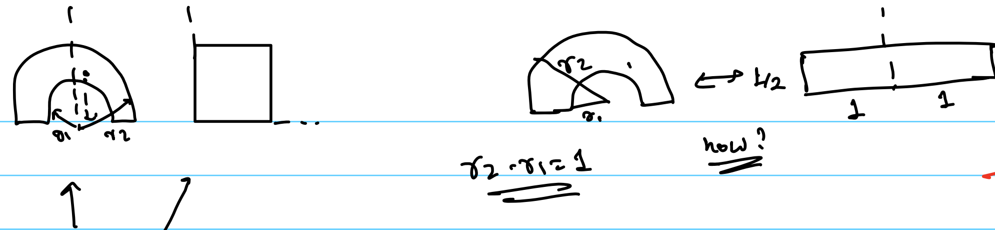


$r_2 - r_1 = 1$

Solution using polar coordinates

Qn 1] $f: Y \to X$ such that

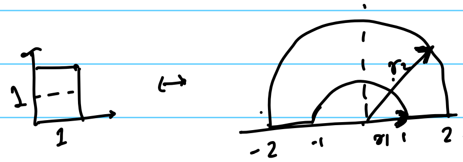
Any point $y \in Y \to$ can be written $r \angle \theta$ $r \in [r_1, r_2]$

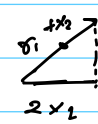
$\theta \in [0, \pi] \Rightarrow$ let $\frac{r - r_1}{2} = y$. & $x = \frac{r\cos\theta}{2}$.

now if $\theta \in [0, \frac{\pi}{2}]$ $y = \frac{r - r_1}{2}$ $x = \frac{r\cos\theta}{2}$.

$\theta \in [\frac{\pi}{2}, \pi] \to y = r - r_1$ $x = -\frac{r\cos\theta}{2}$. $\Big\}$ $\in X$

$f: X \to Y$.





let $(x_1, x_2) \in X$ then if $0 < x_2 \le \frac{1}{2}$ then

$(x_1, x_2) \to \left(2x_1, \sqrt{r_1^2 + x_1^2 + 2r_1x_2 - 4x_2^2}\right)$.

Qn 2] So $\to$ A is near a point $x \in \mathbb{R}^2$ if $\forall$ nbd U of x
$U \cap A \neq \phi$ and is denoted as $x \leftarrow A$

$A, B \subseteq \mathbb{R}^2$ $x \in \mathbb{R}^2$ if $x \leftarrow A$ or $x \leftarrow B \Rightarrow \forall U \ni x$
$U \cap A \neq \phi$ or $U \cap B \neq \phi$
$\Rightarrow (U \cap A) \cup (U \cap B) \neq 0 \Rightarrow U \cap (A \cup B) \neq \phi \to$ hence $x \leftarrow A \cup B$

Say $x \not\leftarrow A$ and $x \not\leftarrow B$ then $\exists$ $U, V \ni x$ such that
$U \cap A = \phi$ & $V \cap B = \phi$ now $W = U \cap V$ which contains x
$\Rightarrow W \cap A = \phi$ & $W \cap B = \phi \Rightarrow W \cap (A \cup B) = \phi \Rightarrow x \not\leftarrow A \cup B$
which is a contradiction.

Qn 3] the first image is not a manifold of $\mathbb{R}^2$ as

(a)

this point does not have nbd homeomorphic to $\mathbb{R}^2$ disk.

the manifold has a few cases depending on the points on the dotted line
depending on if you include it or not.

(b)