other than 250Hz, a new set of filters having equivalent analogue frequency responses similar to what shown in Fig 4 are used.

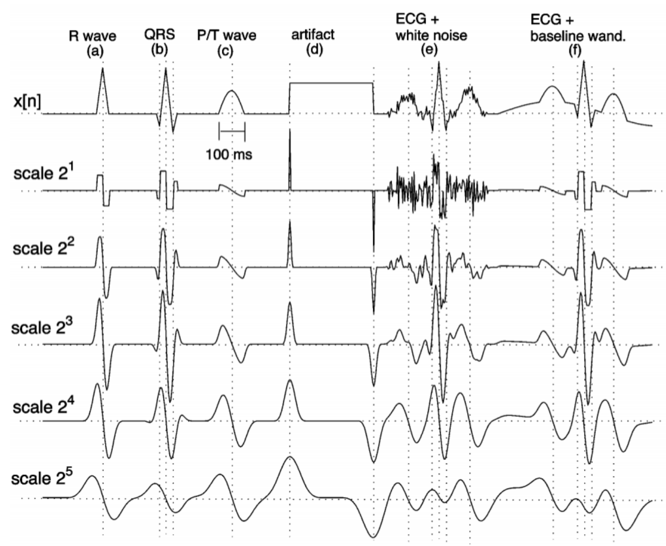

FIG. 5. WT of first five scales of ECG-like simulated waves

B. QRS Detection

From Fig 4, we can conclude that most of the energy of the ECG signal lies within scales 2^1 to 2^5. QRS values are really low on scales higher than 2^4. Fig 5 shows simulated waves similar to ECG. QRS complexes are detected through searches across scales for "maximum modulus lines" exceeding a preset threshold at scales 2^1 to 2^4. The zero crossing of WT at a scale 2^1 between a positive maximum-negative minimum pair is marked as QRS. The detection is not restricted to R wave, but also allows detection of negative waves as well.

C. QRS Delineation

Delineation algorithm looks for start and end of envelope on 2^2 scale for significant maxima , which is local maximum modulus above the threshold defined by previous or subsequent waves. The zero crossings are then assigned on 2^1 scale to name the waves in all possible QRS morphology.

D. T and P wave detection and Delineation

T wave detection happens in a search window relative to the QRS position and recursively computed R-R interval. The algorithm looks for local maxima on 2^4 scale and if at least two exceeds the local threshold then it is marked as a T waves and is named based on the number and polarity of the found maxima. P wave detection algorithm is similar to that of T wave detection. Recursively calculated R-R search window and adequate thresholds are used to detect the waves.