

Summary of academic paper named “A Wavelet-Based ECG Delineator: Evaluation on standard databases”

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I. Introduction

This document is a summary of the academic paper on the usage of wavelet transformation to detect fiducial points on ECG signal (QRS complexes, P and T waves) obtained from a single-lead electrocardiography.

II. Chapters

This summary is composed of four chapters namely Discrete Wavelet Transform, QRS detection, QRS Delineation, P and T waves Detection and delineation Techniques.

A. Discrete Wavelet Transform

Wavelet transform is obtained by multiplying a given signal with a wavelet analysing function. A wavelet is a rapidly decaying wave like oscillation with zero mean IE it exists in finite duration. Wavelet transformation decomposes the signal as a combination of a set of basis functions, obtained through scaling and translation of a prototype wavelet. Wavelet transform of a signal $x(t)$ is

$$W_a x(b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi\left(\frac{t-b}{a}\right) dt, a > 0$$

where a is the scaling factor and $\psi(t)$ is the prototype wavelet.

ECG is composed of slopes and local maxima (or minima) at different scales, occurring at different time instants within the cardiac cycle. If the prototype wavelet $\psi(t)$ is made derivative of a smoothing function say $\theta(t)$ the detection would be made easier. Wavelet transform of a signal $x(t)$ may now be defined as

$$W_a x(b) = -a \frac{d}{db} \int_{-\infty}^{+\infty} x(t) \theta_a(t-b) dt$$

where $\theta_a(t) = \left(\frac{1}{\sqrt{a}}\right) \theta\left(\frac{t}{a}\right)$, which is the scaled version of smoothing function.

Discrete Wavelet Transform (DWT) is computed by passing a signal successively through a high pass and a low pass filter. For each decomposition level, signal convolve with low pass filter $H(z)$ and with high pass filter $G(z)$. The signals would be continuously separated into low frequencies and high frequencies as shown in Fig.1.

The scaling factor a and/or translation parameter b may be discretised based on a dyadic grid on time scale plane to obtain dyadic discrete wavelet transform as shown below. Dyadic wavelet transforms

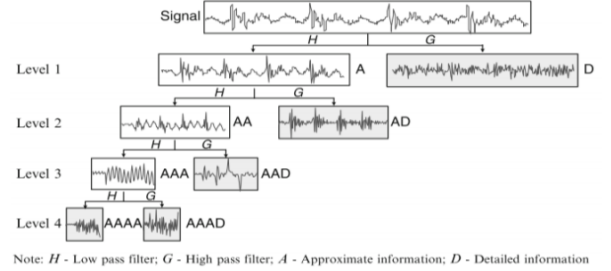


FIG. 1. DWT Process

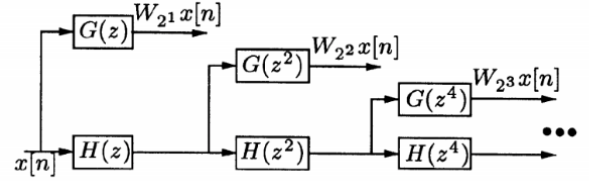


FIG. 2. Algorithme à trous.

are scale samples of wavelet transforms following a geometric sequence of ratio 2. Assuming $a = 2^k$ and $b = 2^k l$

$$\psi_{k,l}(t) = 2^{-k/2} \psi(2^{-k}t - l); k, l \in \mathbb{Z}^+.$$

This discrete transformation is equivalent to a filter bank composed of cascaded Finite impulse response filters. The cascading of filters make the signal time-variant and reduce the temporal resolution for increasing scales. To preserve the time-invariance and temporal resolution same sampling rate is used in all scales using a technique called algorithme à trous. Fig 2 represents the algorithm with coefficients $W_2^k x[2^k l]$. The equivalent frequency response for k th scale can be represented as

$$Q_k(e^{jw}) = \begin{cases} G(e^{jw}) & k=1 \\ G(e^{j2^{k-1}w}), \prod_{l=0}^{k-2} H(e^{j2^l w}) & k \geq 2 \end{cases}$$

