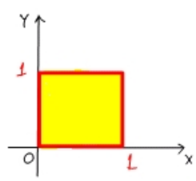
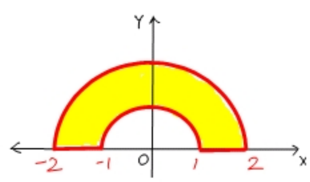
Computational Topology (Spring 2024): Homework 1

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand and scan the pages. Put all the images on a PDF file, though.
- Your file name should identify you in the following manner. If you are Leopold Stotch, you should **name your submission LeopoldStotch_Hw1.pdf**. If you want to add more bits to the title, e.g., Math529, you could name it LeopoldStotch_Math529_Hw1.pdf, for instance. But you should **start the file name with LeopoldStotch. And please avoid white spaces in the file name**.
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, expression, e.g., "LeopoldStotch Hw1 submission".**
- **This homework is due by 5:00 PM on Tuesday, January 23**.

0. (20) Meet with me briefly (in person or on Zoom). Check-in hours work best, but contact me if you want to meet at another time. You do **not** have to contact me if you plan to show up during one of the scheduled check-in hours.

1. (20) Show that the following two sets are homeomorphic by explicitly specifying a homeomorphism, i.e., a continuous function f from one set to the other, and its inverse.

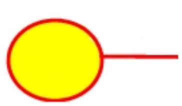




2. (25) Let us define a *neighborhood* of a point as any *open* set that contains the point. Consider a set A that is a subset of $\mathbb{R}^2$. We define a point $\mathbf{x} \in \mathbb{R}^2$ is **near** the set A if every neighborhood of $\mathbf{x}$ contains a point of A, i.e., intersects A. We denote this definition by $\mathbf{x} \leftarrow A$.

   Let A, B be sets in $\mathbb{R}^2$, and $\mathbf{x} \in \mathbb{R}^2$. Prove that if $\mathbf{x} \leftarrow A$ or $\mathbf{x} \leftarrow B$, then $\mathbf{x} \leftarrow A \cup B$. Also prove the converse, i.e., that if $\mathbf{x} \leftarrow A \cup B$, then either $\mathbf{x} \leftarrow A$ or $\mathbf{x} \leftarrow B$, or both.

3. (20) State if each of the following objects is a manifold, a manifold with boundary, or neither. If it is one of the first two cases, specify the dimension of the manifold.

(a) 

(b)