The paper suggests usage of 4th degree spline prototype wavelet as follows

$$\theta(\Omega) = \left(\frac{\sin(\Omega/4)}{\Omega/4}\right)^4$$

whose Fourier transform is given by

$$\Psi(\Omega) = j\Omega \left(\frac{\sin(\Omega/4)}{\Omega/4}\right)^4$$

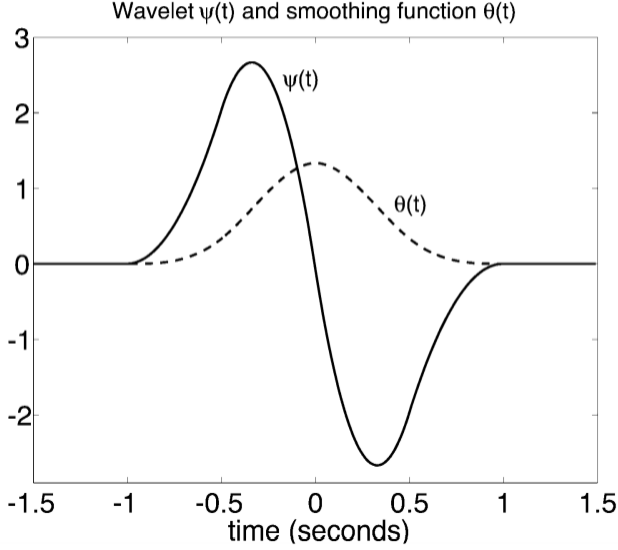


FIG. 3. Prototype wavelet $\psi(t)$ $\theta(t)$

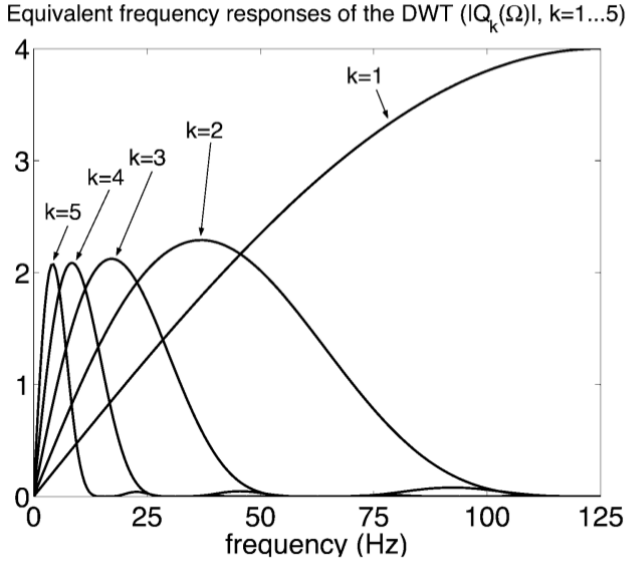


FIG. 4. Equivalent Frequency responses of DWT at scales 2^k , $k = 1, \dots, 5$

Fig 3 represents the prototype wavelet and smoothing function. FIR filters, $H(z)$ and $G(z)$, to implement DWT may be derived as as

$$H(e^{jw}) = e^{jw/2} (\cos(w/2))^3$$

$$G(e^{jw}) = 4je^{jw/2} (\sin(w/2))$$

Using algorithm à trous the frequency responses of first five scales are represented on Fig 4. To obtain frequency responses for sampling rate other than 250Hz, a new set of filters having equivalent analogue frequency responses similar to what shown in Fig 4 are used.

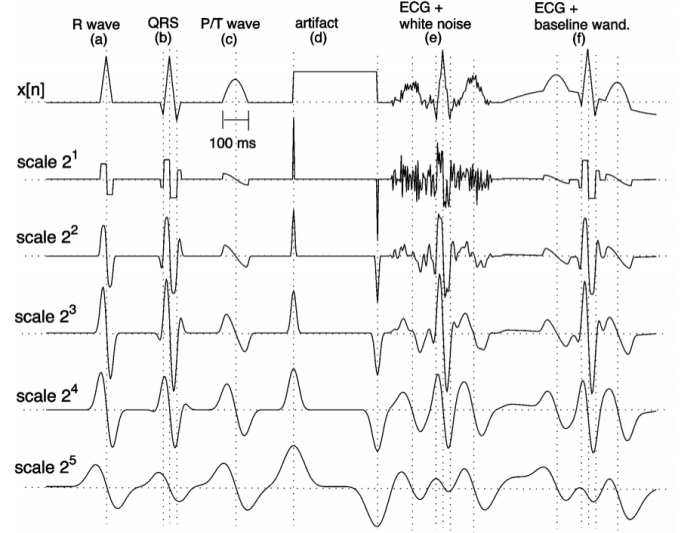


FIG. 5. WT of first five scales of ECG-like simulated waves

B. QRS Detection

From Fig 4, we can conclude that most of the energy of the ECG signal lies within scales 2^1 to 2^5 . QRS values are really low on scales higher than 2^4 . Fig 5 shows simulated waves similar to ECG. QRS complexes are detected through searches across scales for “maximum modulus lines” exceeding a preset threshold at scales 2^1 to 2^4 . The zero crossing of WT at a scale 2^1 between a positive maximum-negative minimum pair is marked as QRS. The detection is not restricted to R wave, but also allows detection of negative waves as well.

C. QRS Delineation

Delineation algorithm looks for start and end of envelope on 2^2 scale for significant maxima, which is local maximum modulus above the threshold defined by previous or subsequent waves. The zero crossings are then assigned on 2^1 scale to name the waves in all possible QRS morphology.

D. T and P wave detection and Delineation

T wave detection happens in a search window relative to the QRS position and recursively computed R-R interval. The algorithm looks for local maxima on 2^4 scale and if at least two exceeds the local threshold then it is marked as a T waves and is named based on the number and polarity of the found maxima. P wave detection algorithm is similar to that of T wave detection. Recursively calculated R-R search window and adequate thresholds are used to detect the waves.